DEVELOPMENT OF A REAL-TIME SOLVER FOR LOCAL EIGENVALUE MODIFICATION PROCEDURE

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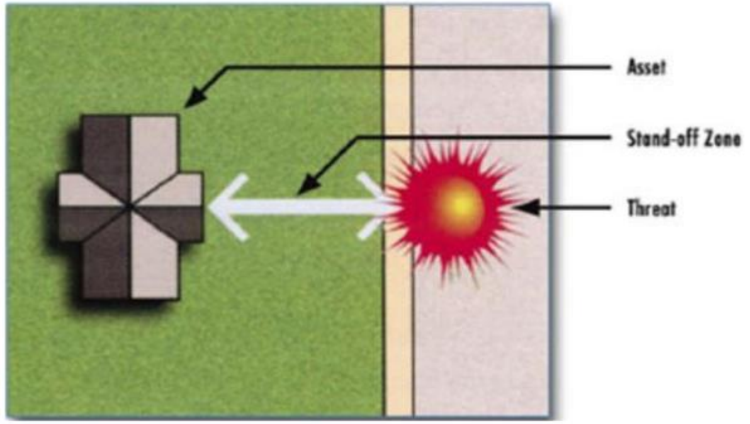
High-rate Dynamics in the Real-world

Civil Structures
Exposed to blast



Samali, B., et al., *Review of the basics of state of the art of blast loading. Asian Journal of Civil Engineering.* (2018).

Automotive safety systems against Collision

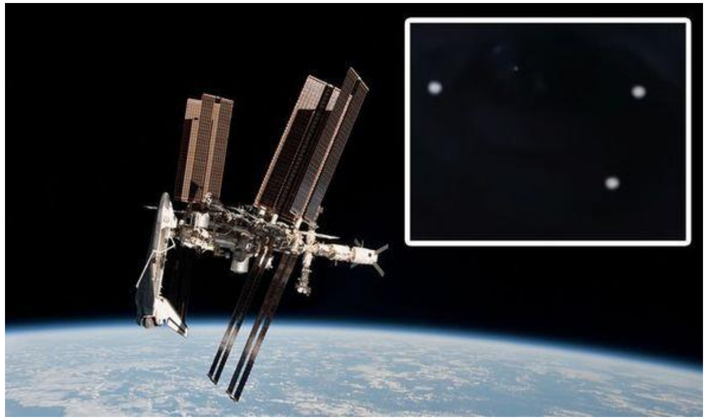
airbag deployment

Space shuttle and Aerial Vehicles Prone to In-Flight Anomalies

Hypersonic vehicles

Ballistic packages

Debris approaching space shuttle

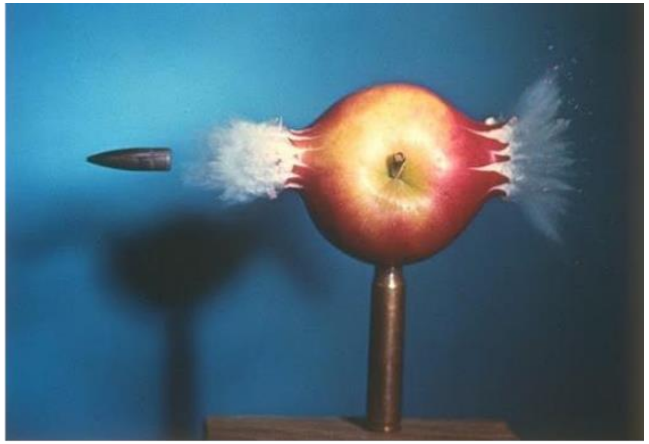


Lightning strikes on aircraft

Fighter jets

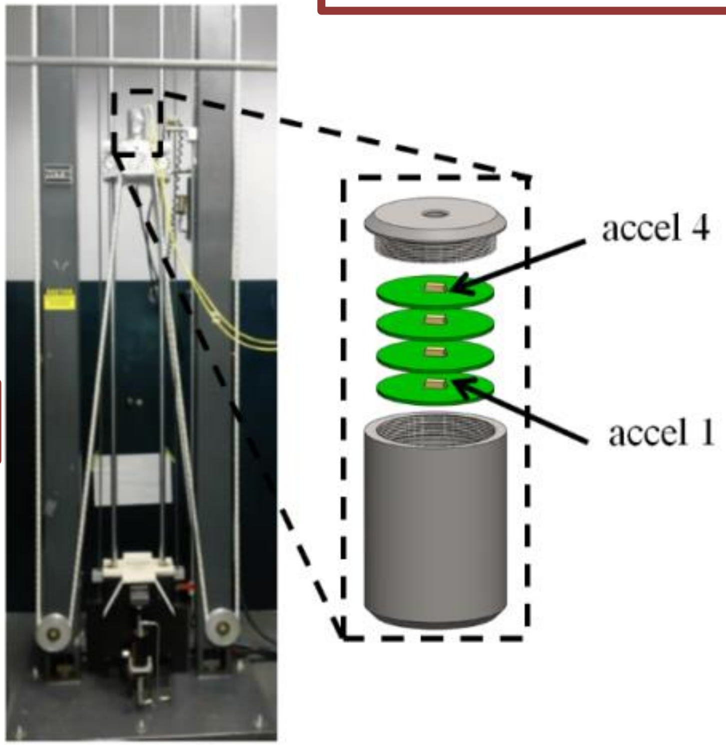
Description of High-rate Dynamics

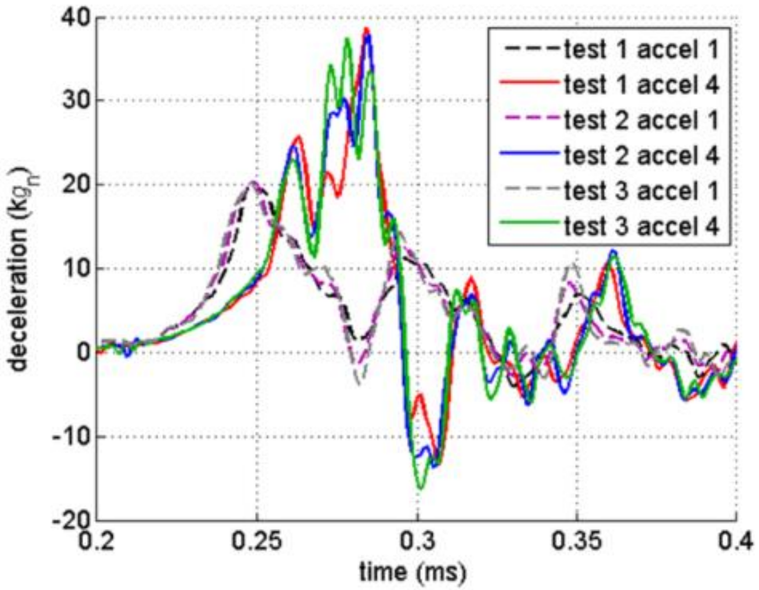
High-rate (<100ms)



High-amplitude (acceleration > 100 *g*)

The deceleration event in drop tower tests typically lasts for 0.5ms





Hong, J. et al,. *Introduction to state estimation of high-rate system dynamics*. Sensors, 18(2):217, Jan 2018

High-rate Dynamics systems

large uncertainties in the external loads

high levels of non-stationarities and heavy disturbances

unmodeled dynamics from changes in system configuration

Dodson, Jacob, et al. "High-rate structural health monitoring and prognostics: an overview." *Data Science in Engineering, Volume 9* (2021): 213-217.

Technical Challenges of Estimating State of High-rate Dynamic system

Adequate sensing

Sensor response

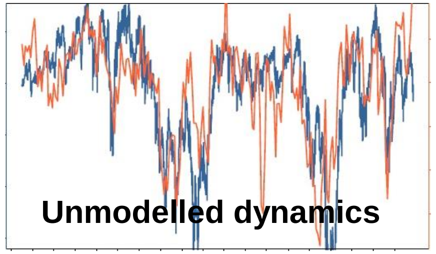
3 µs – 10 µs

Lack of system Knowledge

Duration of event

30 µs – 100 ms

High variability in loading



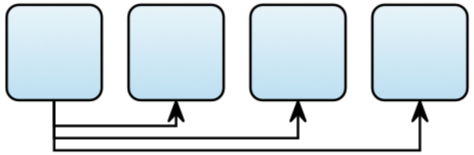
Different behavior regimes

250 µs – 1 sec

Limited resources for algorithm implementation

Algorithm execution and decision making

Current goal

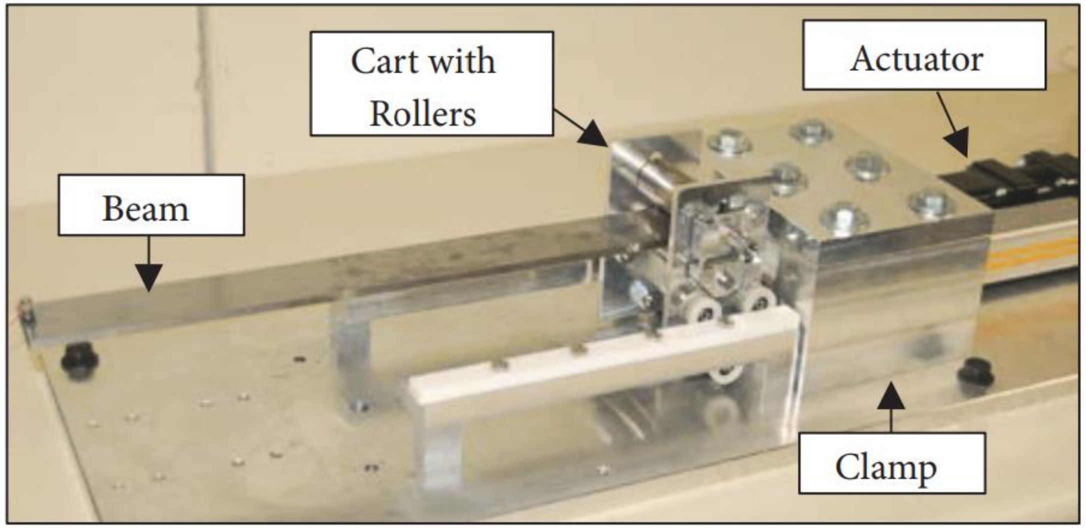


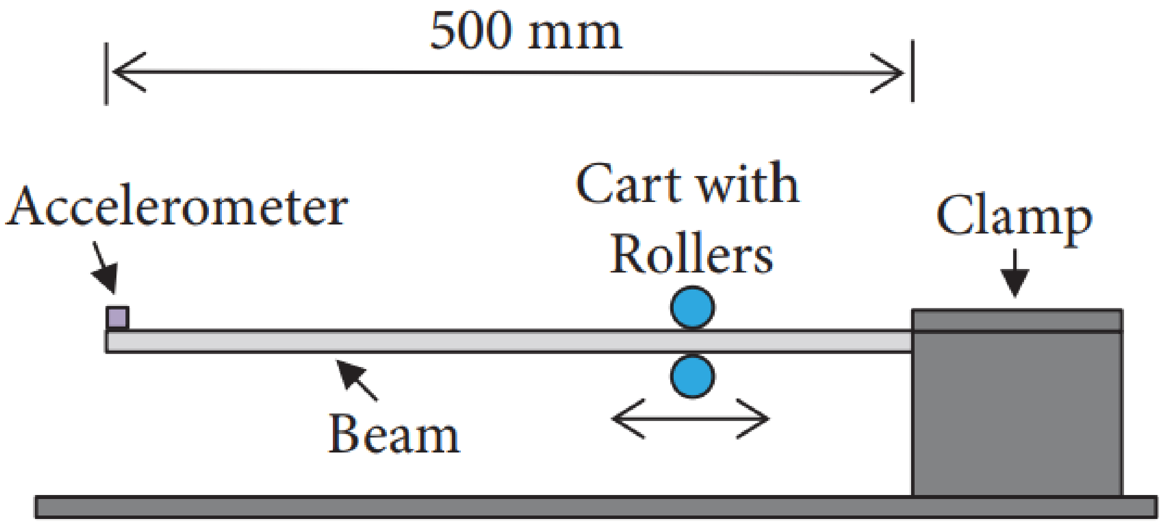
1ms

Dodson, Jacob, et al. "High-rate structural health monitoring and prognostics: an overview." *Data Science in Engineering, Volume 9* (2021): 213-217.

Reproducing High-rate Dynamics in Laboratory

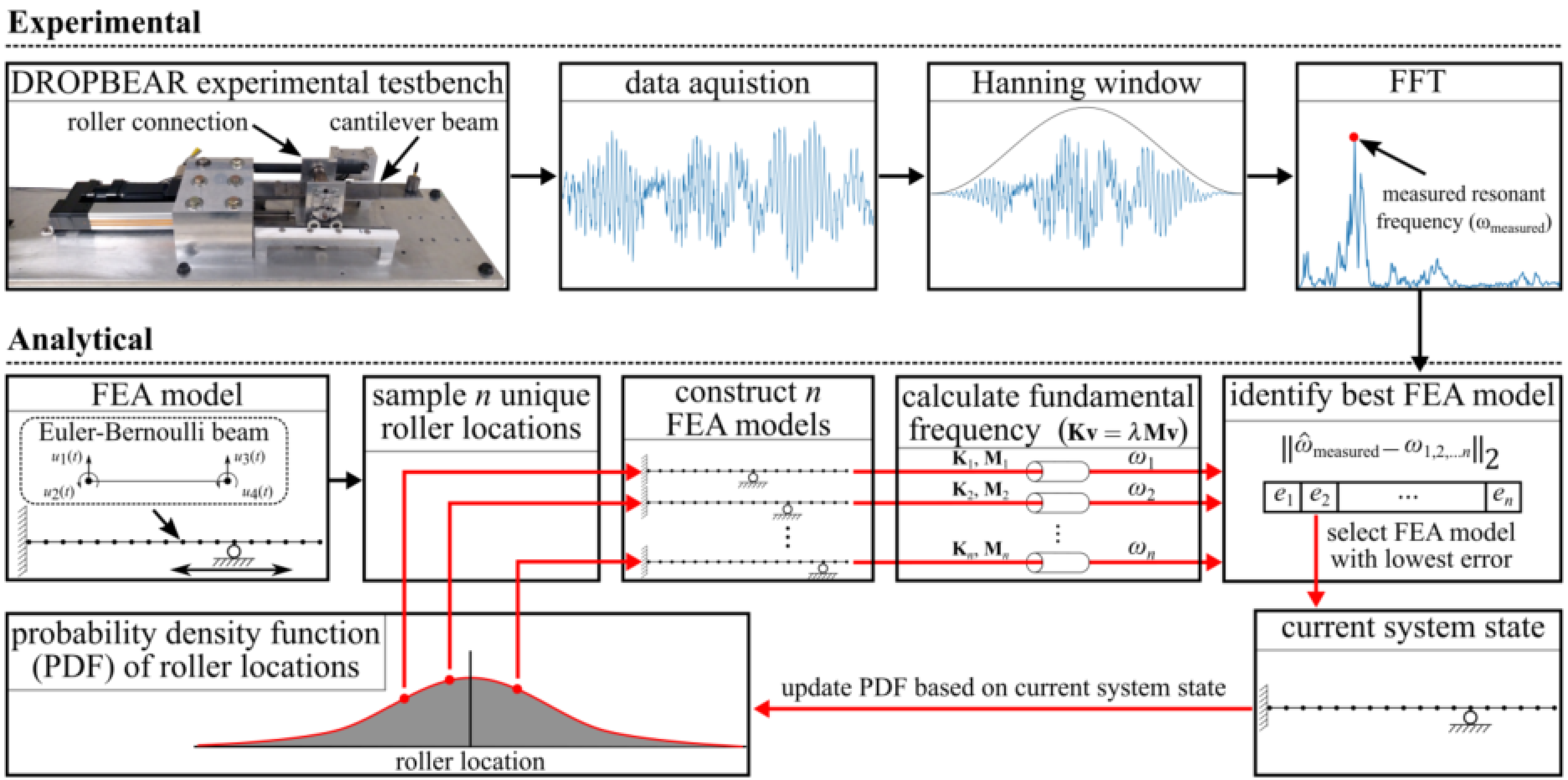
DROPBEAR testbed





Joyce, B., Dodson, J., Laflamme, S., & Hong, J. *An experimental test bed for developing high-rate structural health monitoring methods*. Shock and Vibration, 2018.

Real-time FEA model updating

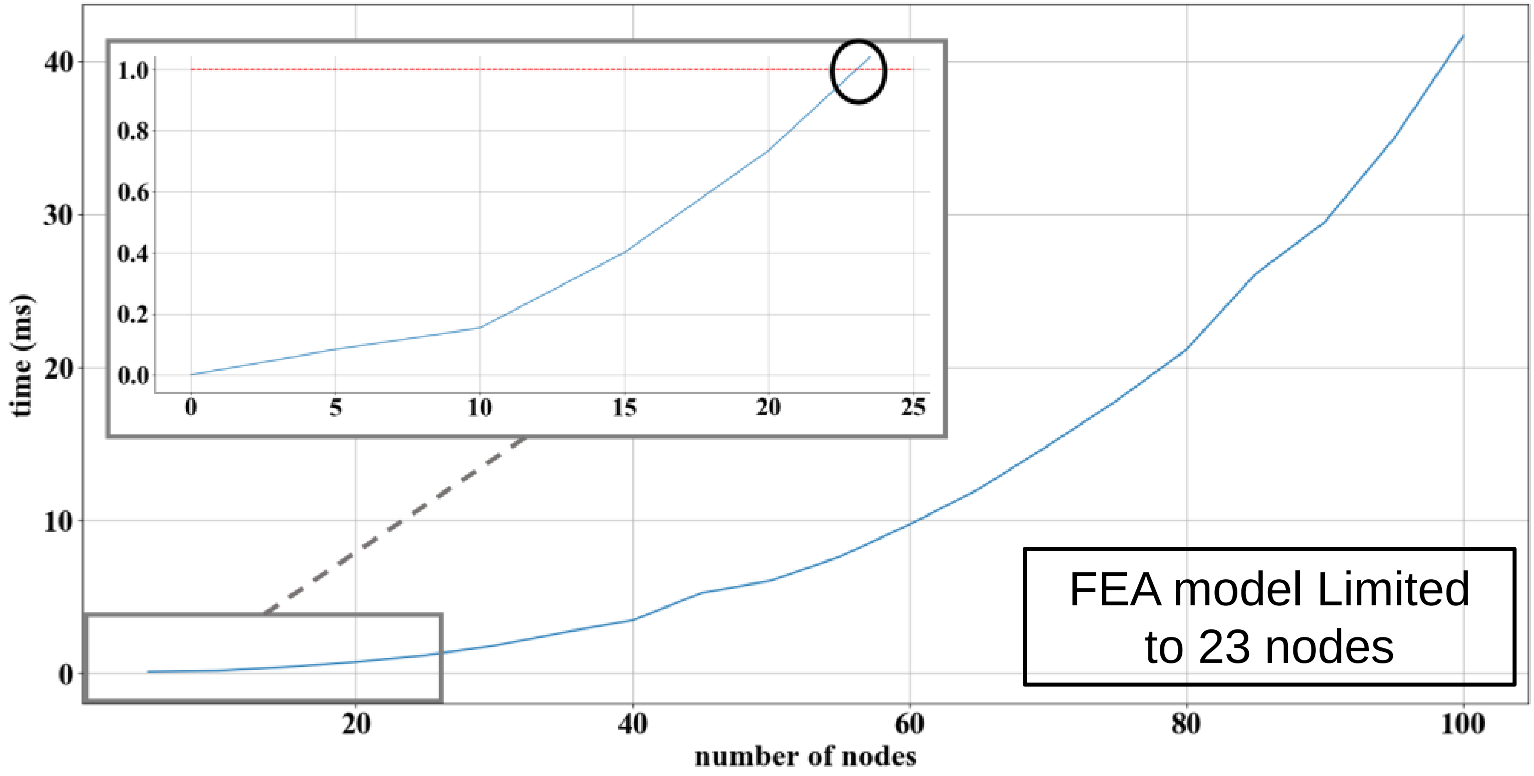


Downey A., et al,. "Millisecond Model Updating for Structures Experiencing Unmodeled High-Rate Dynamic Events" *Mechanical Systems and Signal Processing* ***138***, 2020

FEA Computation speed for the DROPBEAR

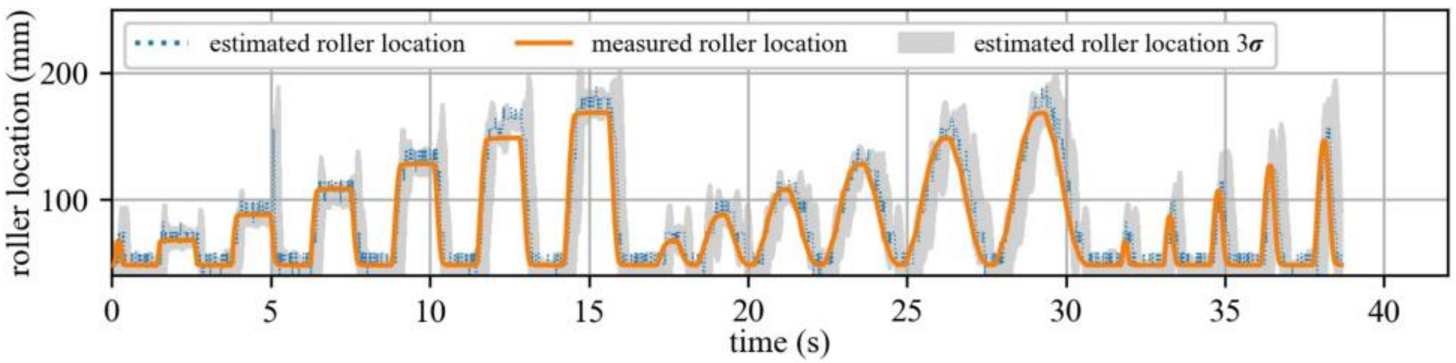
General Eigenvalue solutions accurately estimates the state of the DROPBEAR

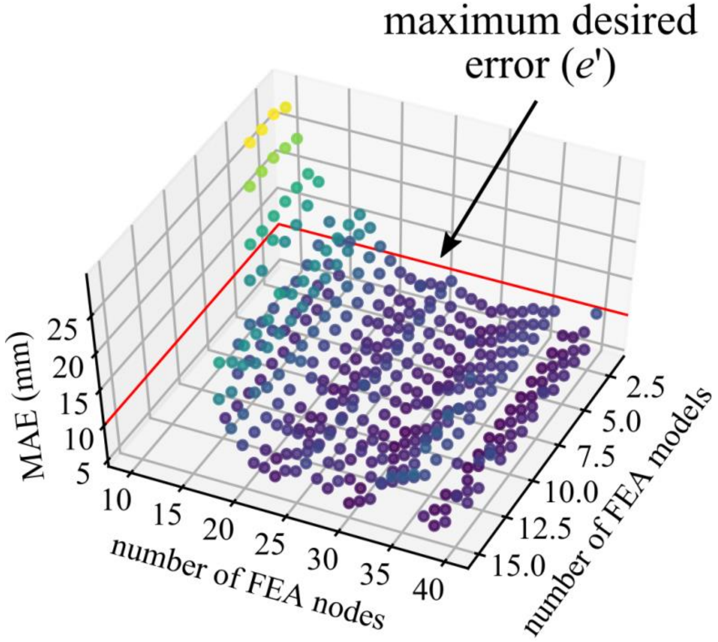
Solving for system's frequencies accounted for 90% of algorithm iteration time

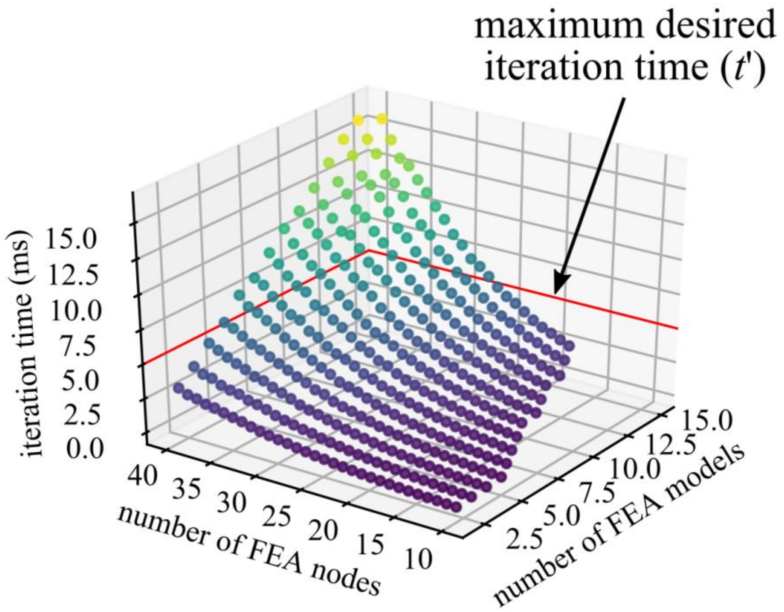


Carroll, M., Downey, A., Dodson, J., Hong, J. and Scheppegrell, J., "Analysis of Computation Speeds of Eigenvalue Solutions for High-Rate Structural Health Monitoring."

Real-time FEA model updating Results





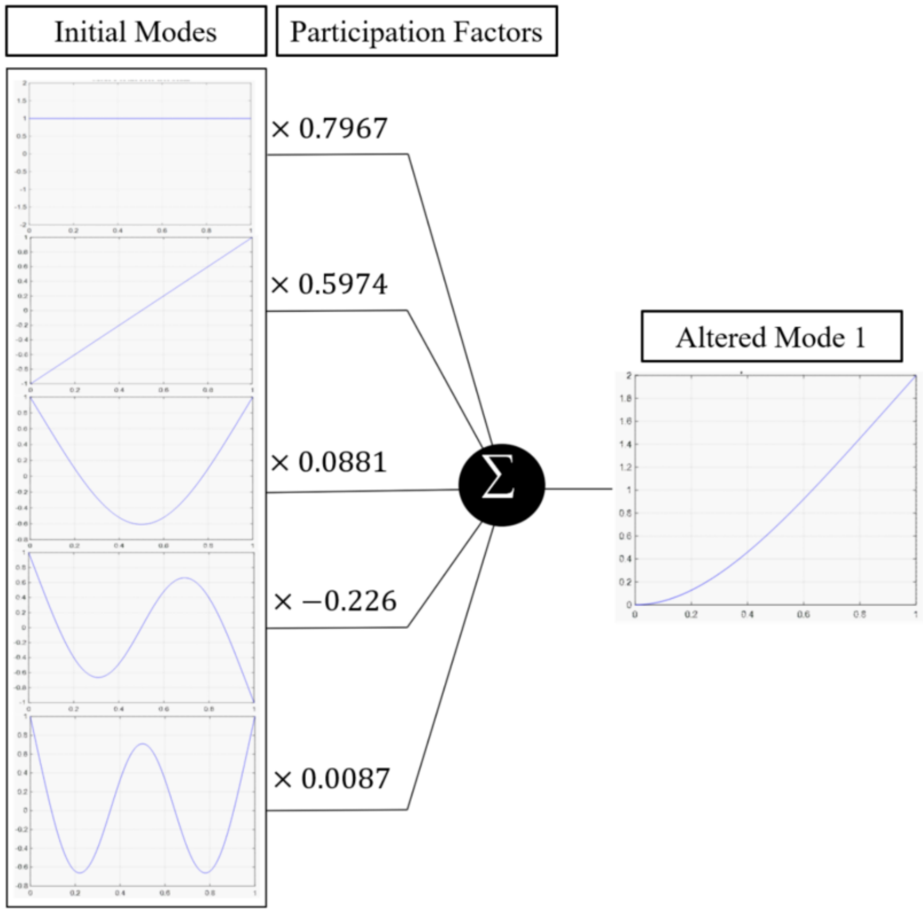


Downey A., et al,. “Millisecond Model Updating for Structures Experiencing Unmodeled High-Rate Dynamic Events” *Mechanical Systems and Signal Processing* ***138***, 2020

Local Eigenvalue Modification Procedure (LEMP)

SUMMARY

- ❑ Developed by Wesseinburger in 1968
- ❑ Identifies physical changes to the system such as mass, stiffness or damping using changes such as frequencies or mode shapes
- ❑ Model the altered state as a mixture of the initial state and changes made to the initial state
- ❑ Reduces the GE equation to a set of second-order equations

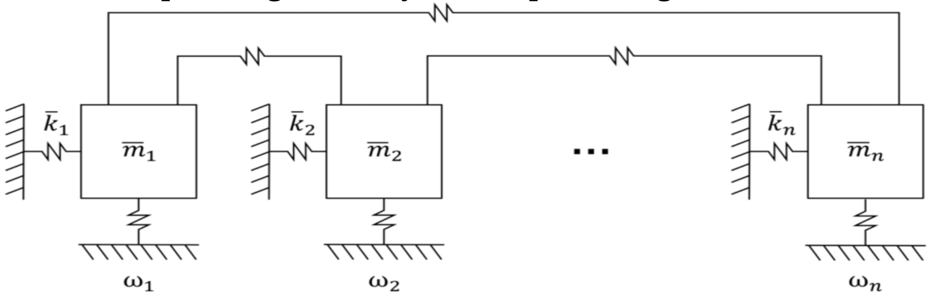


Avitabile, P., "*Twenty Years of Structural Dynamic Modification- A Review*," *Sound and Vibration,* pp. 14-25. 2003
Drnek, C. R., "*Local eigenvalue modification procedure for real-time model updating of structures experiencing high-rate dynamic events*," (2020).

Local Eigenvalue Modification Procedure (LEMP)

n independent single DOF systems representing the initial state

Coupled single DOF systems representing the altered state

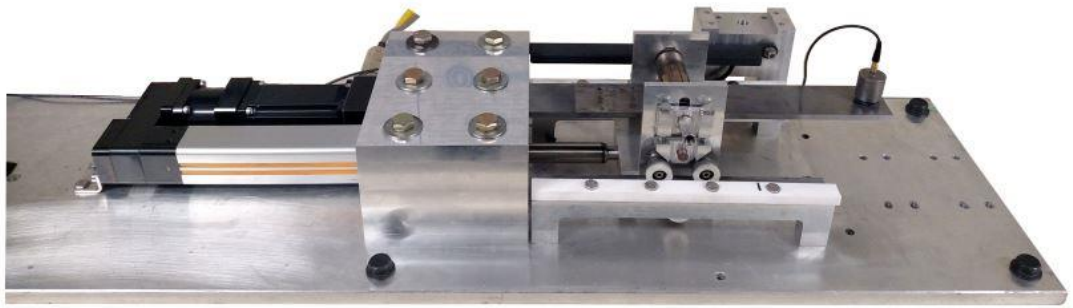


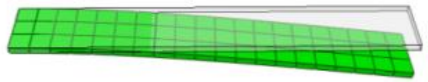
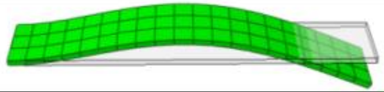
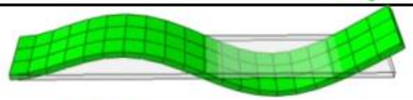
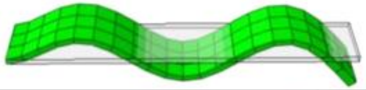
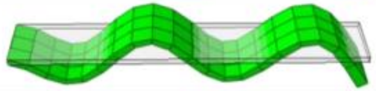
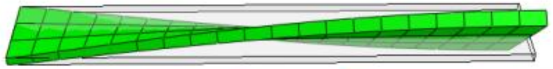
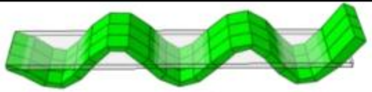
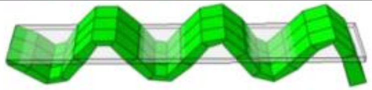
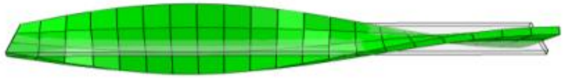
	Original State	Modification	Altered State	
Physical Space	$[\mathbf{M}_1], [\mathbf{K}_1]$	$[\Delta\mathbf{M}_{12}], [\Delta\mathbf{K}_{12}]$	$[\mathbf{M}_2], [\mathbf{K}_2]$	'N' Physical DOF
Modal Transformation	$\{x\} = [U_1]\{p_1\}$	$$\frac{-1}{\alpha} = \sum_{r=1}^{m} \frac{v_r^2}{\omega_r^2 - \Omega_r^2}$$ **Solved using Divide and Conquer method**	$\{x\} = [U_2]\{p_2\}$	M<<N
Modal Space	$[\omega_1^2], [U_1]$	$\{p_1\} = [U_{12}]\{p_2\}$	$[\Omega_2^2], [U_2]$	'M' Modal DOF

Drnek, C. R., "*Local eigenvalue modification procedure for real-time model updating of structures experiencing high-rate dynamic events*," (2020).

Local Eigenvalue Modification Procedure (LEMP)

Additional simplification is achieved by truncating *n* independent single degree of systems to include only the *m* modes of interest.

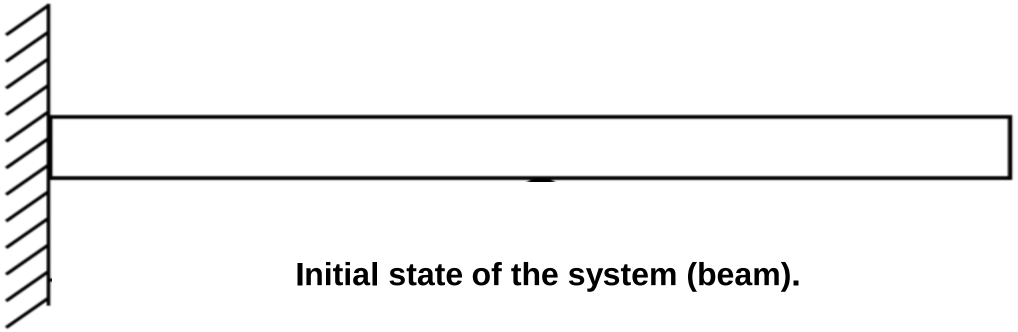


Mode	Frequency (Hz)	Mode type	Shape
1	37.6956	Bending-Y	
2	248.561	Bending-Y	
3	713.463	Bending-Y	
4	1416.4	Bending-Y	
5	2353.62	Bending-Y	
6	3519.66	Bending-Z	
7	4918.5	Torsional	
8	6569.9	Bending-Y	
9	8422.02	Bending-Y	
12	15420.6	Torsional	

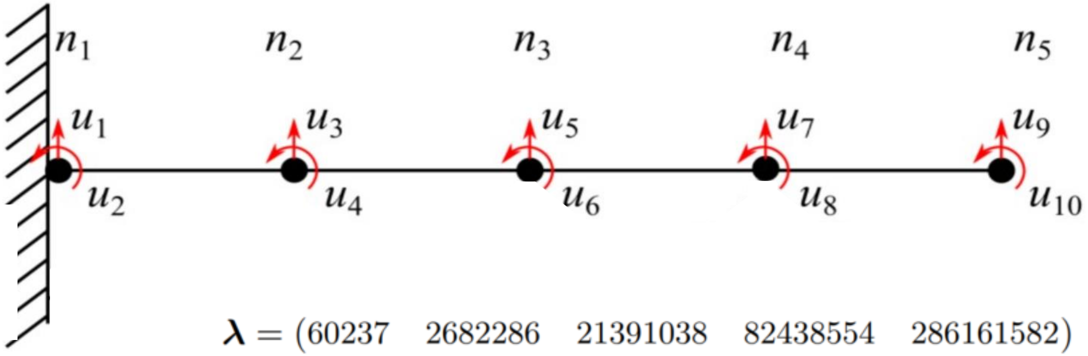
Drnek, C. R., "*Local eigenvalue modification procedure for real-time model updating of structures experiencing high-rate dynamic events*," (2020).

Single-State change Estimation with LEMP

- **Initial state generalized eigenvalue solution**



Initial state of the system (beam).



$$\boldsymbol{\lambda} = (60237 \quad 2682286 \quad 21391038 \quad 82438554 \quad 286161582)$$

Construct the elemental mass and stiffness matrices ($\mathbf{M}_1$ and $\mathbf{K}_1$)

$$\mathbf{U}_1 = \begin{pmatrix} -0.000005 & 0.00011 & 0.00051 & 0.00138 & -0.00340 \\ -0.000001 & 0.000008 & 0.000023 & 0.000046 & -0.000088 \\ -0.184749 & 0.862567 & 1.521322 & 1.535297 & -0.796654 \\ -3.95962 & 13.68743 & 10.37102 & -17.27622 & 68.83187 \\ -0.64779 & 1.52565 & 0.15321 & -1.53923 & 0.260667 \\ -6.37642 & -1.72852 & -3.28287 & -6.21277 & -80.17702 \\ -1.26088 & 0.420824 & -1.25908 & 1.14986 & 0.176068 \\ -7.43893 & -2.20332 & 13.1159 & 21.2392 & 76.3001 \\ -1.92314 & -1.86050 & 1.94283 & -1.87065 & -1.9711 \\ -7.61313 & -27.5937 & 46.6347 & -63.1058 & -96.1961 \end{pmatrix}$$

solve the general eigenvalue problem to obtain the squares of the first n natural frequencies, and the first n modal vectors for the initial state

$$f_1 = (39 \quad 261 \quad 736 \quad 1445 \quad 2692)$$

LEMP Implementation

- **Step 1: Addition of roller condition**

$$\Delta\mathbf{K}_{12} = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1e10 \quad 0 \quad 0)$$

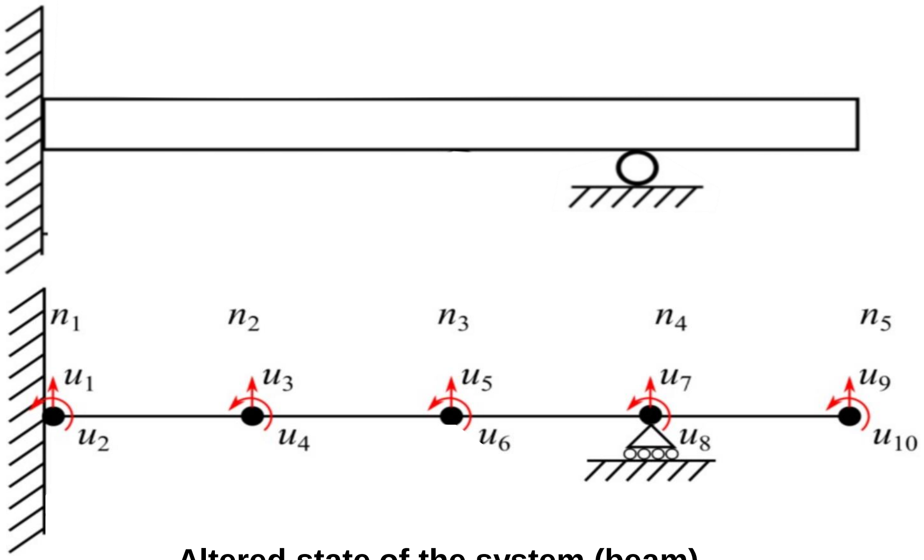
- **Step 2: Spectral decomposition of $\Delta\mathbf{K}_{12}$**

$$\mathrm{T} = (1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1)$$

$$\alpha = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1e10 \quad 0 \quad 0)$$

- **Step 3: Set truncation: include only contributing nodes**

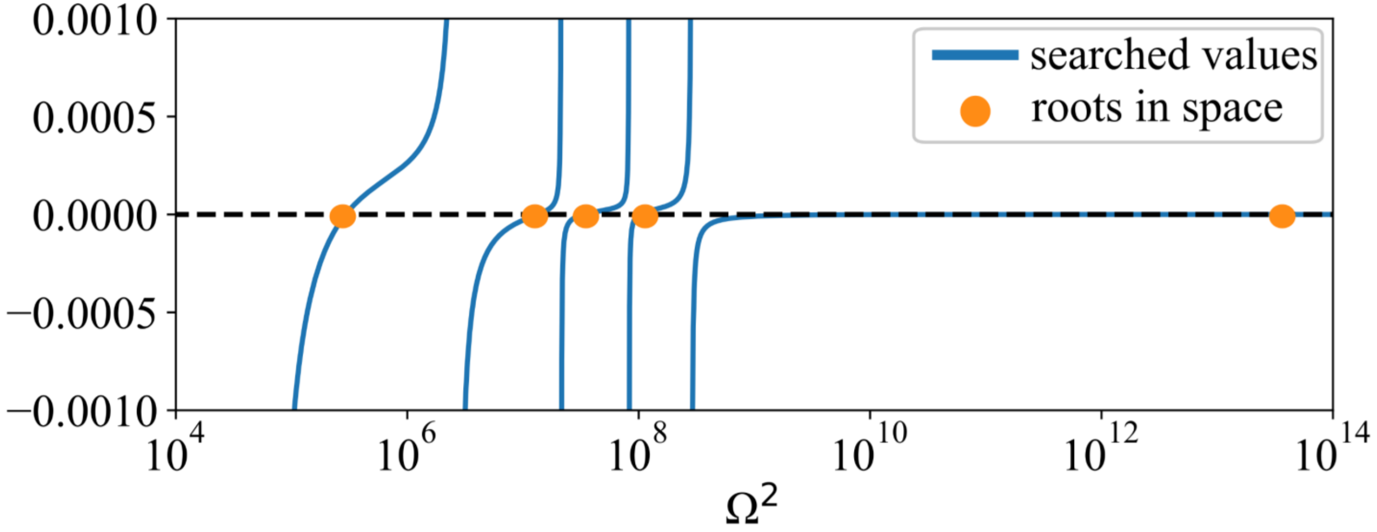
The contributing vectors are reduced to only those values in the 8th row of each matrix.

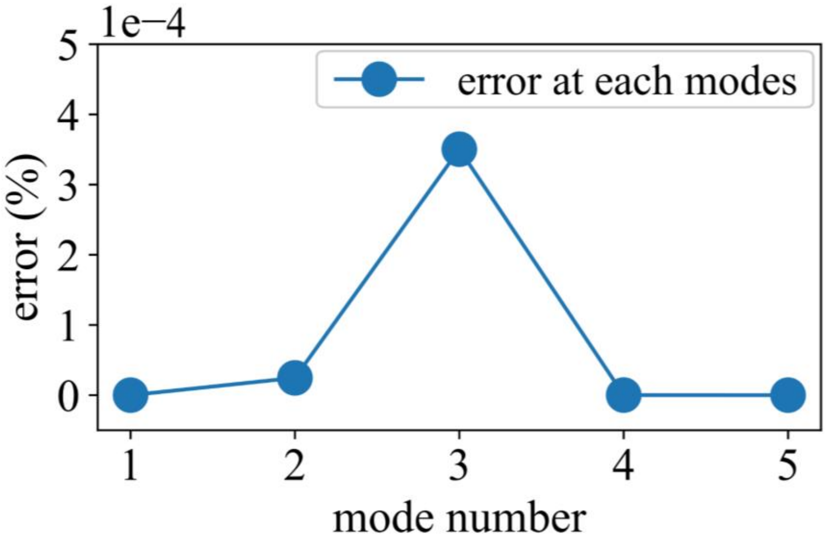


Altered state of the system (beam) with added roller condition (spring)

LEMP Implementation

- **Step 4: Obtain Ω^2 using Divide and conquer**





Ω^2 values using D&C and Sympy function "solveset"

Mode	Frequency (D&C) (Hz)	Frequency (Solveset) (Hz)	Error (Hz)
1	293496.95719048503	293496.95719048500	58.21 x 10^{-12}
2	13405184.4772621	13405181.1772621	33.00 x 10^{-1}
3	33185211.781733	33185095.485877	11.63 x 10^{1}
4	101330615.342713	101330615.250119	92.59 x 10^{-3}
5	69856604350042.539	69856604350042.500	39.06 x 10^{-3}

LEMP Implementation

- **Step 5: Solve for new frequencies**

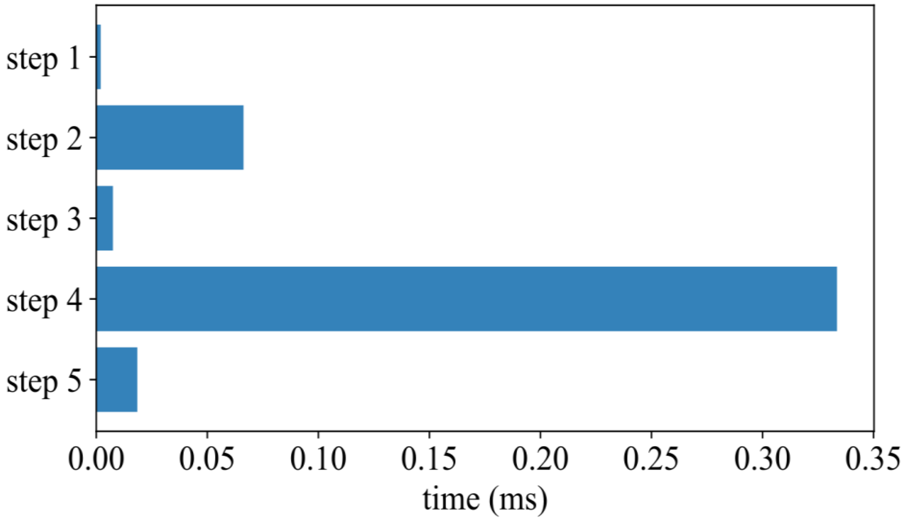
  The new natural frequencies f_2 in Hz are then calculated for the five modes in the model utilized.

  $$f_2 = \begin{pmatrix} 86 & 583 & 917 & 1602 & 1330221 \end{pmatrix}$$

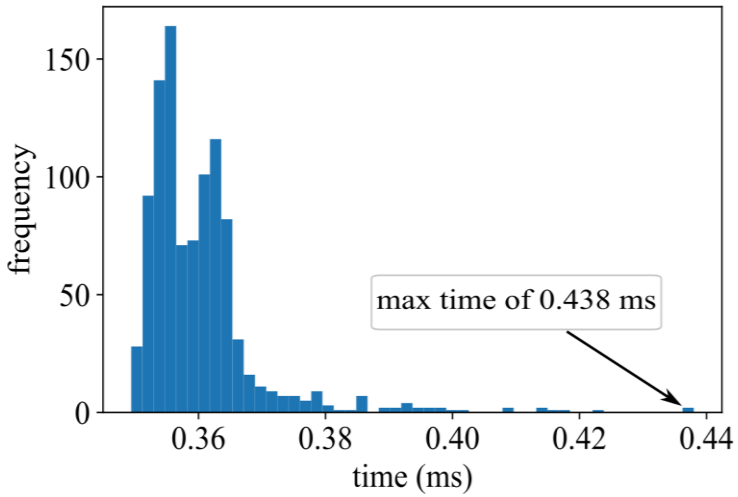
- **Step 6: Update roller position**

  The final step of the process is to use the obtained frequency value to determine the position of the added roller on the beam.

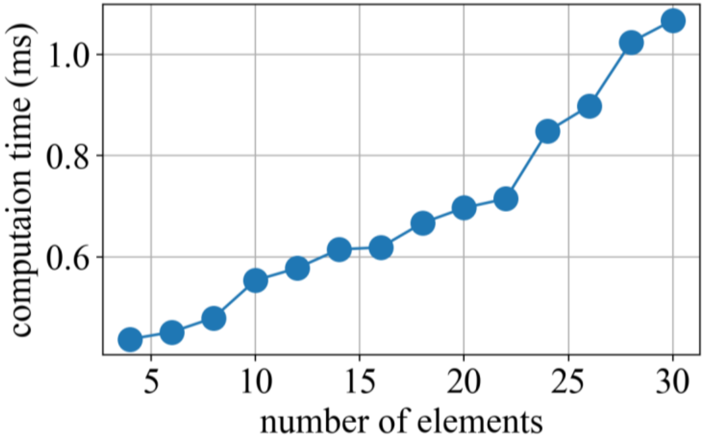
LEMP Algorithm Timing



Timing for each step



Distribution of 1000 simulations

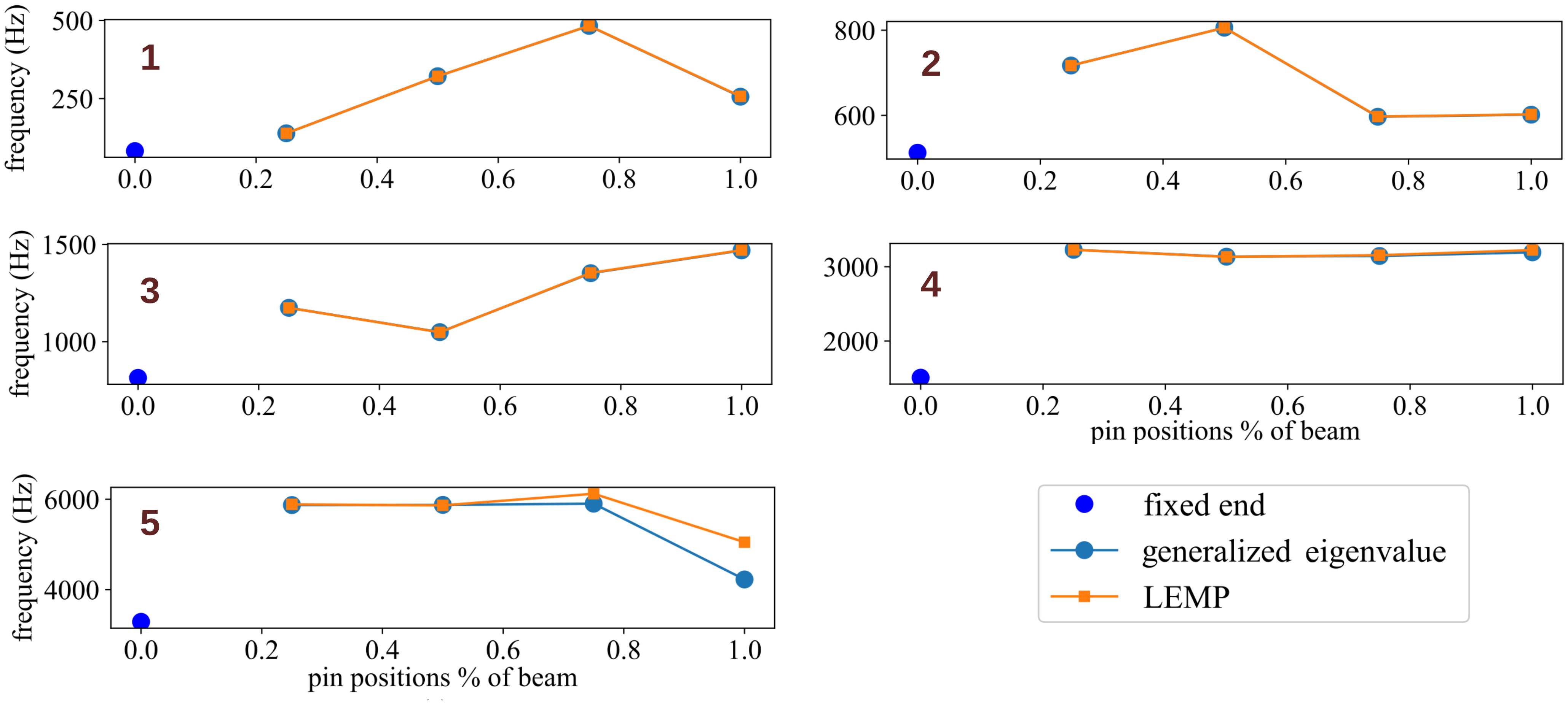


Timing with element number 4 to 30.

- Increasing the nodes increase the accuracy of the model
- <28 nodes achieves the 1ms times constraint

Generalized Eigenvalue and LEMP

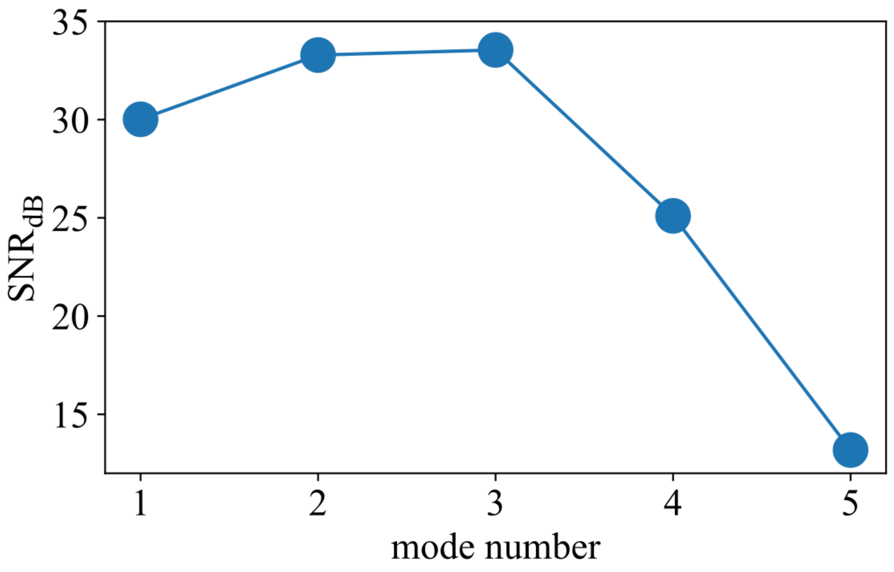
- Beam frequencies at each nodes on addition of roller at node 4 for first five modes

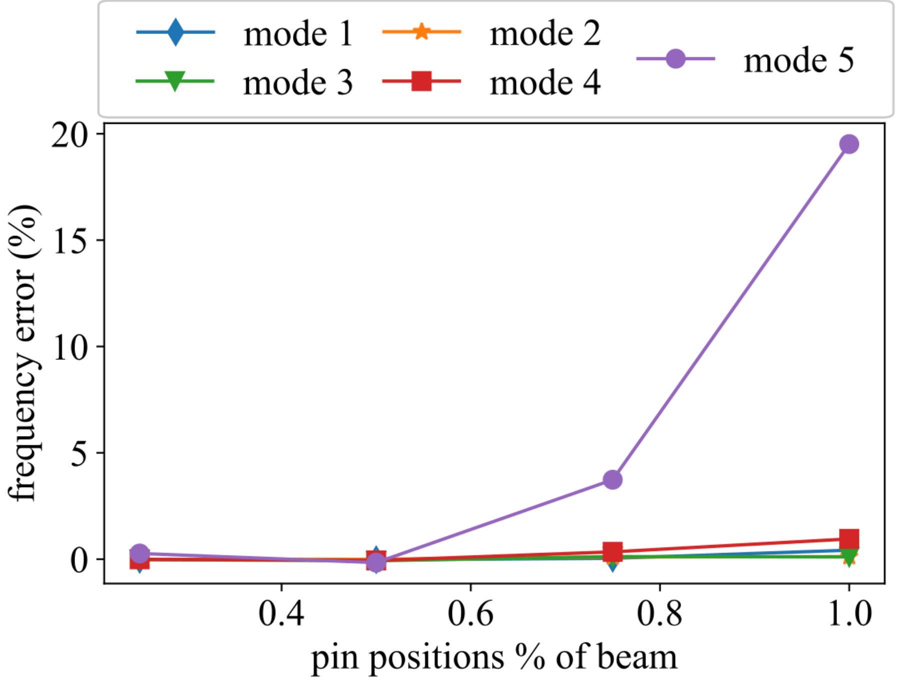


Generalized Eigenvalue and LEMP

- SNR & Error

mode	mean absolute error (Hz)	SNR $_{dB}$
1	0.2989	30.02
2	0.3193	33.38
3	0.5575	33.54
4	9.8136	25.10
5	262.80	13.18





Conclusion

- Experimental results demonstrated an average time of 0.361 ms for single state change updating was achieved using the five nodes beam.
- Results showed that the frequencies obtained for estimation using GE and LEMP are close with high SNR and low error at the nodes. The error at the fifth mode is expected to reduce as the number of nodes in the beam increases.
- The LEMP algorithm has the potential to enable real-time frequency-based model updating of complex systems that would not be achievable using the general eigenvalue approach.

Future Work

- In future work, the LEMP algorithm will be applied to more complex state estimation.

Acknowledgement

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THANKS!

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