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OPTIMIZATION OF RAPID STATE ESTIMATION IN STRUCTURES SUBJECTED TO HIGH-RATE BOUNDARY CHANGE

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ABSTRACT

Many structures are subjected to varying forces, moving boundaries, and other dynamic conditions. Whether part of a vehicle, building, or active energy mitigation device, data on such changes can represent useful knowledge, but also presents challenges in its collection and analysis. In systems where changes occur rapidly, assessment of the system's state within a useful time span is required to enable an appropriate response before the system's state changes further. Rapid state estimation is especially important but poses unique difficulties.

In determining the state of a structural system subjected to high-rate dynamic changes, measuring the frequency response is one method that can be used to draw inferences, provided the system is adequately understood and defined. The work presented here is the result of an investigation into methods to determine the frequency response, and thus state, of a structure subjected to high-rate boundary changes in real-time.

In order to facilitate development, the Air Force Research Laboratory created the DROPBEAR, a testbed with an oscillating beam subjected to a continuously variable boundary condition. One end of the beam is held by a stationary fixed support, while a pinned support is able to move along the beam's length. The free end of the beam structure is instrumented with acceleration, velocity, and position sensors measuring the beam's vertical axis. Direct position measurement of the pin location is also taken to provide a reference for comparison with numerical models.

This work presents a numerical investigation into methods for extracting the frequency response of a structure in real-time. An FFT based method with a rolling window is used to track the frequency of a data set generated to represent the range of the DROPBEAR, and is run with multiple window lengths. The frequency precision and latency of the FFT method is analyzed in each configuration. A specialized frequency extraction technique, Delayed Comparison Error Minimization, is implemented with parameters optimized for the frequency range of interest. The performance metrics of latency and precision are analyzed

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and compared to the baseline rolling FFT method results, and applicability is discussed.

INTRODUCTION

Many systems and structures are in use or presently under development which experiments high-rate dynamic events, defined as changes occurring on the time scale of 100ms less [1]. In these systems, many of the parameters being monitored are part of a process being continuously managed by a control loop and therefore part of a system's normal operation. In many cases, there exists a possibility that these systems will be subjected to conditions or states which are undesirable or will result in damage. Therefore, reacting to an intermediate state before it progresses further allows for an improved outcomes for the system. These applications and others have led to the desire for an observer which can assess the state of these structures and systems, determine the conditions, and make decisions based on the state while the assessment remains relevant. Such observers have potential applications in far-ranging fields, including machinery and automation, blast mitigation, and hypersonic aircraft [2–5]. While the applications may appear disparate, the similarities in their needs have provided motivation to develop general-purpose tools that can be used to create the solutions needed for each case.

The objective of this research is to design, implement, and demonstrate a frequency-based observer which can estimate the state of a complex system with sufficient precision and speed. Achieving the goal of a useful observer in this scenario requires consideration of the structure's properties, determination through modeling and/or testing of what collectible information is most useful, and implementing the computational steps in a way that minimizes processing time on a system which is suitably fast.

One method commonly used for extraction of frequency information from a signal is the Fast Fourier Transform or FFT. While a powerful tool, FFTs are computationally intensive and face other drawbacks. When computing the discrete Fourier Transform using a method such as the Cooley-Tukey FFT algorithm [6], there is a linear relationship between the length of the input sample and the number of output bins generated. It then follows that a requirement for adequately precise bin spacing is also a requirement on time spent collecting the sample, which potentially conflicts with the need for short delay in a system's feedback response. Thus, an FFT based approach will face challenges of lag, difficulty identifying transients, and/or inadequate frequency precision.

When the dynamics of a system are adequately understood, and responses can be expected to fall within a known range, alternatives to the FFT can offer improved performance characteristics and mitigate or avoid the identified drawbacks. What follows is an investigation into a frequency measurement method intended to allow for tracking in high-rate systems, and a comparison to FFT based frequency tracking performance.

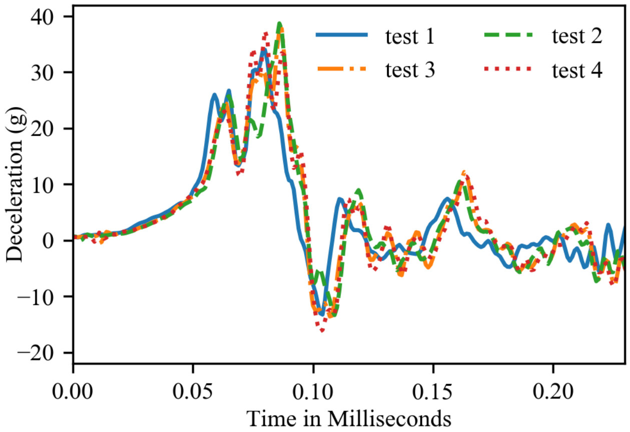

FIGURE 1. Data from a sub-second system showing response from four consecutive tests on the same system.

The contributions of this work are the proposal and demonstration of the Delayed Comparison Error Minimization frequency extraction method. The theory of the method is laid out, a functional implementation is developed, and the performance of the method is investigated beside the FFT approach. The results provide strong evidence for the applicability and advantages of the new method in high-rate state estimation.

BACKGROUND

An event that experiences high-rate dynamics is one that happens on a time-scale of less than 100 ms and is characterized by: 1) large uncertainties in external loads; 2) high levels of nonstationarities and heavy disturbances; and 3) generation of unmodeled dynamics from changes in system configuration [1]. These specific challenges were discussed and demonstrated by Hong et al. using realistic experimental data [7]. In this prior work, the experiment consisted of an electronics unit housing circuit boards and accelerometers ruggedized for shock survivability and tested in an accelerated drop tower to create high impact conditions. For this study, the electronic test unit was subjected to four consecutive accelerated drops. Figure 1 plots a portion of the results. This prior study demonstrates the three challenges that characterize high-rate dynamics: 1) Unknown external loading; 2) A high levels of nonstationarities (e.g., no consistent running mean) and heavy disturbances; and 3) Changes in the structures response for back-to-back tests. This prior study demonstrates the challenges associated with high-rate dynamic events, in particular, the dynamics of the system operate on a very short time-scale and internal damage to the structure causes the dynamics to vary from one event to the next.

While the previously discussed accelerated drop tower ex-

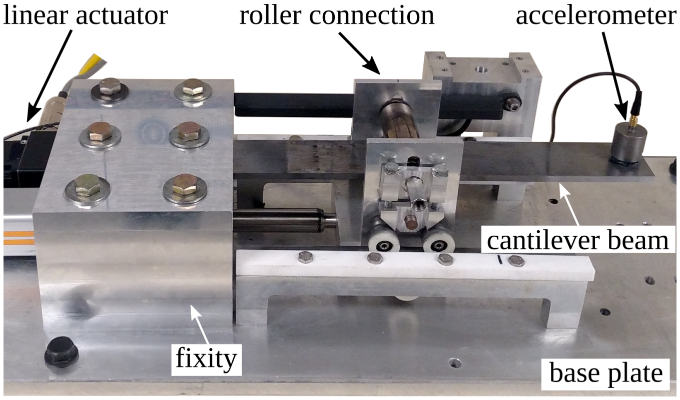

FIGURE 2. The Dynamic Reproduction of Projectiles in Ballistic Environments for Advanced Research (DROPBEAR) experimental testbed with key components annotated.

periments clearly demonstrate the challenges associated with high-rate dynamics, the destructive nature of the tests adds complexity to the development and experimental validation observers designed for the rapid state estimation of structures subjected to high-rate dynamic events. For this reason, the Dynamic Reproduction of Projectiles in Ballistic Environments for Advanced Research (DROPBEAR) testbed was initially introduced and modeled by Joyce et al. [8] and is presented in figure 2. The DROPBEAR is a cantilever beam featuring two time-varying, user-controlled parameters: a continuously movable roller constraint and a detachable mass (not shown in current setup). It is designed such that the movable roller introduces a nonstationary boundary condition into the system, while the mass drop simulates sudden damage to the system. In both cases, the parameter changes were designed to produce repeatable and controllable change in the system dynamics, intended to simulate changes occurring in a structural system (i.e. damage). The nondestructive nature of these changes provides a level of repeatability that is unobtainable in the previously discussed accelerated drop tower experiments. In prior work, the DROPBEAR was used by Downey et al. to develop a millisecond model-updating technique that updated an FEA model of the structural system by minimizing the error between the model and the system in the frequency domain. Experimental results demonstrated that the roller's location updated every 4.04 ms with an accuracy of 2.9% [9].

As discussed before, the DROPBEAR testbed is intended to be representative of the generalized form of many real world systems, possessing a variable parameter that influences the system response in a non-linear fashion. The ability to record the controlled parameter precisely also helps with verification of the output of predictive systems processing data generated by the DROPBEAR. For these reasons, data collected from the DROPBEAR was used to guide the creation of synthetic data sets for initial testing and development. As demonstrated by Downey et al. in Ref. [9], when an accelerometer is used to capture DROPBEAR's response to a dynamic input, and this data set was analyzed using the FFT method described in more detail later, there exists non-trivial challenges of achieving the frequency components of the system with adequate precision and suitably low lag. With knowledge of how this physical system responded, synthetic data sets represent an attempt to isolate challenging aspects in order to better understand what data characteristics will challenge frequency tracking tools.

TABLE 1. Definitions for the components used in the Delayed Comparison Error Minimization technique.

Parameter	Definition
SamplingRate	Sampling rate used to collect data.
Signal	Periodic signal of unknown frequency.
Reference	List of the last 100 samples from Signal.
Comparison	List of 100 delayed samples from Signal.
Difference	List of Reference - Comparison.
DifferenceSquare	Error^2
DifferenceSquareSum	A single value representing the sum of values in the list ErrorSquare.
SumVsDelay	ErrorSquareSum values arranged by delay.
SignalPeriod	Signal period length, in samples.
Frequency	Frequency of Signal in Hz.

METHODOLOGY

This section explains in further detail how each frequency detection method operates, the characteristics and quality of the output that will be generated, and how variations in tuning parameters or data characteristics will affect them.

Rolling FFT

The FFT, or Fast Fourier Transform, is a commonly used method for converting a signal from the time domain to the frequency domain. When the FFT is provided a real input with an even number of samples, the output will include half that many positive frequency bins. The bins will be spaced evenly from DC up to a frequency equal to half the sampling rate of the signal, often called the Nyquist frequency. This means that the frequency spacing between adjacent bins, which can also be interpreted as the frequency precision, will be equal to the sampling rate divided by the number of samples provided by the FFT. The magnitude of the number contained by a given bin is proportional to

Algorithm 1 Pseudocode for the Delayed Comparison Error Minimization Method

```
collect 400 data points from Signal
for 300 Cycles do
    Reference = 100 most recent data points
    Comparison = 100 data points, starting from delay equal to the cycle count
    Difference = point by point difference between Reference and Comparison
    DifferenceSquare = Difference*Difference
    ErrorSquareSum = sum of all points in ErrorSquare
    append ErrorSquareSum value to SumVsDelay
end for
SignalPeriod = position of minimum value between the 100th and 200th point in
SumVsDelay
calculate Frequency using SignalPeriod and SamplingRate
```

the absolute value of the amplitude and duration of the frequency components in the original signal which falls between that bin's upper and lower frequency boundaries. The time at which a frequency component occurred within a sample analyzed by an FFT can not be determined by the FFT's output. Additionally, a component with a high amplitude and short duration can result in a bin magnitude identical to one caused by a component of greater duration and lesser amplitude. While these limitations are well known, [9] they are restated here to emphasize some of the challenges the use of FFTs pose in tracking the frequency of a rapidly time-varying signal.

The desired application for which the FFT is being employed in this paper is determining the response of a time-varying system, with minimal delay. However, analyzing the complete signal does not provide information as to when each frequency component occurred. To solve this challenge, the FFT of the time-series data is computed using a moving window that analyzes the time-series signal section-by-section. Similar to shutter speed in photography or videography, an observation made during a sample could have occurred at any time during it. The shorter each analyzed section is, and the more frequently the analysis is performed, the more closely the frequency can be tracked in the time domain. However, the frequency precision of an FFT decreases with a reduction of the window size. In attempting to use the FFT method to track the frequency of a signal which changes with time, it becomes apparent that compromises are necessary. Therefore, this work uses a selection of FFT lengths to demonstrate the trade-offs between accuracy in the time domain and accuracy in the frequency domain. It is worth noting that increasing the sampling rate increases the Nyquist frequency as well as increasing the number of bins generated by the FFT and therefore has little appreciable effect on the FFT's accuracy in the frequency-domain.

In this work, the fundamental frequency of the system is extracted from the frequency-domain of the FFT output by selecting the frequency, within the relevant/expected frequency range, associated with the bin that contains the highest magnitude. The time coordinate for this frequency measure corresponds to that of the last time-series data point used in the FFT. Therefore, the plots in this work present the time alignment as it would appear if the FFTs were performed in real-time on an idealized system with no computational delay.

Delayed Comparison Error Minimization

Similar to the FFT approach, the Delayed Comparison Error Minimization technique allows for the detection of the natural frequency of the system, but does not require as long window lengths to generate similarly accurate frequency estimations. Therefore, the delayed comparison technique can generate an accurate estimation of the system's fundamental frequency with a reduced lag-time when compared to the FFT-based approach. The principle by which the delayed comparison technique works is to compare sections of samples taken from the same periodic signal, with known time differences between them. By finding what time difference results in the greatest similarity, one can thus determine the period of the signal if it has a periodic component. A similar approach is outlined in [10].

The implementation used in this paper uses the calculation of a point by point difference, point by point square of each difference, and summation of all squared difference values. In a basic implementation of this method, the bandwidth can extend from the frequency possessing a period equal to the time difference, or delay, up to half the sampling rate. The frequency precision will be equal to the sampling rate of the data provided.

Algorithm 1 presents pseudocode for the Delayed Comparison Error Minimization technique while Table 1 provides a definitions for the variables used in this work. Moreover, figures 3-6 report a graphical representation of the technique. A 7 step, step-by-step example of the implementation used for this work is as follows. (1), see figure 3(a). A section of data is selected, starting from the beginning of the data set, with zero on the right-hand side of the figure, and spanning a length comprising the

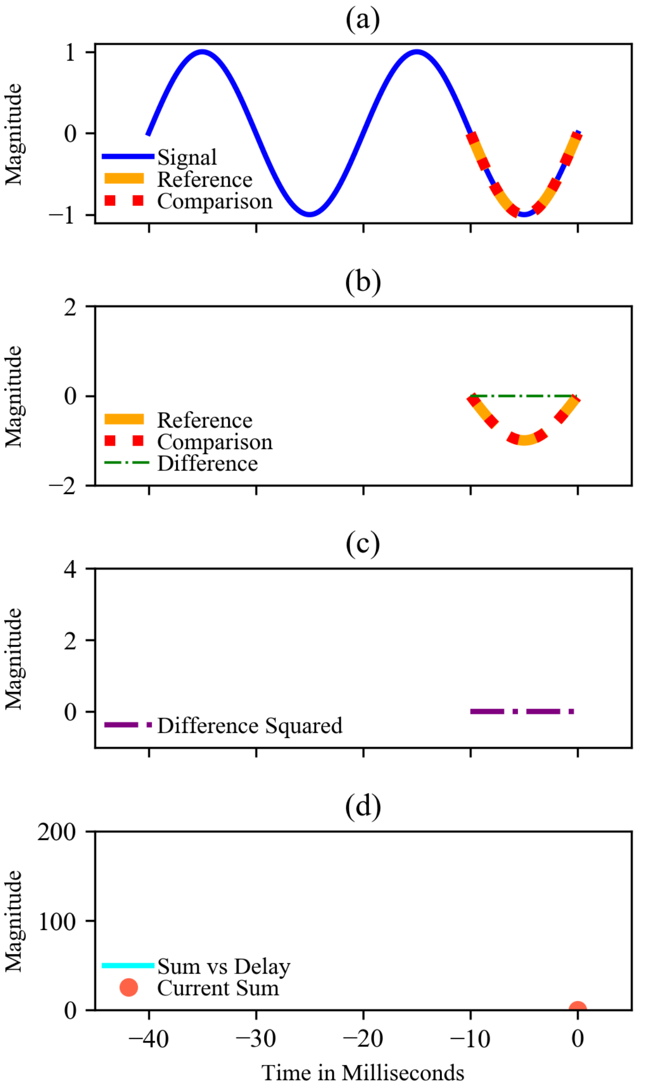

FIGURE 3. DEMONSTRATION OF THE STEPS TO DETERMINE WAVE PERIOD: SUMMED SQUARED ERROR EVALUATED AT 0 DELAY. (a): SIGNAL, REFERENCE AND COMPARISON (b): REFERENCE AND COMPARISON OVERLAPPED, DIFFERENCE (c): DIFFERENCE SQUARED (d): SUM VS DELAY AND CURRENT SUM

sum of two parameters. The parameters are the length of the longest period to be detected, and the length of the "reference" and "comparison" data sections. As implemented, these values are 300 and 100 samples, respectively, or 30ms and 10ms at the 10,000 sample/second sampling rate. (2), again see figure 3(a). The last 100 samples are copied to "reference", and on the first cycle, shown in figure 3, the same 100 values are also copied to "comparison". (3), see figure 3(b). The difference between each of the 100 points in "reference" and their corresponding point in "comparison" is found, resulting in a 100 digit long difference list. (4), figure 3(c). Each of these values is squared, and then (5) the values are summed, see "Current Sum" in figure 3(d). (6) This value is stored as the first value in "sum vs delay", a list of difference squared sums versus delay values, representing the difference at 0 delay. Steps (2) through (6) in this process are repeated, with the samples for "comparison" being collected from a position one sample earlier in the data set each time, and the value accordingly being appended to the "sum vs delay" list, visible as "Sum vs Delay" in figure 3(d), representing a delay 1

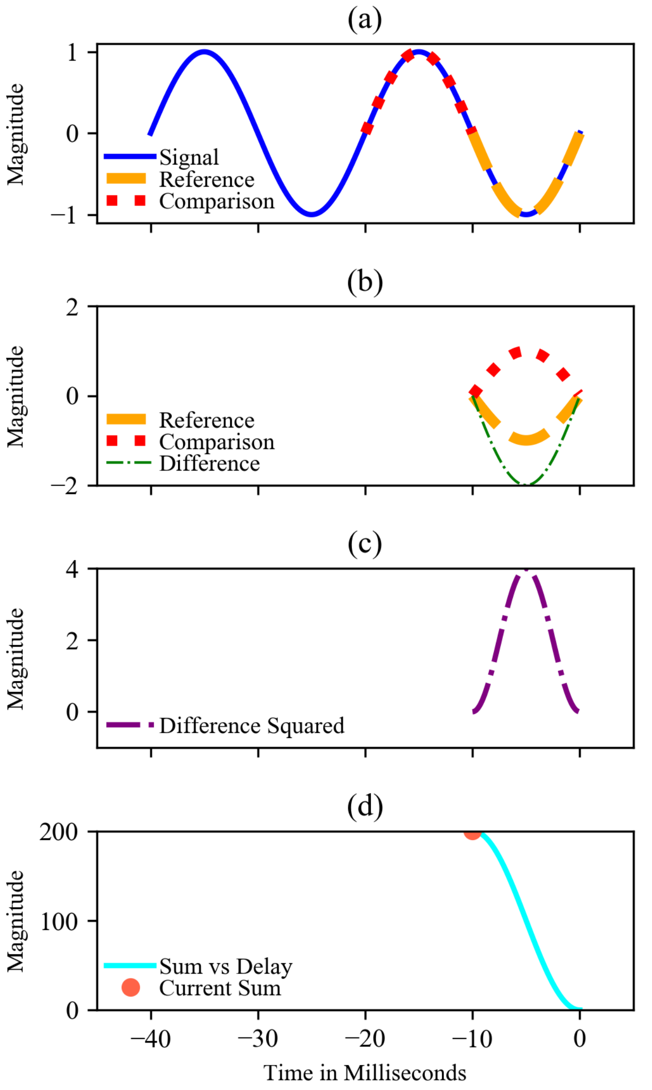

FIGURE 4. DEMONSTRATION OF THE STEPS TO DETERMINE WAVE PERIOD: SUMMED SQUARED ERROR EVALUATED AT 10ms OR 100 SAMPLES DELAY

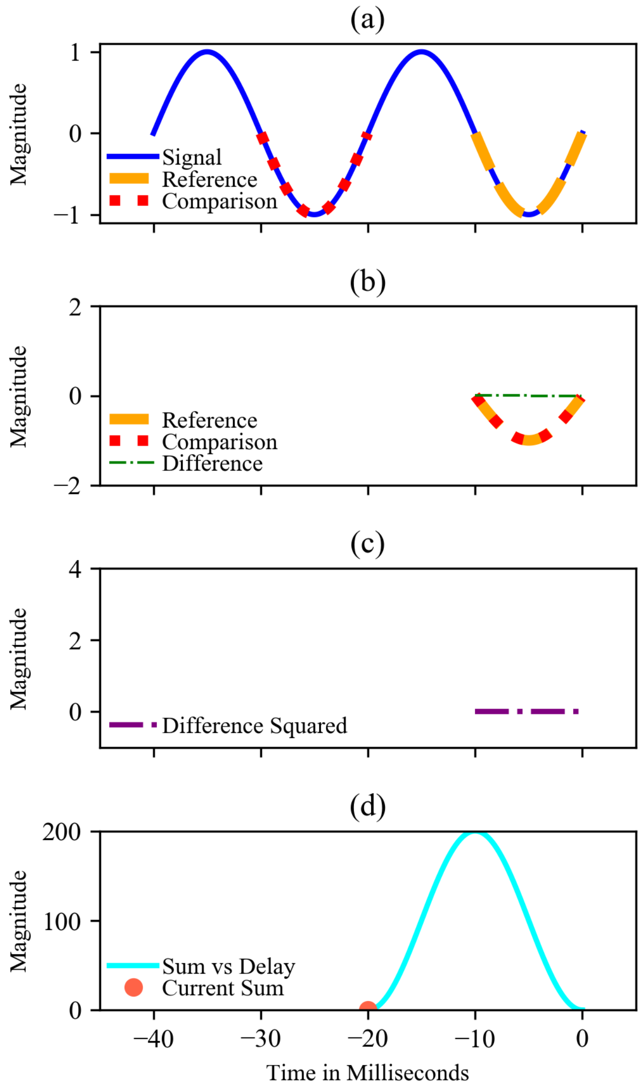

FIGURE 5. DEMONSTRATION OF THE STEPS TO DETERMINE WAVE PERIOD: SUMMED SQUARED ERROR EVALUATED AT 20ms OR 200 SAMPLES DELAY

sample space greater each time. This repeats until the last cycle, shown in figure 6, when the data points for “comparison” are collected from 300 points offset from the points in “reference”. (7) After the “sum vs delay” list is completed, the local minimum within the expected range of wave periods is selected, and the position of that value in the list, interpreted to indicate the period of the signal, is copied to a Signal Period list, with a time stamp matching the last data point used in this cycle of comparisons. If desired, the frequency can be calculated from the period. Each step in the process as described up to this point is repeated in order to analyze longer signals piece by piece.

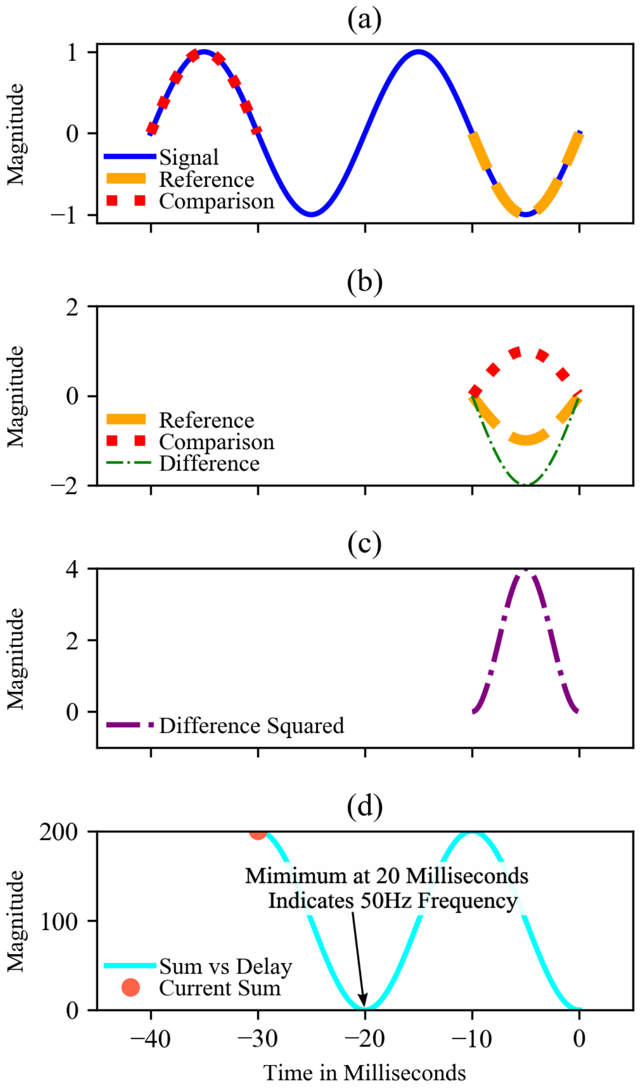

FIGURE 6. DEMONSTRATION OF THE STEPS TO DETERMINE WAVE PERIOD: SUMMED SQUARED ERROR EVALUATED AT 30ms OR 300 SAMPLES DELAY, WITH MINIMUM AT 20ms OR 200 SAMPLES INDICATED

RESULTS

Each of the methods described was compared by running them over a well-defined data set and then analyzing their results to assess performance. This data set is presented in figure 7. The data set used for testing consists of a half-second of 50hz, a phase-coherent transition to a half-second linear sweep from 50hz to 100hz, and then a half-second of 100hz. The data is sampled at 1000 samples/second. Only a single frequency component is present at any time during this set. A maximum theoretical delay, defined here as the time difference between the

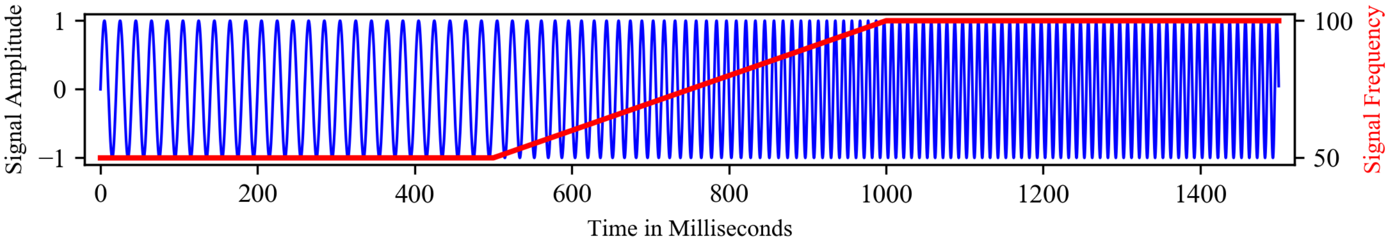

FIGURE 7. PHASE COHERENT FREQUENCY SWEEP FOR EVALUATION

most recent sample and the sample farthest into the past which was used for calculations, will be calculated. An observed delay will also be estimated so that each parameter can be compared between the varying frequency tracking methods.

When the delayed comparison and rolling FFT methods of primary frequency identification and tracking were applied to the frequency sweep, the results were as shown in figure 8 for FFT lengths of 100ms, 200ms, and 400ms, the precision of each pass is 10hz, 5hz, and 2.5hz, respectively. Each FFT point is generated using data that spans from the current time through an FFT length in the past. The "binning" effect, or precision limitations, are clearly visible in the FFT outputs as steps in the traces. As the FFT length is increased, the precision and number of bins in the FFT increase accordingly, and the steps become shorter in height. However, this increase in FFT length results in an increase in the lag, as demonstrated by the offset of the estimated frequency vs the target frequency. As the age of the oldest data in each FFT calculation directly corresponds to the FFT window length, the maximum theoretical delay resulting from the FFT is equal tol the FFT length as well. Visual inspection of tracking performance suggests that the induced lag is around half of each FFT's window length on the data set used, where amplitude is constant and frequency varies at a linear rate.

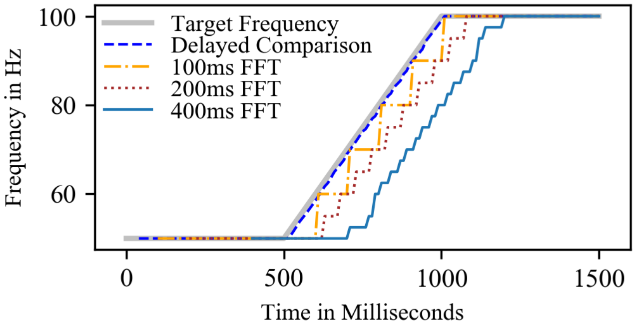

FIGURE 8. PLOT SHOWING TRACKING OF FFT AND DELAYED COMPARISON METHODS.

Because it was known that the lowest frequency in the dataset was 50hz, the frequency tracker was tuned beforehand to extend only slightly below that range. It was configured such that the oldest data used was equal to the period of the lowest frequency (20ms, in the case of 50hz), plus a 10ms margin, plus the length of the samples used for comparison (10ms, in the case where samples are half the period of the lowest expected frequency). This adds up to a maximum theoretical delay of 40ms. Inspection of the results suggests a lag of 10ms on the dataset provided. Precision and accuracy of the system appear to be significantly better than the FFT method at any of the settings used, there is some jitter visible in the results which is not well understood at this time.

CONCLUSION

On signals which are rapidly changing, or otherwise need to be characterized in a relatively short number of cycles, Delayed Comparison Error Minimization can provide higher precision and/or be completed in fewer cycles than an FFT. However, it is a special purpose tool that requires the signal to fall within some expected range in order to work correctly. Without narrow limits on the max and min values to look for, the advantages of such a technique are diminished.

ACKNOWLEDGMENT

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