Real-time state estimation of nonstationary systems using fast topological data analysis features

Graduate Research Group

Arman Razmarashooli

Daniel A. Salazar

Principal Investigators:

Dr. Simon Laflamme

Dr. Chao Hu

UCONN UNIVERSITY OF CONNECTICUT

School of Engineering

IOWA STATE UNIVERSITY

Department of Civil, Construction and Environmental Engineering

FAST FOUNDATIONS OF ADAPTIVE SYSTEMS AND TECHNOLOGIES

Foundations of Adaptive Systems and Technologies

High-Rate Structural Health Monitoring (HRSHM)

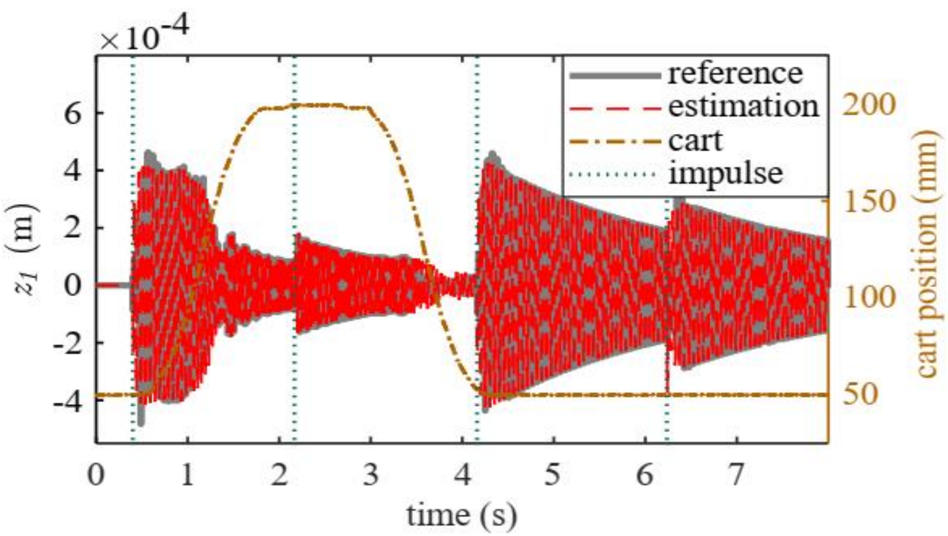
Definition

- 10 kg TNT takes 0.3 to 100 ms to 100 ms to travel over 1 to 40 m distance.
- 100 μs at Mach 5 corresponds to 150 mm. 1.5 MHz sampling rate gives a 1 mm. resolution.

High-Rate Structural Health Monitoring (HRSHM)

Definition

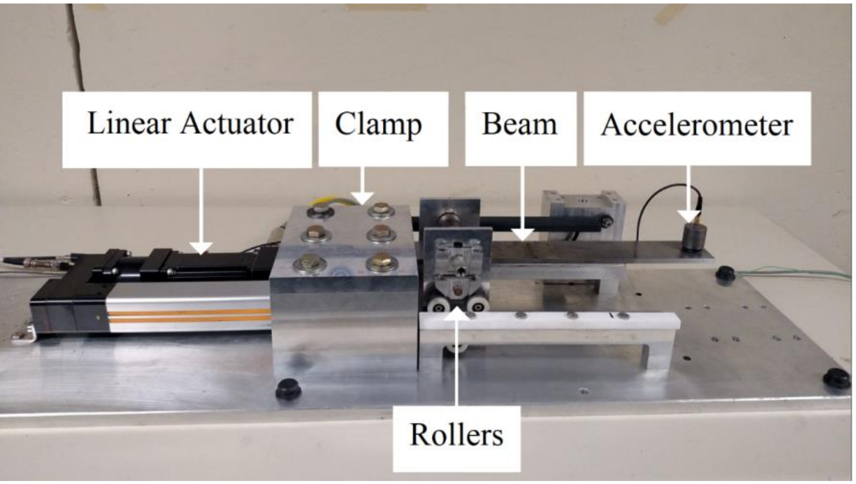
- **Systems experiencing high-rate dynamics**
  - Accelerations higher than100 g_n ($g_n = 9.81$ m/s^2) in less than 1 ms.
- **Characterized by**
  - Large uncertainties in external loading.
  - High levels of nonstationary and heavy disturbance.
  - Generations of unmodeled dynamics from changes in mechanical configuration.

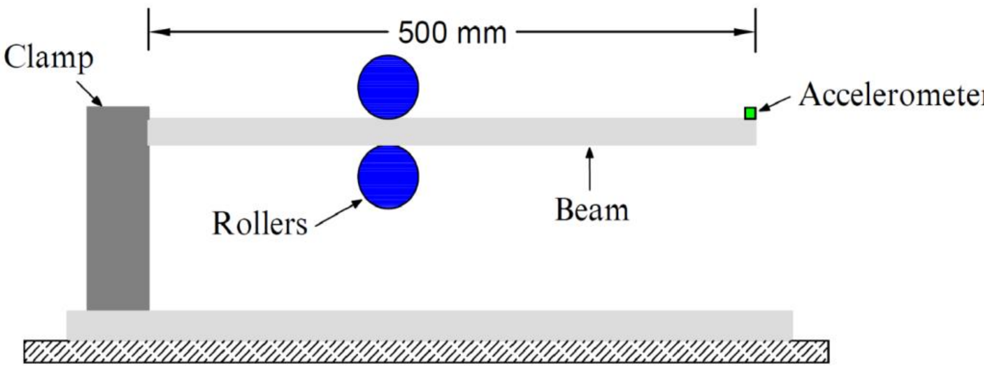


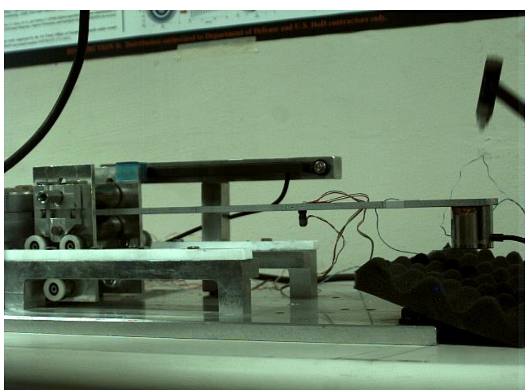
Motivation & Objectives

Fast Model Reference Adaptive System

- Dynamic Reproduction of Projectile Ballistic Environments for Advanced Research (DROPBEAR).



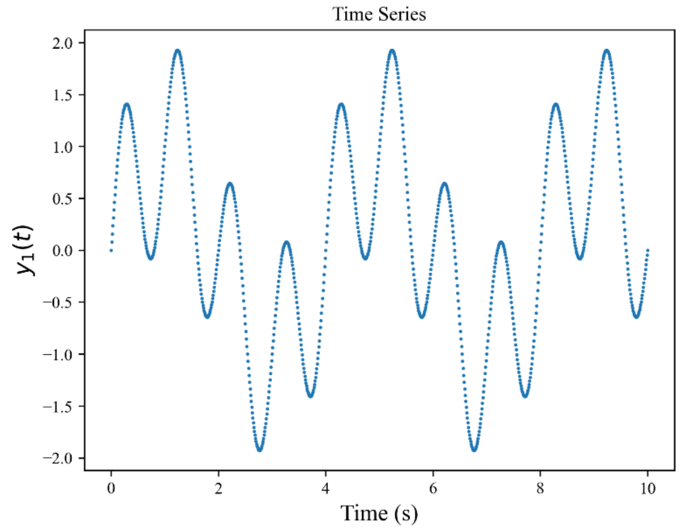


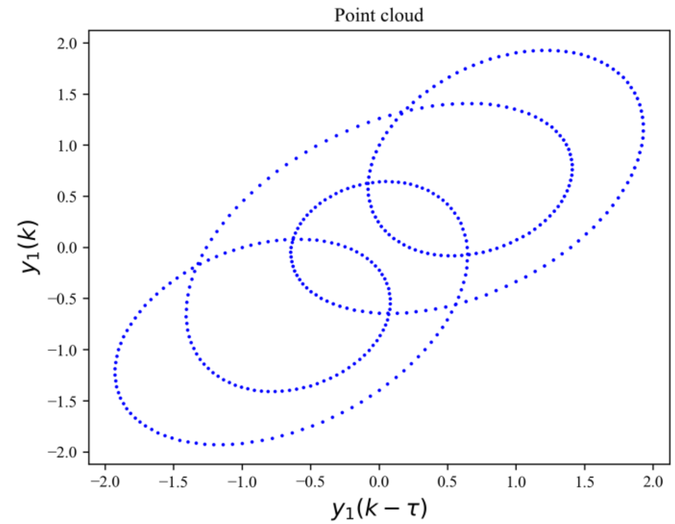


Tests	Pin position	Frequency (Hz)	Estimated frequency (Hz)	Convergence time (ms)
1-5	50 mm	17.7	17.67	780
6-10	100 mm	21.0	21.00	400
11-15	150 mm	25.0	24.99	160
16-20	200 mm	31.0	31.01	100

Data Pre-Processing

Embedding Theorem





$$\mathbf{s}(k) \equiv \mathbf{x}(k) = [y_1(k), y_1(k-\tau), y_1(k-2\tau), \ldots, y_1(k-(d-1)\tau]$$

Key Remarks:

- 1-to-1 mapping exists between the state vector **s** and delay vector **x.**
- The delay vector **x** preserves the essential dynamics.
- Minimal representation can be obtained using the essential dynamics as inputs.

Topological Data Analysis

Introduction:

- Characteristics of data that do not depend on certain details of the representation.
- Infer relevant topological features from these spaces.
- Using these features for further processing (data classification).
- TDA has never been used for time series prediction.

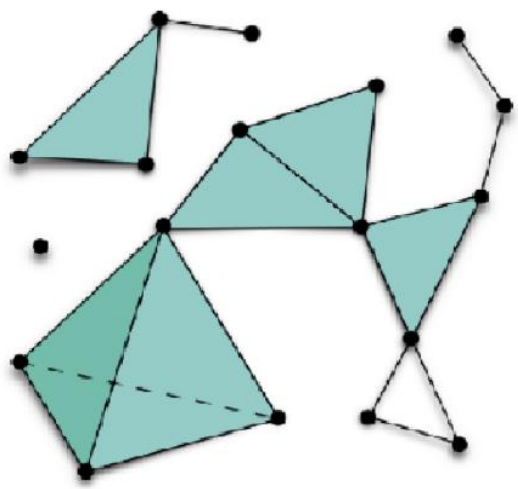
Challenges:

- No direct access to topological information.
- Need for topological construction (simplicial complexes).
- Distinguish topological signal from noise.
- Find a way to incorporate TDA features within neural network.
- Find a fast way (shortcut) to implement TDA features.

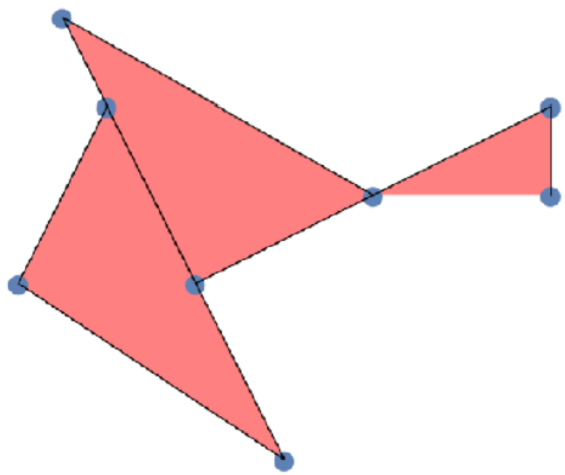
Topological Data Analysis

Simplicial Complexes

- A generalization of a graph.
- A 0-simplicial complex is a set of points, a 1-simplicial complex is a graph.
- An n-simplicial complex contains up to n-dimensional simplices.



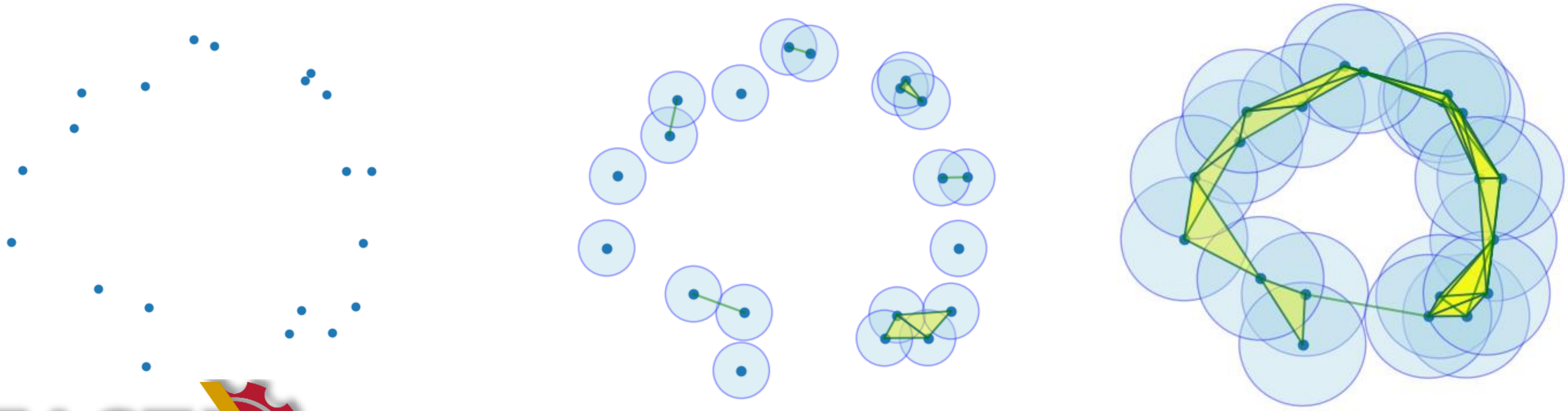
A simplicial complex



Not a simplicial complex

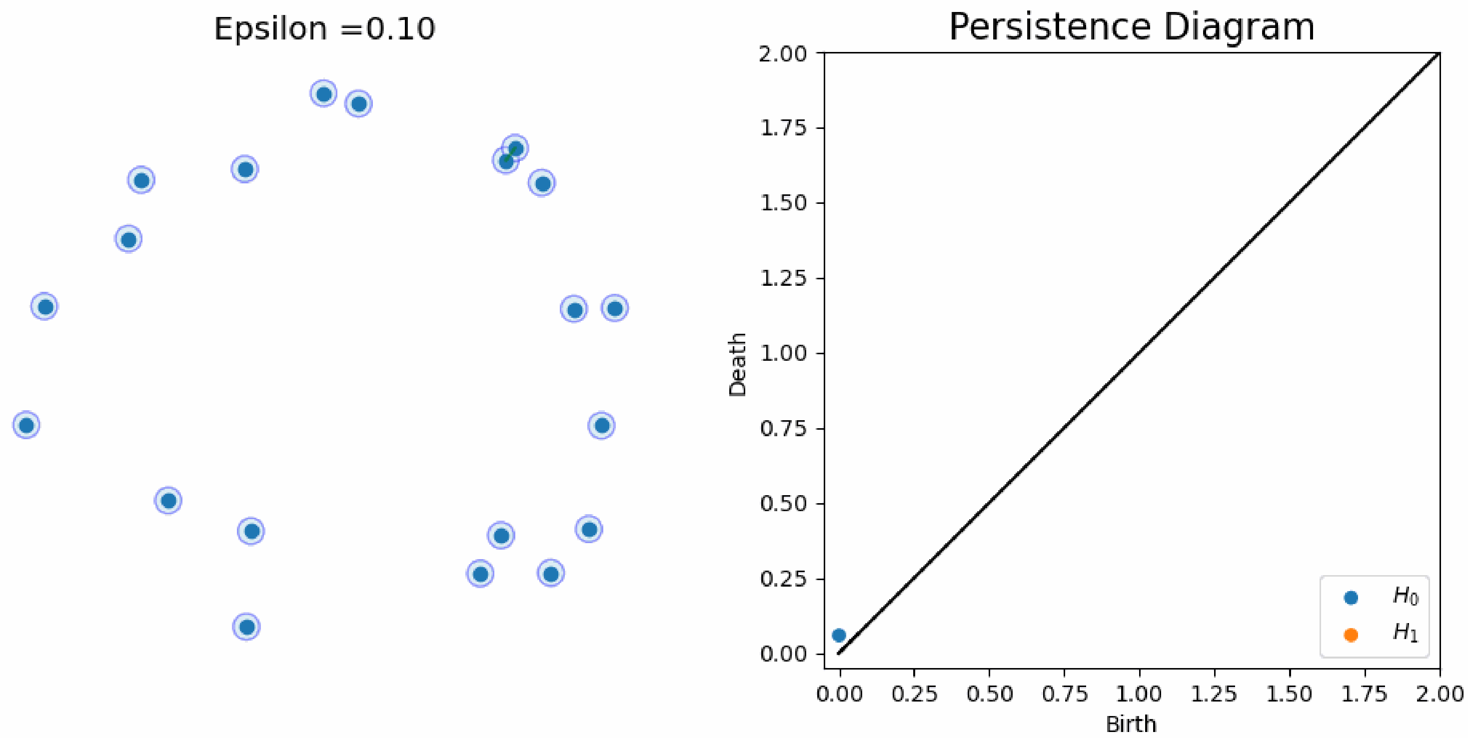
Simplicial Complexes

- It is not obvious what the correct radius is for the construction of our simplicial complex.
- Persistent homology solved this problem by measuring topological features which persist while growing radii.
- Persistence diagram keeps track of the increase/decrease in each Betti number, representing the birth and death of features as radii increase.
- Informally, the kth Betti number refers to the number of k-dimensional holes on a topological surface.



Persistence Diagram

- Record the changes when increasing the threshold into a plot known as the persistence diagram.
- Each point represents a hole in the point cloud.



Topological Data Analysis

- TDA features of Interest:
  - Maximum Persistence.
  - Bottleneck Distance.
  - Wasserstein Distance.
  - Persistence Landscape.
  - Persistence Silhouette.
  - Number of Off-Diagonal Points.

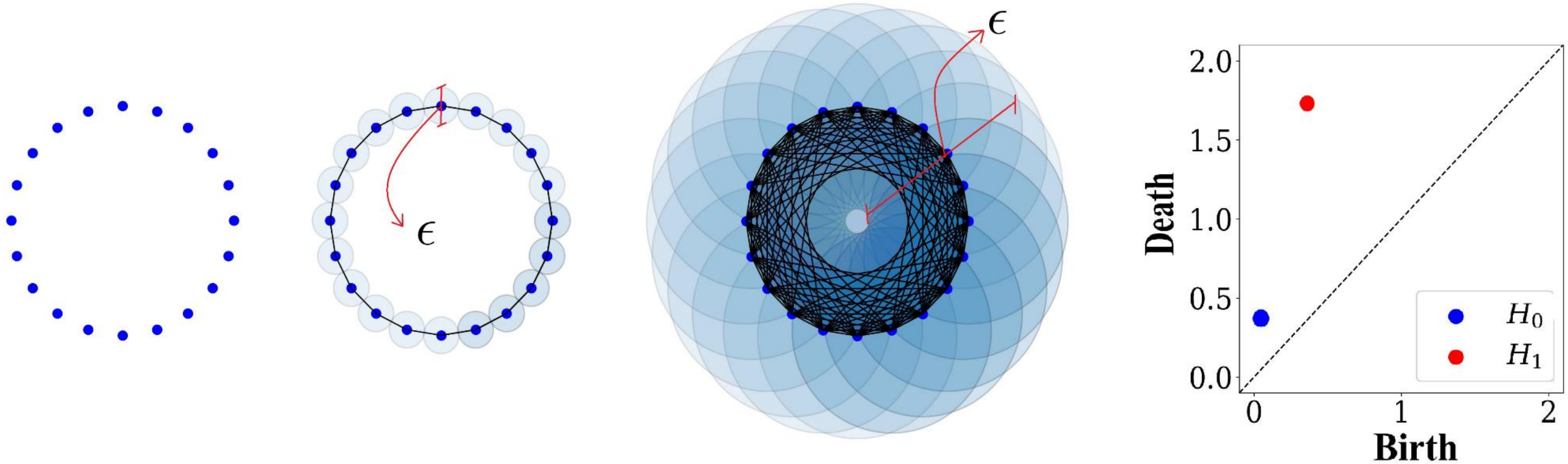
- TDA of DROPBEAR
  - TDA features on a physical context.
  - Application: cantilever beam with a fast-moving boundary condition [1].
  - $$\omega_j = 4\pi^2 \sqrt{\frac{EI}{\rho A}} \left(\frac{4j+1}{4L}\right)^2$$
  - $$x(t) = A\cos(\omega t) = \cos(2\pi f t)$$

[1] Eduardo Kausel. Advanced Structural Dynamics. Cambridge University Press, 2017.

FAST FOUNDATIONS OF ADAPTIVE SYSTEMS AND TECHNOLOGIES

Topological Data Analysis

- The meaning of TDA features for a single-harmonic time series.
- Suggested optimal embedding dimension is 2.
- To account for noise dimension 3 is selected
- Containing information about zero-dimensional hole (H_0) and one-dimensional hole (H_1) and two dimensional hole (H_2)
- Maximum Persistence of H_1 and H_0 relates to the frequency of harmonic signals.



(a) (b) (c) (d)

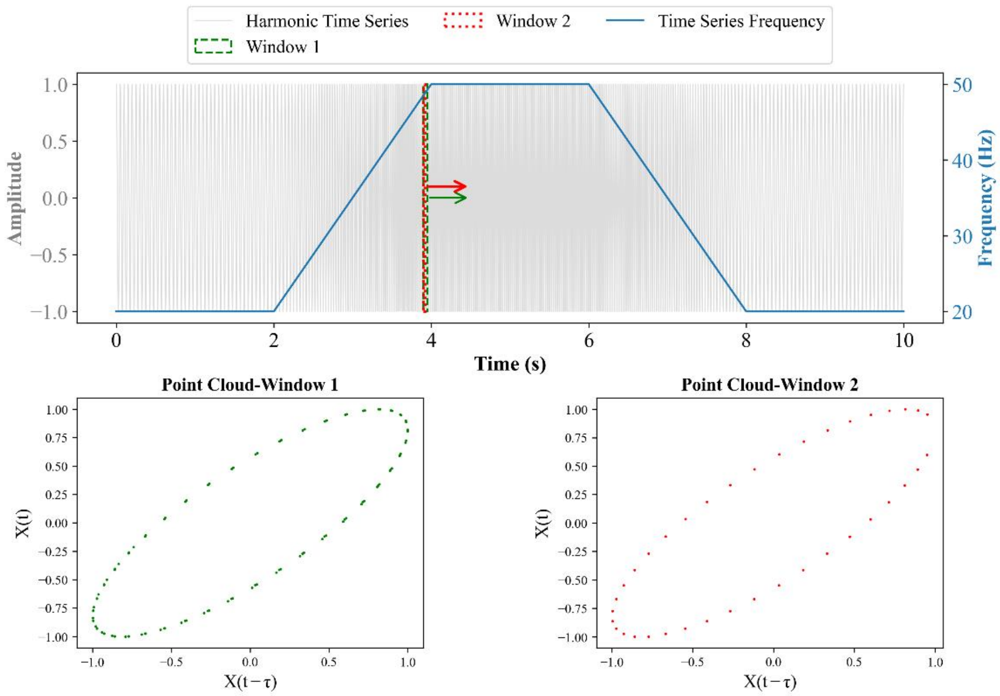
Topological Data Analysis

Challenges:

- The embedding theorem is applicable only to stationary systems.
- Our dynamics are highly non-stationary.

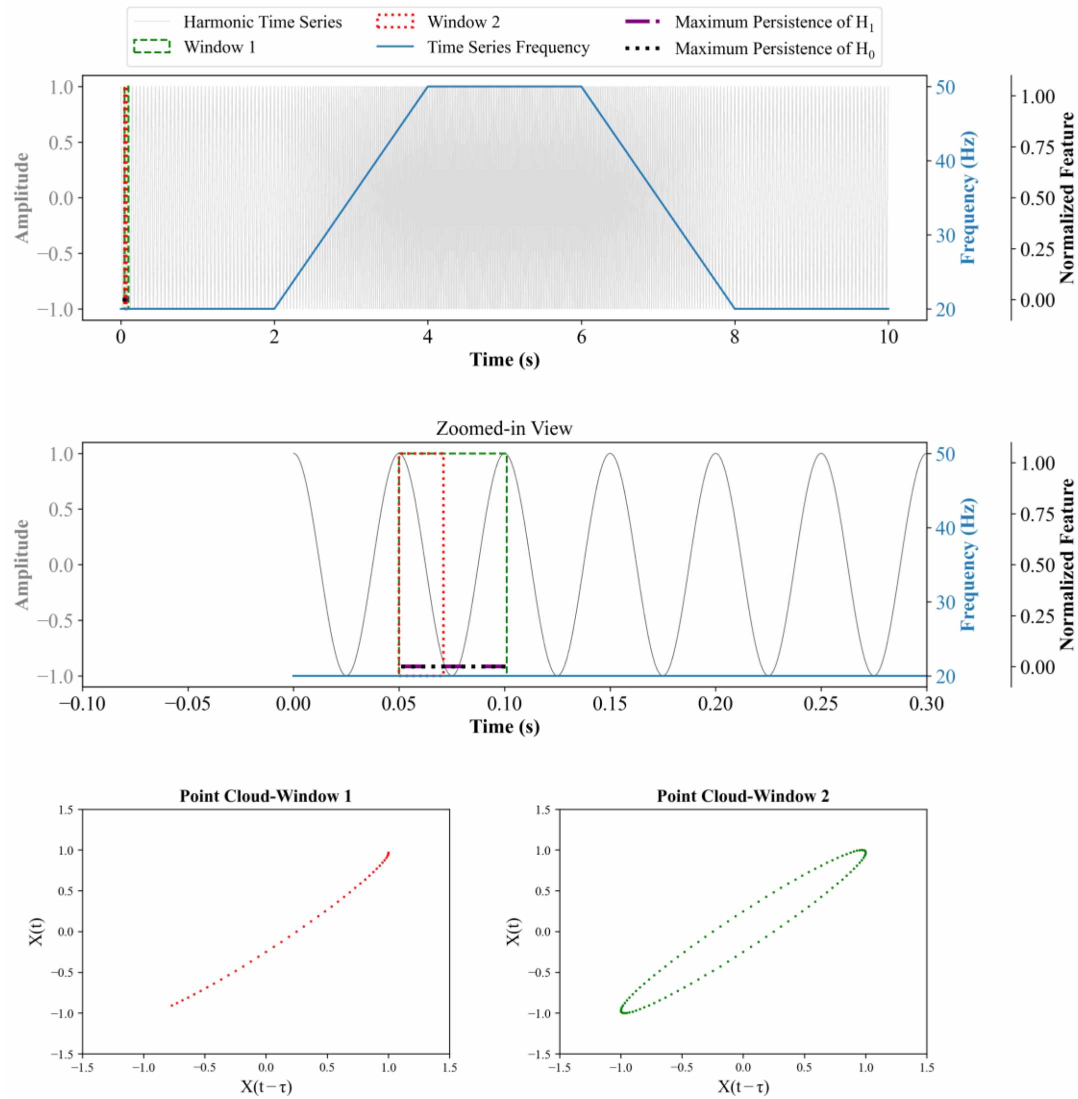
Strategy: Multi-Resolution Windowing

- Applying a sliding window over the dataset to extract local values for H_1.
- Maximum allowable time delay $(\tau) = \dfrac{0.25}{f_{max}}$
- Size of window 1 = $1/f_{min} + \tau$
- Size of window 2 = $1/f_{max} + \tau$,



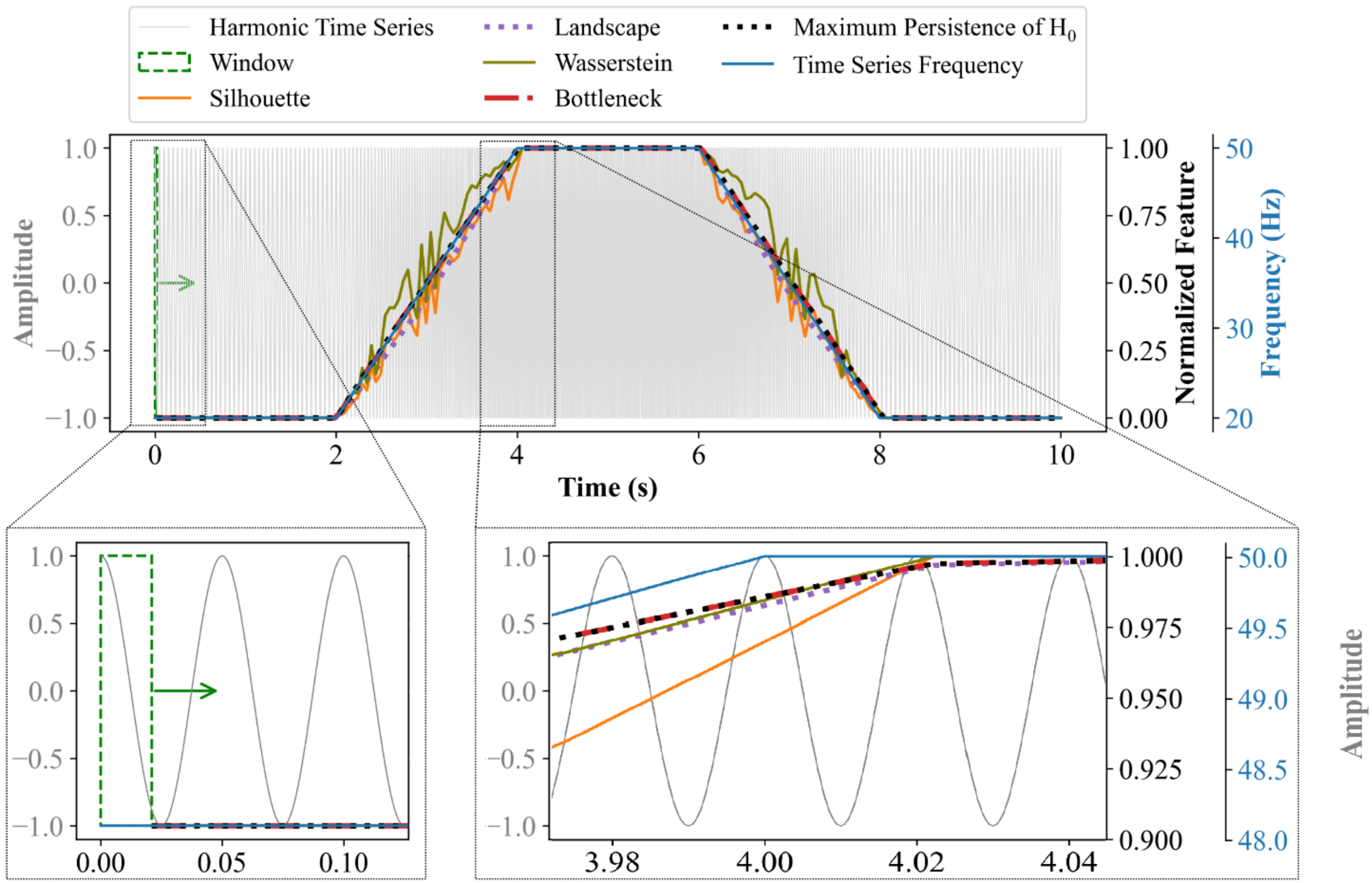
Case Study #1: Synthetic Cosine Data

- $x(t) = Cos(2\pi f(t)t)$
- Moving window size:
- Window 1 = 0.052 s
- Window 2 = 0.022 s
- Time delay = 0.03 s



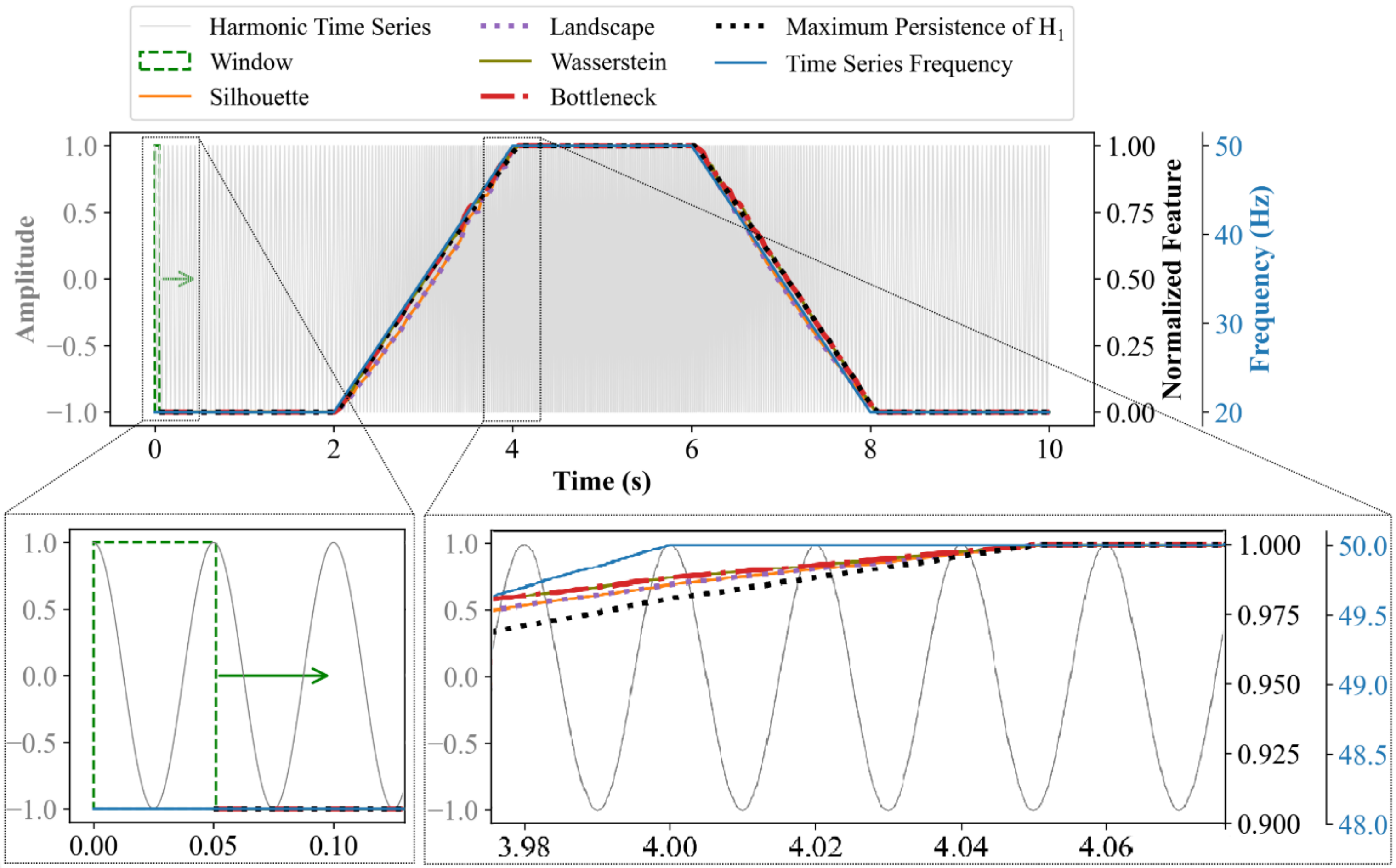
Case Study #1: Synthetic Cosine Data

Maximum Persistence of H_0 correlates with Cart Location



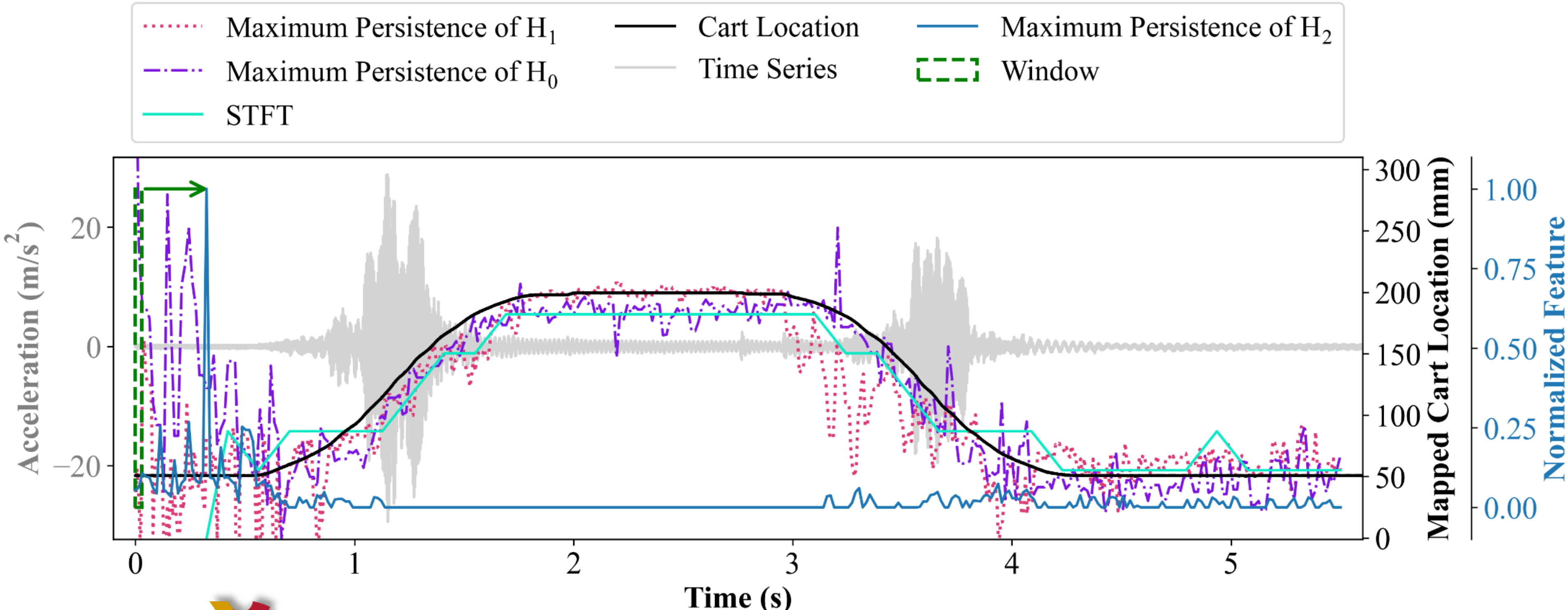
Case Study #1: Synthetic Cosine Data

Maximum Persistence of H_1 correlates with Cart Location



Case Study #2: Experimental Data from DROPBEAR Testbed

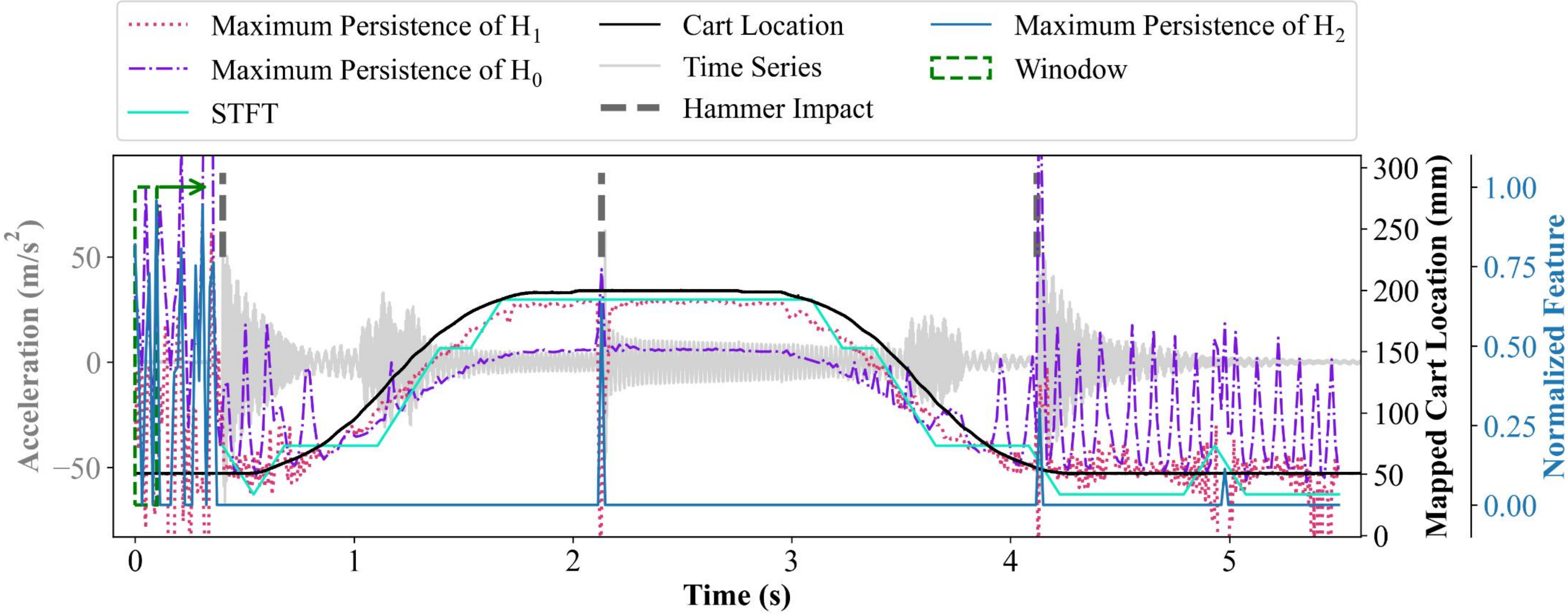
- DROPBEAR without Impact Hammer
- Assumption: TDA features are linearly related to the frequency
- Linear Regression: $x^2 = a_0 + a_1 H_i,$



FAST FOUNDATIONS OF ADAPTIVE SYSTEMS AND TECHNOLOGIES

Case Study #2: Experimental Data from DROPBEAR Testbed

- DROPBEAR without Impact Hammer
- Assumption: TDA features are linearly related to the frequency
- Linear Regression: $x^2 = a_0 + a_1 H_i,$



Performance Metrics:

J_1 is the mean absolute error

J_2 is the ratio of incorrect estimation within a defined threshold (5, 10, and 20 mm)

J_3 is the mean absolute error within a defined threshold (5, 10, and 20 mm)

Performance results for cart localization.

Test	Feature	J_1 (mm)	$J_{2,5}$ (%)	$J_{2,10}$ (%)	$J_{2,20}$ (%)	$J_{3,5}$ (mm)	$J_{3,10}$ (mm)	$J_{3,20}$ (mm)
Test 1	H_1	14.1	65	50	25	2.2	3.8	7.5
	STFT	11.9	81	48	14	2.8	5.6	10.6
Test 2	H_1	16	88	68	18	2.4	5.6	10.5
	STFT	12.1	84	52	14	2.4	5.6	10.8

Results from LSE

Test	Feature	a_0	a_1	R^2
Test 1	H_0	-0.41	3.17	0.95
	H_1	-0.8	1.85	0.95
	STFT	-0.13	1.07	0.98
Test 2	H_0	-0.07	5.25	0.63
	H_1	-1.05	2.31	0.91
	STFT	-0.11	1.059	0.96

FAST Topological Data Analysis

Challenges:

- Chaotic and complex environments in **high-rate dynamic systems**.
- High computational complexity/cost in TDA algorithms.

What is fast TDA?

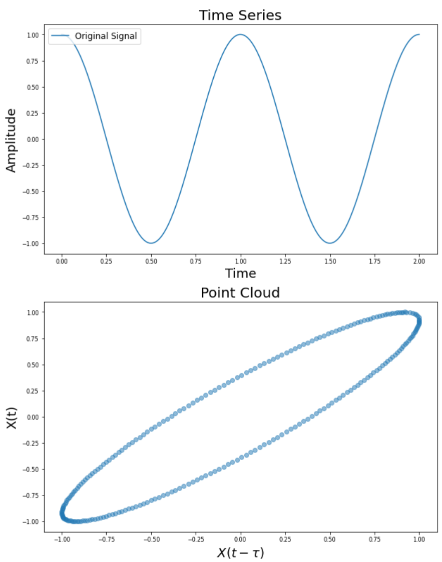
We coined fast TDA as the geometric feature extraction from a point cloud inspired by conventional TDA features obtained from persistence homology.

- In each window, an ellipse is fitted through the Least Square Optimization.
- Plot the ratio of the minor axis to the major axis as an indication of the persistence of the ellipse.

FAST Topological Data Analysis

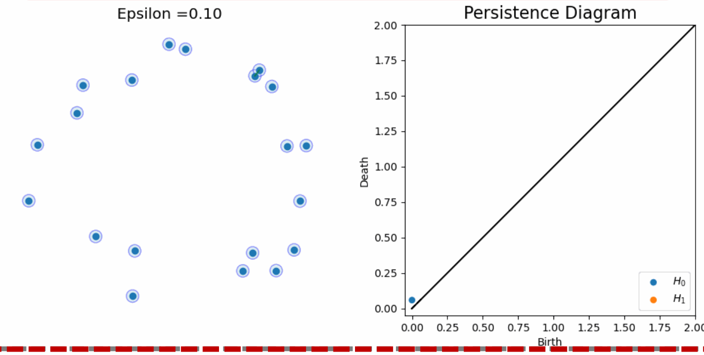
Real-time estimation of nonstationary Systems

Takens' Embedding Theorem



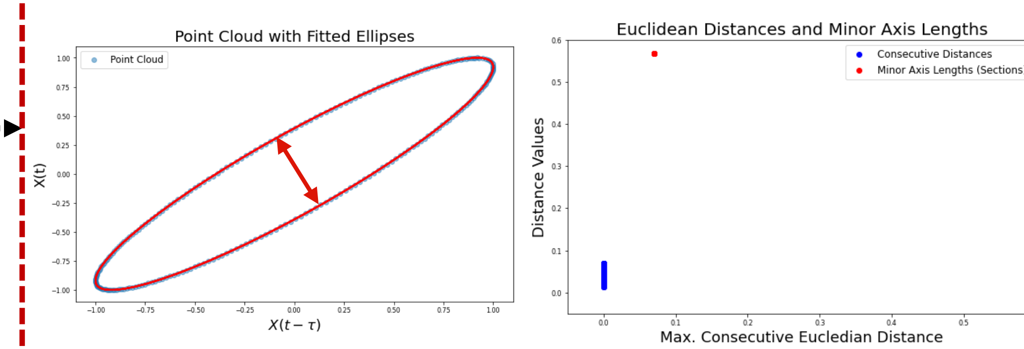
TDA

Persistence Homology



Fast TDA

Proposed Framework (Fast TDA)



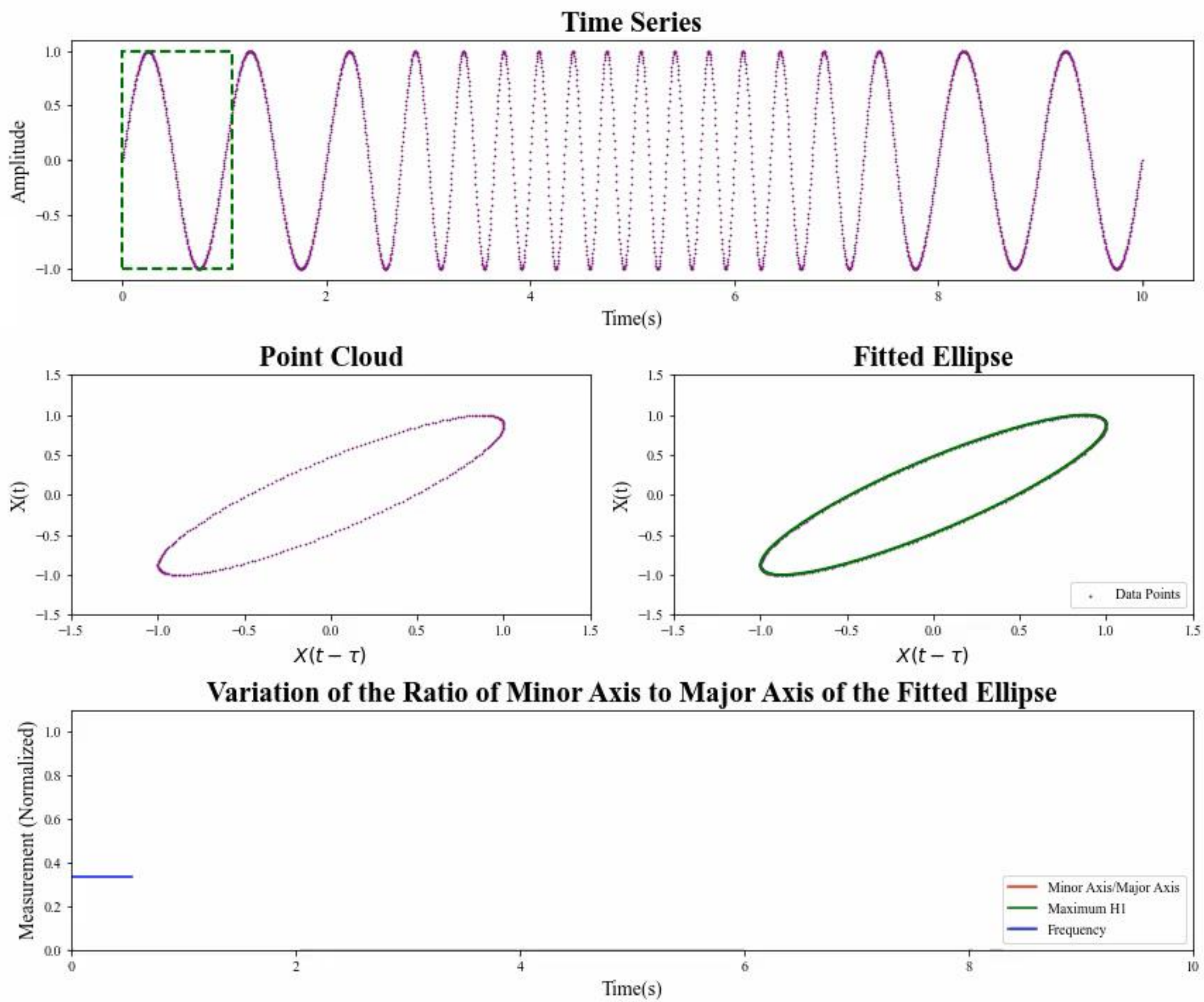
FOUNDATIONS OF ADAPTIVE SYSTEMS AND TECHNOLOGIES

FAST Topological Data Analysis

- $x(t) = \cos(2\pi f(t)t)$
- Moving window size:

$$\tau + \frac{1}{f_{min}} = 1 + 0.03 = 1.03$$

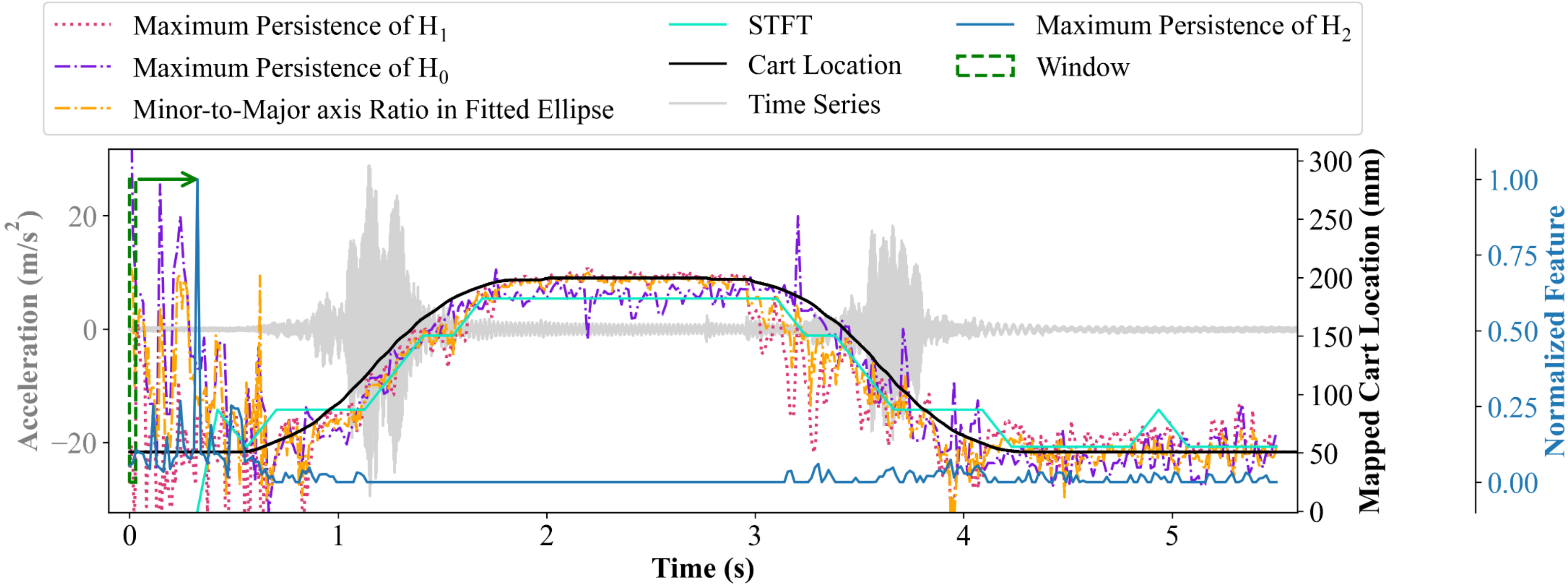
- Time delay = 0.03 s



FOUNDATIONS OF ADAPTIVE SYSTEMS AND TECHNOLOGIES

Case Study #2: Experimental Data from DROPBEAR Testbed

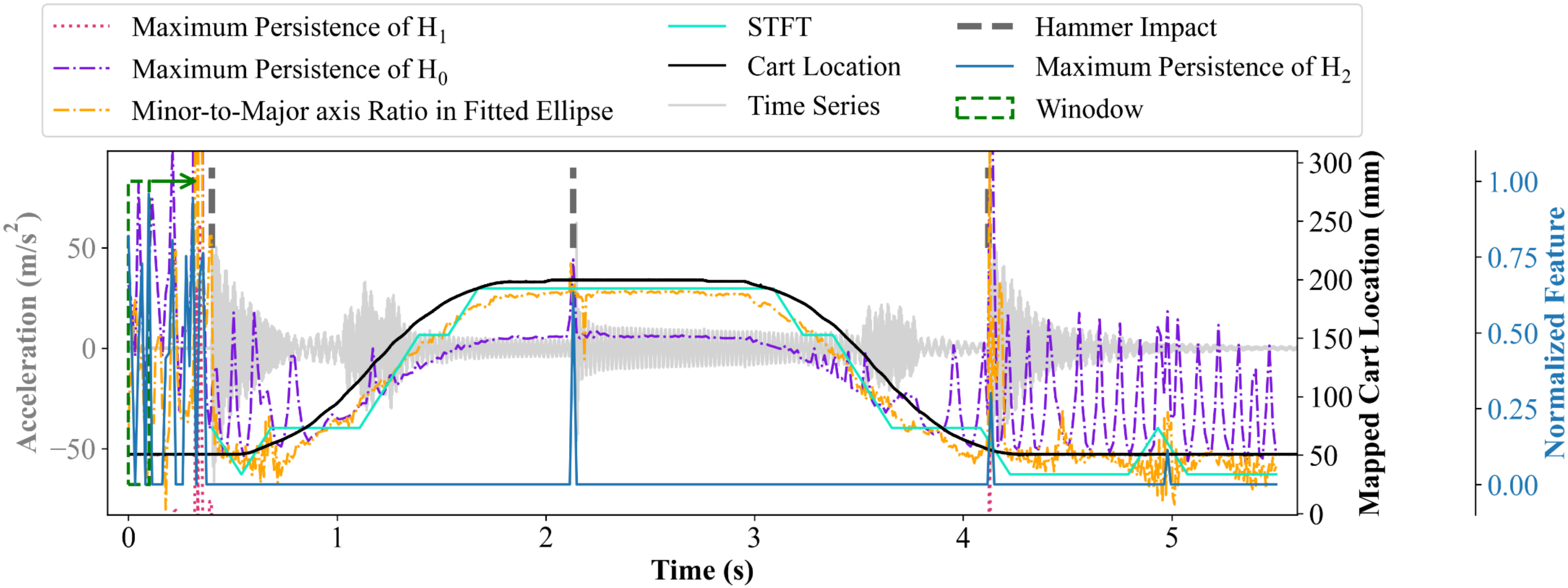
- DROPBEAR without Impact Hammer (Fast TDA)
- Assumption: TDA features are linearly related to the frequency
- Linear Regression: $x^2 = a_0 + a_1 H_i,$



FOUNDATIONS OF ADAPTIVE SYSTEMS AND TECHNOLOGIES

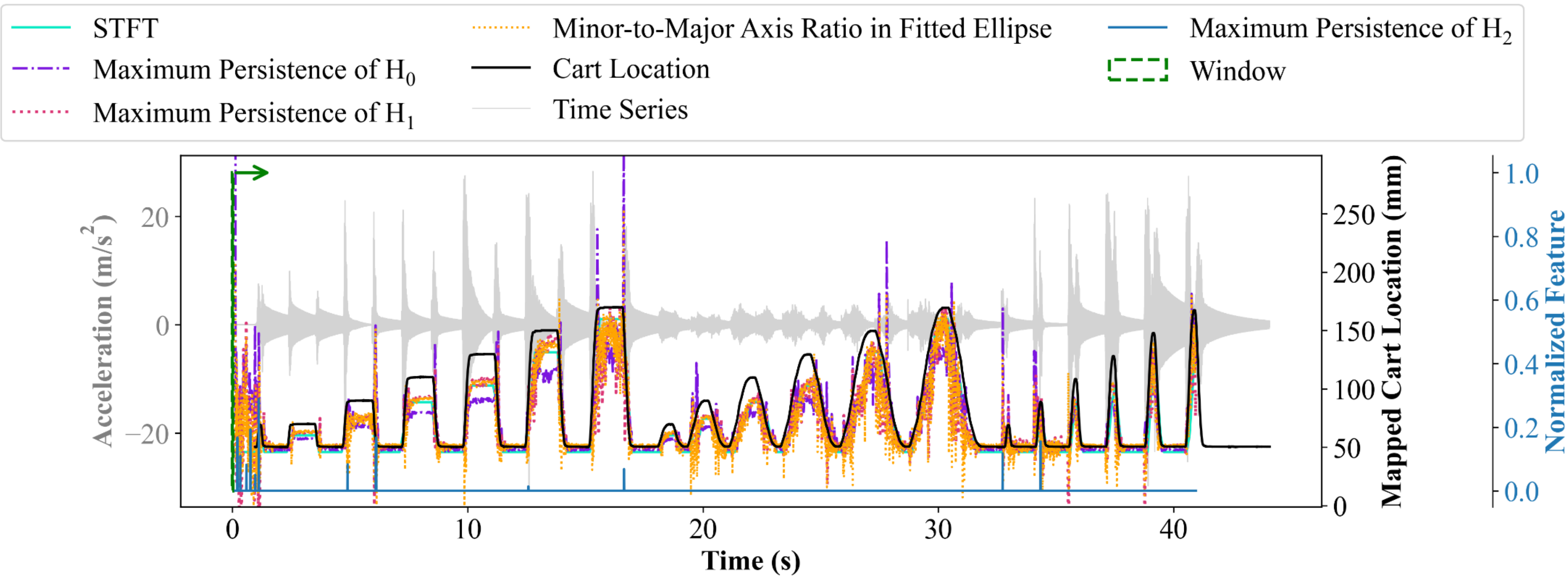
Case Study #2: Experimental Data from DROPBEAR Testbed

- DROPBEAR without Impact Hammer (Fast TDA)
- Assumption: TDA features are linearly related to the frequency
- Linear Regression: $x^2 = a_0 + a_1 H_i,$



Case Study #2: Experimental Data from DROPBEAR Testbed

- DROPBEAR without Impact Hammer (Fast TDA)
- Assumption: TDA features are linearly related to the frequency
- Linear Regression: $x^2 = a_0 + a_1 H_i,$



FAST
FOUNDATIONS OF ADAPTIVE SYSTEMS AND TECHNOLOGIES

Comparison TDA – Fast TDA Results

Computational Time		
STFT	**Fast TDA**	**TDA**
10 ms	84 ms	960 ms

Table 1: Results from LSE

Test	Feature	a_0	a_1	R^2
Test 1	H_0	-0.41	3.17	0.95
	H_1	-0.8	1.85	0.95
	STFT	-0.13	1.07	0.98
	Fast-TDA (H_1)	-0.52	1.57	0.98
Test 2	H_0	-0.07	5.25	0.63
	H_1	-1.05	2.31	0.91
	STFT	-0.11	1.059	0.96
	Fast-TDA (H_1)	-0.31	4.22	0.95
Test 3	H_0	-0.15	4.25	0.86
	H_1	-0.92	2.47	0.87
	STFT	-0.034	0.93	0.95
	Fast-TDA (H_1)	-0.46	2.16	0.91

Comparison TDA – Fast TDA Results

Table 2: Performance results for cart localization.

Test	Feature	J_1 (mm)	$J_{2,5}$ (%)	$J_{2,10}$ (%)	$J_{2,20}$ (%)	$J_{3,5}$ (mm)	$J_{3,10}$ (mm)	$J_{3,20}$ (mm)
Test 1	H_1	14.1	65	50	25	2.2	3.8	7.5
	STFT	11.9	81	48	14	2.8	5.6	10.6
	Fast-TDA	8.5	53	33	10	2.1	3.5	6.3
Test 2	H_1	16	88	68	18	2.4	5.6	10.5
	STFT	12.1	84	52	14	2.4	5.6	10.8
	Fast-TDA	10.9	81	48	20	2.3	5.8	8.3
Test 3 (sect. 1)	H_1	8.7	46	29	12	1.4	2.9	4.9
	STFT	6.3	38	13	3	3.4	4.6	5.4
	Fast-TDA	8.25	42	30	10	1.9	4.6	5.0
Test 3 (sect. 2)	H_1	10.2	56	34	15	1.9	3.8	6.0
	STFT	7.3	50	23	6	3.5	4.6	6.4
	Fast-TDA	9.2	54	31	11	2.1	3.9	6.2
Test 3 (sect. 3)	H_1	5.7	30	18	8	1.3	2.1	3.4
	STFT	5.9	21	12	5	4	4.4	5.1
	Fast-TDA	5.7	28	17	7	1.9	2.6	3.8

FAST
FOUNDATIONS OF ADAPTIVE SYSTEMS AND TECHNOLOGIES

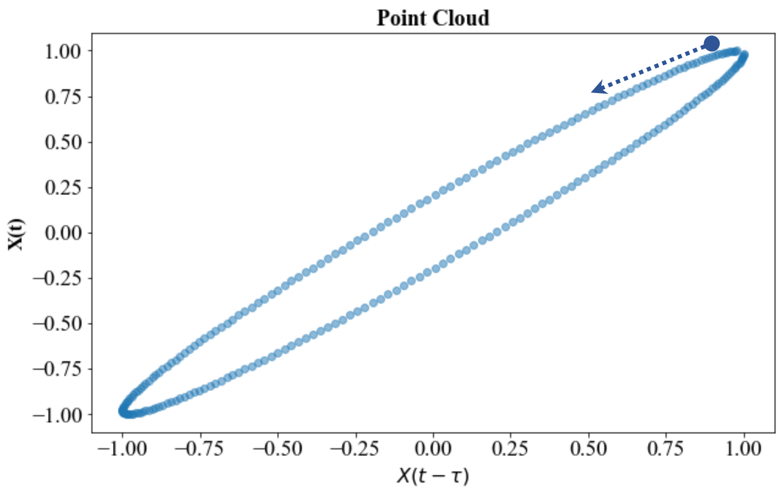
Major findings (Increasing frequency Implementation)

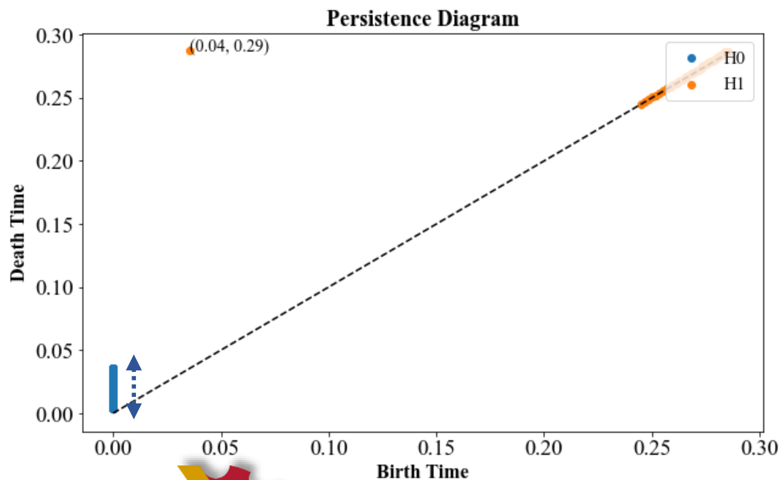
	TDA	Fast TDA
Advantages	• Online available resources • Mechanistic Implementation	• Direct Implementation for maximum H1. • Low Computational cost
Disadvantages	• High Computational cost	• No available documentation • Application under development

FAST FOUNDATIONS OF ADAPTIVE SYSTEMS AND TECHNOLOGIES

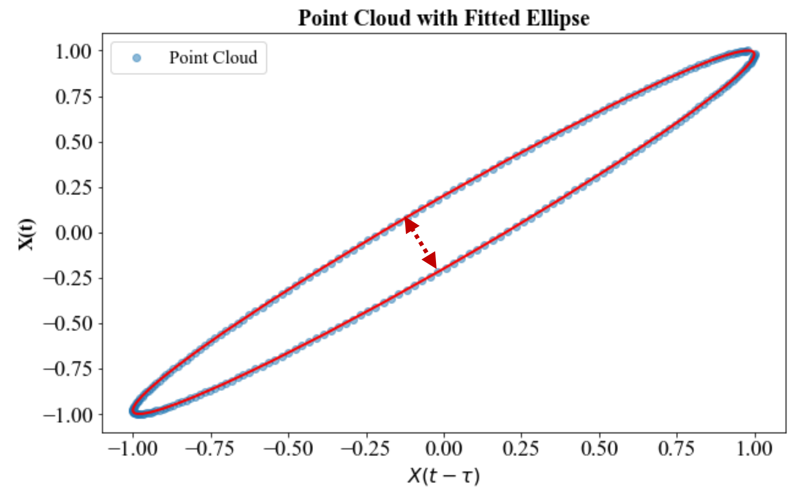
Persistence Diagram Analysis and reconstruction

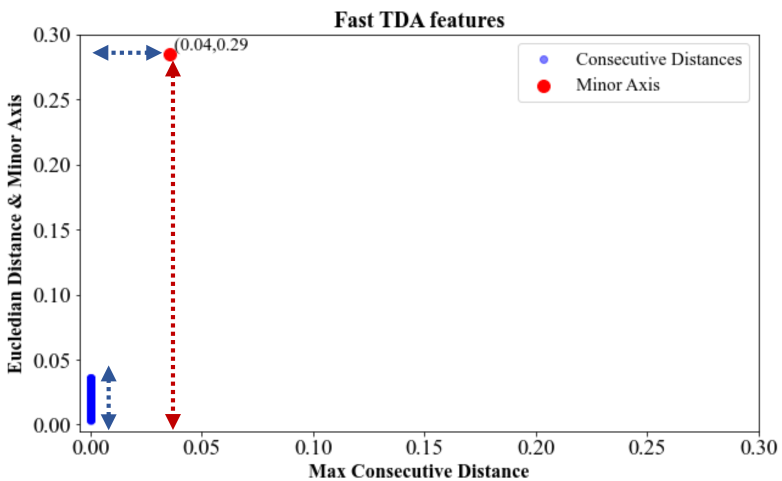
Persistence Homology Results





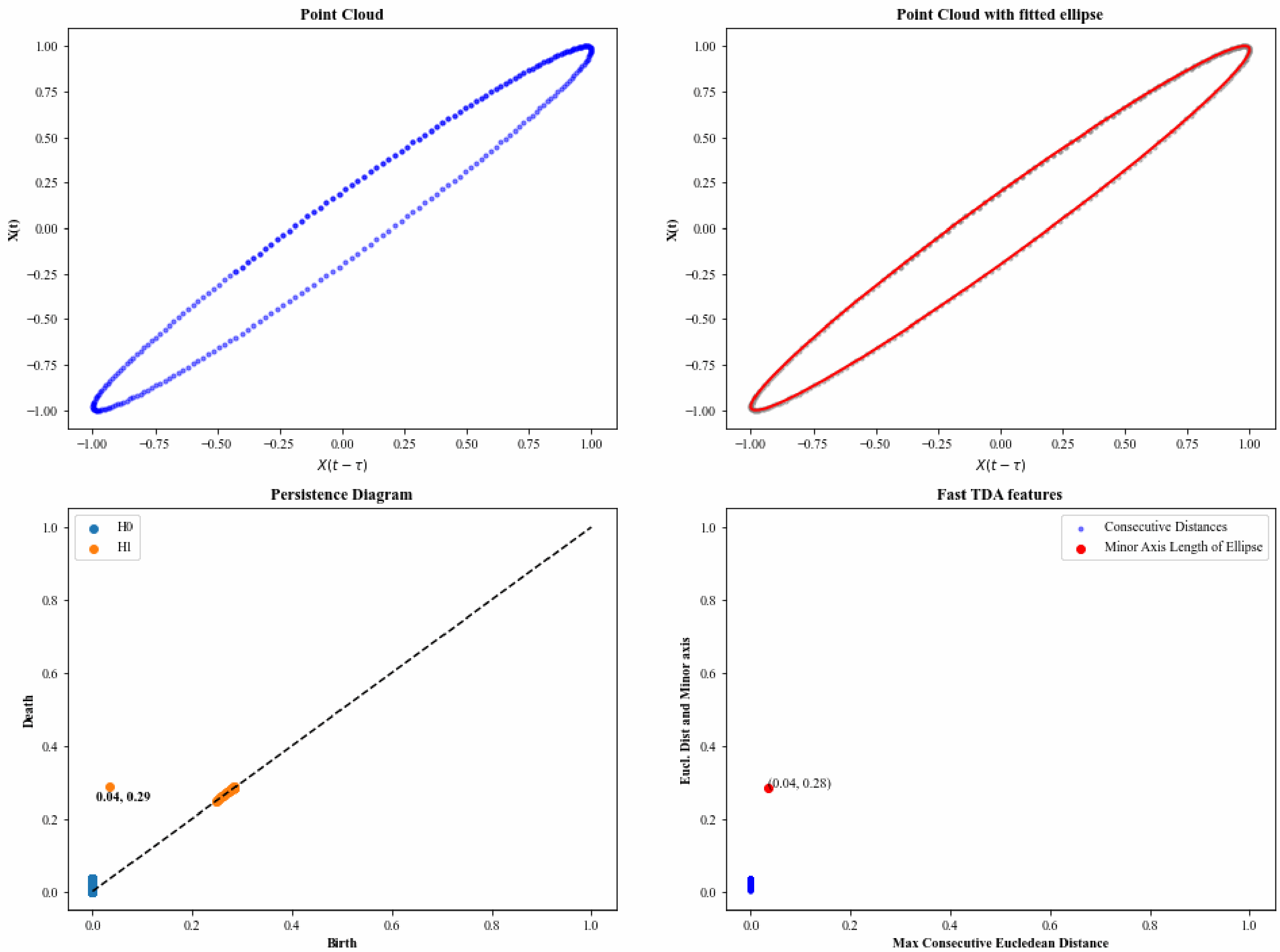
Fast TDA results





FAST
FOUNDATIONS OF ADAPTIVE
SYSTEMS AND TECHNOLOGIES

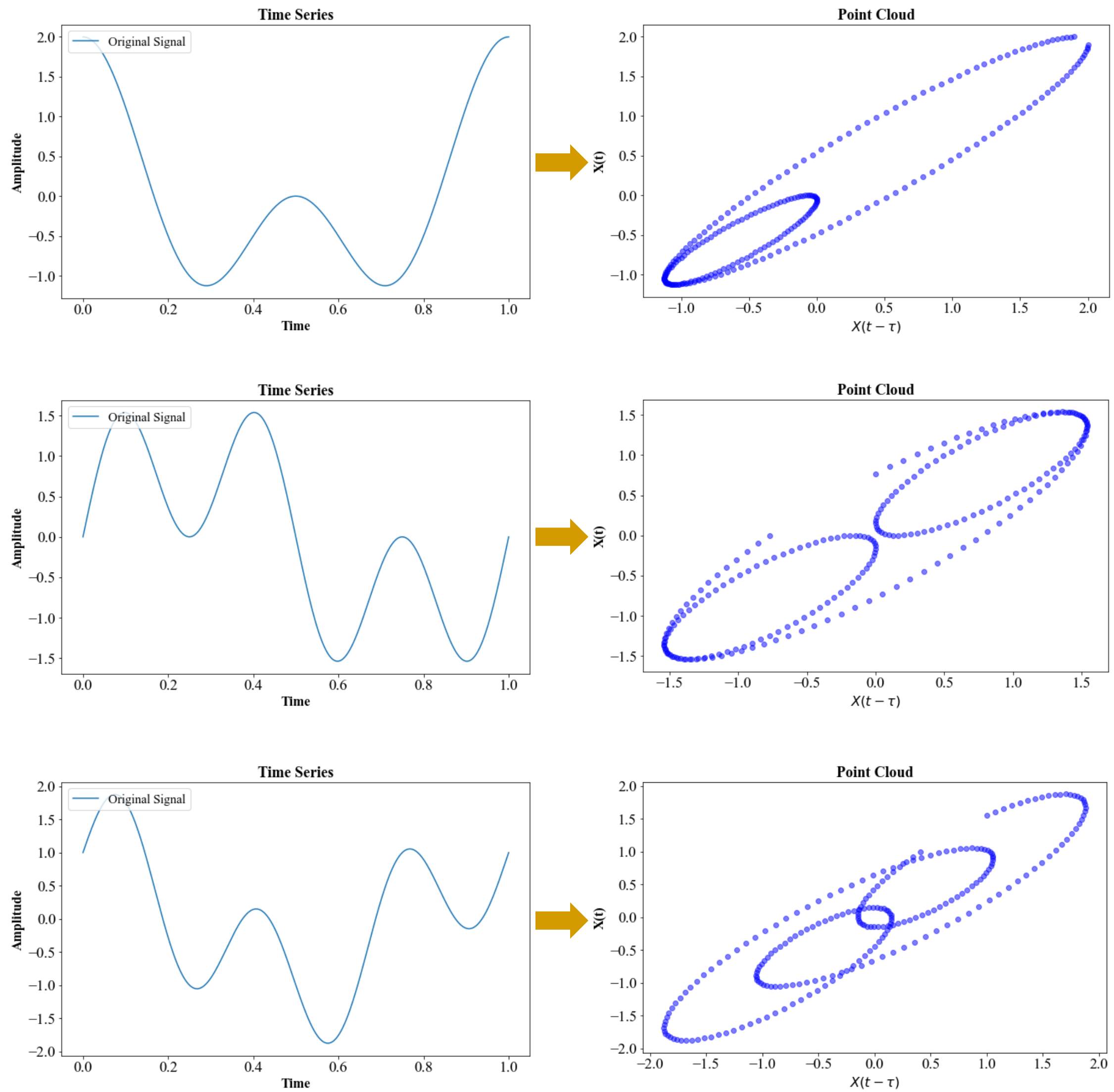
Synthetic data with Fast TDA Diagram



Fast TDA (Multiple Frequency Implementation)

Challenges:

- Few training data.
- Our dynamics are highly non-stationary.
- Sub-millisecond computations.
- Complexity of the Point Cloud.
- Feature interpretation.



Ellipse Identification Methods

Method I:

- Minimum pairwise distance.
- Implementing a threshold (10%) to find possible links between points in the point cloud.

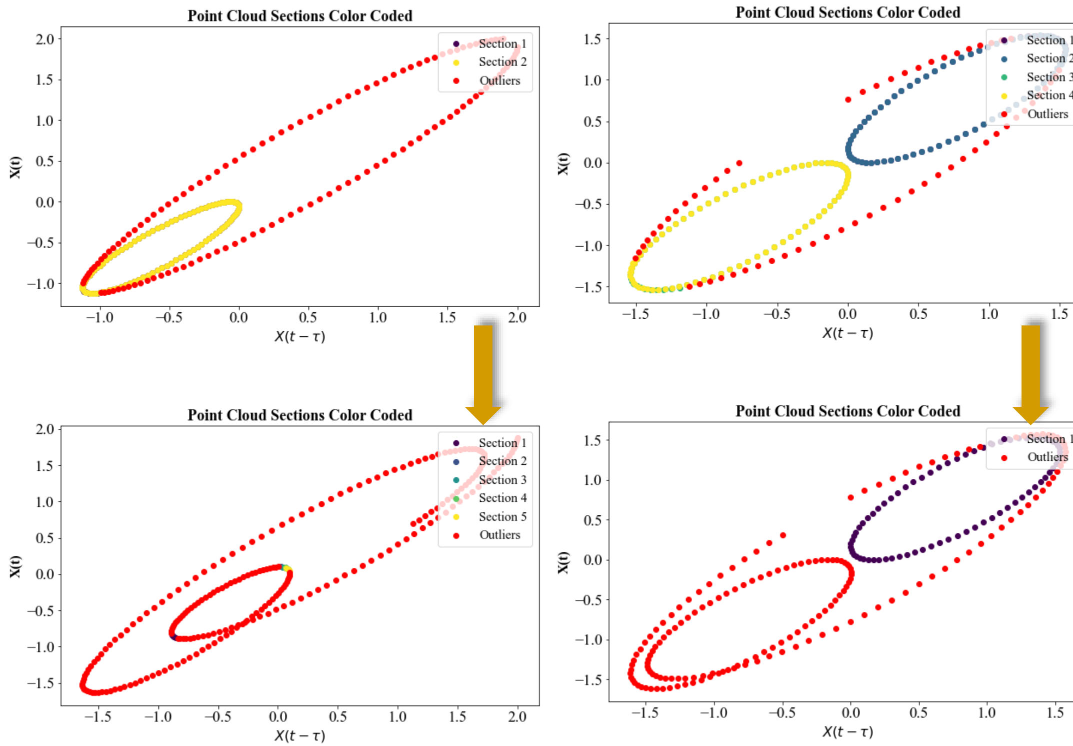
$$\boldsymbol{dmin}\ (\boldsymbol{d_{pairwise}} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2})$$

$$\boldsymbol{T} = d_{min} \cdot 1.1$$

$$\textbf{Condition} \Rightarrow d_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \leq T$$

Applications:

- Good for multiple section identification based on minimum Threshold.
- Good for identifying intersections in noisy data.



Ellipse Identification Methods

Method II:

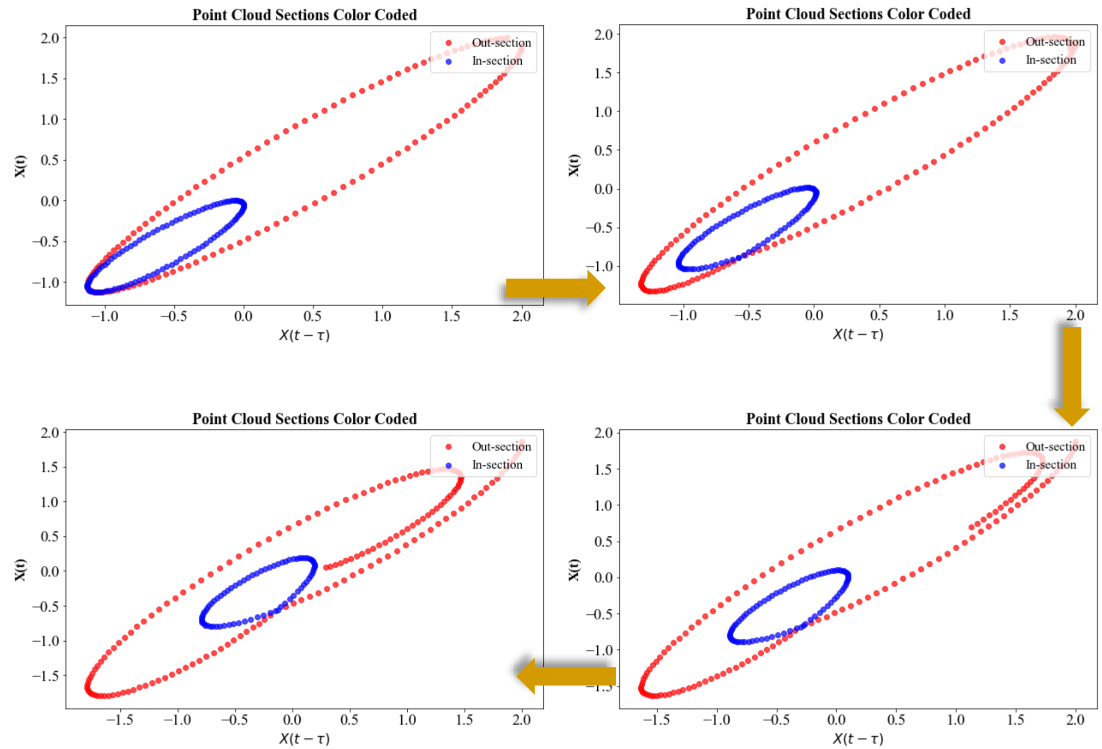
- Minimum pairwise distance identified between non-consecutive points.

$$\boldsymbol{dmin}(\boldsymbol{d}_{i-j\_ignored\ range} =)\ \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\textbf{Condition} \Rightarrow dmin$$

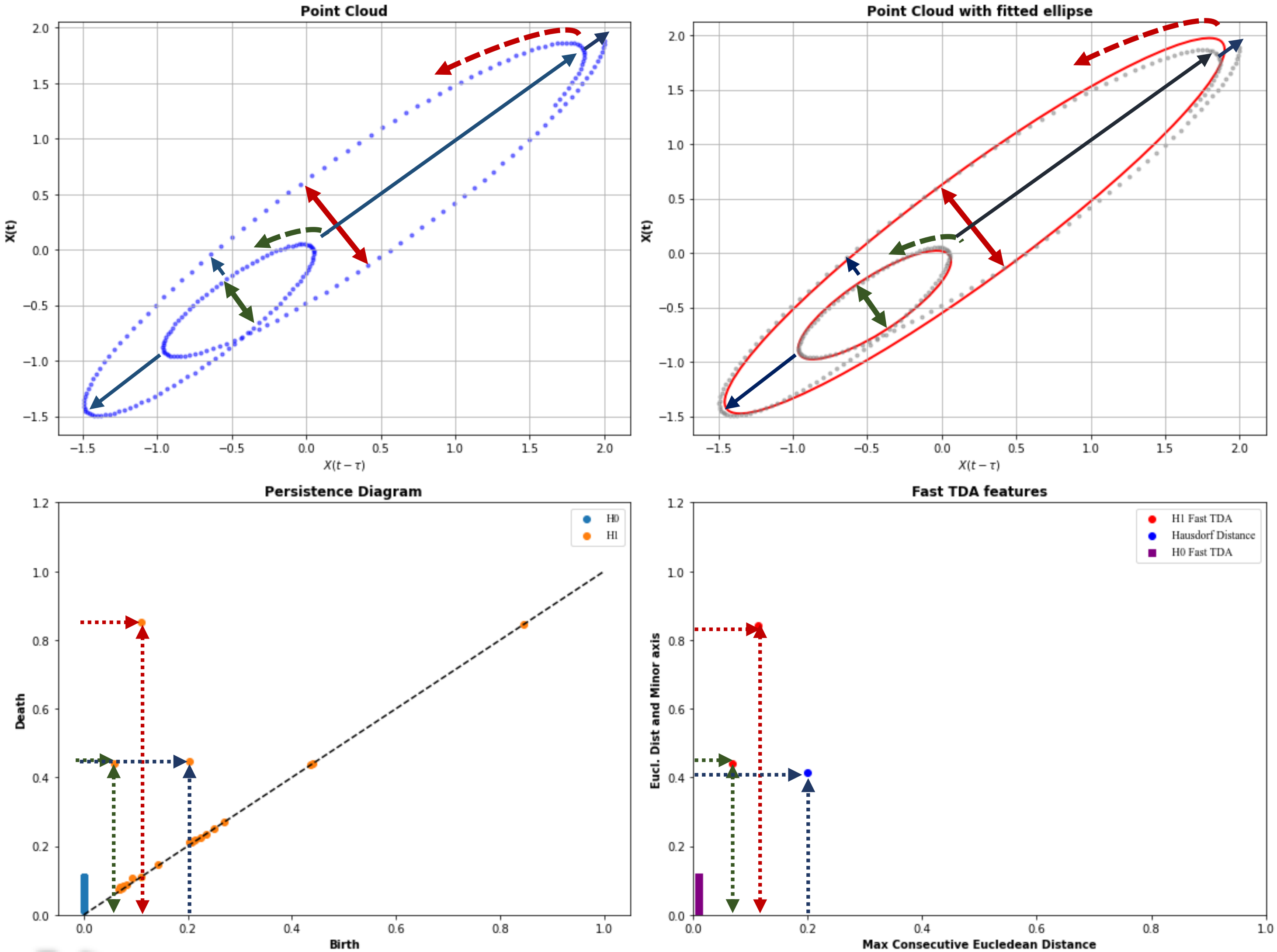
Applications:

- Efficient for singular node tracking.
- Optimal for maximum two ellipse identification.
- Better identification between sections and outliners.



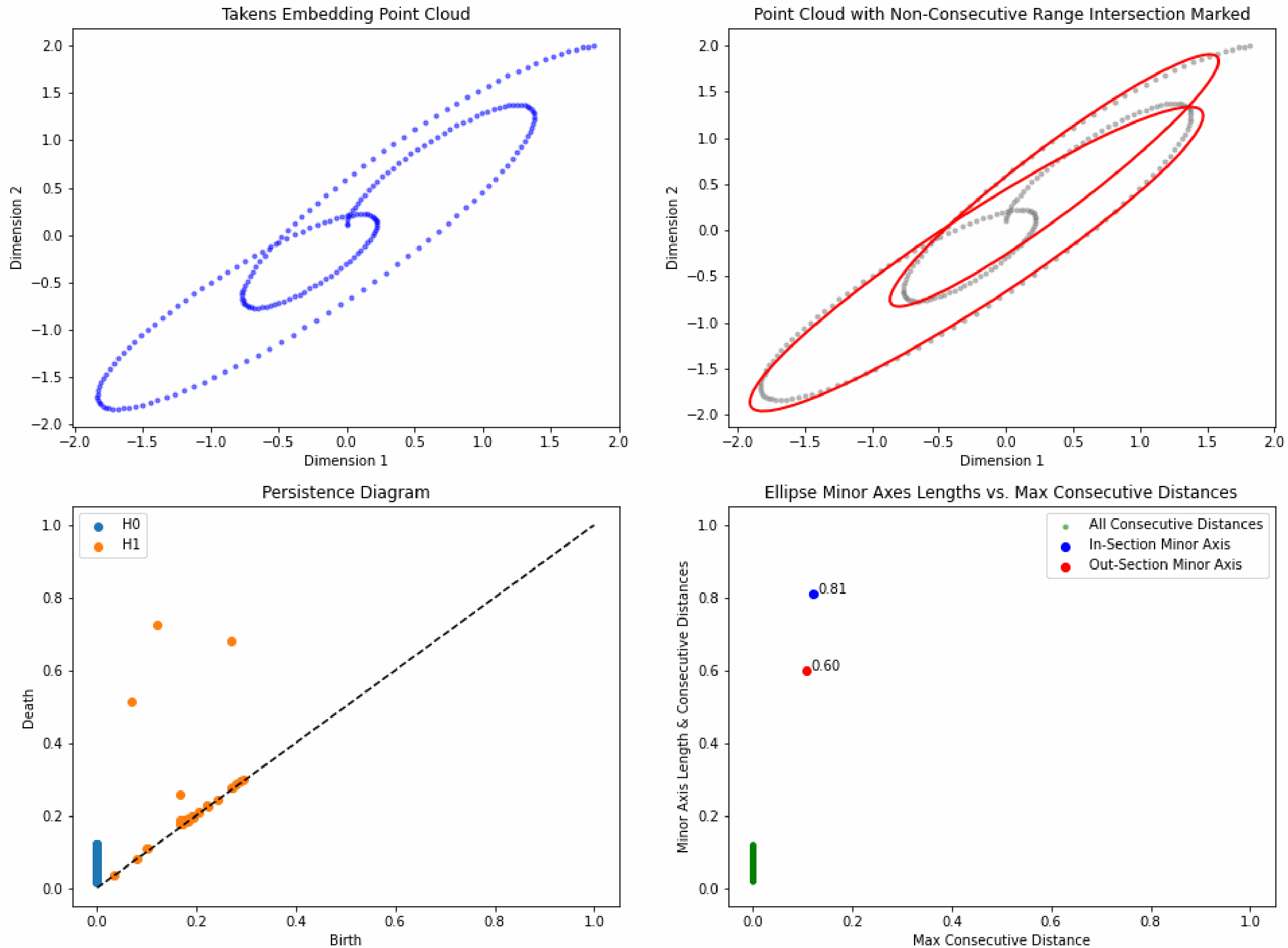
FAST
FOUNDATIONS OF ADAPTIVE SYSTEMS AND TECHNOLOGIES

Persistence Diagram Analysis and reconstruction



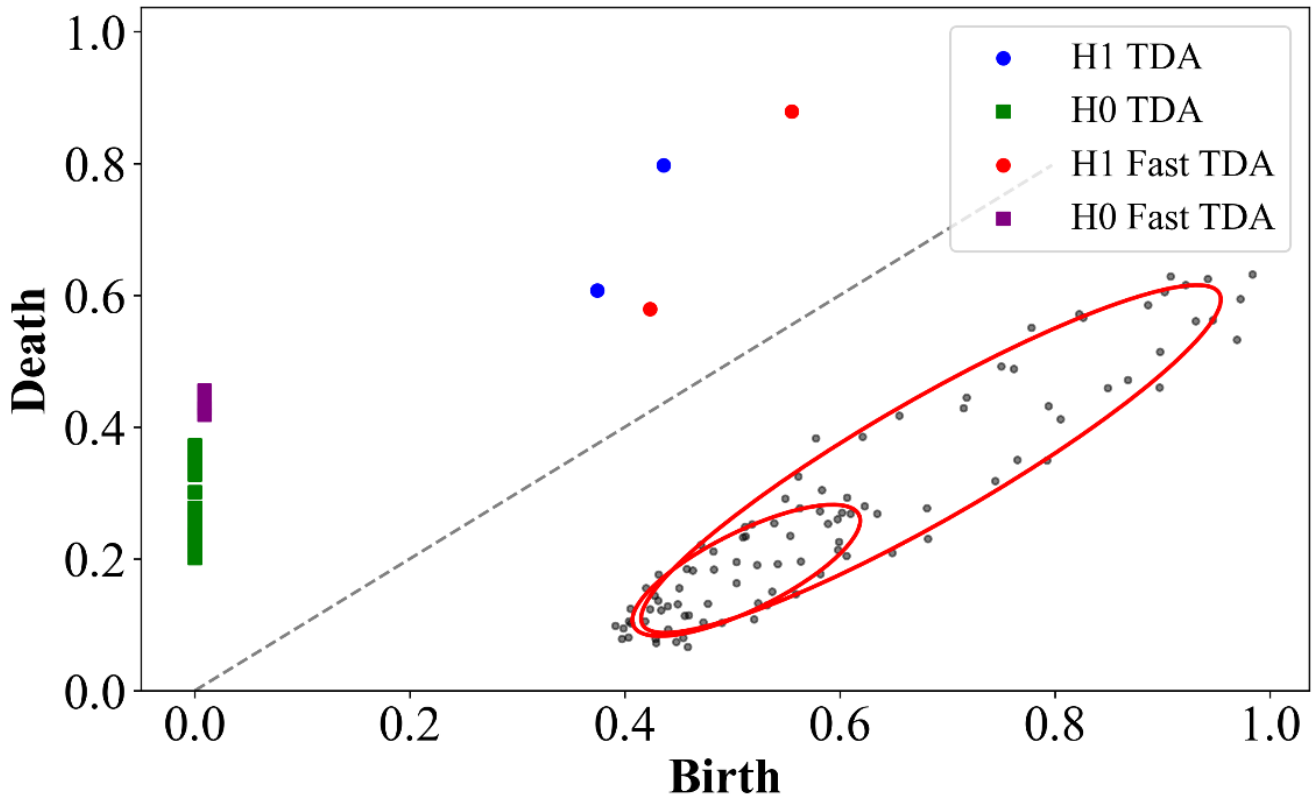
FAST
FOUNDATIONS OF ADAPTIVE SYSTEMS AND TECHNOLOGIES

Synthetic Data with Fast TDA Diagram



TDA and Fast TDA with Noisy Data

- Better identification of hidden characteristics in noisy environments.



FAST FOUNDATIONS OF ADAPTIVE SYSTEMS AND TECHNOLOGIES

Summary

- **Key Outcomes:**
  - Fast TDA feature extractions based on persistence homology diagram.
  - Fast TDA homology diagram implementation for multiple frequencies.
  - Implementation of Fast TDA for noisy environments.
- **Upcoming Work/Challenges:**
  - Improve two dimensions of multiple ellipse identification.
  - Optimize metrics for multiple H1 features identification.
  - Relate Multiple frequency topological features to time series characteristics.
  - Implementation of three dimensions and feature extraction in multiple frequency point cloud.

Acknowledgements

- Air Force Office of Scientific Research (AFOSR).
- Defense Established Programs to Stimulate Competitive Research (DEPSCoR).
- Air Force Research Laboratory Munitions Directorate.

Thank you for your time

Questions?