OPTIMAL SAMPLING METHODOLOGIES FOR HIGH-RATE STRUCTURAL TWINNING

Alexander B. Vereen, Emmanuel A. Ogunniyi, Dr. Austin R.J. Downey
Dr. Erik Blasch, and Jason D. Bakos, and Dr. Jacob Dodson

UNIVERSITY OF
South Carolina

Presenter: Alexander B. Vereen

MOTIVATION

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HIGH-RATE STRUCTURAL HEALTH MONITORING

- Health monitoring of structures operating in high-rate dynamic environments behavioral interventions in response external stimuli.
- Examples of structures operating in high-rate dynamic environments include:
  - hypersonic vehicles
  - space craft
  - ballistic packages
- Intelligent reactions require an up-to-date model of the structure's state.

STRUCTURES EXPERIENCING HIGH-RATE DYNAMIC EVENTS

Applications:

1. Vehicle collision
2. Blast mitigation
3. Ballistic packages
4. Hypersonic vehicles
5. Hard Target Penetrating Weapons

Vehicle Collision

Active Blast Mitigation

Ballistics Packages

Hypersonic Vehicles

Hard Target Penetrating Weapons

HIGH-RATE STRUCTURAL HEALTH MONITORING

- Due to the timescale of relevance to these structures means that the model must be continuously updated with a time step of 1 millisecond or less.
- However, traditional frequency-based methods for updating the finite element model online require solving the generalized eigenvalue problem a computationally expensive process.

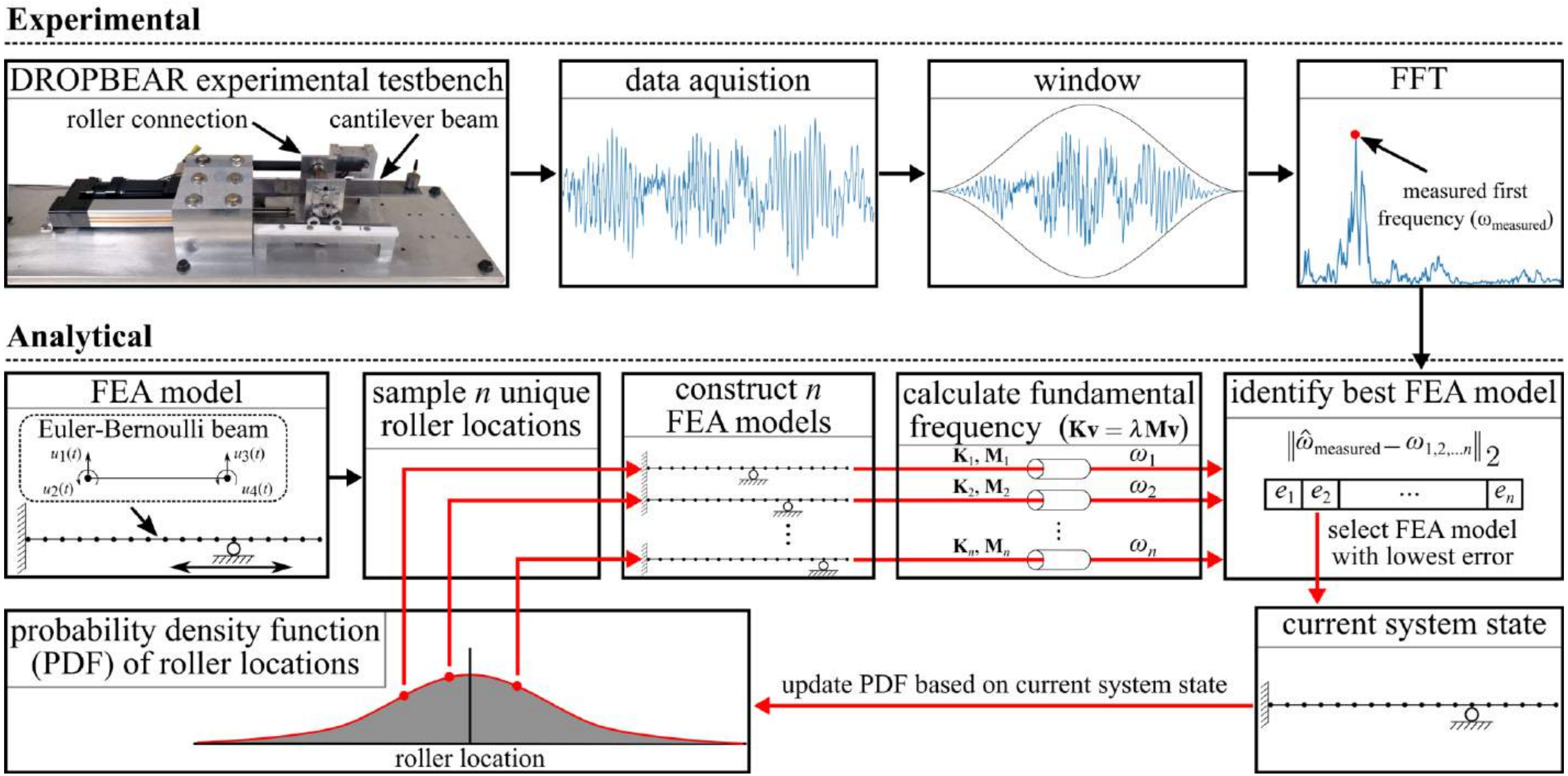
Automotive impact and crashes

High Speed aircraft and airframes

BACKGROUND

Real-Time Model Updating Through Error Minimization

A frequency-based model updating technique was developed to update an FEA model of the system.



Downey A., et al,. "Millisecond Model Updating for Structures Experiencing Unmodeled High-Rate Dynamic Events" *Mechanical Systems and Signal Processing* ***138***, 2020

Background: Modal Analysis

Modal analysis is used to find the mode shapes and frequencies of a structure during free vibration.

Starting with the equation of motion:

$$\mathbf{M}\ddot{x} + \mathbf{C}\dot{x} + \mathbf{K}x = 0$$

the damping coefficient can be ignored as its effect on the natural frequency is less than 0.0005%, resulting in the expression:

$$\mathbf{M}\ddot{x} + \mathbf{K}x = 0$$

assuming a temporal solution:

$$x(t) = \Phi\big(A_n \cos(\omega_n t) + B_n sin(\omega_n t)\big)$$

yields the following expression:

$$\Big(-\mathbf{\Omega}_n^2\mathbf{M}\Phi + \mathbf{K}\Phi_n\Big)q_n(t) = 0$$

where $q_n(t)=0$ is a trivial solution, therefore the eigenvalues and eigenvectors are solved for using the general eigenvalue problem formulation:

$$\boxed{\mathbf{K}\Phi_n = \lambda_n\mathbf{M}\Phi_n}$$

where:

$$\lambda_n = \mathbf{\Omega}_n^2$$

and:

$$\omega_n = \sqrt{\lambda_n}$$

TIME CONSUMING

COMPETITION

WHY A LIVE MODEL UPDATE

- The logical consideration is that solving for the position at all will always be slower than a look up table
- Model Updating holds promise for:
  - 2D systems such as thin plates
  - Multiple sequential modifications such as crack progation or multi damage sources
- The look up table would grow impractically large as the dimensionality of the problem increases as pre-calculated solutions are required every potential case and its branching evolutions

LEMP usage in the ‘70s and ’80s

LEMP enabled these calculations to be done very efficiently on very slow desktop computers.

- Structural Measurements Systems (SMS) sold a custom hardware and software setup.
- This was before the “personal computer” stage.

HP1000/A700 w/DIFA modal analysis system

SMS modal software called SDM used LEMP

HP5423 first dedicated FFT/Modal system - 1979

HP3000 desktop running “Rocky Mountain BASIC”

All images and knowledge curtesy of Peter Avitabile Professor Emeritus, Co-Director - Structural Dynamics & Acoustic Systems Laboratory at the University of Massachusetts Lowell

LEMP

- The Local Eigenvalue Modification Procedure (LEMP) is put forward to accelerate the extraction of natural frequencies from finite element models updated online.
- LEMP:
  1. presolve for the eigenvalue solution to a reference state of the system
  2. computes the single (i.e., local) change in the modal domain from the reference state to the current state online. The modal domain update in the local eigenvalue modification procedure bypasses the general eigenvalue problem, which is the most expensive computational step.

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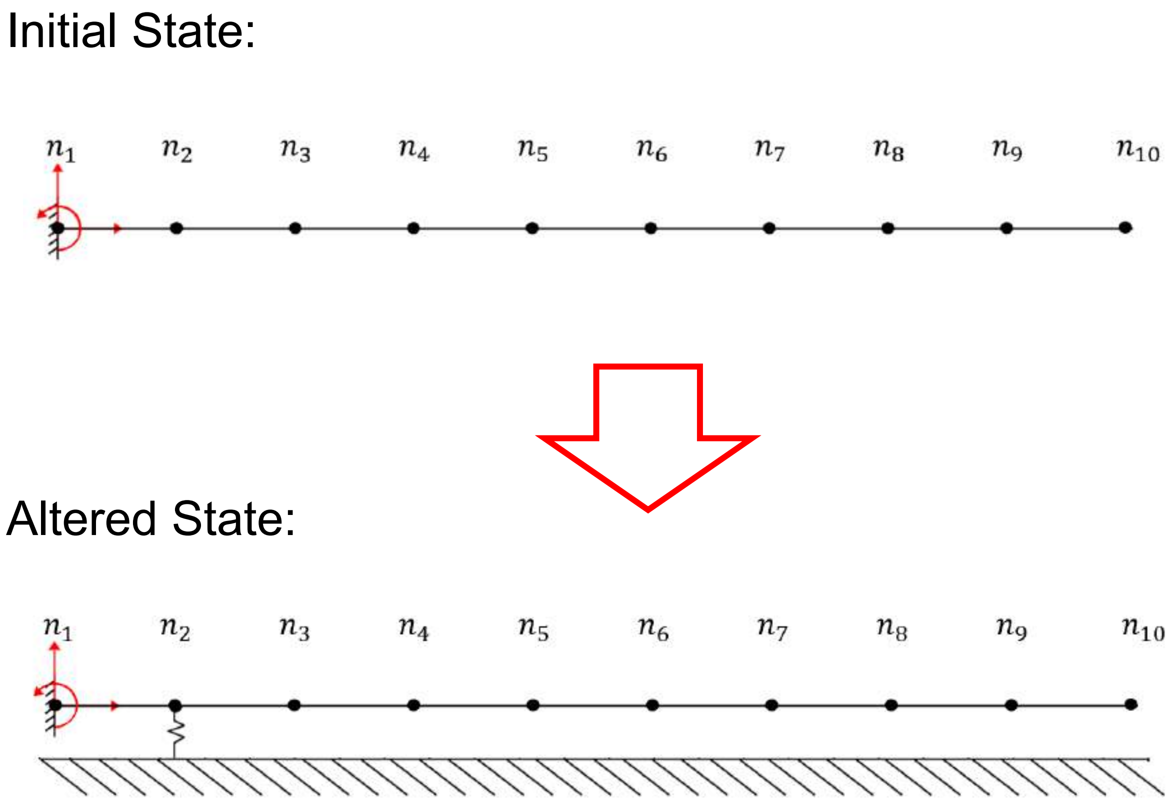
Changing States

- LEMP models one change in the system at a time.
- Still need to solve the GE problem once, then it can be updated with each successive step.



Single-State Change Estimation using LEMP

LEMP requires:

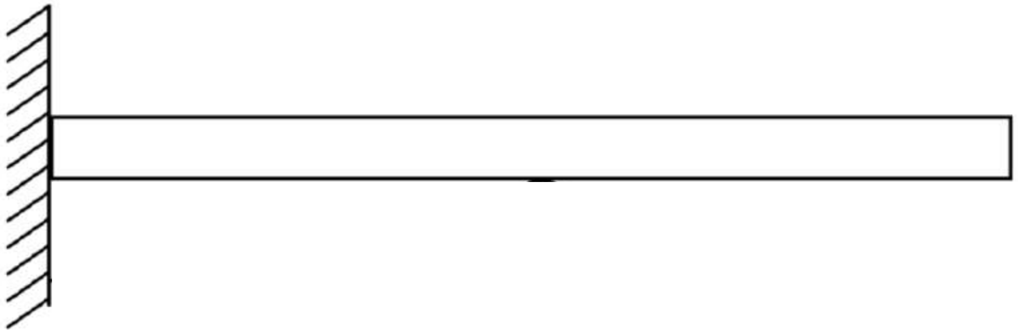
- Initial state general eigenvalue solution.
- Construct the elemental mass and stiffness matrices ($\mathbf{M}_1$ and $\mathbf{K}_1$).
- Solve the general eigenvalue problem to obtain the squares of the first n natural frequencies, and the first n modal vectors for the initial state.

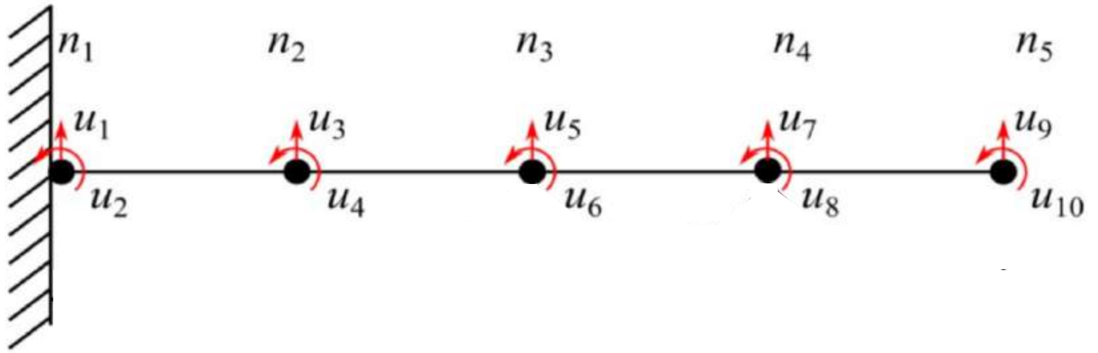
$$\boldsymbol{\lambda} = (60237 \quad 2682286 \quad 21391038 \quad 82438554 \quad 286161582)$$

$$\mathbf{U}_1 = \begin{pmatrix} -0.000005 & 0.00011 & 0.00051 & 0.00138 & -0.00340 \\ -0.000001 & 0.000008 & 0.000023 & 0.000046 & -0.000088 \\ -0.184749 & 0.862567 & 1.521322 & 1.535297 & -0.796654 \\ -3.95962 & 13.68743 & 10.37102 & -17.27622 & 68.83187 \\ -0.64779 & 1.52565 & 0.15321 & -1.53923 & 0.260667 \\ -6.37642 & -1.72852 & -3.28287 & -6.21277 & -80.17702 \\ -1.26088 & 0.420824 & -1.25908 & 1.14986 & 0.176068 \\ -7.43893 & -2.20332 & 13.1159 & 21.2392 & 76.3001 \\ -1.92314 & -1.86050 & 1.94283 & -1.87065 & -1.9711 \\ -7.61313 & -27.5937 & 46.6347 & -63.1058 & -96.1961 \end{pmatrix}$$

$$f_1 = (39 \quad 261 \quad 736 \quad 1445 \quad 2692)$$

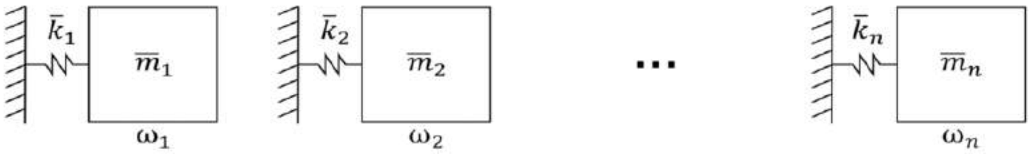
Initial state of the system (beam).



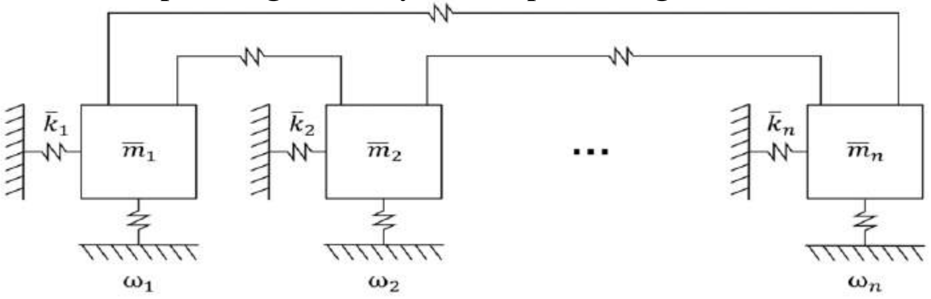


Local Eigenvalue Modification Procedure (LEMP)

n independent single DOF systems representing the initial state



Coupled single DOF systems representing the altered state



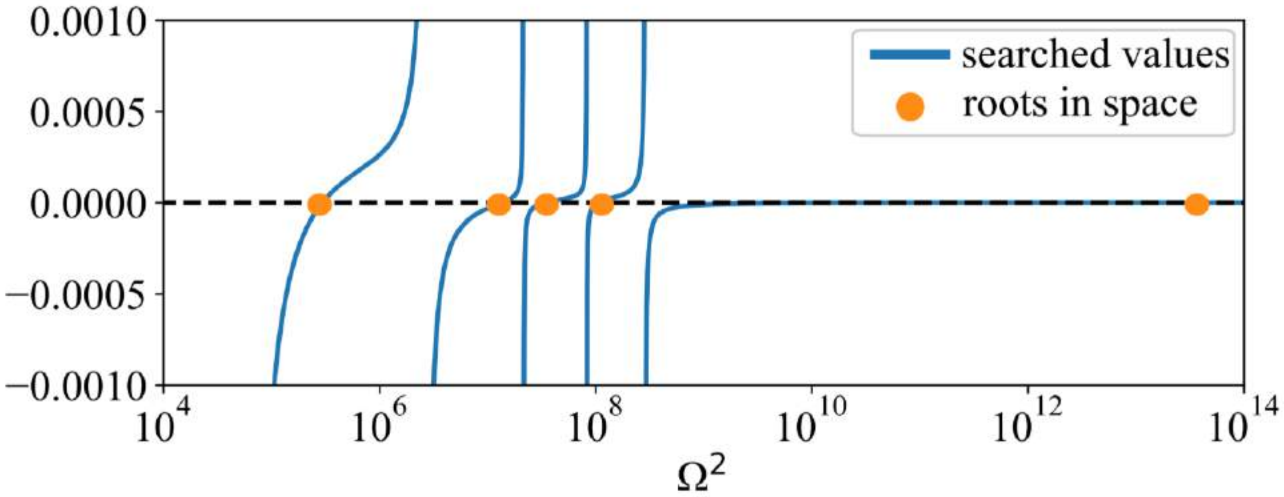
	Initial State	Modification →	Altered State	
Physical Space	$[\mathbf{M}_1], [\mathbf{K}_1]$ →	$[\Delta\mathbf{M}_{12}], [\Delta\mathbf{K}_{12}]$ →	$[\mathbf{M}_2], [\mathbf{K}_2]$	‘*n*’ Physical DOF
Modal Transformation	↓ $\{x\} = [U_1]\{p_1\}$ ↓	$\frac{-1}{\alpha} = \sum_{r=1}^{m} \frac{v_r^2}{\omega_r^2 - \Omega_r^2}$ **Solved using Divide and Conquer method**	↓ $\{x\} = [U_2]\{p_2\}$ ↓	*m*<<*n*
Modal Space	$[\omega_1^2], [U_1]$ →	$\{p_1\} = [U_{12}]\{p_2\}$ →	$[\Omega_2^2], [U_2]$	‘*m*’ Modal DOF

Avitabile, P., “*Twenty Years of Structural Dynamic Modification- A Review*,” *Sound and Vibration,* pp. 14-25. 2003
Drnek, C. R., “*Local eigenvalue modification procedure for real-time model updating of structures experiencing high-rate dynamic events*,” (2020).

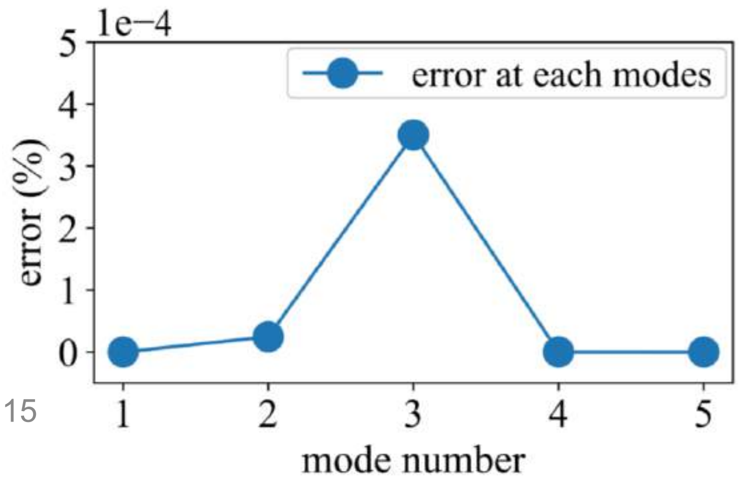
Real-time Secular Equation Solver

To solve LEMP, we need to find Ω^2, or the roots of our secular equation. We do this with a 6-step divide and conquer method.

Step 1: Adding roller condition
Step 2: Spectral decomposition of ΔK_{12}
Step 3: Set truncation: include only contributing nodes
Step 4: Obtain Ω^2 using Divide and Conquer
Step 5: Solve for new frequencies
Step 6: Update roller position



Mode	Eigenvalues (D&C)	Eigenvalues (Solveset)	Error
1	293496.95719048503	293496.95719048500	58.21 x
2	13405184.4772621	13405181.1772621	33.00 x
3	33185211.781733	33185095.485877	11.63 x
4	101330615.342713	101330615.250119	92.59 x
5	69856604350042.539	69856604350042.500	39.06 x



Code for SPIE conference paper at: https://github.com/ARTS-Laboratory/Paper-Development-of-a-Real-time-solver-for-the-Local-Eigenvalue-Modification-Procedure

PREVIOUS WORK

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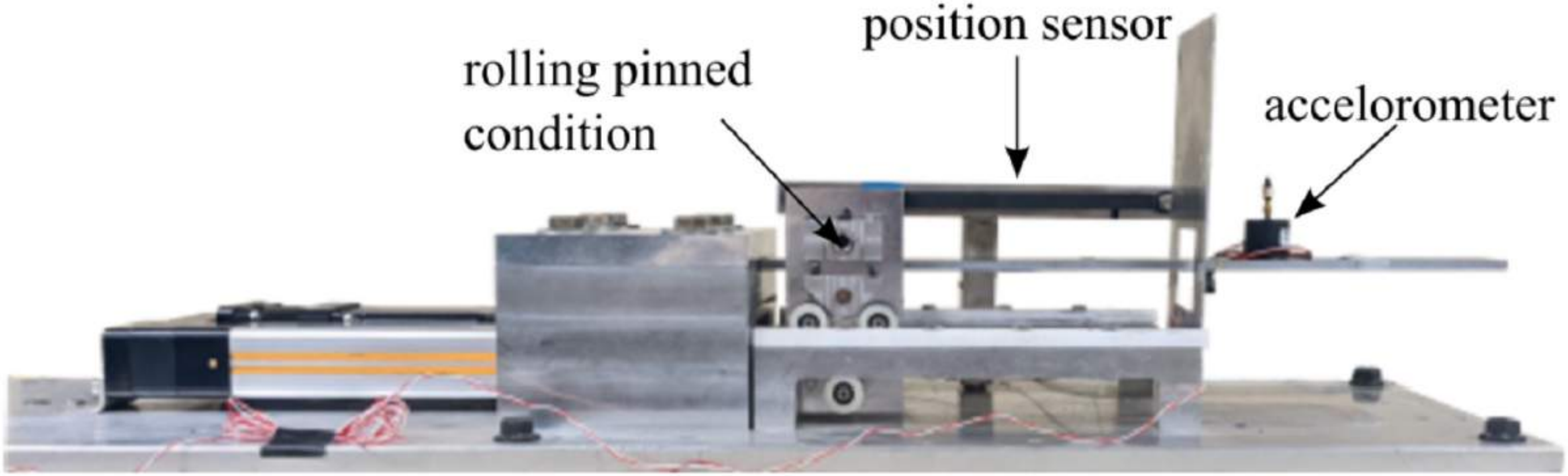
DROPBEAR

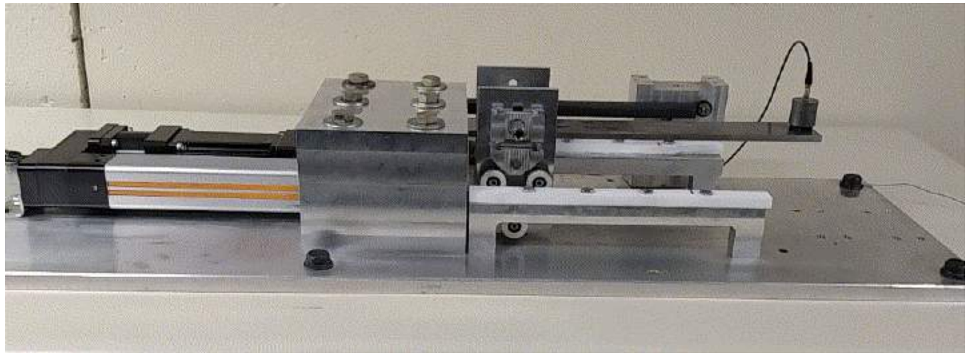
- Dynamic Reproduction of Projectiles in Ballistic Environments for Advanced Research (DROPBEAR) testbed

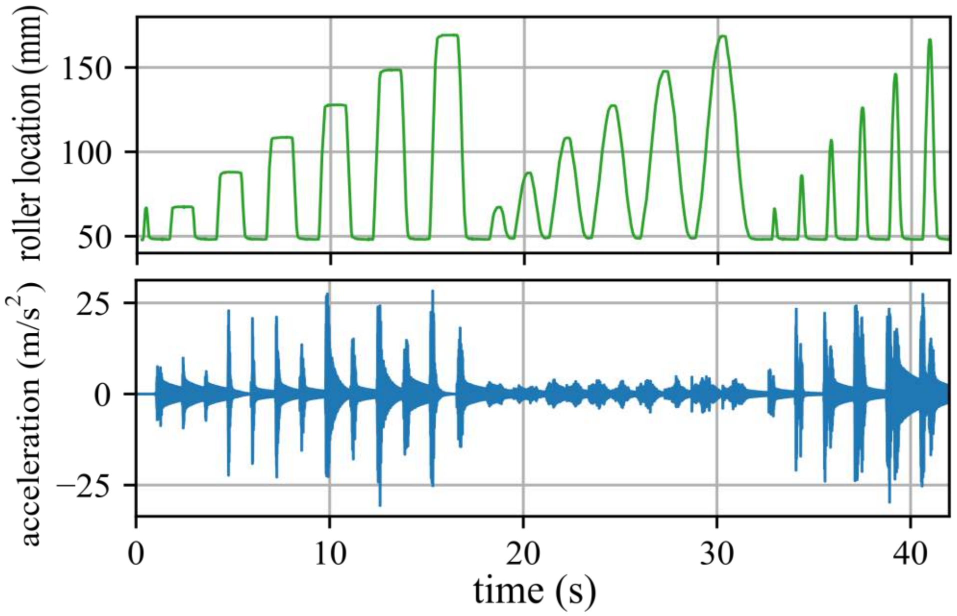


DROPBEAR

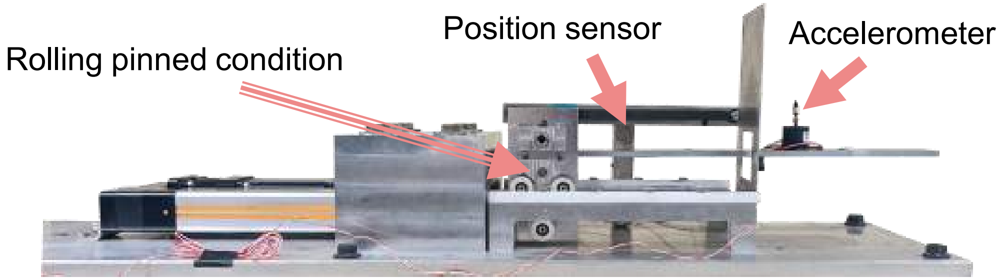
DROPBEAR experimental testbed:

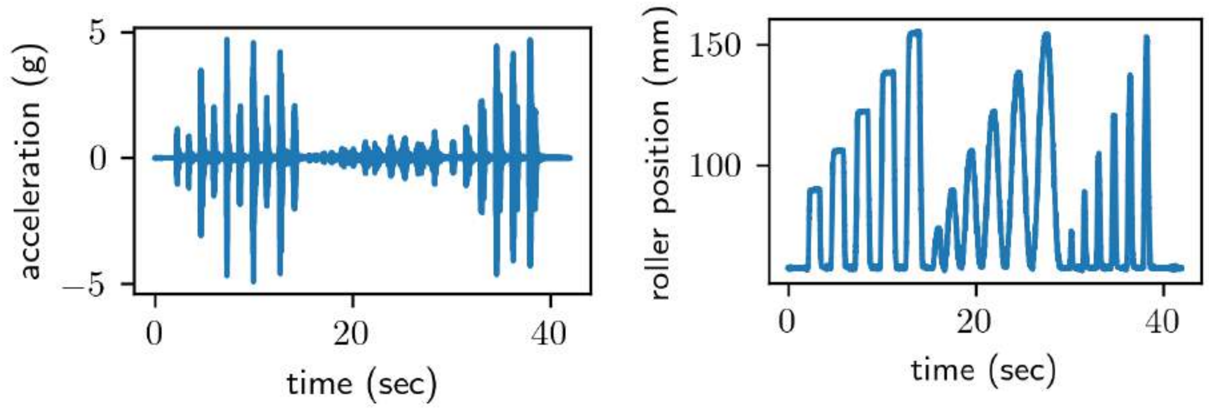
- The Dynamic Reproduction of Projectiles in Ballistic Environments for Advanced Research (DROPBEAR) was used to generate the experimental data in this work.
- Cantilever beam with a controllable roller to alter the state.
- Acceleration and pin location are recorded.
- Dataset available on GitHub at: https://github.com/High-Rate-SHM-Working-Group/Dataset-2-DROPBEAR-Acceleration-vs-Roller-Displacement

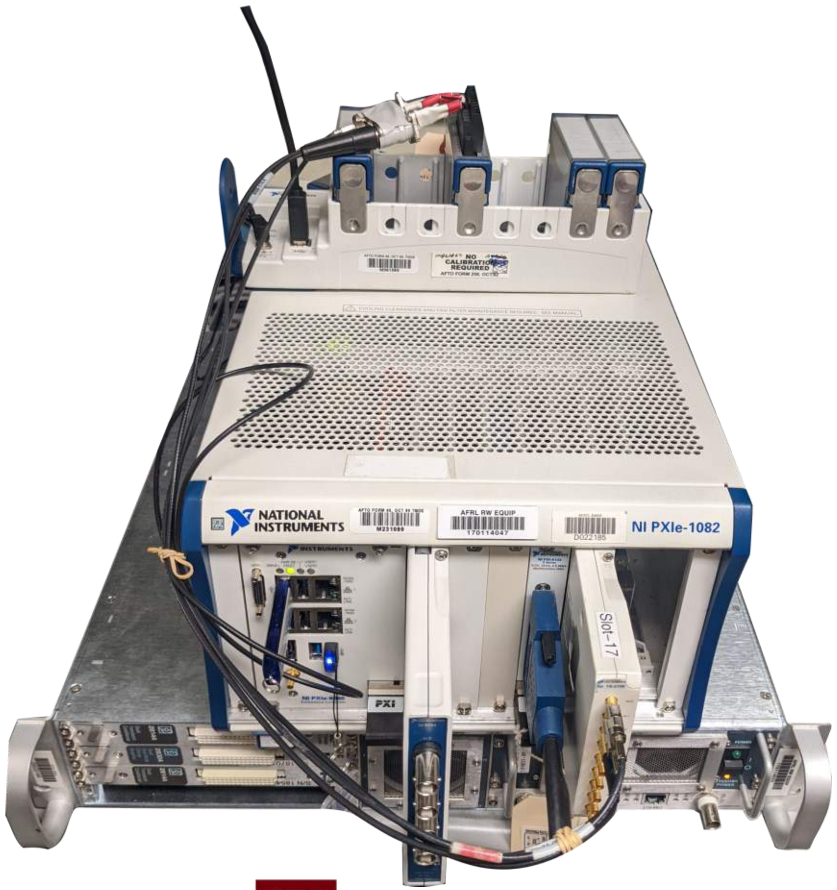




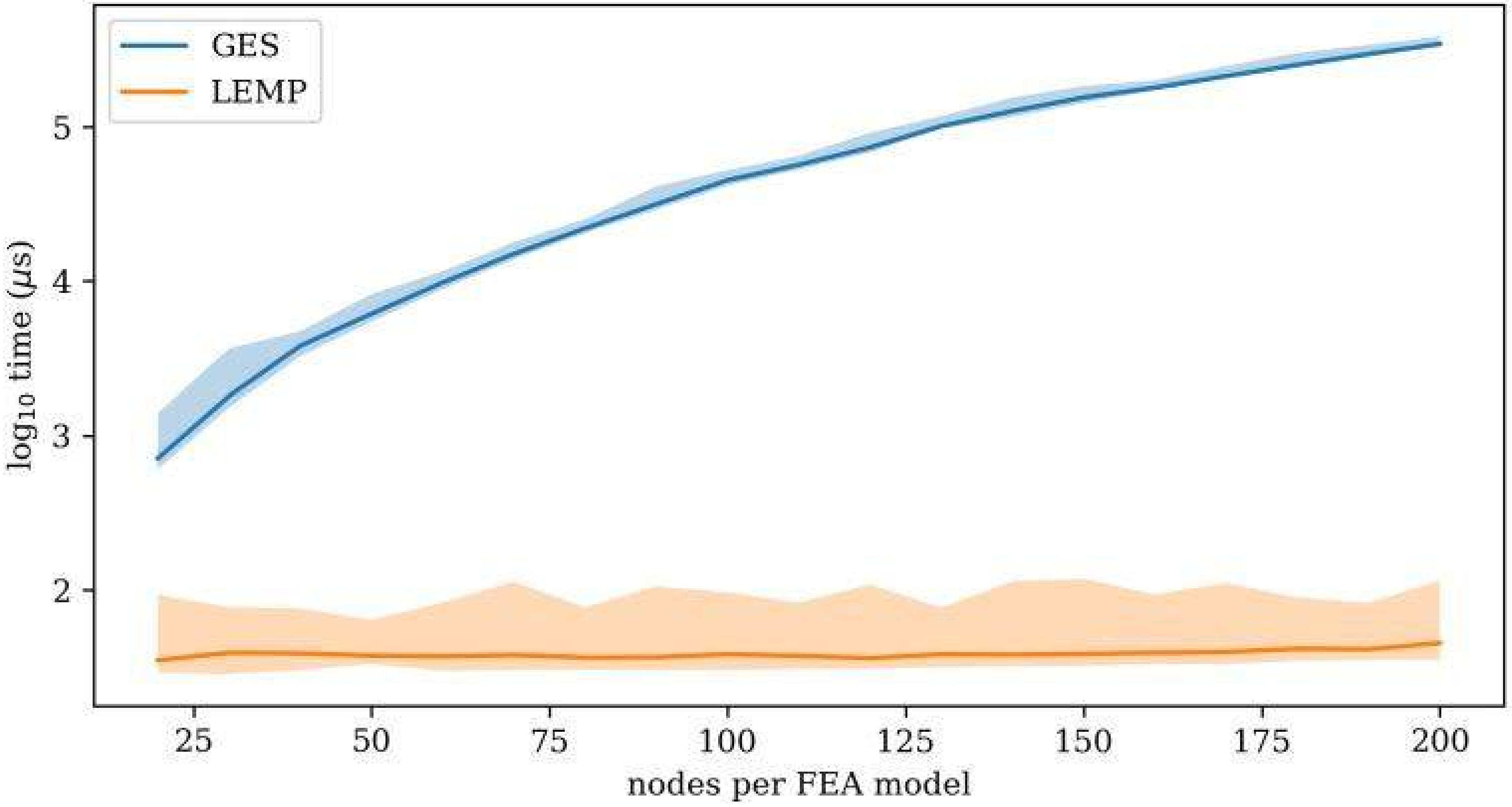
CYBER PHYSICAL EQUIVALENT

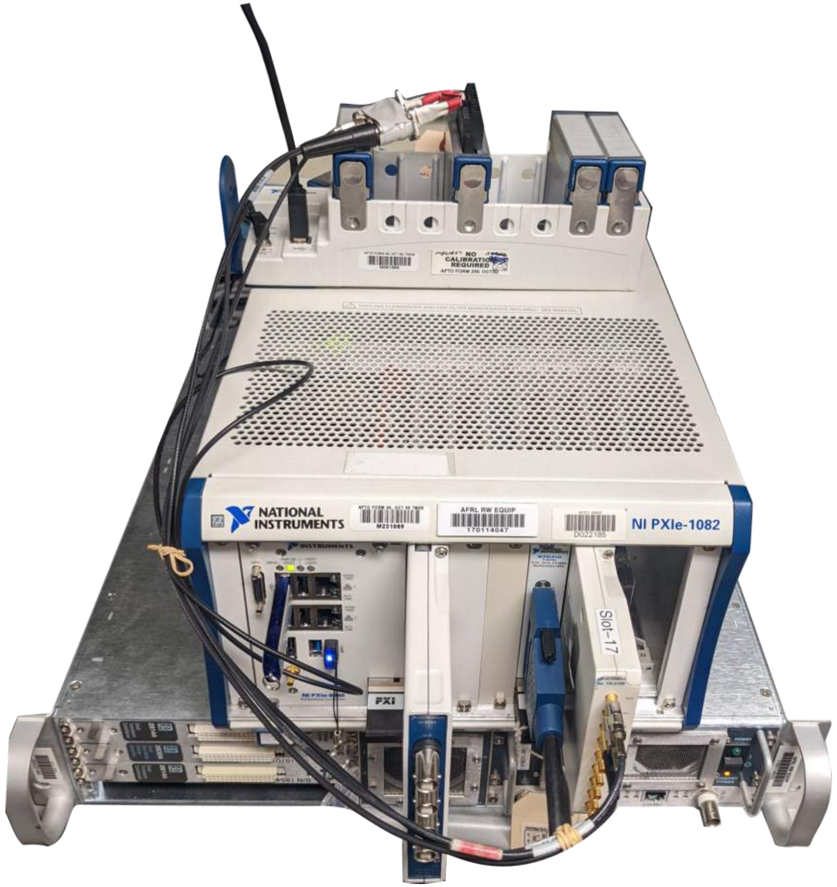






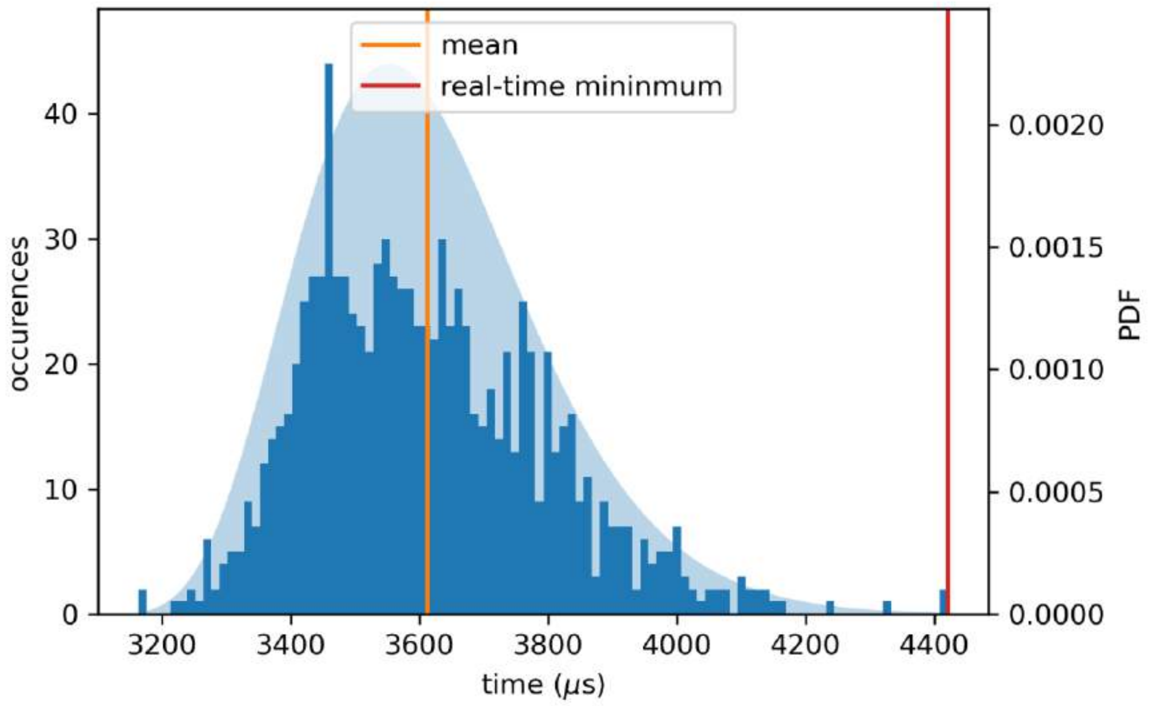
ALGORITHMIC TIMING TARGETS



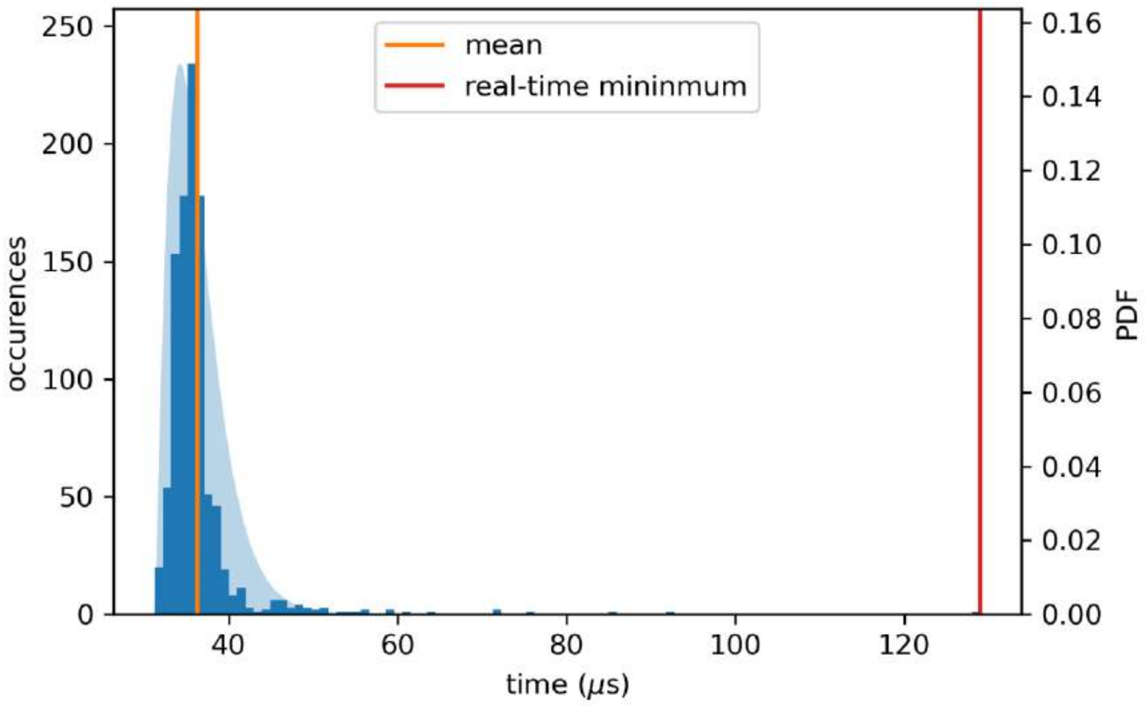


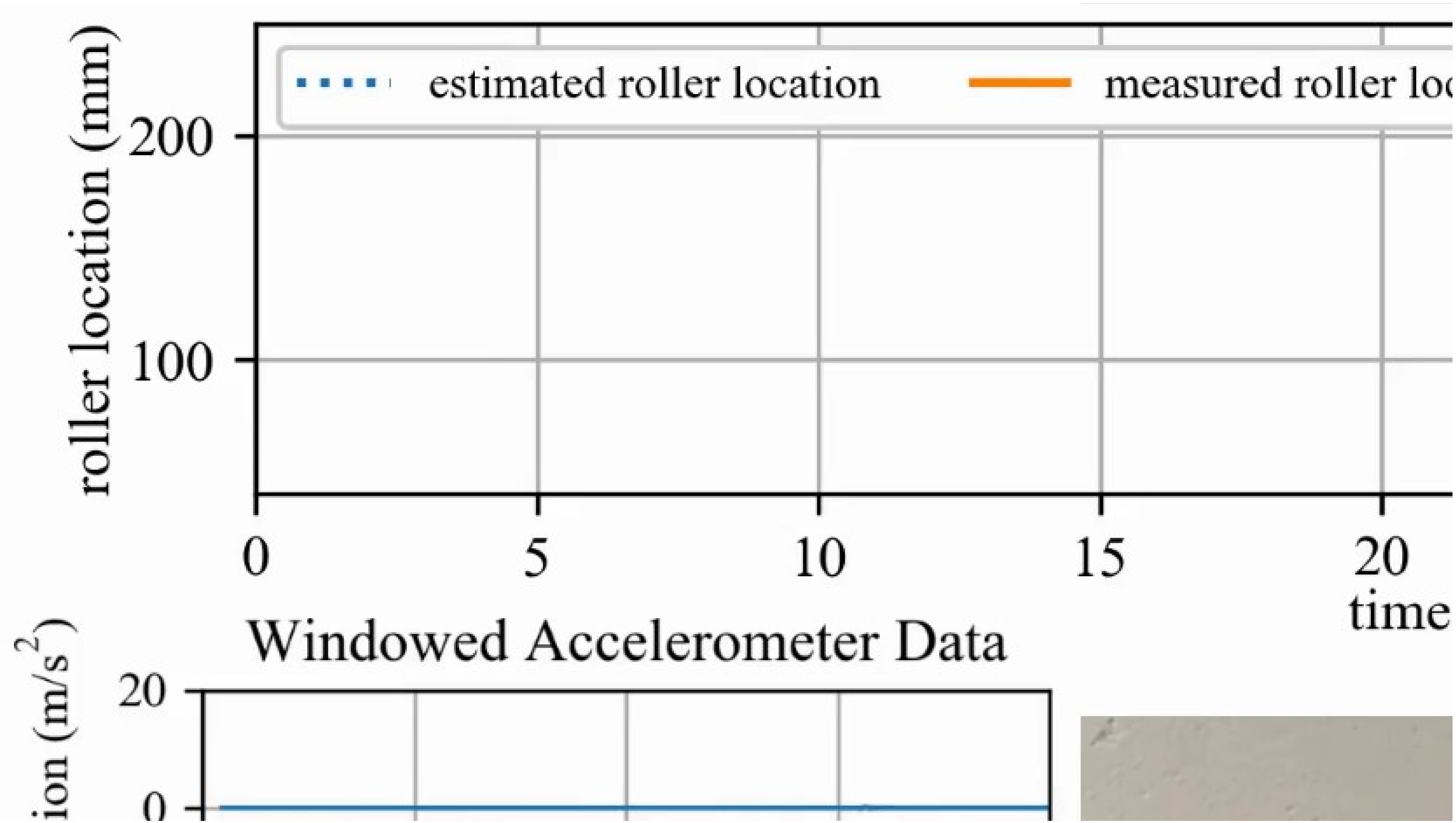
PERFORMANCE: TIMING OUTSIDE THE LOOP

General Eigenvalue Solver



Local Eigenvalue Modification Procedure

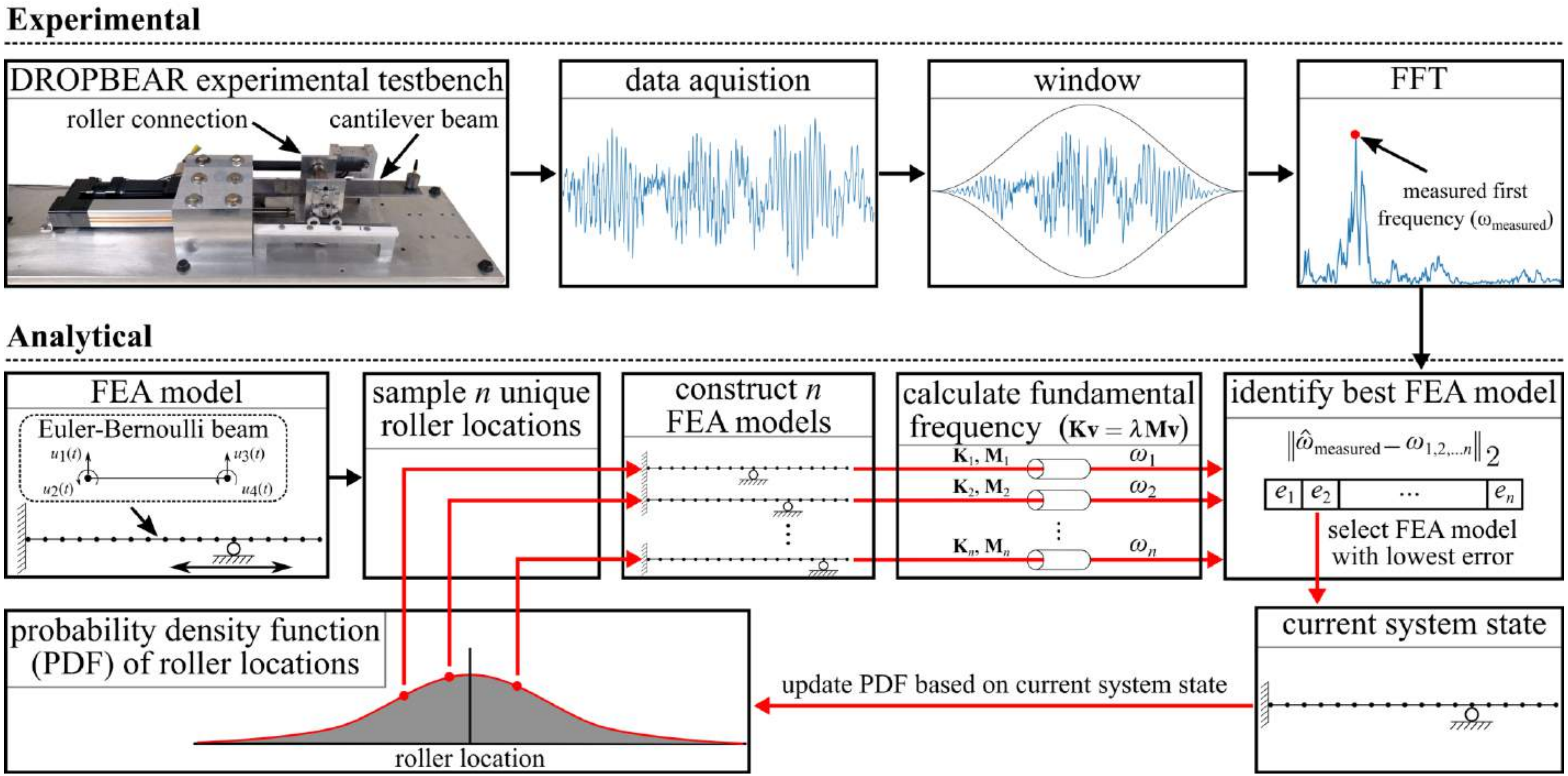



estimated roller location
measured roller loc
roller location (mm)
200
100
0
5
10
15
20
time
Windowed Accelerometer Data
ion (m/s²)
20
0

DATA FUSION

Real-Time Model Updating Through Error Minimization

A frequency-based model updating technique was developed to update an FEA model of the system.



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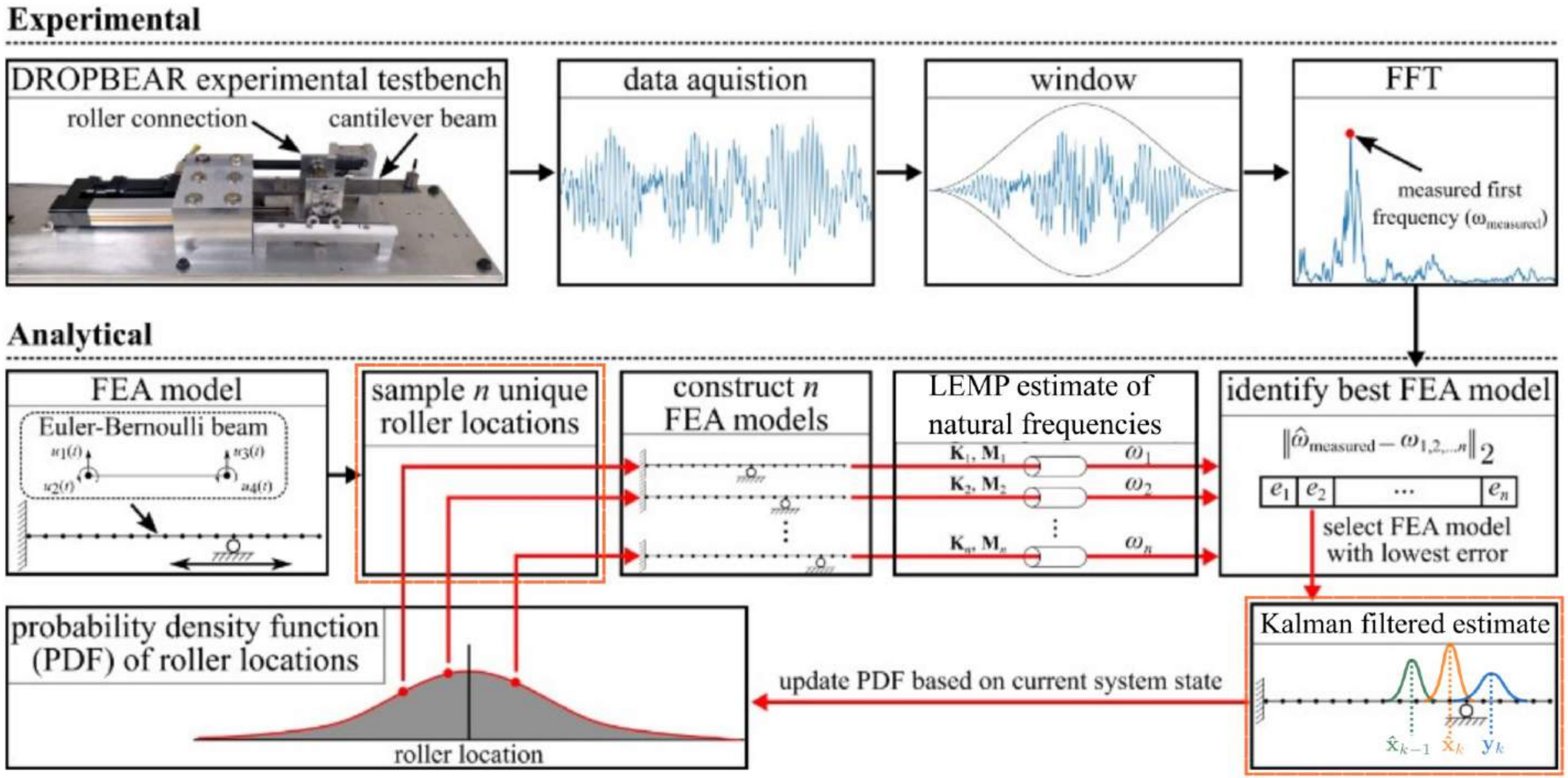
Real-Time Model Updating with data fusion techniques

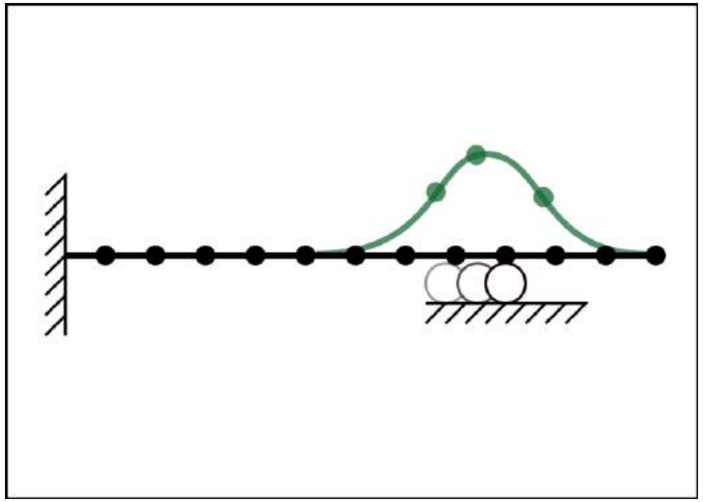
A frequency-based model updating technique was developed to update an FEA model of the system.

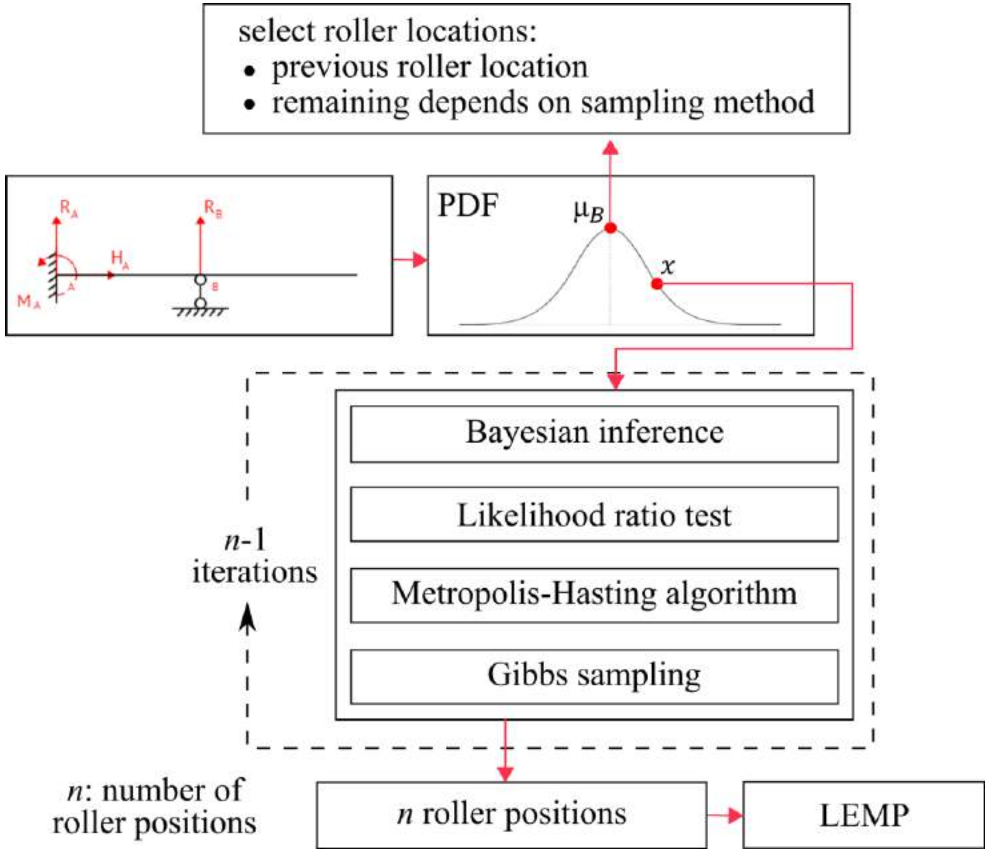


Downey A., et al,. “Millisecond Model Updating for Structures Experiencing Unmodeled High-Rate Dynamic Events” *Mechanical Systems and Signal Processing* ***138***, 2020

SAMPLING

Four sampling methods are used for selecting an appropriate roller location on which LEMP is applied for roller location estimation.





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KALMAN FILTER

a priori estimate

$$\hat{\mathbf{x}}_{a,k} = \mathbf{A}_k \hat{\mathbf{x}}_{s,k-1} \tag{1}$$

$$\hat{\mathbf{P}}_{a,k} = \mathbf{A}_k \hat{\mathbf{P}}_{s,k-1} \mathbf{A}_k^T + \mathbf{Q}_k \tag{2}$$

measurement innovation and update

$$\tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{C}_k \hat{\mathbf{x}}_{a,k} \tag{3}$$

$$\mathbf{S}_k = \mathbf{C}_k \hat{\mathbf{P}}_{s,k} \mathbf{C}_k^T + \mathbf{R}_k \tag{4}$$

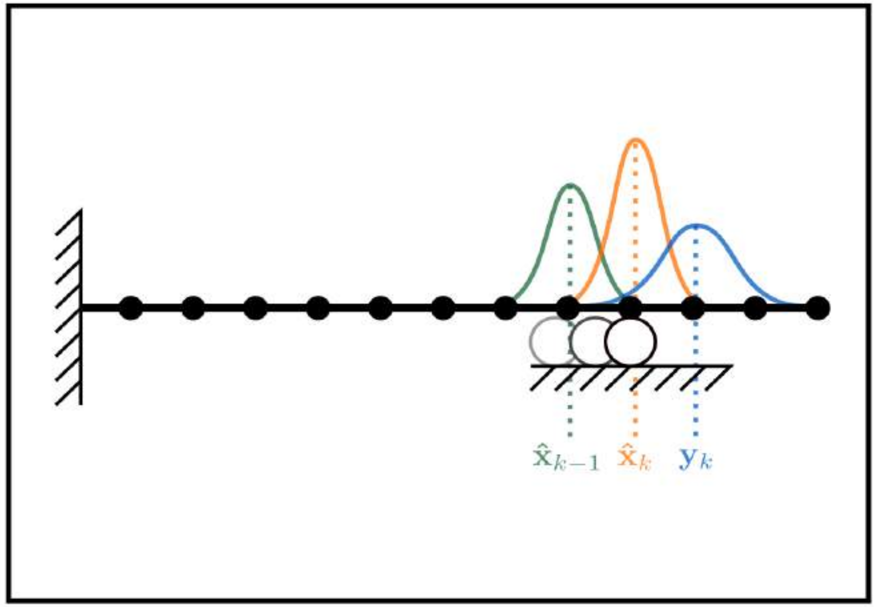
$$\boldsymbol{\epsilon}_k = \tilde{\mathbf{y}}_k^T (\mathbf{S}_k) \tilde{\mathbf{y}}_k \tag{5}$$

$$\mathbf{L}_k = \hat{\mathbf{P}}_{a,k} \mathbf{C}_k^T \mathbf{S}_k^{-1} \tag{6}$$

a posteriori state estimate

$$\hat{\mathbf{x}}_{s,k} = \hat{\mathbf{x}}_{a,k} + \mathbf{L}_k \tilde{\mathbf{y}}_k \tag{7}$$

$$\hat{\mathbf{P}}_{s,k} = (\mathbf{I} - \mathbf{L}_k \mathbf{C}_k) \hat{\mathbf{P}}_{a,k} \tag{8}$$



NORMALIZED INNOVATION ERROR SQUARED

Consistency ensures two desirable properties in a Kalman filter:

(i) the filter is ‘aware’ of how wrong it could be

(ii) the filter blends the right amount of information from its process model and measurements to recursively correct its state estimate

NORMALIZED INNOVATION ERROR SQUARED

We assume R_k and H_k (measurement and process noise matrices) to be close to the actual noise of the measurement and the process however, model can be mismatched.

In this case, we seek to satisfy the weaker condition of *covariance consistency.*

NORMALIZED INNOVATION ERROR SQUARED

S_k and P_k (innovation and process covariace) of the chosen model should be *consistent* with the R_k and H_k (measurement and process noise)

$$\hat{\mathbf{x}}_{i|j} \approx \mathrm{E}\left[\mathbf{x}_i | \mathbf{z}_{1:j}\right]$$

$$\mathbf{P}_{i|j} \geq \mathrm{E}\left[\left(\mathbf{x}_i - \hat{\mathbf{x}}_{i|j}\right) | \mathbf{z}_{1:j} \left(\mathbf{x}_i - \hat{\mathbf{x}}_{i|j}\right) | \mathbf{z}_{1:j}{}^{\top}\right]$$

NORMALIZED INNOVATION ERROR SQUARED

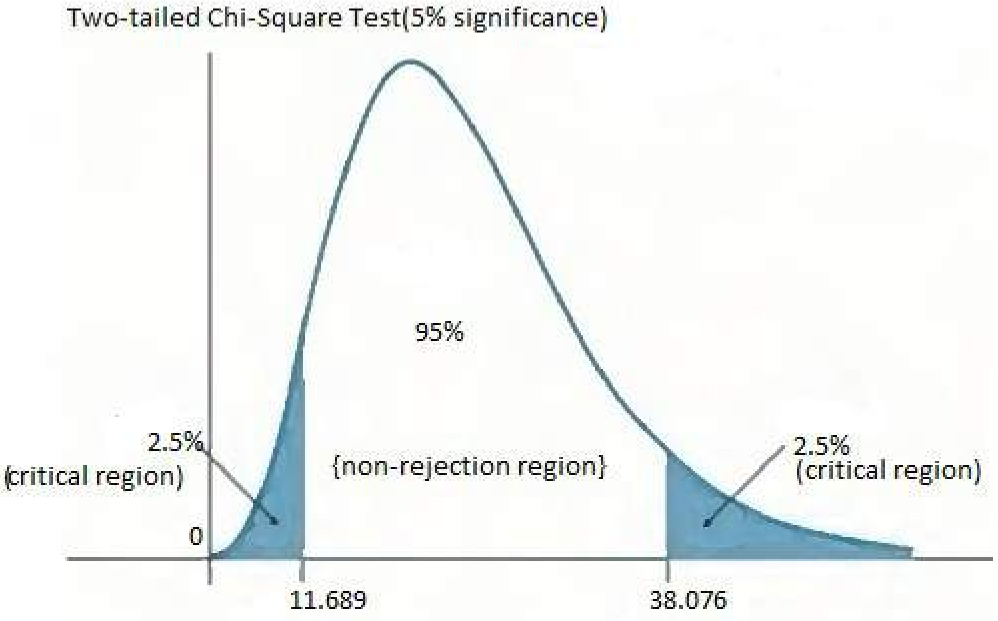
These conditions can be assessed by examining the normalized scalar magnitudes of the random variable $\epsilon_{z,k}$

$$normalized\ innovation\ error\ squared$$

$$\epsilon_{\mathbf{z},k} = \mathbf{e}_{\mathbf{z},k}^{T} \mathbf{S}_{k|k-1}^{-1} \mathbf{e}_{\mathbf{z},k}$$

NORMALIZED INNOVATION ERROR SQUARED

$\epsilon_{z,k}$ is a χ^2 random variable with and n degrees of freedom where n is the number of measurements made of the system



NORMALIZED INNOVATION ERROR SQUARED

In conclusion IF you:

1. Suspect the sensor to be faulty infrequently, outside of explanations of noise (i.e. 1/1000 chance of reading max or min voltage, clear outliers)
2. Expect the innovation covariance (S_k) to be explanatory of the measurement prediction error ($\tilde{y}_k$)

THEN rejecting the a posteriori update step a when the NIS is exceeds the confidence interval will effectively ignore likely sensor faults

NORMALIZED INNOVATION ERROR SQUARED

a priori estimate

$$\hat{\mathbf{x}}_{a,k} = \mathbf{A}_k \hat{\mathbf{x}}_{s,k-1} \quad (1)$$

$$\hat{\mathbf{P}}_{a,k} = \mathbf{A}_k \hat{\mathbf{P}}_{s,k-1} \mathbf{A}_k^T + \mathbf{Q}_k \quad (2)$$

measurement innovation and update

$$\tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{C}_k \hat{\mathbf{x}}_{a,k} \quad (3)$$

$$\mathbf{S}_k = \mathbf{C}_k \hat{\mathbf{P}}_{s,k} \mathbf{C}_k^T + \mathbf{R}_k \quad (4)$$

$$\boldsymbol{\epsilon}_k = \tilde{\mathbf{y}}_k^T (\mathbf{S}_k) \tilde{\mathbf{y}}_k \quad (5)$$

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a posteriori state estimate

$$\hat{\mathbf{x}}_{s,k} = \hat{\mathbf{x}}_{a,k} + \mathbf{L}_k \tilde{\mathbf{y}}_k \quad (7)$$

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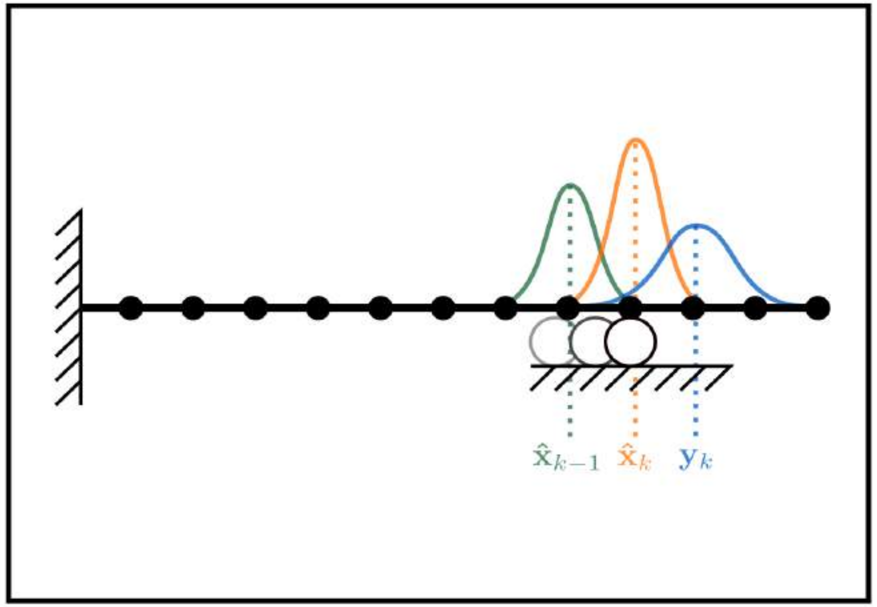
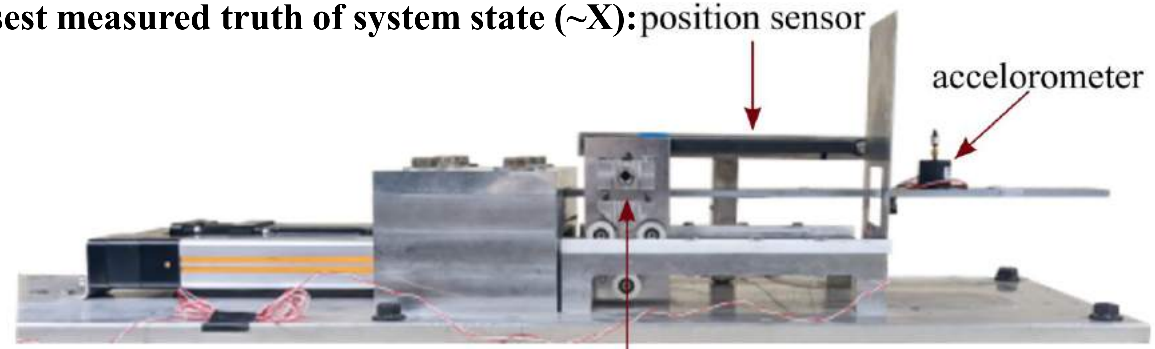


MODELING

Closest measured truth of system state (~X): position sensor

accelorometer

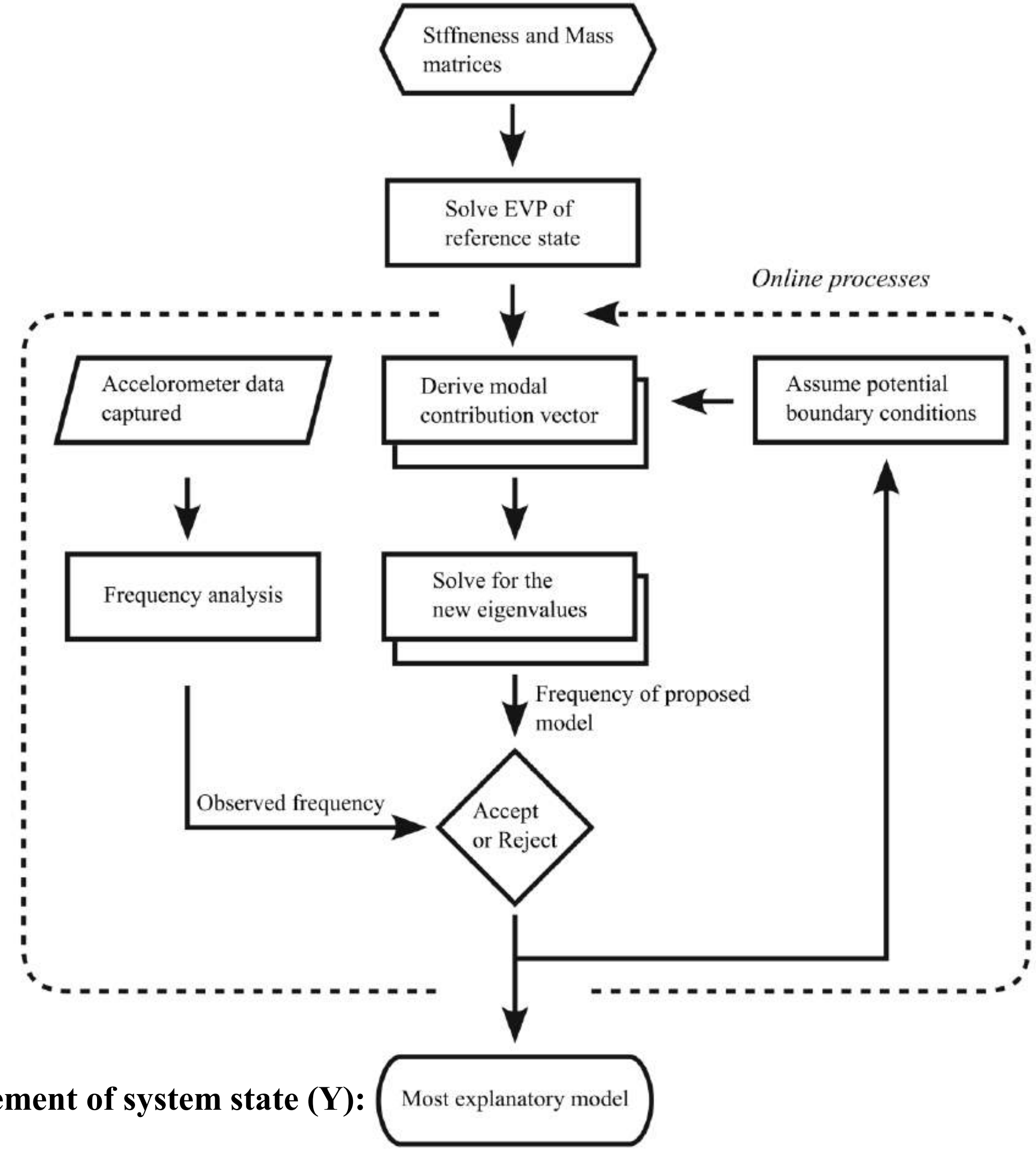


Ground truth of system state (X): rolling pinned condition

Assumed to be Constant Velocity Model:

$$\dot{x} = Ax + \Omega_p$$

$$y = Cx + \Omega_m$$



Measurement of system state (Y):

MODELING

Discrete Constant Velocity Model:

$$x_k = Ax_{k-1} + \Omega_p$$

$$y_k = Cx_k + \Omega_m$$

Where we assume that between the (k − 1) and k timestep, uncontrolled forces cause a constant velocity

$$x = \begin{bmatrix} p \\ v \end{bmatrix}, \qquad A = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

RESULTS

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METRICS

SNR compares the level of a desired signal to the level of background noise measured in Decibels (dB)

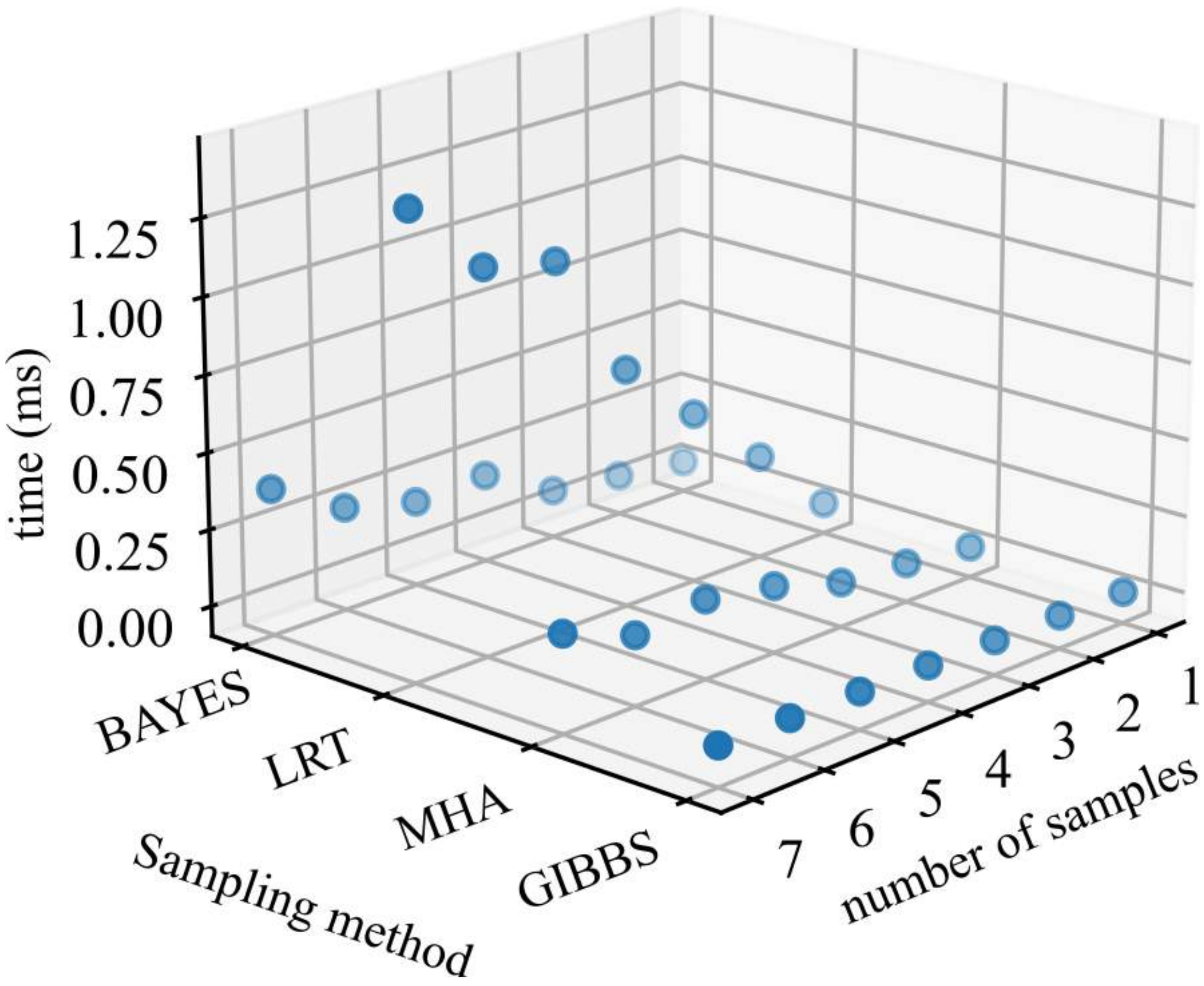
TRAC is a method that quantifies between zero and one the similarity between two signals in time

$$\mathrm{SNR_{dB}} = 10\,\log_{10}\left(\frac{P_{\text{signal}}}{P_{\text{noise}}}\right)$$

$$\mathrm{TRAC} = \frac{[\{t_m\}^{\mathrm{T}}\{t_e\}]^2}{[\{t_m\}^{\mathrm{T}}\{t_m\}][\{t_e\}^{\mathrm{T}}\{t_e\}]}$$

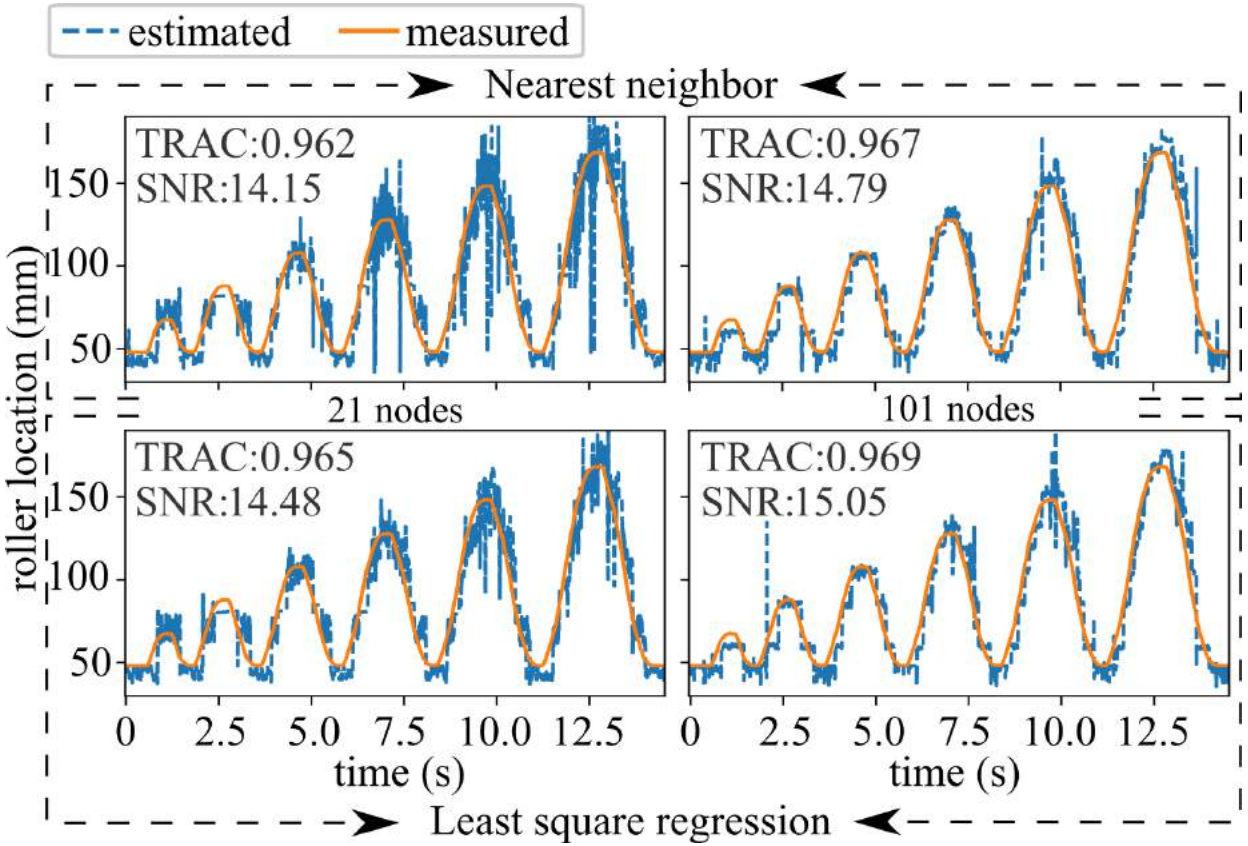
SAMPLING

Estimation results obtained using LEMP with a 21-node and 101-node model of the beam and the previously investigated Gaussian sampling technique without the use of a Kalman filter; termed the “base state”.



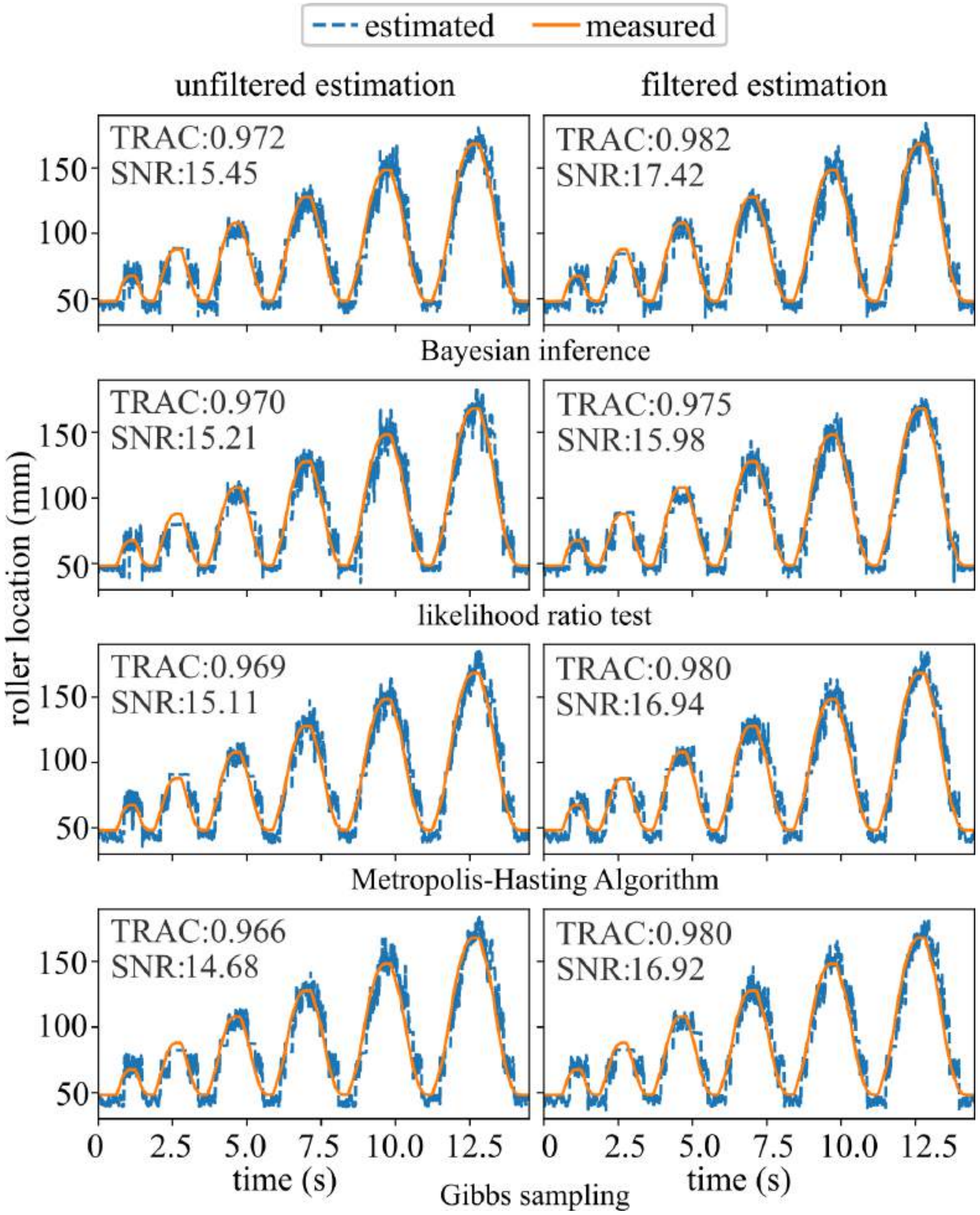
BASE STATE

Estimation results obtained using LEMP with a 21-node and 101-node model of the beam and the previously investigated Gaussian sampling technique without the use of a Kalman filter; termed the “base state”.



21-NODE MODEL

State estimation results on a 21-node model of the beam with LEMP in the filtered and unfiltered configuration where Bayes inference, Likelihood ratio test, Metropolis-Hasting Algorithm, and Gibbs sampling are used to sample roller location.



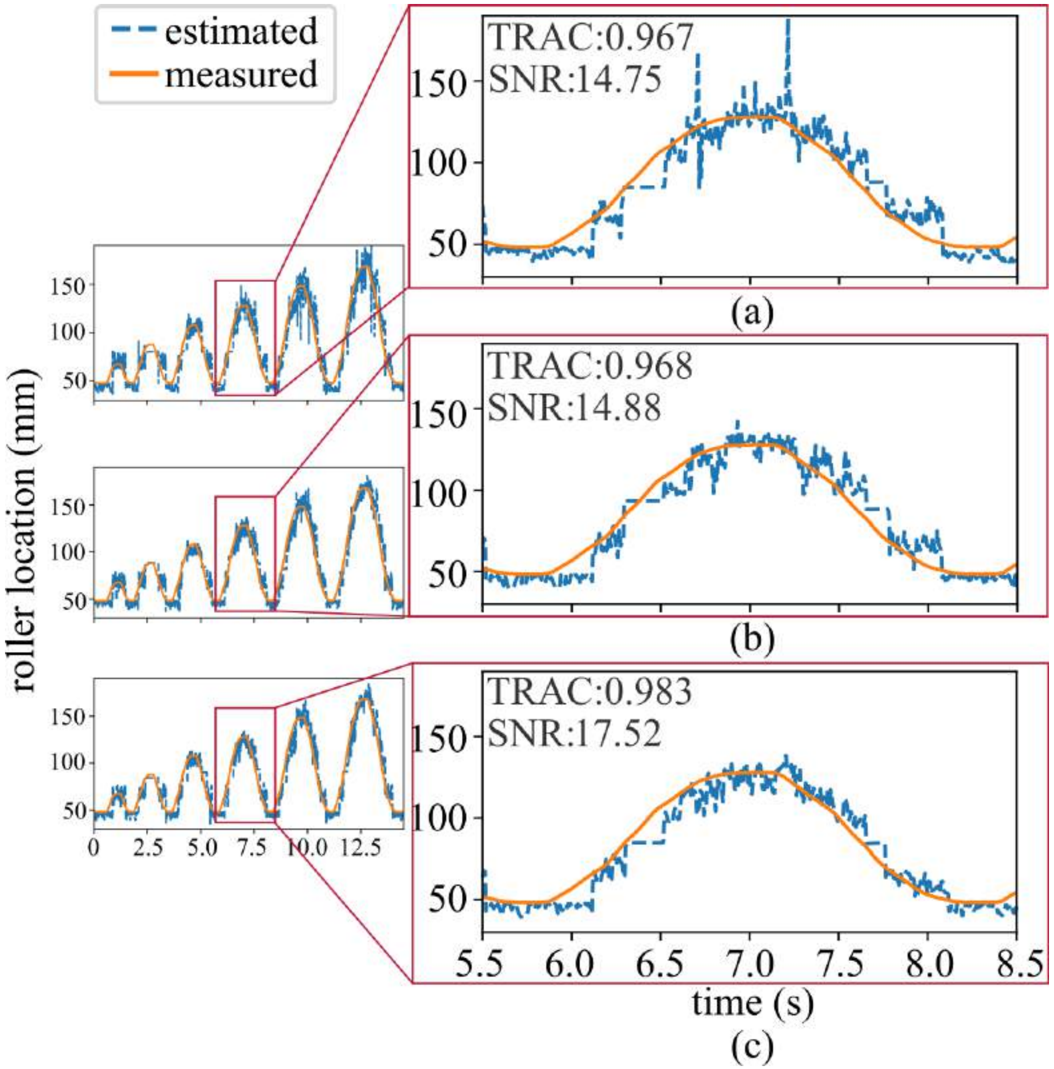
21-NODE MODEL

Roller position estimation using a 21-node beam model for

(a) LEMP estimate with no sampling or Kalman filter methodology

(b) LEMP estimate where roller positions are sampled using Bayesian search space

(c) improved LEMP estimate where roller positions are sampled using the Bayesian search space and filtered with the Kalman filter.



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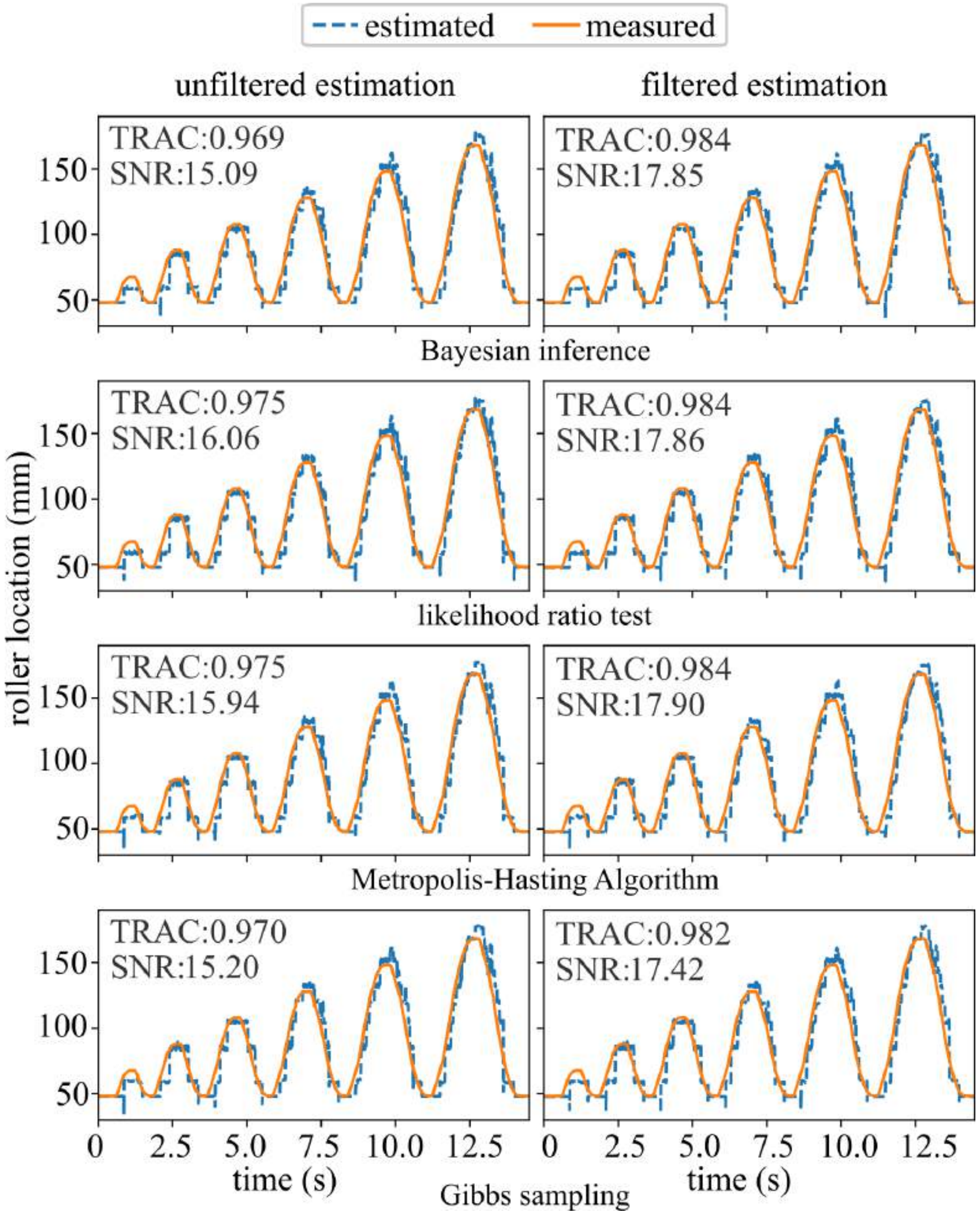
101-NODE MODEL

State estimation results on a 101-node model of the beam with LEMP in the filtered and unfiltered configuration where Bayes inference, Likelihood ratio test, Metropolis-Hasting Algorithm, and Gibbs sampling are used to sample roller location.



RESULTS

Average percentage improvement in SNR_{dB} compared to estimation without sampling and Kalman filter at 21 and 101 nodes for three particle models over 100 trials.

	SNR_{dB} improvement			
	21 nodes		101 nodes	
sampling method	unfiltered	filtered	unfiltered	filtered
Bayesian inference	3.03%	15.86%	0.95%	13.93%
likelihood ratio test	5.30%	**17.18%**	2.64%	14.69%
Metropolis-Hasting Algorithm	0.87%	13.27%	-0.18%	13.47%
Gibbs sampling	0.72%	12.53%	0.33%	14.28%
Gaussian sampling	base case	13.14%	base case	14.69%

CONCLUSION

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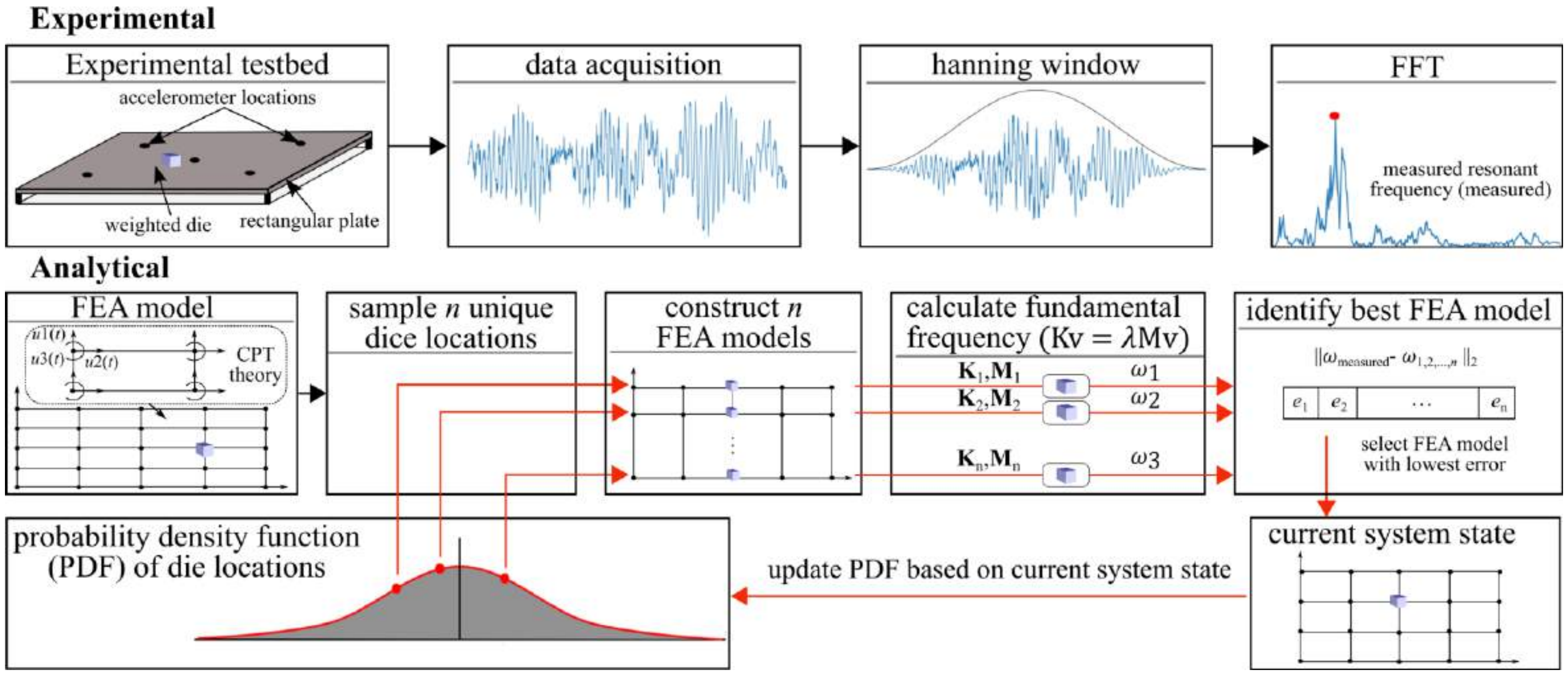
CONCLUSION

- The study found that the likelihood ratio test alongside the linear Kalman filter effectively produced accurate results, with an ~17% increase in accuracy for a 21-node model of the considered structure.
- The study also highlighted the importance of filtering outliers, as demonstrated by using the Normalized Innovation Squared (NIS) metric.
- This study successfully improved accuracy over the previous model updating methods, especially for lightweight models with low node counts on all the methodologies tested.

FUTURE WORK

In future studies, the methodology would be expanded to include two-dimensional analysis and sequential damage cases, emphasizing the need for intelligent model selection and outlier filtering.



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THANKS!

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