Physics Informed Machine Learning Part I: Different Strategies to Incorporate Physics into Engineering Problems

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Physics-Based Approaches vs. Data-Driven Strategies

**Physics-based strategies** most popular approaches used to characterize complex phenomena focusing on the use of mathematical models to describe the physical laws and principles governing the behavior of a dynamic system.

- interpretable and generalizable to systems with similar characteristics
- flexibility to incorporate prior knowledge and constraints into the model
- often have large and time-varying modeling errors
- heavy computational burden
- complex dynamics

**Data-driven strategies** has seen remarkable advancements due to the abundance of data and computing resources. These methods utilize data to learn the system dynamics and control without the need for an explicit model.

- flexible, adaptive, and scalable
- handle uncertainties and disturbances in the data
- can limitedly represent only the datasets they were trained to learn, without any flexibility or inference capability towards unseen conditions
- low level of interpretability and explainability.
Going Hybrid

What if the physics become too complicated?

What if we don’t have enough meaningful data?

Measured Quantities

time history
When and How?
Variety of strategies classified according to three major characteristics:

1. the **amount** and **quality** of data that is utilized to describe a given model

2. the **strategy** chosen to incorporate the physics into the problem

3. the level of **physical knowledge** and understanding representing the phenomena of interest


A comprehensive review of digital twin — part 1: modeling and twinning enabling technologies


Literature first!
Strategies
Strategies

Data Augmentation

Delta - Learning Physics-Corrector

Physics-Constrained

Physics-Guided

Transfer Learning

Delta-Learning Unknown Physics

Physics-Encoded

1. Physics-Constrained

This approach is centered around the strict enforcement of physical laws on models (physically consistent).

Physics-constrained neural networks are the most popular implementation of this strategy.

2. Physics Guided

Domain-specific physical knowledge into the machine learning process, but rather than enforcing strict constraints, it uses this knowledge as a **guide**

Learning algorithms are employed to capture the discrepancy between an explicitly defined model based on prior knowledge and the true system from which data is attained. The goal is to fine-tune the overall model’s parameters (i.e. the prior and model) in a way that the physical prior knowledge steers the training process toward the desired direction.

- **Incorporating Prior Knowledge**: Prior knowledge on the physics of the system is integrated into the network architecture, or as part of the model

- **Capturing Discrepancy**: Deep learning models excel in learning from data, even when this contradicts prior knowledge.

Physics-guided Deep Markov Model (PgDMM) for inferring the characteristics and latent structure of nonlinear dynamical systems from measurement data.

Introduce a physical prior model into the DMM to guide the training process: simplified linear model that excludes the cubic term.

Figure 5. Predictions vs exact solutions of displacement (top) and velocity (bottom) using the PgDMM applied to the working example. Displacement is assume to be the only measurement. The gray dash-dot line is the physical prior model and the blue bounding boxes represent the estimated $2\sigma$ range.

Physics-Encoded ML framework that embeds physical knowledge into the architecture or design of machine learning models

Via selection of operators, kernels, or transforms such as convolutional layers and recurrent layers (physics-inspired layers or modules)

Constrained Gaussian Process uses prior knowledge to modify the kernel function, the likelihood function, or the posterior distribution of the GP model.

\[ f(X) \sim \mathcal{GP}(m(x), K(x, x')) \]

3. Physics Encoded

Duffing Oscillator

\[ K_{SE}(t, t') = \exp\left( -\frac{1}{2\ell^2} (t - t')^T (t - t') \right) \]

Scaled squared-exponential kernel

\[ K_{SDOF}(t, t') = \frac{\sigma_f^2}{4m^2\omega_n^3} e^{-\xi \omega_n |\tau|} \left( \cos(\omega_d \tau) + \frac{\xi \omega_n}{\omega_d} \sin(\omega_d |\tau|) \right), \quad \tau = t - t' \]

Assume a Gaussian white noise force input kernel

**Figure 10.** Predicted vs exact solutions of displacement estimation using a GP applied to a subsample of the working example, with (top) no physics embedded and (bottom) constrained GP. The blue bounding boxes represent the estimated 2σ range.

4. Delta – Learning Physics-Corrector

- Use a physics-based model to generate training data for an ML model
- The predictions of this physics-trained model will be used to inform a second ML model together with experimental data to learn the residuals
- The final predictions are the sum of the initial predictions and residuals to compensate for missing physics

5. Delta – Learning Unknown Physics

Represents the set of strategies where a data-driven ML model is used as a surrogate to learn and recover the unmodeled physics.

The result is a cumulative damage model where the physics-informed layers are used to model the relatively well-understood physics (L10 fatigue life) and the data-driven layers account for the hard to model components (i.e., grease degradation).

6. Transfer Learning

Strategies focused on using a model already trained on one problem to help solve another problem that is similar but not the same.

This strategy can save time and resources by using existing models instead of training new ones and can also improve the performance of models when there is not enough data for the new problem.

By pre-training the ML model on the synthetic data, the model can learn general and robust features and representations that capture the underlying physical mechanisms.
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**Physics-informed domain adaptation** is a technique that combines physics-based models with machine learning to improve the accuracy of predictions in new or unseen environments. The goal is to adapt a model trained on one domain to another domain with different characteristics, such as different physical properties or environmental conditions.

Physical laws governing the domain shifts and use a small amount of source-domain and target-domain data to fit the physical law.
6. Physics-Informed Transfer Learning

Transfer learning can be used to leverage information across related domains. The authors propose utilizing the Modal Assurance Criterion (MAC) between modes of healthy structures as a measure of data similarity to identify features that minimize conditional distribution shift.

Transfer feature criterion that incorporates MAC-discrepancy into a feature selection criterion to address the challenge of selecting features with high cross-domain similarity.

\[ L = -\frac{1}{n_S} \sum_{n=1}^{n_S} L(f_s(x_{sd}), y_s) + \lambda d_{MAC}(\Phi_s, \Phi_t) - \mu C \]


Figure 1. The experimental setup to perform modal testing on a metal (right) and composite (left) blade simultaneously.

Figure 5. PCA visualization of the TFC-selected frequencies, corresponding to the fourth and fifth modes, for M→C (panel (a)) and C→M (panel (b)), representing 66% and 64% of the variance respectively.
6. Physics-Informed Transfer Learning

Graphical domain is presented as an objective way of assessing structural similarity, with distance metrics utilised for assessing data-space similarities.

Knowledge transfer is performed using a branch of transfer learning called domain adaptation.

The authors demonstrate a methodology for transferring knowledge within a heterogeneous population (a group of non-identical structures).

Transfer localisation labels from a Gnat aircraft wing to an unlabelled Piper Tomahawk aircraft wing dataset, resulting in 100% classification accuracy.

7. Data Augmentation

Set of techniques to artificially increase the amount of data by generating **new data** points from existing data.

Improve the performance and generalization of machine learning models, especially when the original **data is insufficient** or noisy.

When an accurate and robust knowledge of the first principle is available, it is possible to leverage this information and run **first-principle simulations** to generate data at various states and operating conditions of a physical system.
7. Data Augmentation

The authors generated different damage scenarios using first-principle simulations to augment a training dataset for an ML classifier used for damage detection of a bar structure.

Open Challenges
Open Questions and Challenges

**Generalization**
How well do the integrated models generalize?

**Uncertainty Quantification**
Incorporating physical laws and constraints into machine learning models can make the quantification of the uncertainty for the integrated model a challenge.

**Scalability**
Are the integrated models scalable to large datasets? Is it computationally more efficient?

**Interpretability and explainability**
Machine learning models are often considered as black boxes, making it difficult to interpret and explain their predictions. Is their introduction helping?

**Data quality**
How integrated models perform in the case of changes in measurements data quality and unbalanced datasets.