Physics Informed Machine Learning Part II: Applications in Structural Response Forecasting

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ABSTRACT
Physics-informed machine learning is a methodology that combines principles from physics with machine learning techniques to enhance the accuracy and interpretability of predictive models. By incorporating physical laws and constraints into the learning process, physics-informed machine learning enables more robust predictions and reduces the need for large amounts of training data. In part II of this two-part series, the authors present structural response forecasting using a physics-constrained methodology to solve the homogeneous second-order differential equations that constitute the equation of motion of a linear structural system. This forward problem is formulated to allow the incorporation of numerical methods into the training process while using segmented training to circumvent intrinsic stability limitations to the physics-informed machine learning problem. The ability of physics-informed machine learning to make generalizations for limited training data is discussed.

Keywords: Physics-informed, physics-constrained, machine learning, structural, forecasting, time series

INTRODUCTION

Physics-Informed Machine Learning (PIML) introduces a synthesis between foundational physical principles and the advanced capabilities of machine learning algorithms [1]. The novel strategy not only elevates the accuracy of predictive models, but substantially curtails the reliance on exhaustive training datasets. The focus of this second part in the series revolves around utilizing this potent blend for the targeted forecasting of structural responses, particularly salient in structural control contexts.

In practical applications, especially in civil structures, the data collected is often riddled with noise due to various factors. The strength of PIML lies in its ability to not just handle this noise but to generalize effectively in its presence. This quality ensures that, regardless of the environmental and instrumental noise, the forecasting and parameter identification remain consistent and reliable. With PIML, there's the capability to reconstruct intricate displacement fields, ensuring engineers have a clear and accurate representation of the system's state, even from minimal data points.

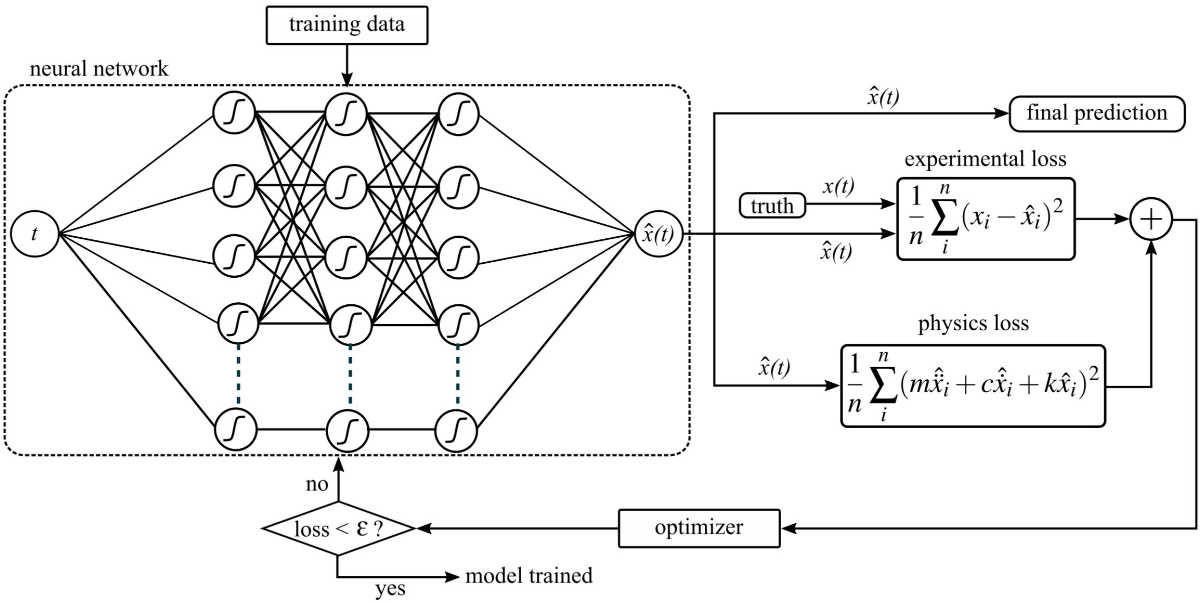

Figure 1: Layout of the physics-constrained approach where the "experimental loss" and "physics loss" together form the total loss function used for training the neural network.

METHODOLOGY

At the heart of structural modeling lies the equation of motion, represented primarily as homogeneous and non-homogeneous second-order differential equations. In its homogeneous form, the equation of motion is as follows

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{1}$$

Where m is the mass of the system, c is the damping related coefficient and k is the stiffness of the beam. $x(t)$ is the displacement of the beam as a function of time t.

The challenge in accurately forecasting structural responses for a real-world system stems from the intrinsic stability limitations present in using typical machine learning approaches. Moreover, significant challenges arise when trying to learn the response from experimental data that contains all the complexities of real data [2, 3]. PIML overcomes this by integrating the assumed numerical model of a system (the equation of motion shown in equation 1) directly into the training regimen of the machine learning approach.

In this work, we present a physics-constrained approach for training a neural network on experimental data to forecast the predicted displacement of a structure $\hat{x}$ at any time t while constraining the training of the neural network with the physics that is contained within the equation of motion, as written in equation 1 [4]. To enumerate, this equation constrains the training of the neural network with the following physics of a vibrating system [5], including the following:

1. The system is linear.
2. The system is time invariant.
3. The system does not experience outside forces.

Figure 1 diagrams the training process for the physics-constrained approach. This figure shows how a neural network has a singular input t and inferences a singular predicted response at that time $(\hat{x}(t))$. Similar to the training of a normal neural network, the error between the predicted response $(\hat{x}(t))$ and the real response $(x(t))$ is computed and used to adjust the weights of the neural network during training, using a process called "backpropagation". However, unlike traditional neural network approaches, the physics-constrained approach has a loss function made up of two constituent parts. The first is termed the experimental loss and is obtained by computing the mean-squared error between the predictions and the truth values for each data point in the training set with n data points. The second constituent part of the loss function is the loss from the physical constraint, obtained by feeding in $\hat{\ddot{x}}$, $\hat{\dot{x}}$, and $\hat{x}$ as inferred from the neural networks into the equation of motion to obtain a residual that is than squared. Together, the total loss (L_{total}) function for the physics-constrained approach is expressed in equation 2 where the first term is the "experimental loss" and the second term is the "physics loss".

$$L_{\text{total}} = \frac{1}{n}\sum_{i}^{n}(x_i - \hat{x}_i)^2 + \frac{1}{n}\sum_{i}^{n}(m\hat{\ddot{x}}_i + c\hat{\dot{x}}_i + k\hat{x}_i)^2 \tag{2}$$

EXAMPLE

A simple numerical example is presented here. One degree of freedom structural dynamics problem that consists of the cantilever beam shown in figure 2 represented as a spring-mass-dashpot model is used. For this example, $k = 625$ N/m, $m = 1$ kg, $c = 25kg/s$, $\omega_0 = 4$ rad/sec, and $\zeta = 0.080$. Where the damping coefficient $0 < \zeta < 1$ and computed as $\zeta = c/(2\sqrt{km})$ and the natural frequency of the system ω_0 is calculated as $\omega_0 = \sqrt{k/m}$ [5].

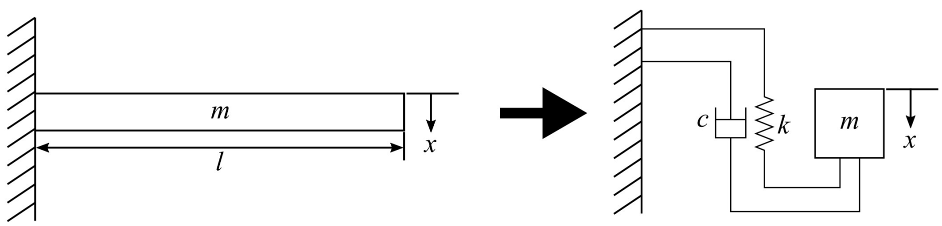

Figure 2: A cantilever beam modeled as a one-degree freedom spring-mass-dashpot model.

For this example, a traditional neural network is trained on simulated experimental data out to 1 second under the assumption that this is all the data that is available. The training phase consists of two parallel processes on the same network; one for the experimental data (out to 1 second) and the other for the physics data (to and beyond 1 second). After obtaining results from both processes, the two loss functions are calculated accordingly. During optimization, both loss functions are considered together as shown in equation 2 to train the model.

Then the trained model is used to predict the structural response past the training data. Figure 3 provides an overview of the considered example, comparing neural network predictions with and without physics loss. As shown in figure 3(a), the neural network prediction without the physics constraint is not capable of tracking the system-level dynamics out past the training data. In comparison, when the physics-constrained loss term is added to the experimental data, as shown in figure 3(b), the training can continue past the end of the data; resulting in an accurate prediction. In this scenario, the training is extended to consider the time steps past what experimental data is available by using the knowledge added to the system by the physical constraints. By constraining the training process to the known physics of the problem at hand, the predicted response of the system is significantly improved; despite having the same amount of information. This simple example demonstrates the potential for physics-constrained neural networks to enable advanced structural response forecasting on limited or sparse datasets.

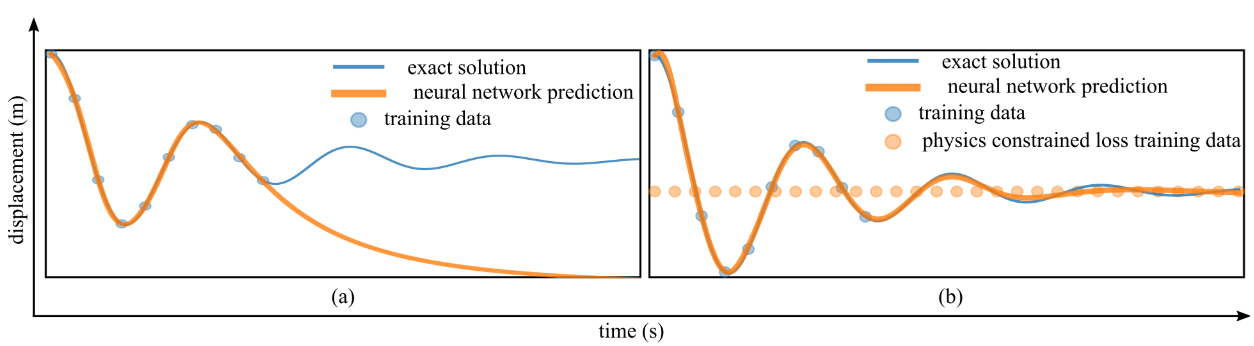

Figure 3: Results for the one degree of freedom cantilever beam model as: (a) without physics constrained loss, and; (b) with physics constrained loss.

CONCLUSION

The amalgamation of physics and machine learning, as presented in this series, paints a promising picture for the future of structural modeling and monitoring. For civil applications, where the safety and longevity of structures are paramount, the abilities of PIML to forecast responses and identify hidden system parameters with unparalleled accuracy can potentially revolutionize the field. As the intersection of traditional engineering and modern computational techniques continues to evolve, PIML stands out as a beacon of innovation, bridging the gap between theoretical physics and real-world applications.

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