REDUCED-ORDER MODAL FRAMEWORK FOR MICROSECOND UPDATING OF 2D STRUCTURAL SYSTEMS USING THE LOCAL EIGENVALUE MODIFICATION PROCEDURE

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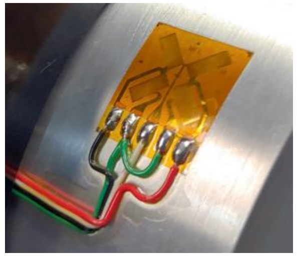
Technical Challenges of Estimating State of High-rate Dynamic system

Adequate sensing



Sensor response

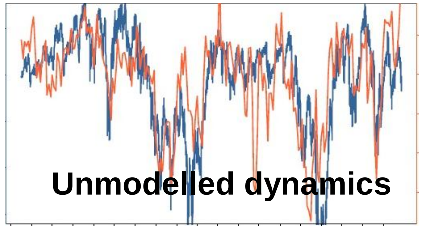
3 µs – 10 µs

Lack of system Knowledge

Duration of event

30 µs – 100 ms

High variability in loading



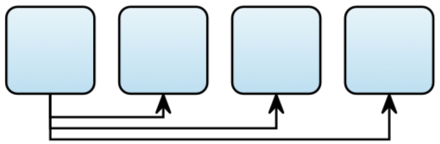
Different behavior regimes

250 µs – 1 secs

Limited resources for algorithm implementation

Algorithm execution and decision making

Main Objective



100µs-1ms

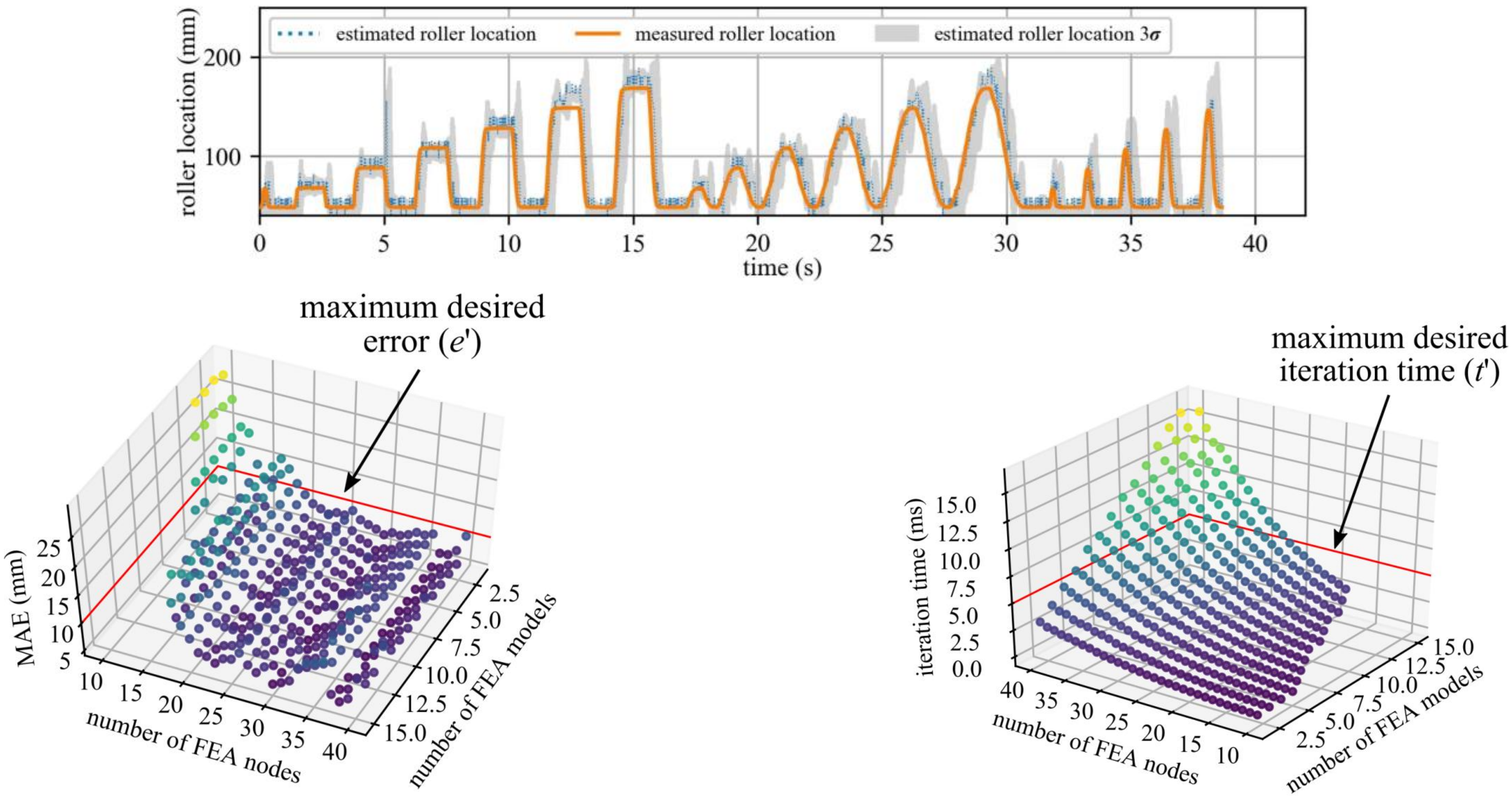
Dodson, Jacob, et al. "High-rate structural health monitoring and prognostics: an overview." *Data Science in Engineering, Volume 9* (2021): 213-217.

Methodologies for high-rate state estimation

- Physics-enhanced machine learning (PEML) models
- Real-time fusion of high-speed dynamic data augmented by model-based data
  - ✓ Model reduction and model-updating (offline and real-time) approaches
    - ❖ Finite Element model updating
- Uncertainty quantification (UQ) methods to enable decisions connected to confidences

Dodson, Jacob, et al. "High-rate structural health monitoring and prognostics: an overview." *Data Science in Engineering, Volume 9* (2021): 213-217.

Real-time FEA model updating (1D)



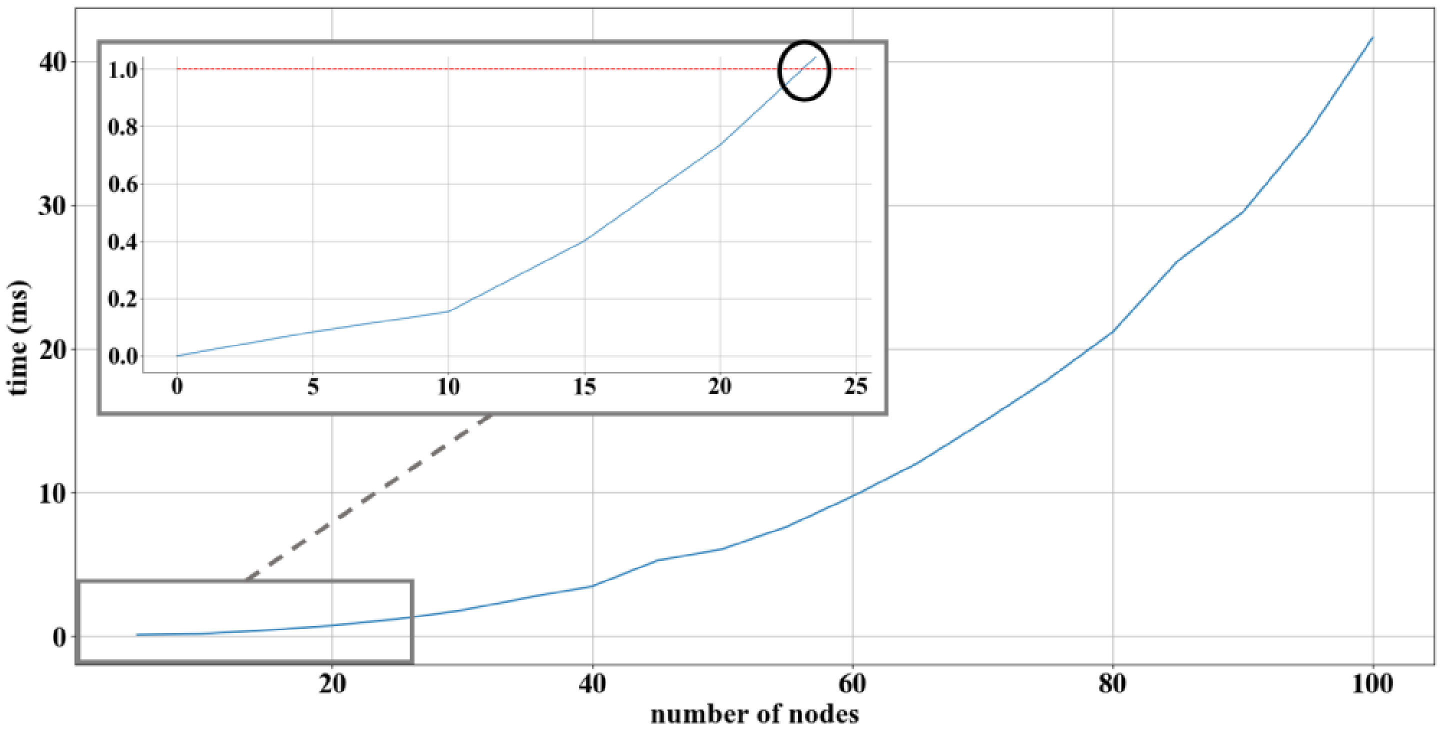
Downey A., et al,. "Millisecond Model Updating for Structures Experiencing Unmodeled High-Rate Dynamic Events" *Mechanical Systems and Signal Processing* ***138***, 2020

FEA Computation speed for the DROPBEAR

General Eigenvalue solutions accurately estimates the state of the DROPBEAR

However, FEA model is limited to 23 nodes to achieve 1ms model updating time

Solving for system's frequencies accounted for 90% of algorithm iteration time



Carroll, M., Downey, A., Dodson, J., Hong, J. and Scheppegrell, J., "Analysis of Computation Speeds of Eigenvalue Solutions for High-Rate Structural Health Monitoring."

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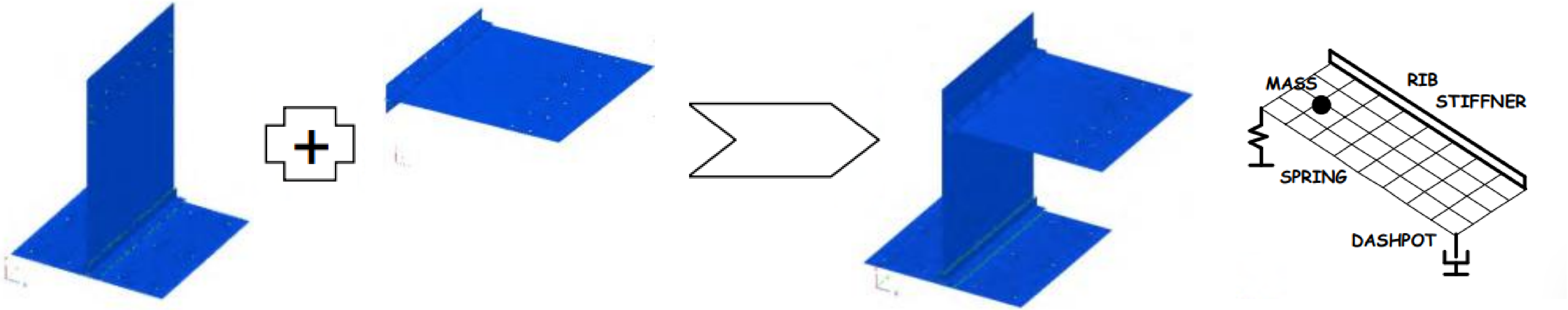
Structural Dynamic Modification



Avitabile, P., “*Twenty Years of Structural Dynamic Modification- A Review*,” *Sound and Vibration,* pp. 14-25. 2003

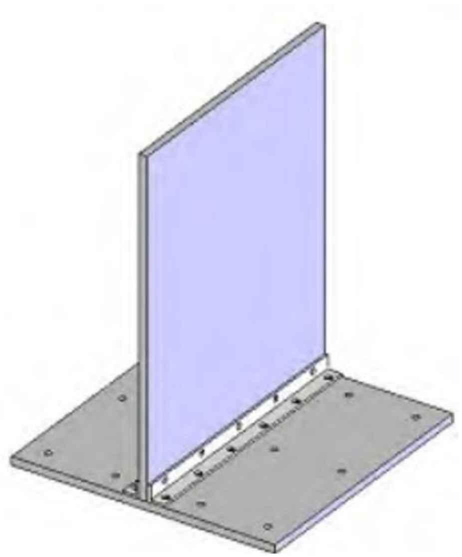
Eigenvalue Modification Procedure

Can the existing frequencies and mode shapes be used to predict new frequencies and mode shapes dues to changes in mass and stiffness?

$$\left[\begin{bmatrix} \ddots & & \\ & \overline{M}_1 & \\ & & \ddots \end{bmatrix} + [\Delta\overline{M}_{12}]\right]\{\ddot{p}_1\} + \left[\begin{bmatrix} \ddots & & \\ & \overline{K}_1 & \\ & & \ddots \end{bmatrix} + [\Delta\overline{K}_{12}]\right]\{p_1\} = [0]$$

$$[\Delta\overline{M}_{12}] = [U_1]^T[\Delta M_{12}][U_1]$$

$$[\Delta\overline{K}_{12}] = [U_1]^T[\Delta K_{12}][U_1]$$



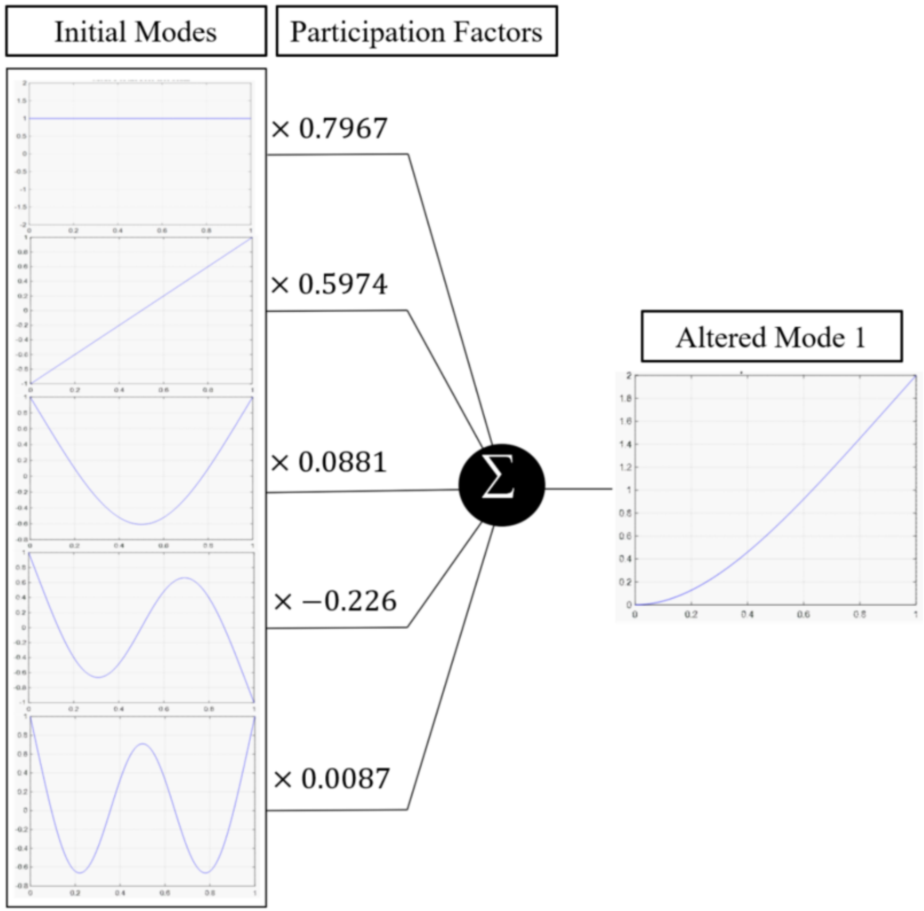
$$\left[\left[\begin{bmatrix} \ddots & & \\ & \overline{K}_1 & \\ & & \ddots \end{bmatrix} + [U_1]^T[\Delta K_{12}][U_1]\right] - \lambda\left[\begin{bmatrix} \ddots & & \\ & \overline{M}_1 & \\ & & \ddots \end{bmatrix} + [U_1]^T[\Delta M_{12}][U_1]\right]\right]\{p_1\} = \{0\}$$

Avitabile, P., "*Twenty Years of Structural Dynamic Modification- A Review*," *Sound and Vibration,* pp. 14-25. 2003

Local Eigenvalue Modification Procedure (LEMP)

What is LEMP?

- Identifies physical changes to the system such as mass, stiffness or damping representing them in terms of frequencies or mode shapes
- Model the altered state as a mixture of the initial state and changes made to the initial state
- Reduces the GE equation to a set of second-order equations



Avitabile, P., "*Twenty Years of Structural Dynamic Modification- A Review*," *Sound and Vibration,* pp. 14-25. 2003

Local Eigenvalue Modification Procedure (LEMP)

If only one change of mass or stiffness is considered, then this equations can be reduced to

Stiffness change

$$\left[\left[\begin{bmatrix} \ddots & & \\ & \overline{K}_1 & \\ & & \ddots \end{bmatrix} + \{v_{1k}\}\alpha_{1k}\{v_{1k}\}^T\right] - \lambda\left[\begin{bmatrix} \ddots & & \\ & \overline{M}_1 & \\ & & \ddots \end{bmatrix}\right]\right]\{p_1\} = \{0\}$$

Mass change

$$\left[\left[\begin{bmatrix} \ddots & & \\ & \overline{K}_1 & \\ & & \ddots \end{bmatrix}\right] - \lambda\left[\begin{bmatrix} \ddots & & \\ & \overline{M}_1 & \\ & & \ddots \end{bmatrix} + \{v_{1m}\}\alpha_{1m}\{v_{1m}\}^T\right]\right]\{p_1\} = \{0\}$$

The solution then reduces to a second order equation for each of the 'm' modes of the system

A solver for the equation was formulated using Divide and Conquer method

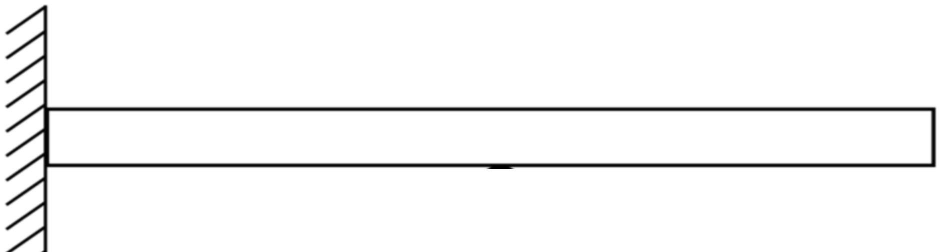
$$\frac{1}{\alpha_k} = \sum_{i=1}^{m} \frac{\left(\left\{u^{(i)}\right\}^T \{t_k\}\right)^2}{\Omega_2^2 - \omega_i^2}$$

$$\frac{1}{\Omega^2\alpha_m} = \sum_{i=1}^{m} \frac{\left(\left\{u^{(i)}\right\}^T \{t_m\}\right)^2}{\Omega_2^2 - \omega_i^2}$$

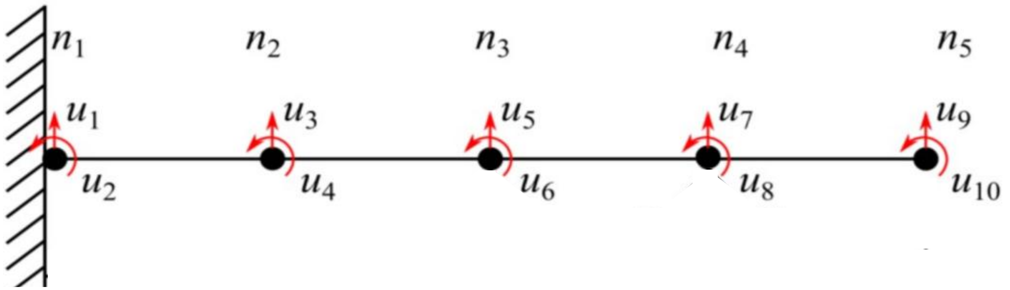
Avitabile, P., "*Twenty Years of Structural Dynamic Modification- A Review*," *Sound and Vibration,* pp. 14-25. 2003

Single-State change Estimation

- [ ] **Initial state general eigenvalue solution**



Initial state of the system (beam).



$$\boldsymbol{\lambda} = (60237 \quad 2682286 \quad 21391038 \quad 82438554 \quad 286161582)$$

Construct the elemental mass and stiffness matrices ($\mathbf{M}_1$ and $\mathbf{K}_1$)

$$\mathbf{U}_1 = \begin{pmatrix} -0.000005 & 0.00011 & 0.00051 & 0.00138 & -0.00340 \\ -0.000001 & 0.000008 & 0.000023 & 0.000046 & -0.000088 \\ -0.184749 & 0.862567 & 1.521322 & 1.535297 & -0.796654 \\ -3.95962 & 13.68743 & 10.37102 & -17.27622 & 68.83187 \\ -0.64779 & 1.52565 & 0.15321 & -1.53923 & 0.260667 \\ -6.37642 & -1.72852 & -3.28287 & -6.21277 & -80.17702 \\ -1.26088 & 0.420824 & -1.25908 & 1.14986 & 0.176068 \\ -7.43893 & -2.20332 & 13.1159 & 21.2392 & 76.3001 \\ -1.92314 & -1.86050 & 1.94283 & -1.87065 & -1.9711 \\ -7.61313 & -27.5937 & 46.6347 & -63.1058 & -96.1961 \end{pmatrix}$$

Solve the general eigenvalue problem to obtain the squares of the first n natural frequencies, and the first n modal vectors for the initial state

$$f_1 = (39 \quad 261 \quad 736 \quad 1445 \quad 2692)$$

LEMP Implementation

- **Step 1: Addition of roller condition (change)**

$$\Delta\mathbf{K}_{12} = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1e10 \quad 0 \quad 0)$$

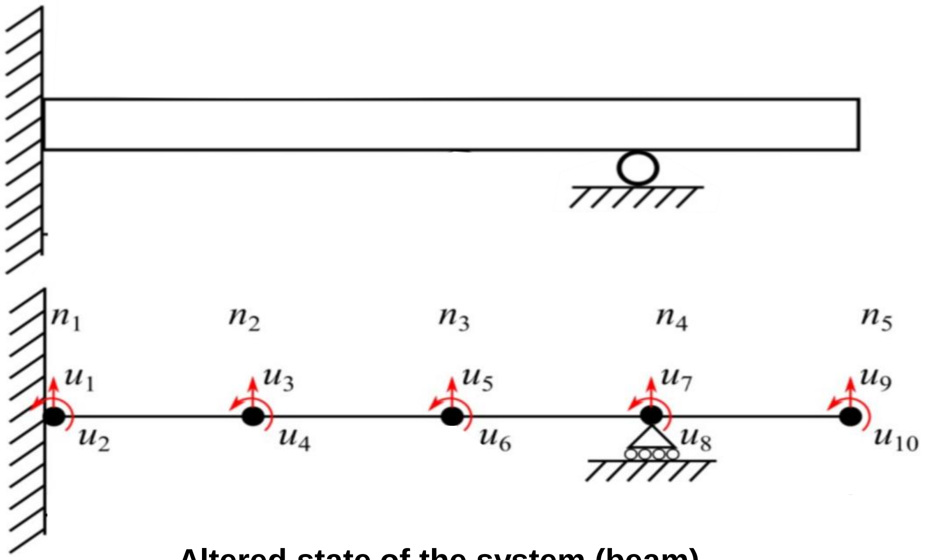
- **Step 2: Spectral decomposition of $\Delta\mathbf{K}_{12}$**

$$\mathrm{T} = (1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1)$$

$$\alpha = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1e10 \quad 0 \quad 0)$$

- **Step 3: Set truncation: include only contributing modes**

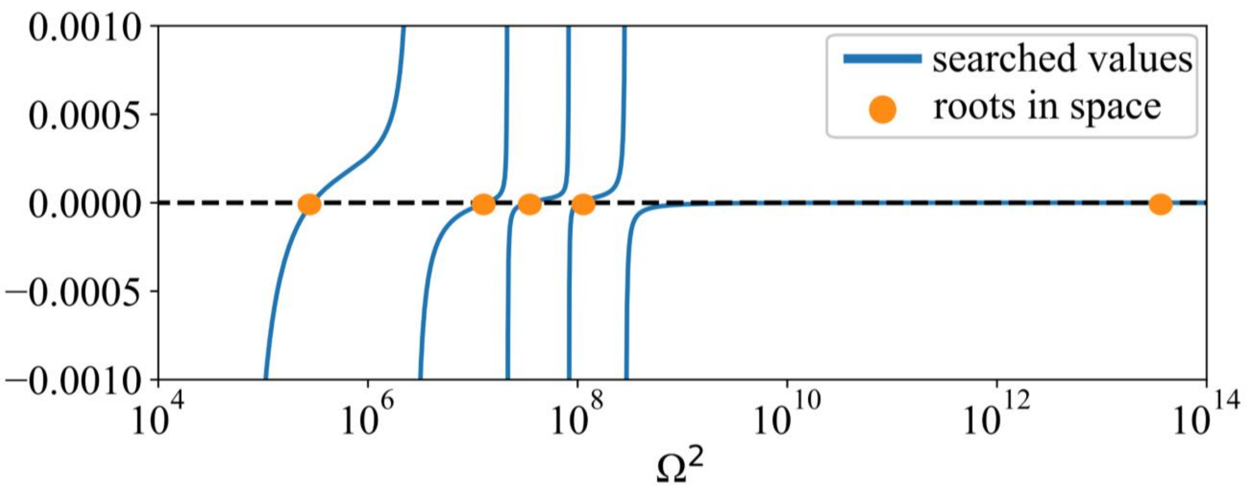
The contributing vectors are reduced to only those values in the 8th row of each matrix.



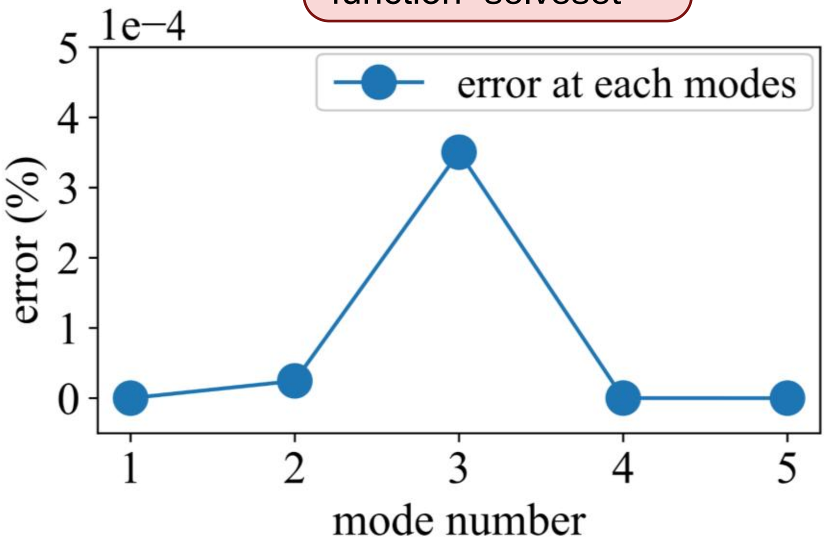
Altered state of the system (beam) with added roller condition (spring)

LEMP Implementation

- **Step 4: Obtain Ω^2 using Divide and conquer**



Ω^2 values using D&C and Sympy function "solveset"



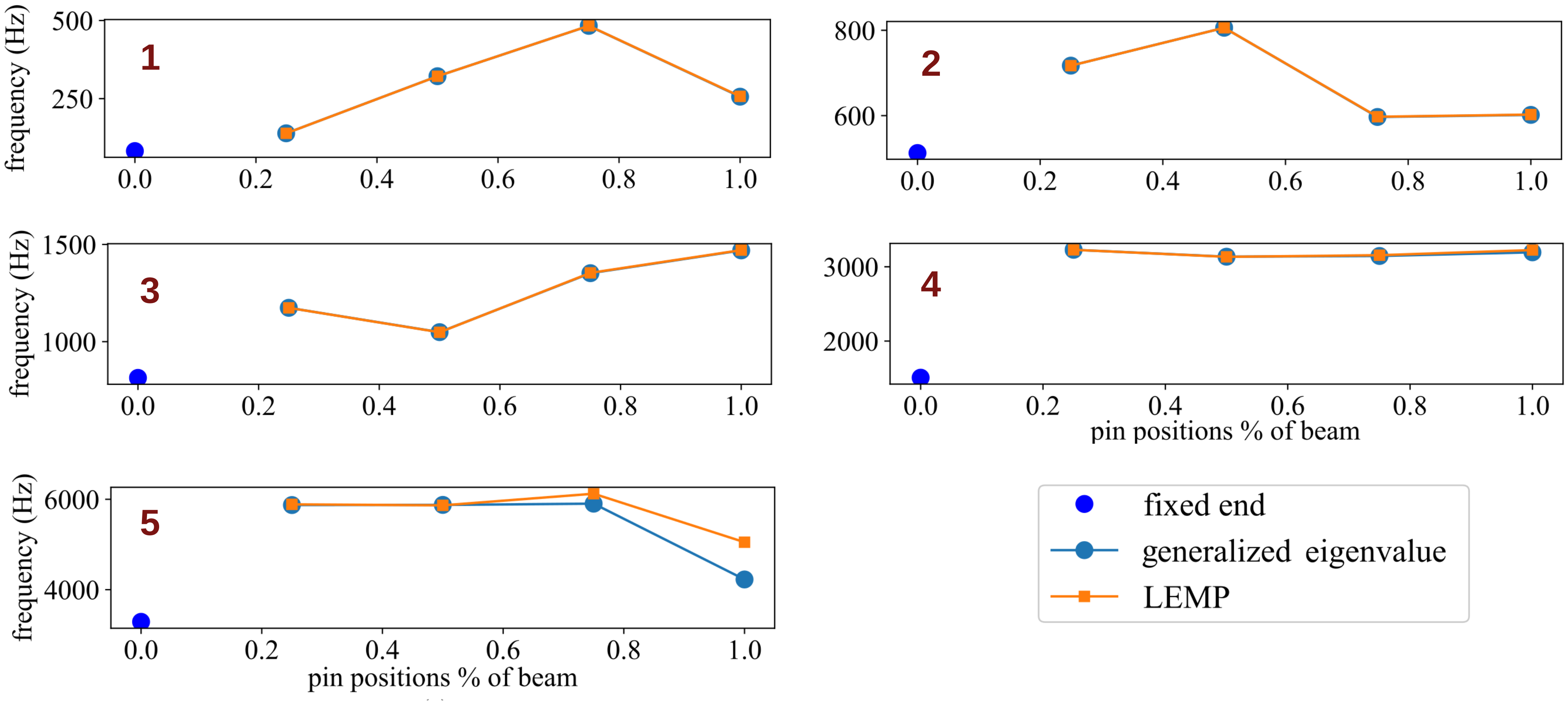
- **Step 5: Solve for new frequencies**

The new natural frequencies f_2 in Hz are then calculated for the five modes in the model utilized.

$$f_2 = \begin{pmatrix} 86 & 583 & 917 & 1602 & 1330221 \end{pmatrix}$$

Generalized Eigenvalue and LEMP

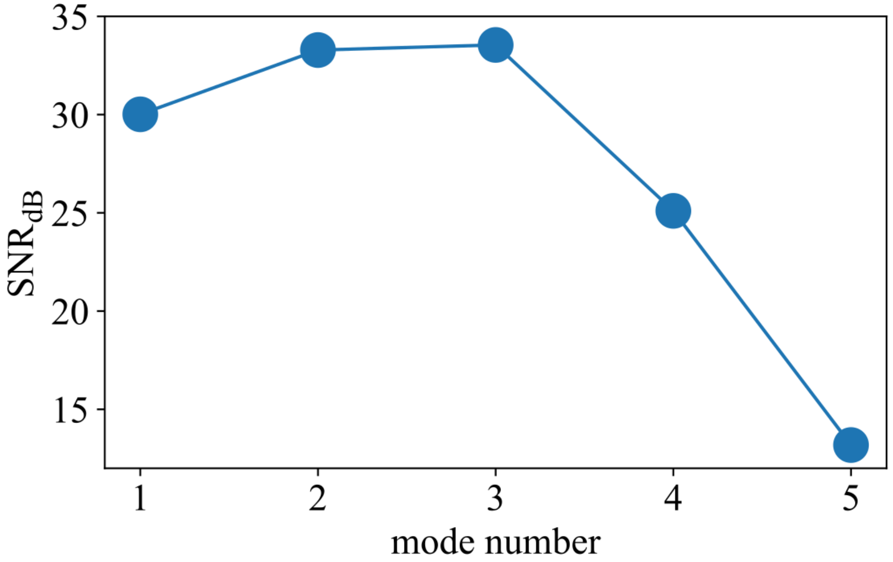
- Beam frequencies at each nodes on addition of roller at node 4 for first five modes

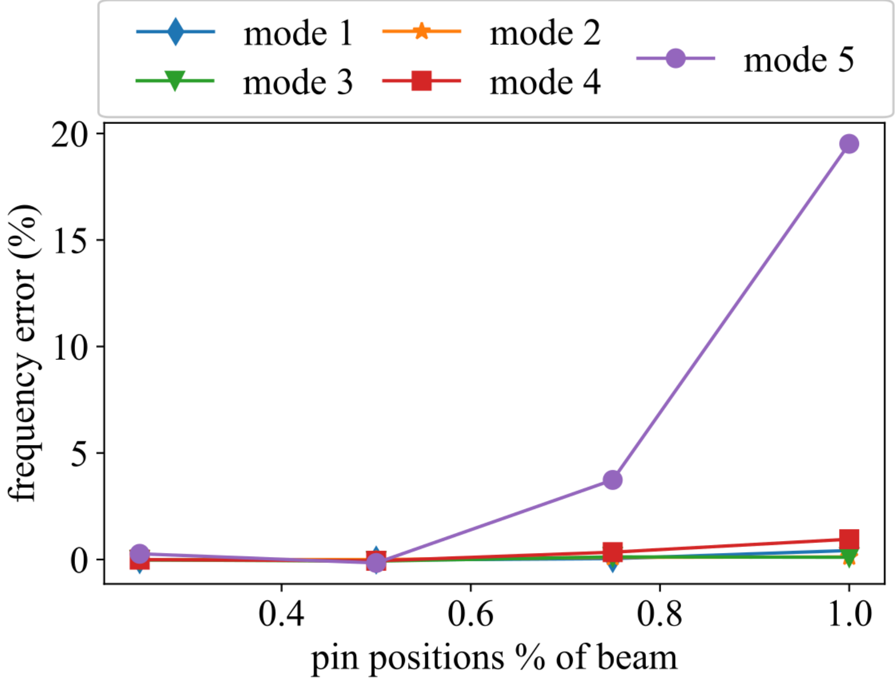


Generalized Eigenvalue and LEMP

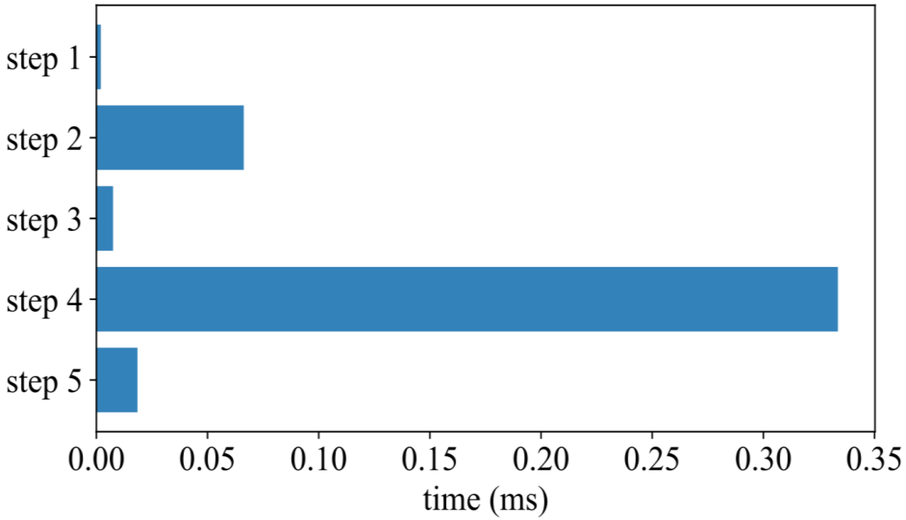
- SNR & Error

mode	mean absolute error (Hz)	SNR $_{dB}$
1	0.2989	30.02
2	0.3193	33.38
3	0.5575	33.54
4	9.8136	25.10
5	262.80	13.18

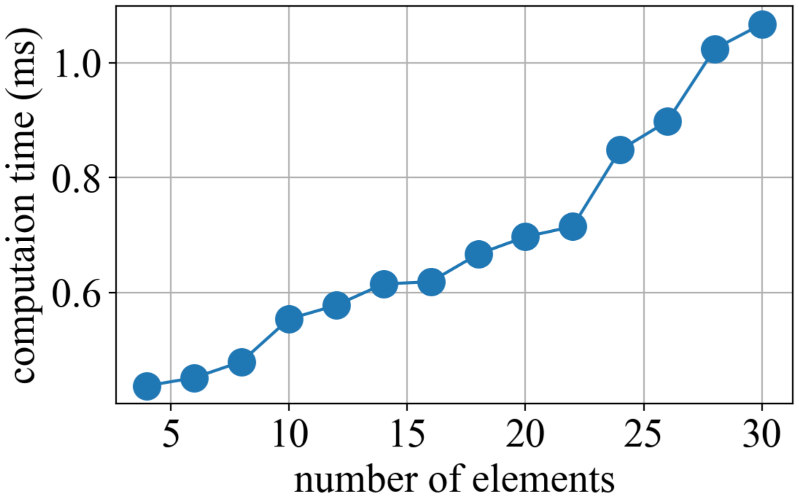




LEMP Algorithm Timing study



Timing for each step



Timing with element number 4 to 30.

- Increasing the nodes increase the accuracy of the model
- <29 nodes achieves the 1ms times constraint

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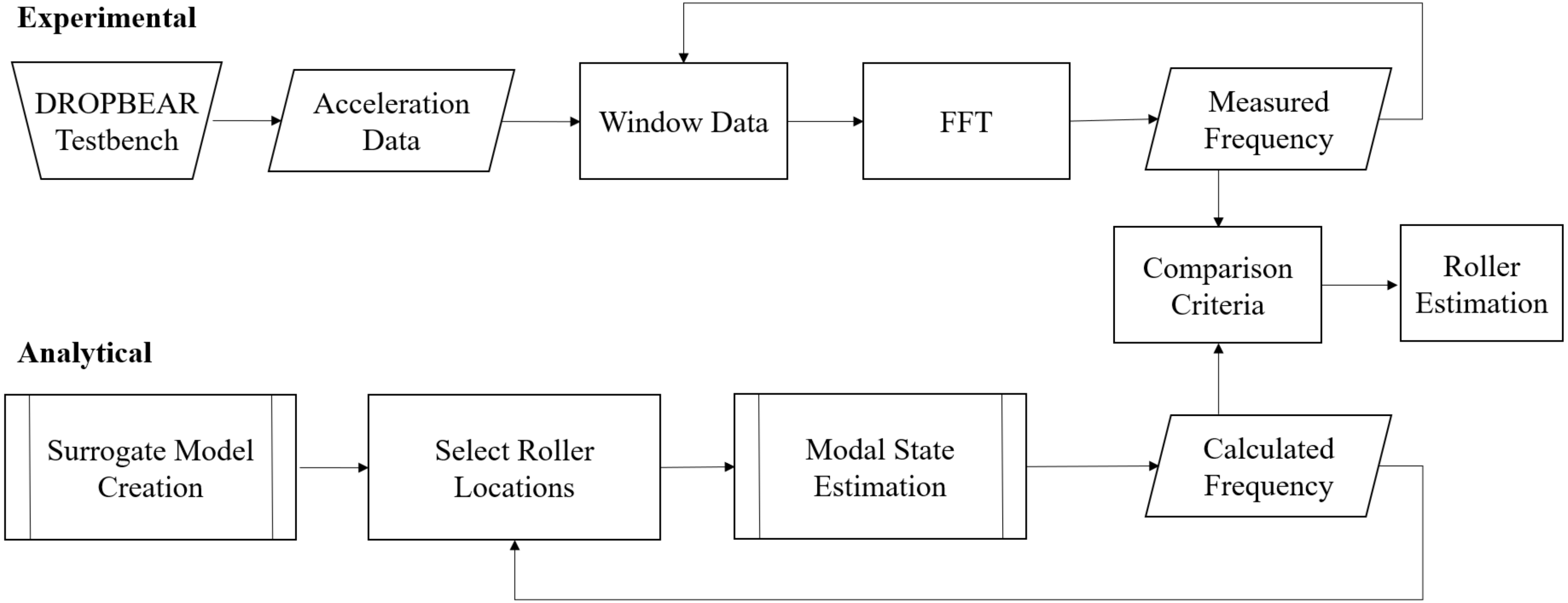
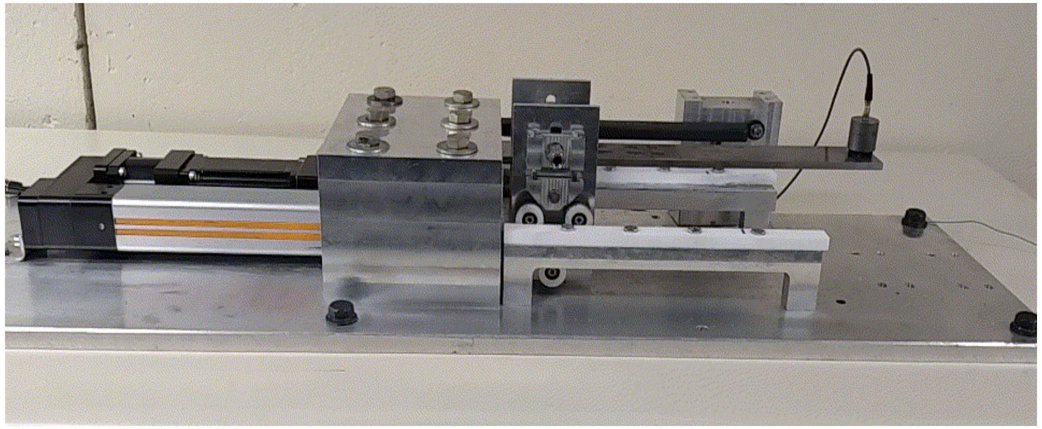
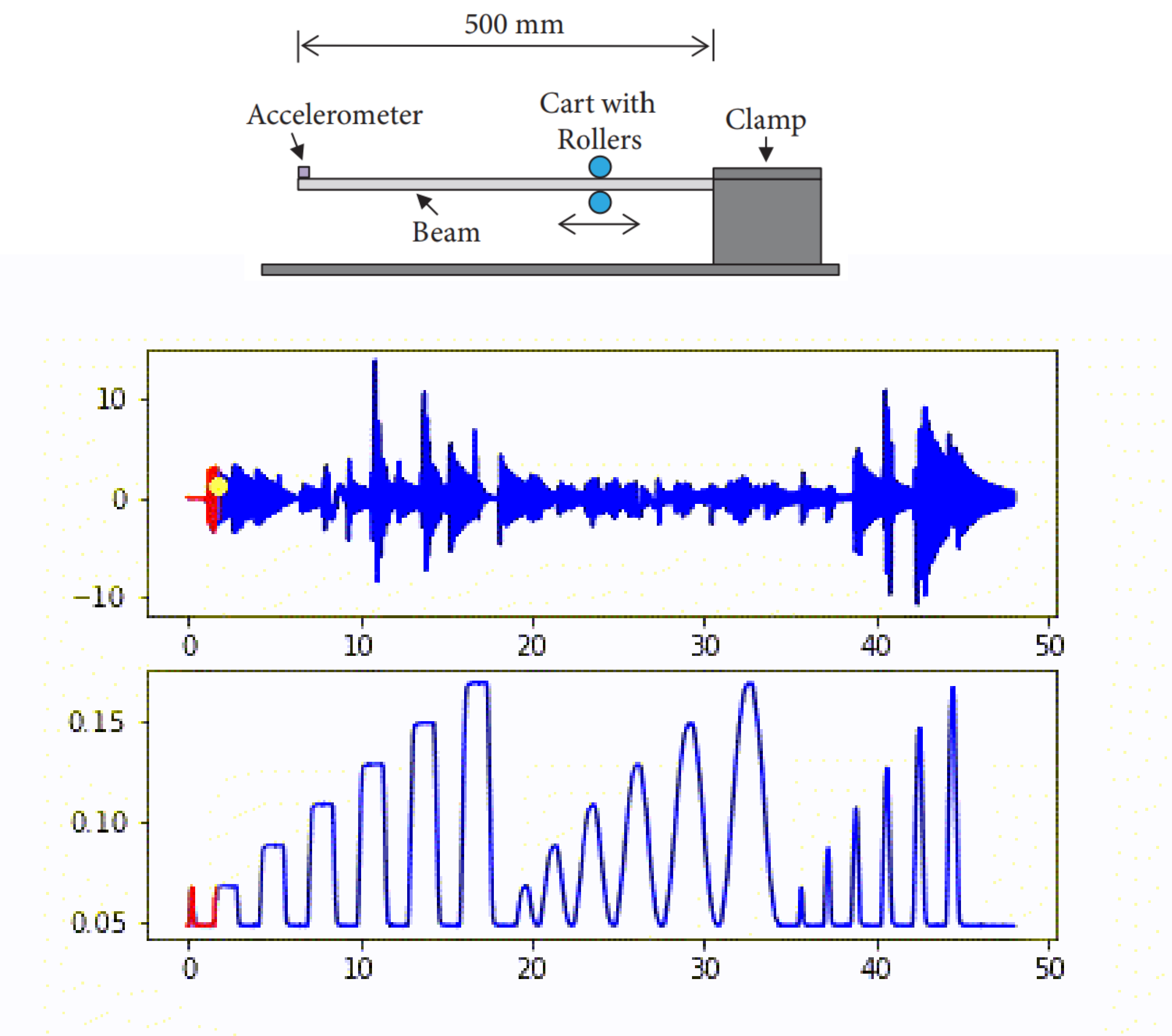
Model Updating Process



Experimental System used for Validation

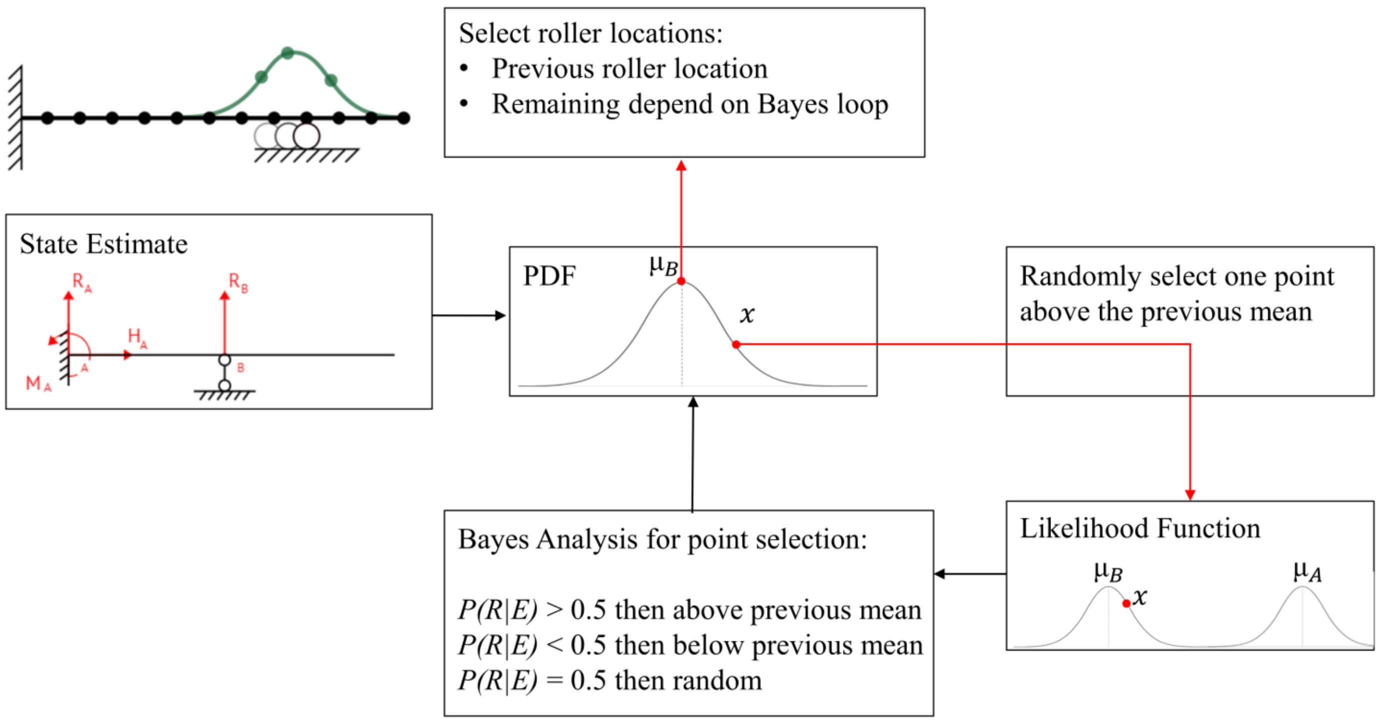
- The Dynamic Reproduction of Projectiles in Ballistic Environments for Advanced Research (DROPBEAR) was used to generate the experimental data in this work.





Joyce, B., Dodson, J., Laflamme, S., & Hong, J. *An experimental test bed for developing high-rate structural health monitoring methods*. Shock and Vibration, 2018.

Analytical: Probabilistic Roller Location Selection (Sampling)



Comparison Criteria

selected roller locations

$$X = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{bmatrix}$$

Error Minimization

min

$$Y = \begin{bmatrix} \omega_1 - \omega_{\text{true}} \\ \omega_2 - \omega_{\text{true}} \\ \omega_3 - \omega_{\text{true}} \end{bmatrix}$$

Least square regression

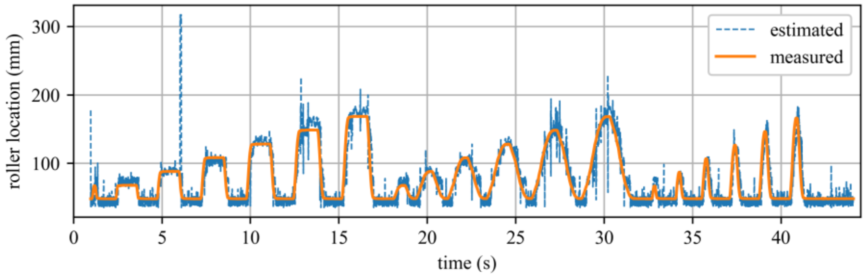
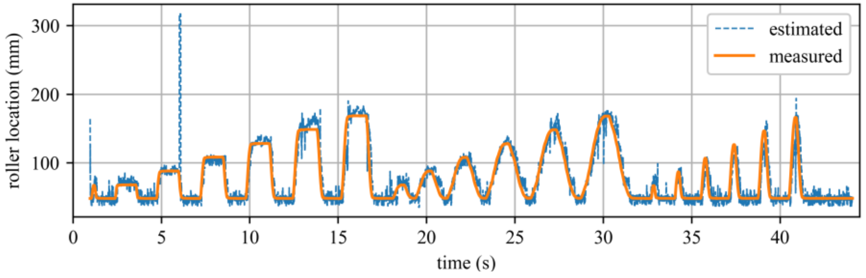
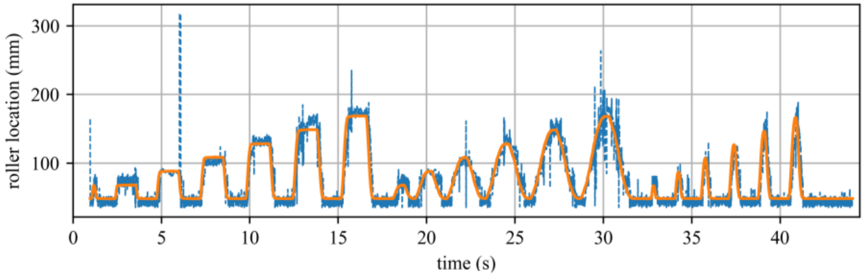
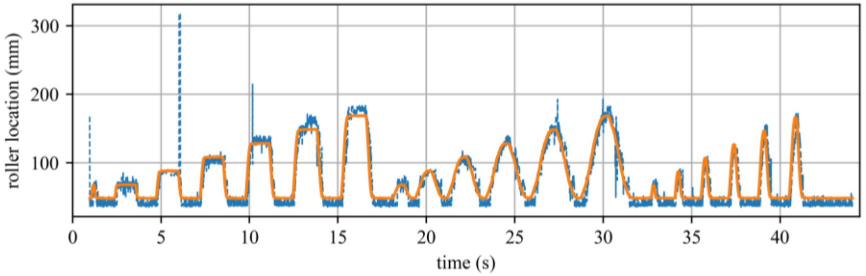
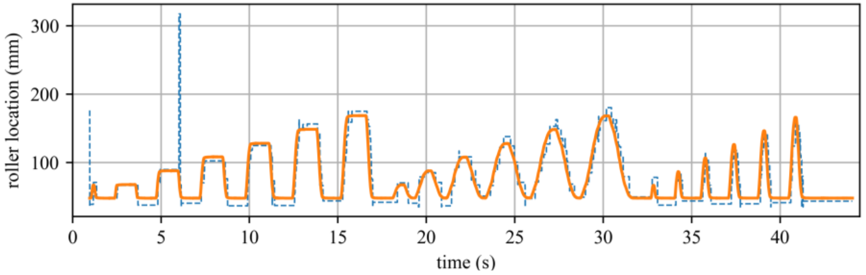
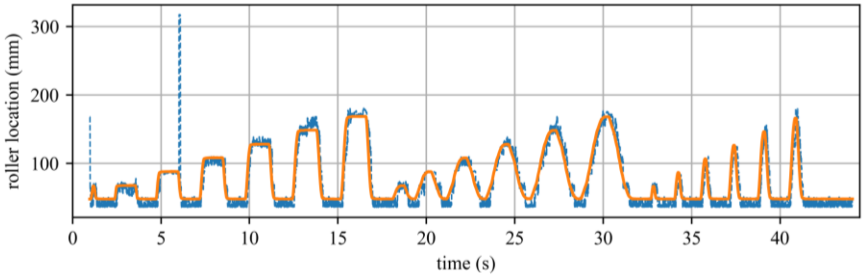
$$\begin{bmatrix} a \\ b \end{bmatrix} = (X^T X)^{-1} X^T Y$$

$$x_c = \begin{cases} x_{\min} & -{}^{b}/_{a} < x_{\min} \\ x_{\max} & -{}^{b}/_{a} > x_{\max} \\ -{}^{b}/_{a} & \text{elsewhere} \end{cases}$$

Where a and b are regression parameters such that $\omega - \omega_{\text{true}} = ax + b$. Therefore, $\omega = \omega_{\text{true}}$ when $x = -{}^{b}/_{a}$.

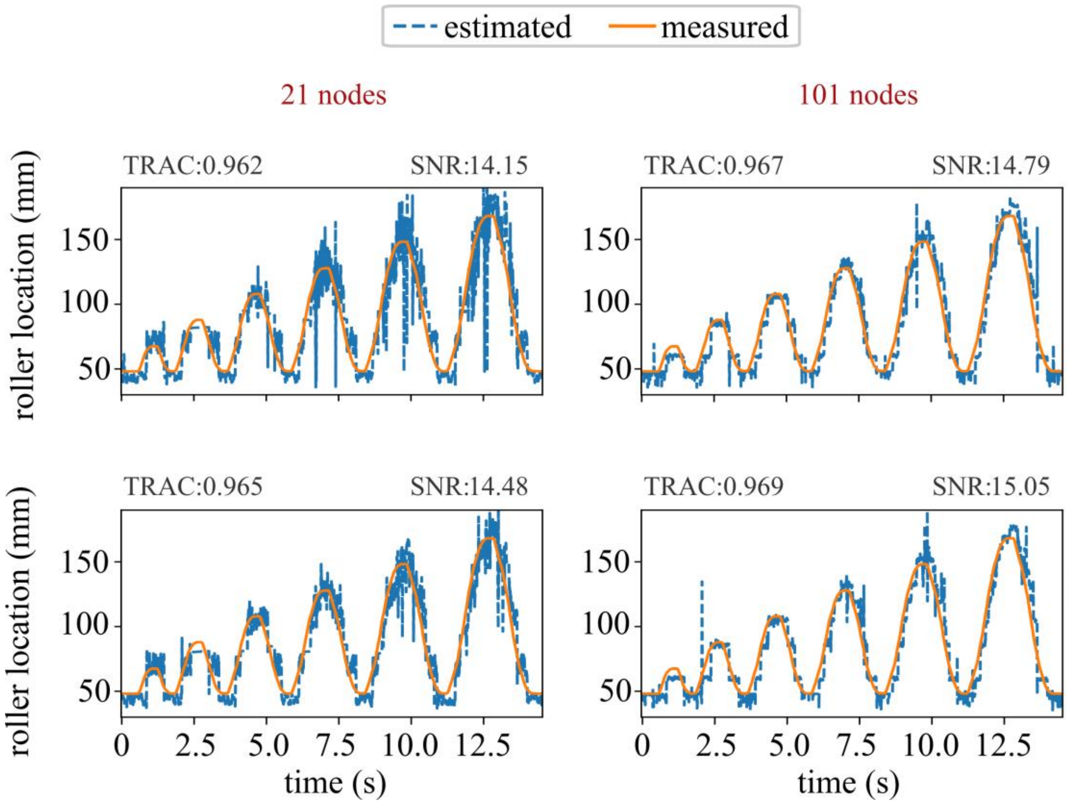
DROPBEAR Roller Location Estimation

21-node beam model

	Error Minimization	Least square regression
Generalized Eigenvalue		
LEMP estimations		
LEMP estimations alongside Bayesian search space.		

DROPBEAR Roller Location Estimation

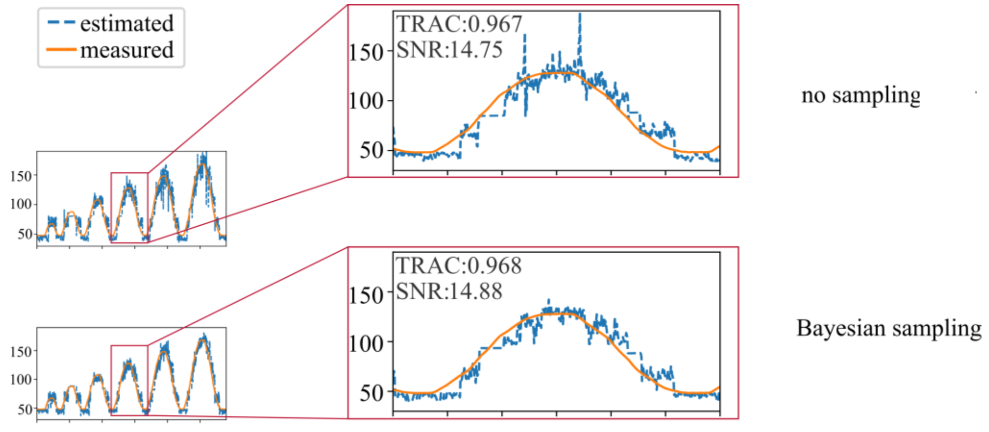
LEMP estimate



Error Minimization

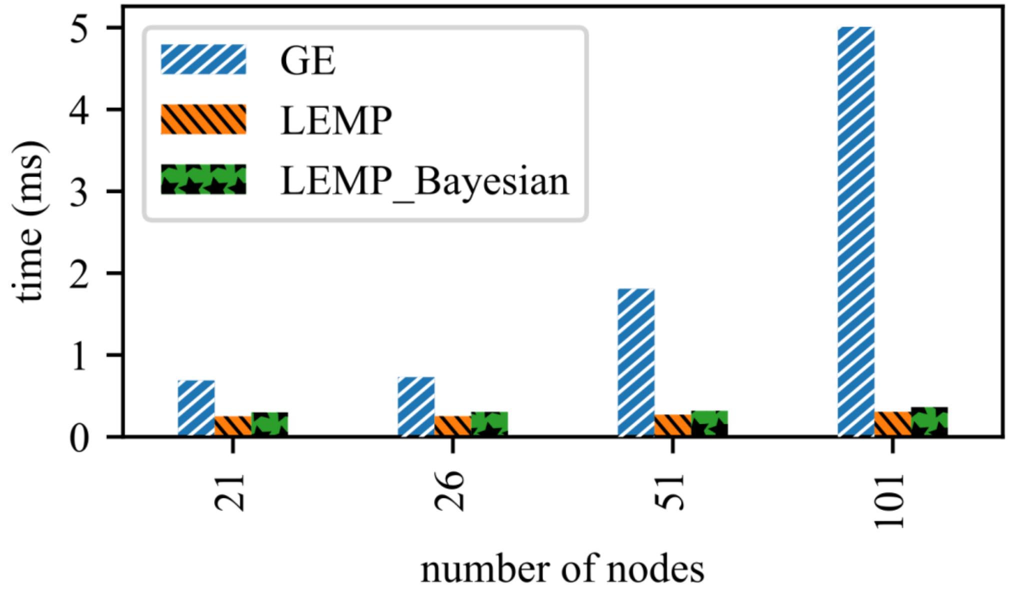
Least square regression

21-node beam model extended view

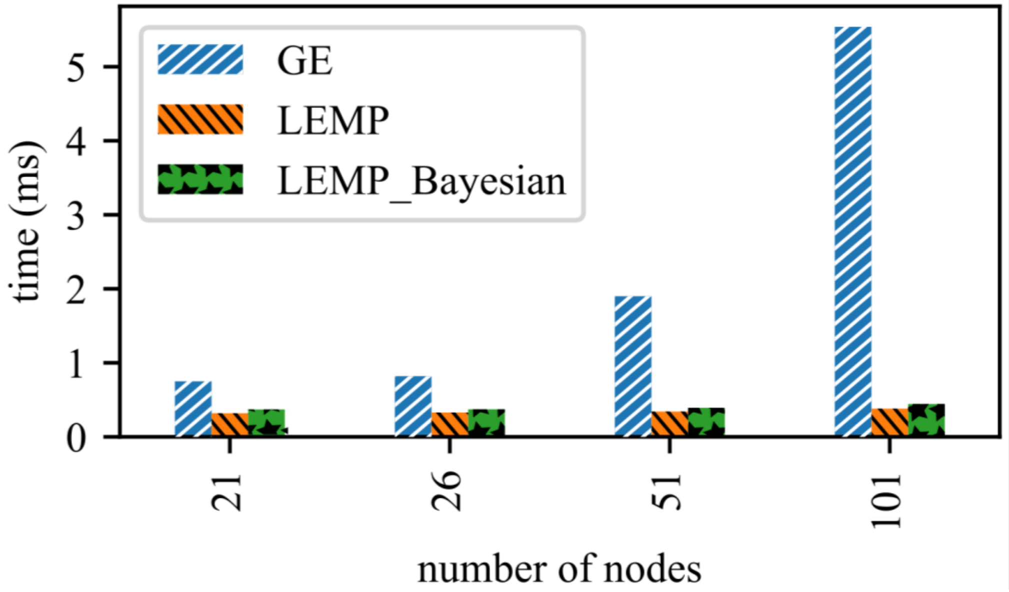


Timing Results

Error Minimization

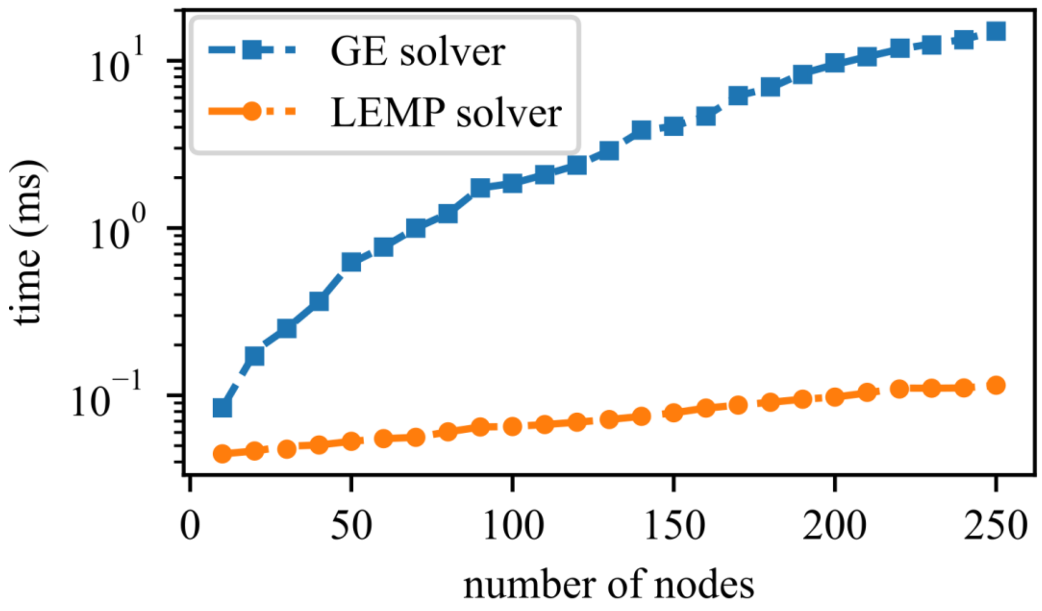


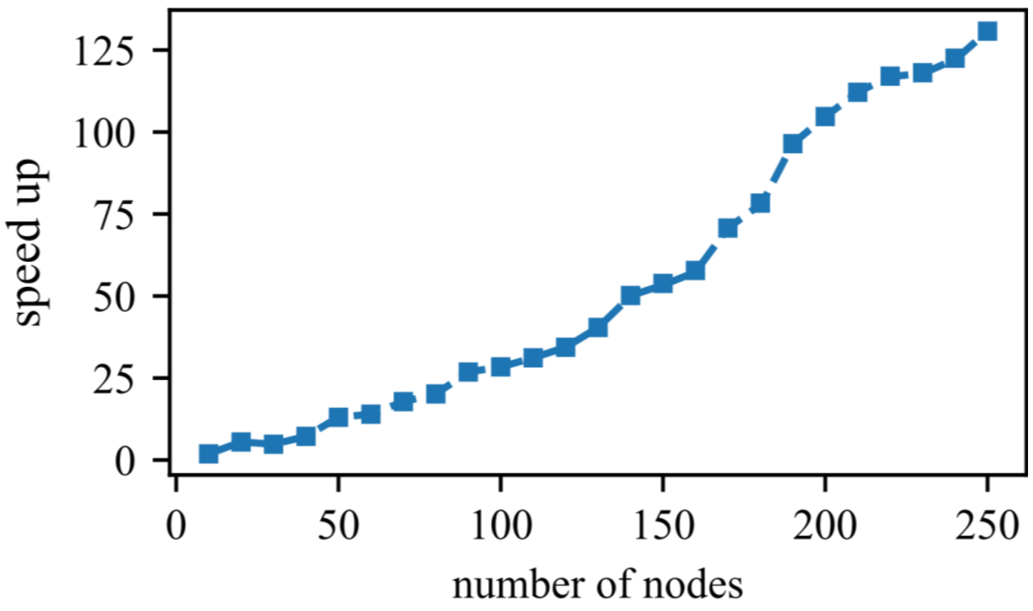
Least square regression



Timing Results

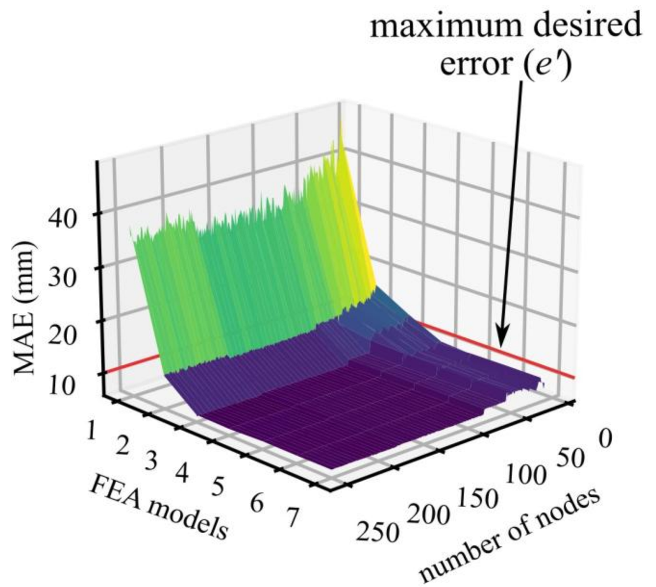
Solver time for the Generalize Eigenvalue and LEMP



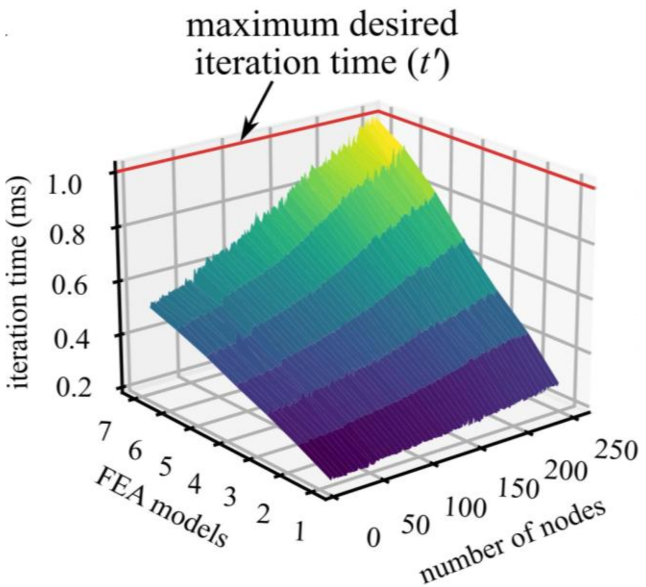


Optimal modal Configuration

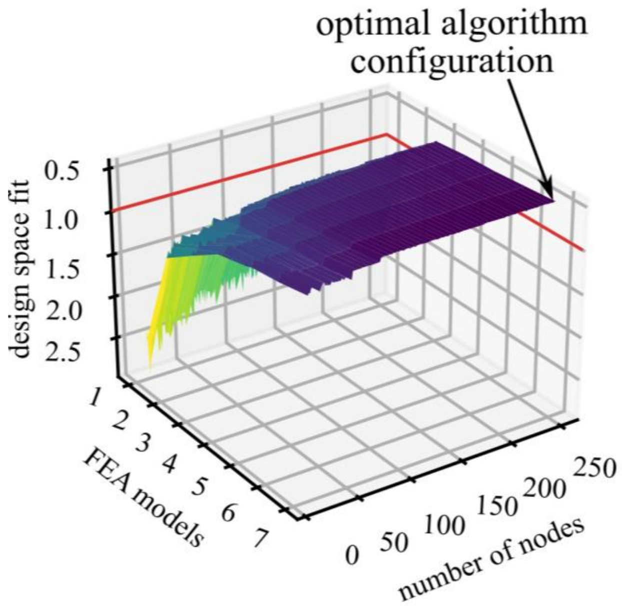
MAE



iteration time



Optimal configuration



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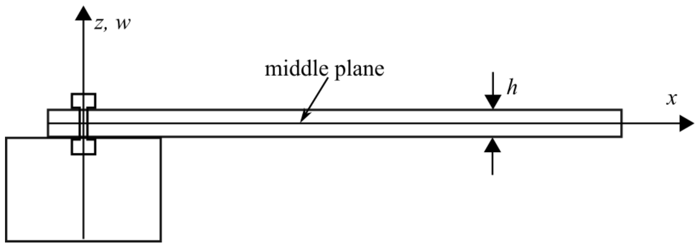
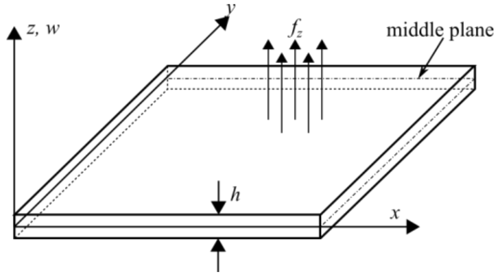
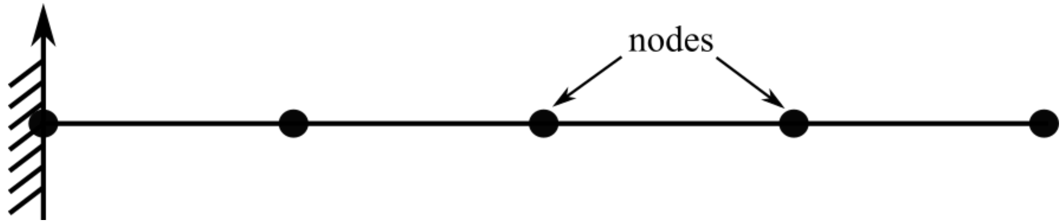
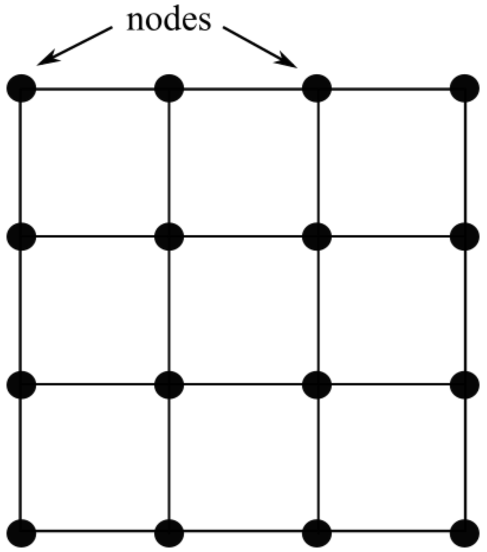
1D vs 2D Node construction



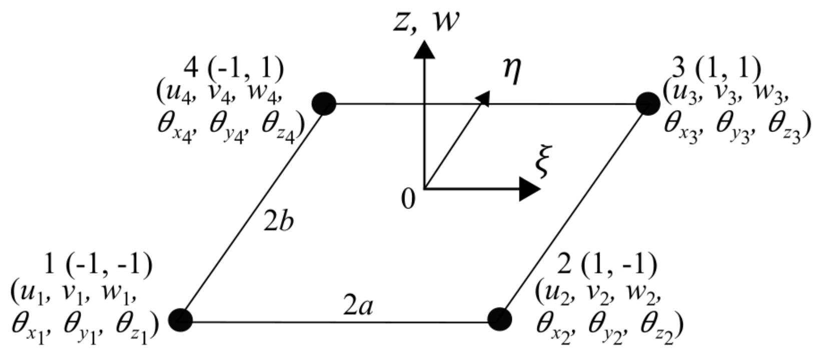






2D Model Formulation

Shell element = solid element + plate element



Three translational displacements in the *x*, *y*, and *z* directions, and three rotational deformations with respect to the *x*, *y*, and *z* axes.

$$\mathbf{d_e} = \begin{array}{l} \left\{ \begin{matrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \mathbf{d}_3 \\ \mathbf{d}_4 \end{matrix} \right\} \end{array} \begin{array}{l} \text{node 1} \\ \text{node 2} \\ \text{node 3} \\ \text{node 4} \end{array}$$

where **d***i* (*i*=1, 2, 3, 4) are the displacement vector at node *i*:

$$\mathbf{d}_i = \left\{ \begin{matrix} u_i \\ v_i \\ w_i \\ \theta_{xi} \\ \theta_{yi} \\ \theta_{zi} \end{matrix} \right\} \begin{array}{l} \text{displacement in } x \text{ direction} \\ \text{displacement in } y \text{ direction} \\ \text{displacement in } z \text{ direction} \\ \text{rotation about } x\text{-axis} \\ \text{rotation about } y\text{-}axis \\ \text{rotation about } z\text{-axis} \end{array}$$

2D Model Formulation

Modeling steps

1. Construction of shape functions matrix **N**

2. Formulation of the strain matrix for **2D element** B, and **2D plate**, B_I and B_O.

3. Calculation of k_e and m_e using shape functions N and strain matrix in step 2.

2D Model Formulation

STEP 1. Construction of shape functions matrix N that satisfies Eqs. 1 and 2

<u>Subscript</u>

e – 2D element
p – 2D plate

2D element

$$\mathbf{N}_e = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \qquad (1)$$

2D plate

$$\mathbf{N}_p = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 \end{bmatrix} \qquad (2)$$

2D Model Formulation

STEP 2. Formulation of the strain matrix for 2D element B, Eq. 3 and 2D plate, B_I and B_O shown in Eqs. 4 and 5.

2D element

$$\mathbf{B} = \mathbf{LN} = \frac{1}{4}\begin{bmatrix} -\frac{1-\eta}{a} & 0 & \frac{1-\eta}{a} & 0 & \frac{1+\eta}{a} & 0 & -\frac{1+\eta}{a} & 0 \\ 0 & -\frac{1-\xi}{b} & 0 & -\frac{1+\xi}{b} & 0 & \frac{1+\xi}{b} & 0 & \frac{1-\xi}{b} \\ -\frac{1-\xi}{b} & -\frac{1-\eta}{a} & -\frac{1+\xi}{b} & \frac{1-\eta}{a} & \frac{1+\xi}{b} & \frac{1+\eta}{a} & \frac{1-\xi}{b} & -\frac{1+\eta}{a} \end{bmatrix} \qquad (3)$$

2D plate

$$\mathbf{B}^{\mathrm{I}} = \begin{bmatrix} \mathbf{B}_1^{\mathrm{I}} & \mathbf{B}_2^{\mathrm{I}} & \mathbf{B}_3^{\mathrm{I}} & \mathbf{B}_4^{\mathrm{I}} \end{bmatrix}, \qquad \mathbf{B}_{\mathrm{j}}^{\mathrm{I}} = \begin{bmatrix} 0 & 0 & -\partial N_j/\partial x \\ 0 & \partial N_j/\partial x & 0 \\ 0 & \partial N_j/\partial y & -\partial N_j \partial y \end{bmatrix} \qquad (4)$$

$$\mathbf{B}^{\mathrm{O}} = \begin{bmatrix} \mathbf{B}_1^{\mathrm{O}} & \mathbf{B}_2^{\mathrm{O}} & \mathbf{B}_3^{0} & \mathbf{B}_4^{0} \end{bmatrix}, \qquad \mathbf{B}_{\mathrm{j}}^{\mathrm{O}} = \begin{bmatrix} \partial N_j/\partial x & 0 & N_j \\ \partial N_j/\partial y & -N_j & 0 \end{bmatrix} \qquad (5)$$

2D Model Formulation

STEP 3. Calculation of k_e and m_e using shape functions N and strain matrix in step 2. to obtain Eqs. 6 and 7.

mass matrix

$$\mathbf{m}_{\mathrm{e}} = \int_A h\rho \mathbf{N}^{\mathrm{T}}\mathbf{N}\mathrm{dA}, \qquad \mathbf{m}_{\mathrm{p}} = \int_{A_p} \mathbf{N}^{\mathrm{T}}\mathbf{I}\mathbf{N}\mathrm{dA} \quad (6)$$

$$\mathbf{I} = \begin{bmatrix} \rho h & 0 & 0 \\ 0 & \rho h^3/12 & 0 \\ 0 & 0 & \rho h^3/12 \end{bmatrix}$$

stiffness matrix

$$\mathbf{k}_{\mathrm{e}} = \int_A h\mathbf{B}^{\mathrm{T}}\mathbf{c}\mathbf{B}\mathrm{dA}, \qquad \mathbf{k}_{\mathrm{p}} = \int_{A_p} \frac{h^3}{12}\left[\mathbf{B}^{\mathrm{I}}\right]^{\mathrm{T}}\mathbf{c}\mathbf{B}^{\mathrm{I}}dA + \int_{A_p} \kappa h\left[\mathbf{B}^{\mathrm{O}}\right]^{\mathrm{T}}\mathbf{c}_s\mathbf{B}^{\mathrm{O}}dA \quad (7)$$

2D Model Formulation

Elements in the global coordinate system

$$\mathbf{K}_e = \mathbf{T}^T \mathbf{k}_e \mathrm{T}$$

$$\mathbf{M}_e = \mathbf{T}^T \mathbf{m}_e \mathbf{T}$$

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 \end{bmatrix}_{24\times 24}$$

T is the transformation matrix

$$\mathbf{T}_3 = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix}_{3\times 3}$$

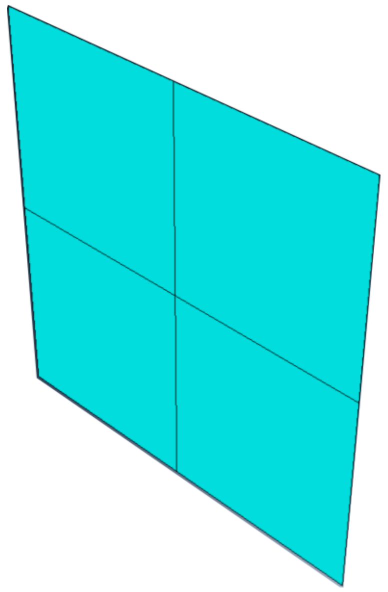
where l_k, m_k and n_k ($k=x, y, z$) are direction cosines

Contents

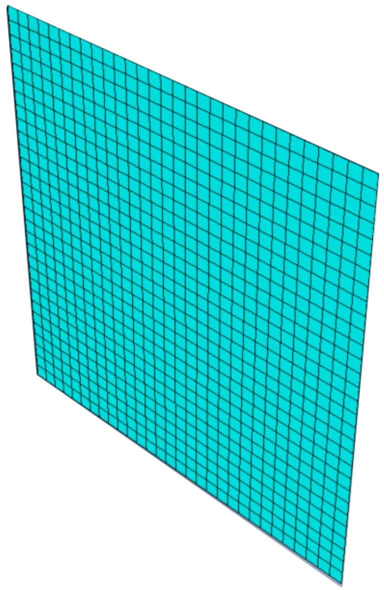
2D Matrices Validation

Type	Poisson's ration	Young's modulus	density	length	width	thickness
Steel	0.3	200e9	7700 kg/m3	0.3 m	0.3 m	0.006 m

4 elements – 9 nodes

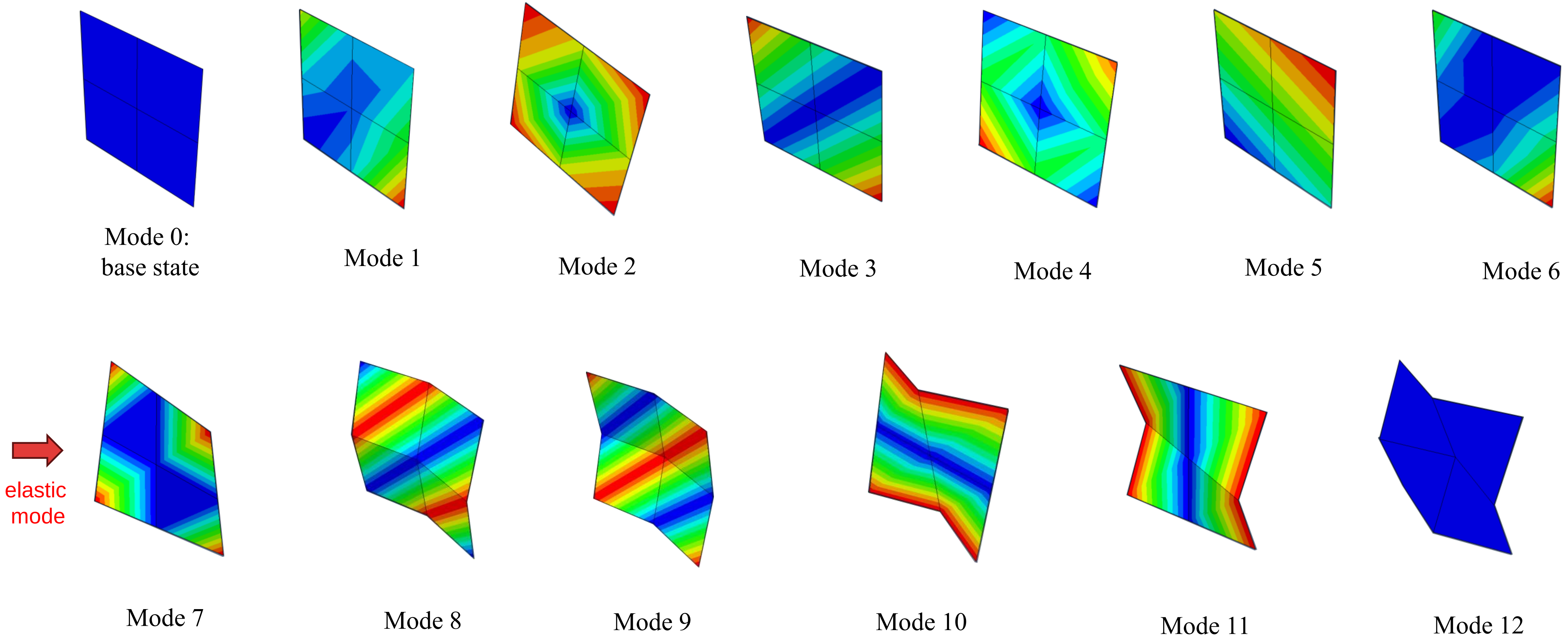


900 elements – 961 nodes

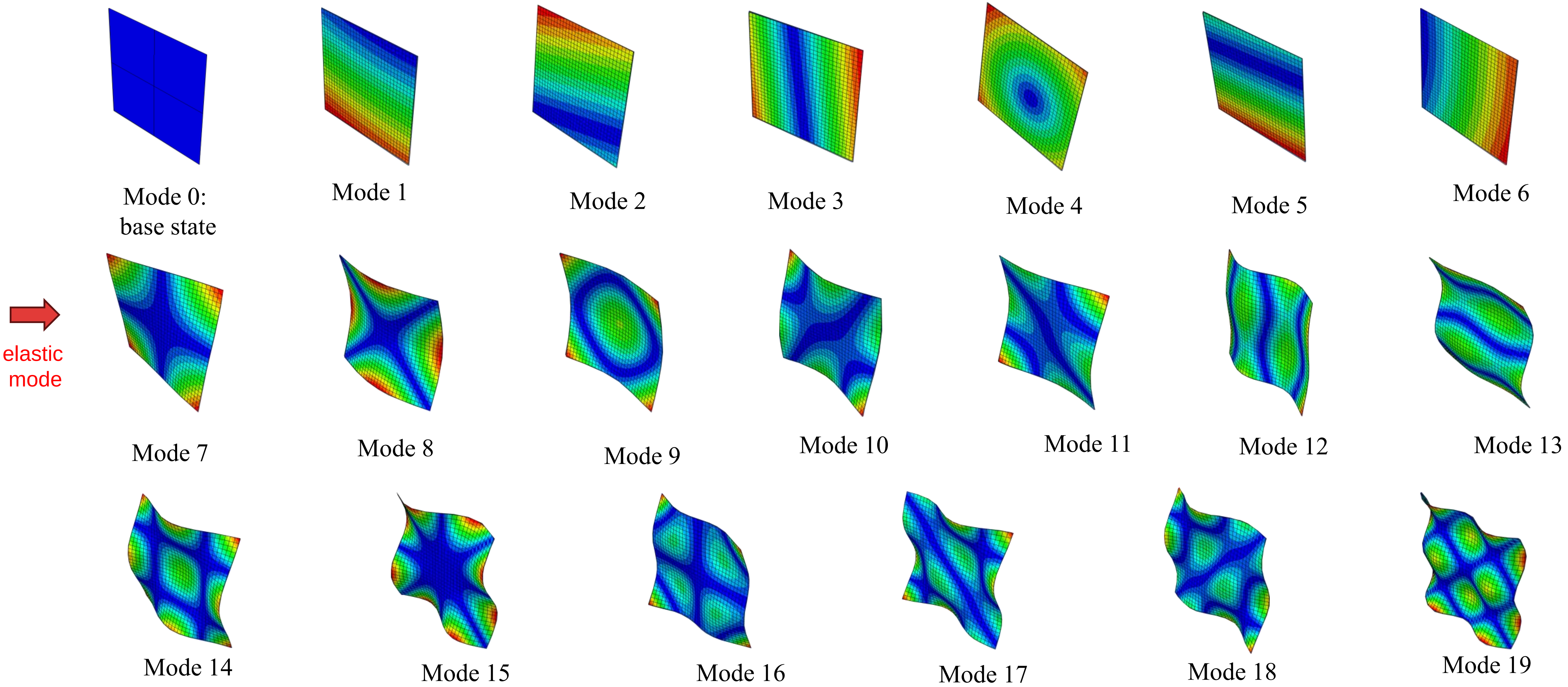


The plate was modeled in a free-free mode

FEA Simulations (4 elements plate)



FEA Simulations (900 elements plate)



Modal frequencies

Step/Frame

Step Name	Description
Step-1	

4 elements

Frame

Index	Description
0	Increment 0: Base State
1	Mode 1: Value = -3.19909E-07 Freq = 0.0000 (cycles/time)
2	Mode 2: Value = -2.69152E-07 Freq = 0.0000 (cycles/time)
3	Mode 3: Value = -1.24332E-07 Freq = 0.0000 (cycles/time)
4	Mode 4: Value = -8.33534E-08 Freq = 0.0000 (cycles/time)
5	Mode 5: Value = -4.33065E-08 Freq = 0.0000 (cycles/time)
6	Mode 6: Value = -3.72529E-09 Freq = 0.0000 (cycles/time)
7	Mode 7: Value = 2.12713E+06 Freq = 232.12 (cycles/time)
8	Mode 8: Value = 5.66377E+06 Freq = 378.77 (cycles/time)
9	Mode 9: Value = 1.05068E+07 Freq = 515.89 (cycles/time)
10	Mode 10: Value = 1.41477E+07 Freq = 598.64 (cycles/time)
11	Mode 11: Value = 1.41477E+07 Freq = 598.64 (cycles/time)
12	Mode 12: Value = 3.52346E+07 Freq = 944.72 (cycles/time)

Step/Frame

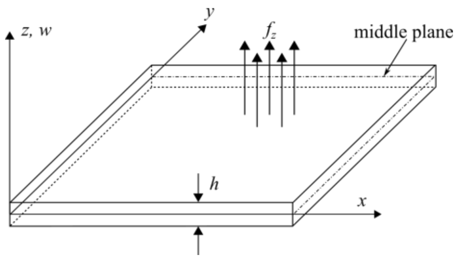
Step Name	Description
Step-1	

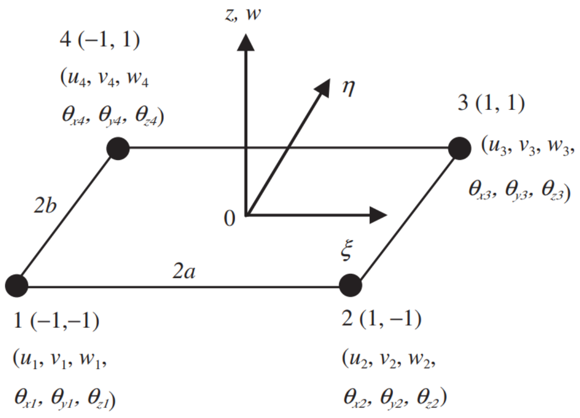
900 elements

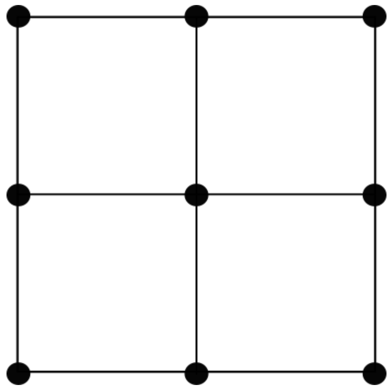
Frame

Index	Description
0	Increment 0: Base State
1	Mode 1: Value = 2.11708E-06 Freq = 2.31573E-04 (cycles/time)
2	Mode 2: Value = 3.40977E-06 Freq = 2.93888E-04 (cycles/time)
3	Mode 3: Value = 5.05996E-06 Freq = 3.58009E-04 (cycles/time)
4	Mode 4: Value = 6.18608E-06 Freq = 3.95847E-04 (cycles/time)
5	Mode 5: Value = 7.60294E-06 Freq = 4.38845E-04 (cycles/time)
6	Mode 6: Value = 1.44800E-05 Freq = 6.05625E-04 (cycles/time)
7	Mode 7: Value = 1.89263E+06 Freq = 218.95 (cycles/time)
8	Mode 8: Value = 4.05830E+06 Freq = 320.62 (cycles/time)
9	Mode 9: Value = 6.23002E+06 Freq = 397.25 (cycles/time)
10	Mode 10: Value = 1.26330E+07 Freq = 565.68 (cycles/time)
11	Mode 11: Value = 1.26330E+07 Freq = 565.68 (cycles/time)
12	Mode 12: Value = 3.95886E+07 Freq = 1001.4 (cycles/time)
13	Mode 13: Value = 3.95886E+07 Freq = 1001.4 (cycles/time)
14	Mode 14: Value = 4.20637E+07 Freq = 1032.2 (cycles/time)
15	Mode 15: Value = 5.01417E+07 Freq = 1127.0 (cycles/time)
16	Mode 16: Value = 6.26389E+07 Freq = 1259.6 (cycles/time)
17	Mode 17: Value = 1.15204E+08 Freq = 1708.3 (cycles/time)
18	Mode 18: Value = 1.15204E+08 Freq = 1708.3 (cycles/time)
19	Mode 19: Value = 1.46137E+08 Freq = 1924.0 (cycles/time)

2D Matrices Validation

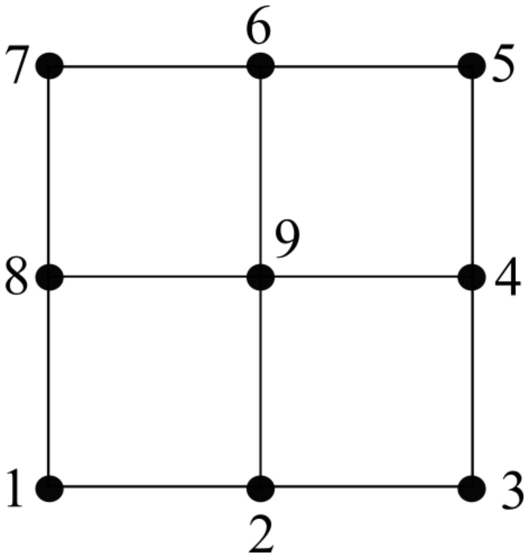


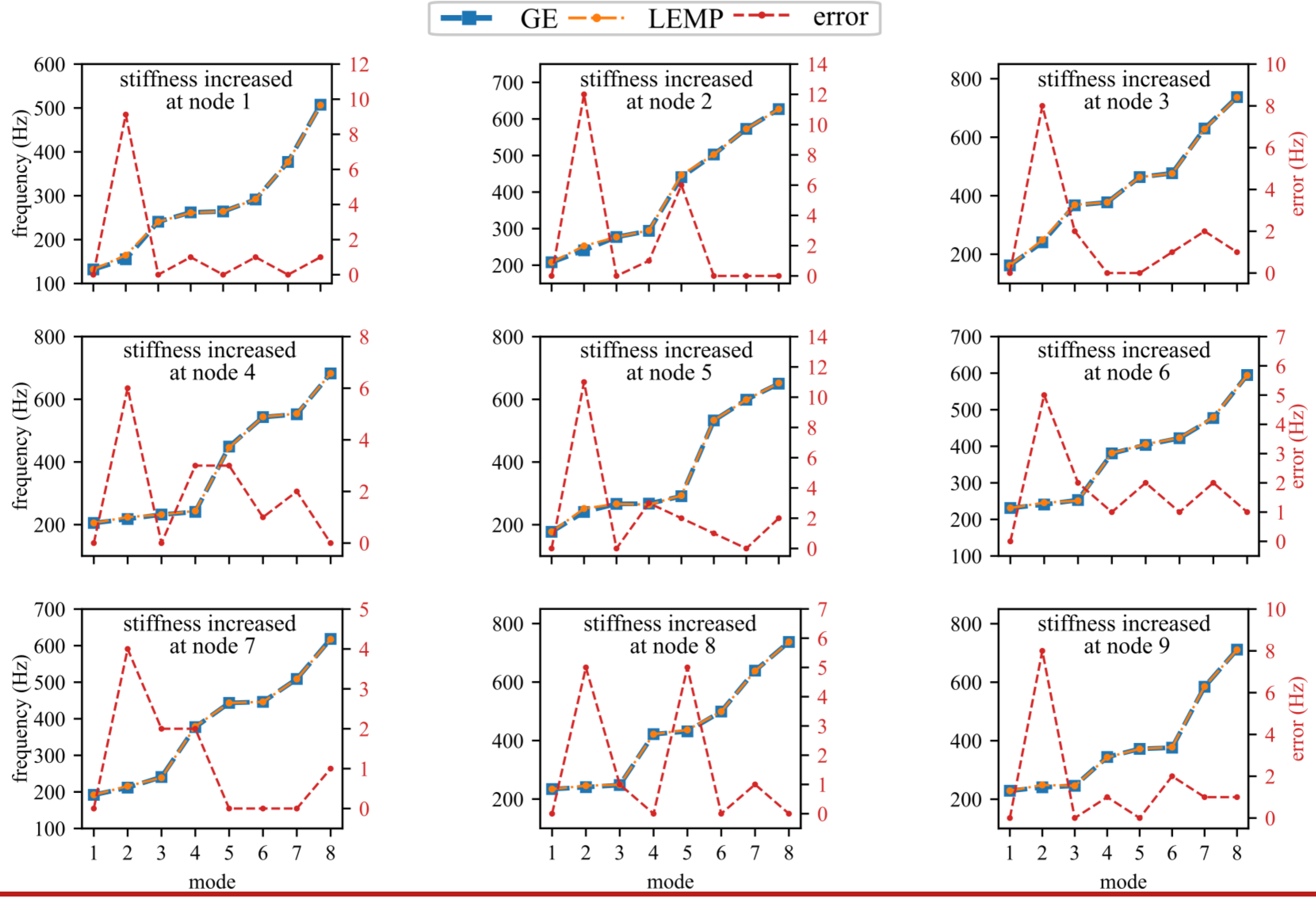




Mode	FEA	GE	Error (abs)
7	232.12	232.027	0.0093
8	378.77	379.044	0.274
9	515.89	515.983	0.0093
10	598.64	598.768	0.128
11	598.64	598.768	0.128
12	944.72	945.03	0.31

Single state change with GE and LEMP





Single state change with GE and LEMP

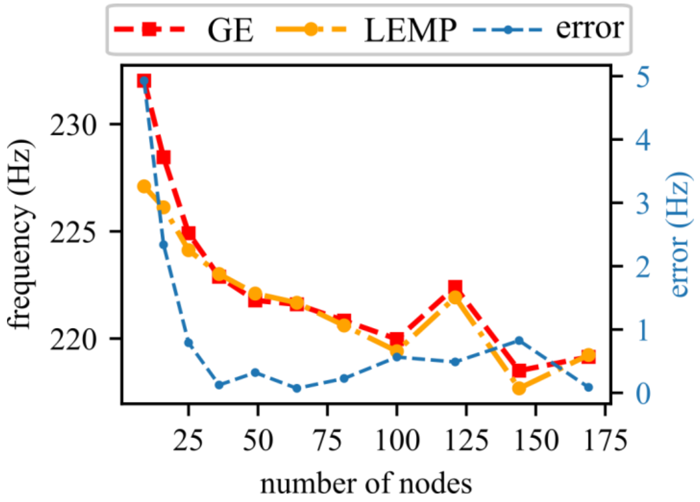
Estimation Timing for GE and LEMP

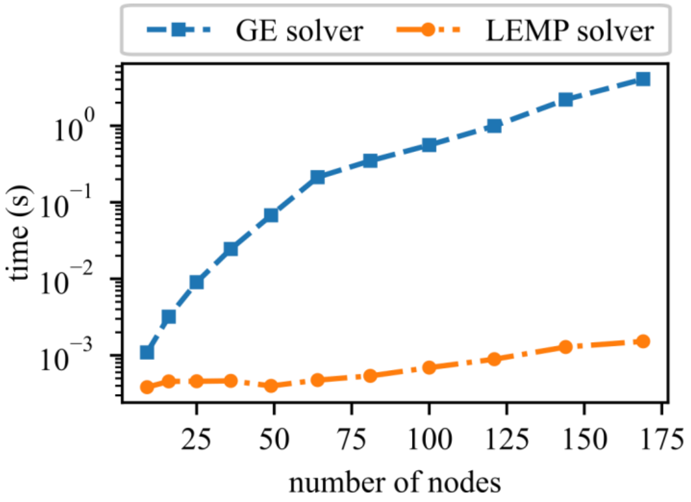
single change calculated using:				generalized eigenvalue		LEMP		
no. of nodes	no. of element	DOF	matrix size	freq (Hz)	time GE (s)	freq (Hz)	time LEMP (s)	error (Hz)
9	4	54	54 x 54	232.027	0.001093	227.099	0.000384	4.928
16	9	96	96 x 96	228.458	0.00320	226.120	0.000456	2.338
25	16	150	150 x 150	224.914	0.009031	224.123	0.000458	0.791
36	25	216	216 x 216	222.886	0.024529	223.01	0.000464	-0.124
49	36	294	294 x 294	221.78	0.067579	222.1	0.000399	-0.32
64	49	384	384 x 384	221.599	0.212773	221.67	0.000475	-0.071
81	64	486	486 x 486	220.837	0.348656	220.610	0.000539	0.227
100	81	600	600 x 600	219.975	0.559744	219.41	0.000691	0.565
121	100	726	726 x 726	222.409	0.994675	221.919	0.000890	0.488
144	121	864	864 x 864	218.505	2.197694	217.68	0.001285	0.825
169	144	1014	1014 x 1014	219.147	4.075451	219.234	0.001523	-0.087

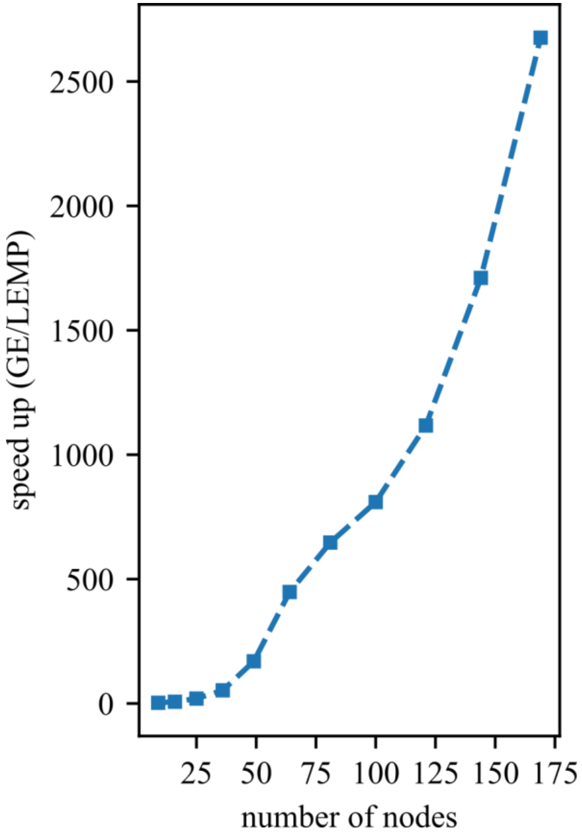
Up to 100 nodes, the LEMP algorithm can still achieve 691 μs while GE is already at 0.56 s.

Single state change with GE and LEMP

Model update accuracy and timing

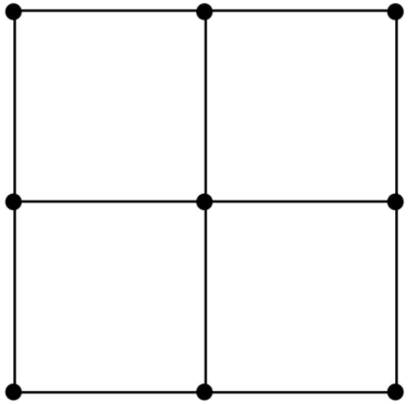




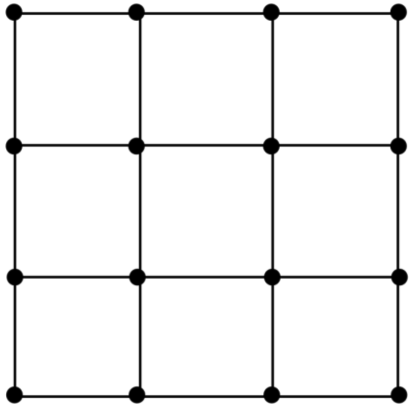


Only the first elastic mode was used for the frequency plot

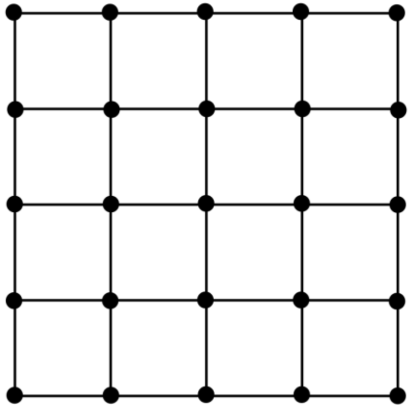
Optimal Reduced model



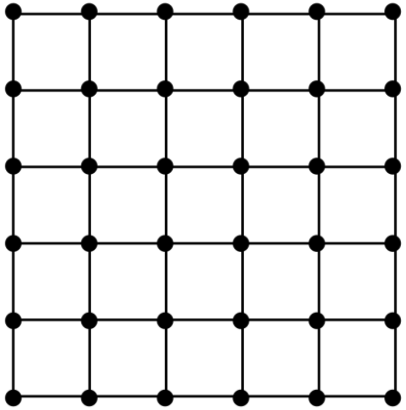
9 nodes



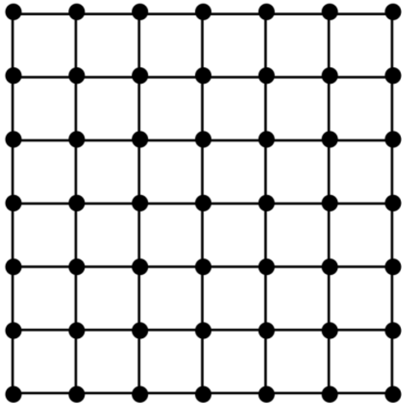
16 nodes



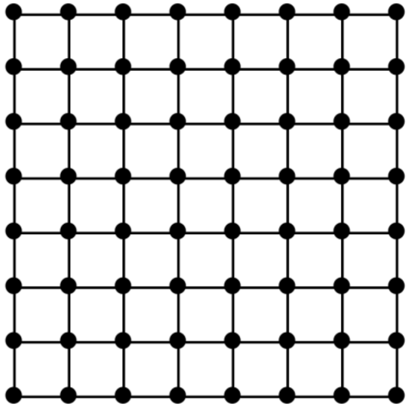
25 nodes



36 nodes



49 nodes

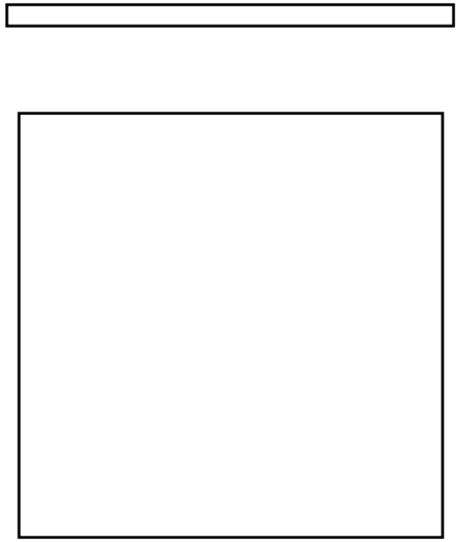


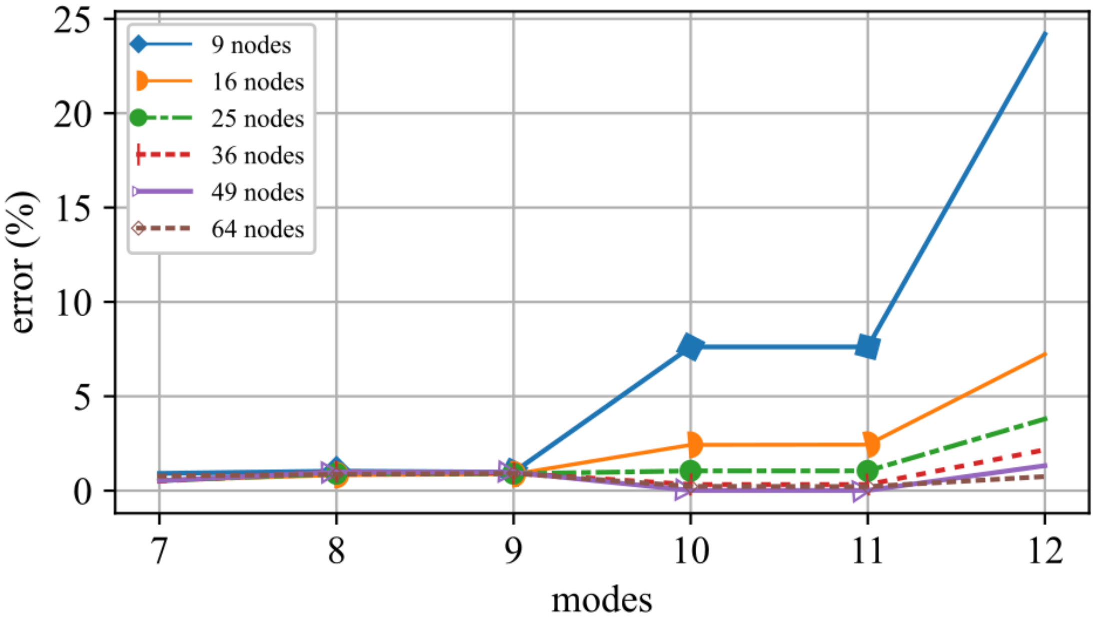
64 nodes

Free plate

Each reduced model are compared to a perfectly meshed free plate

Free plate

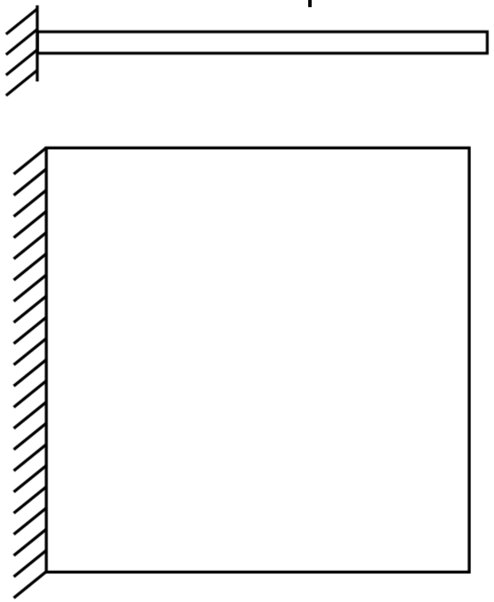


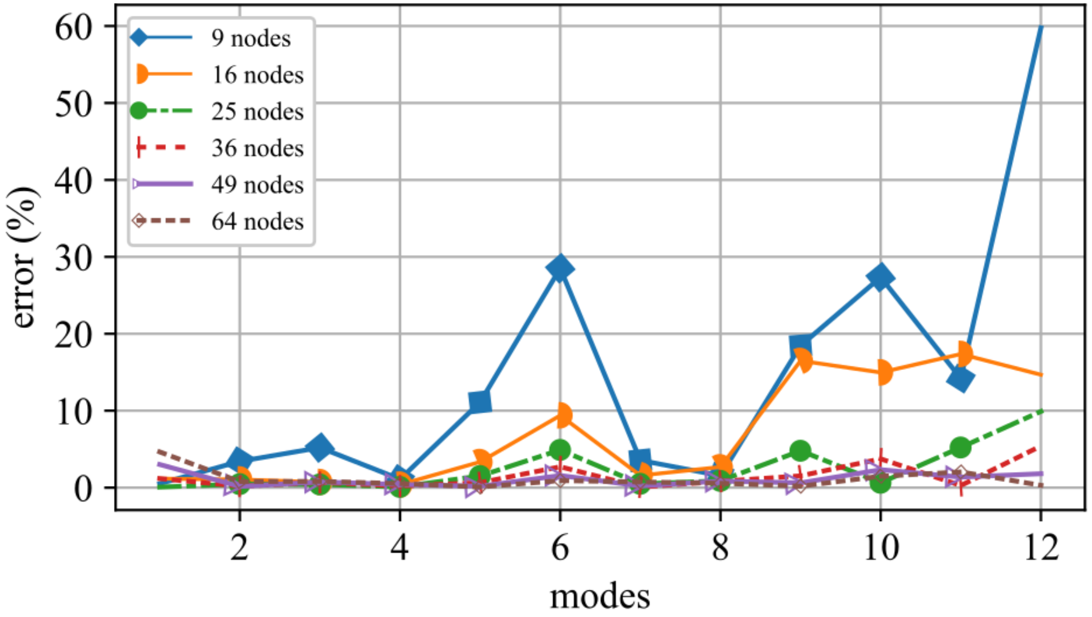


Cantilever plate

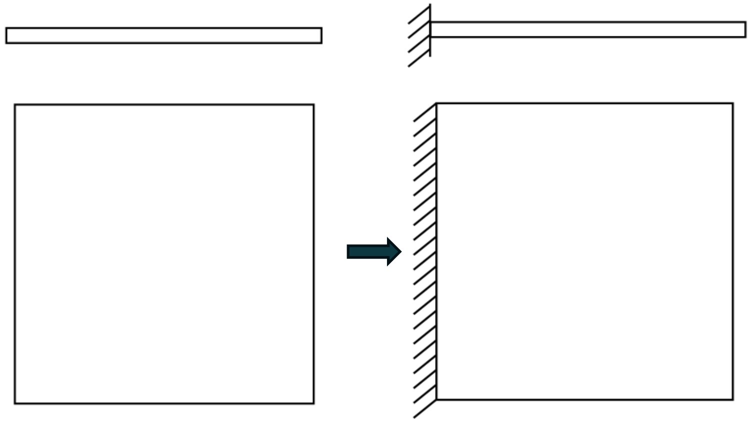
Each reduced model are compared to a perfectly meshed cantilever plate

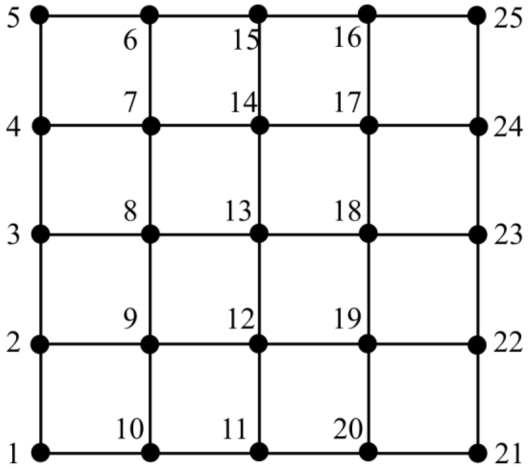
Cantilever plate



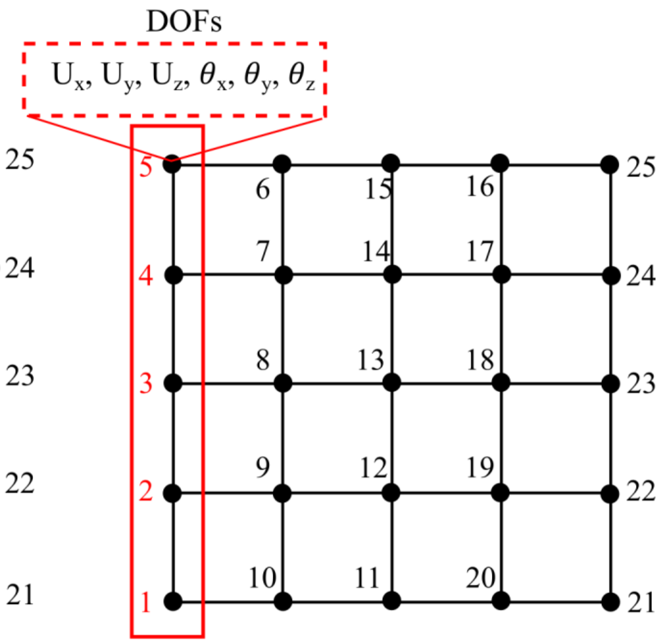


Free Plate to Cantilever Plate



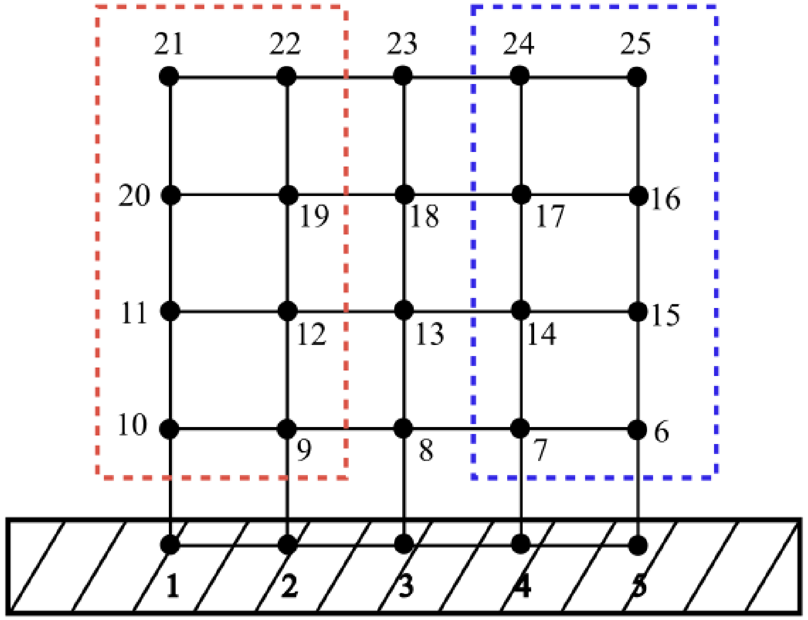


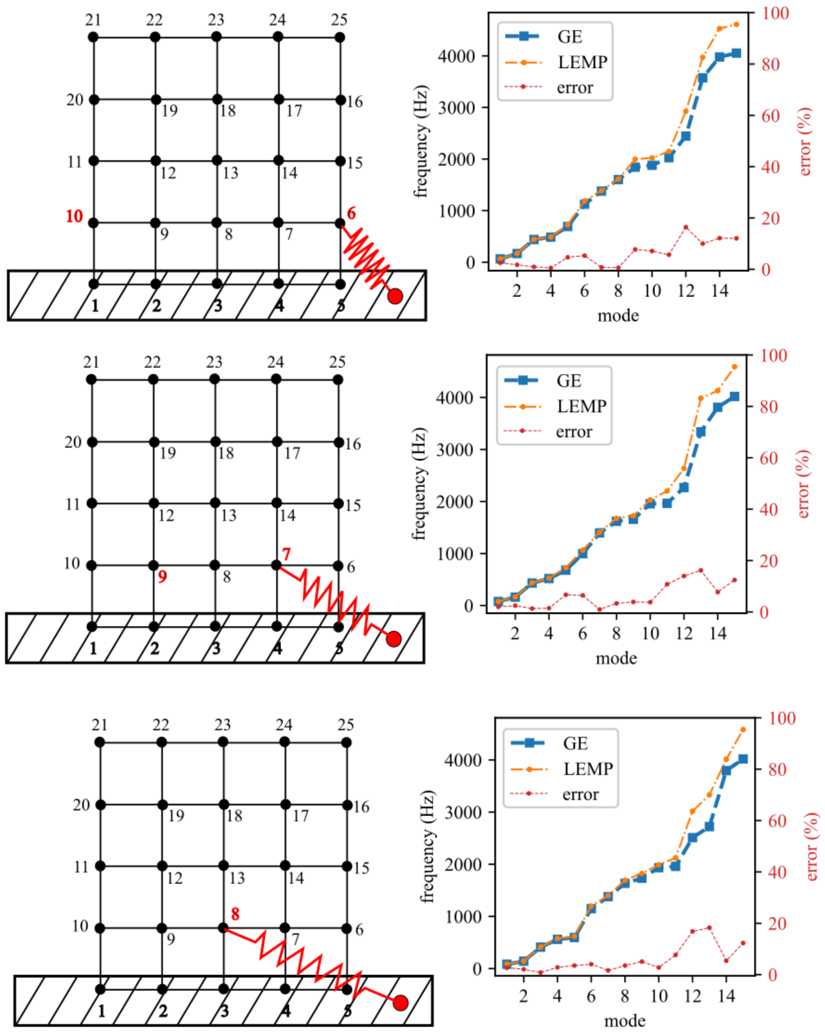
free plate nodal construction



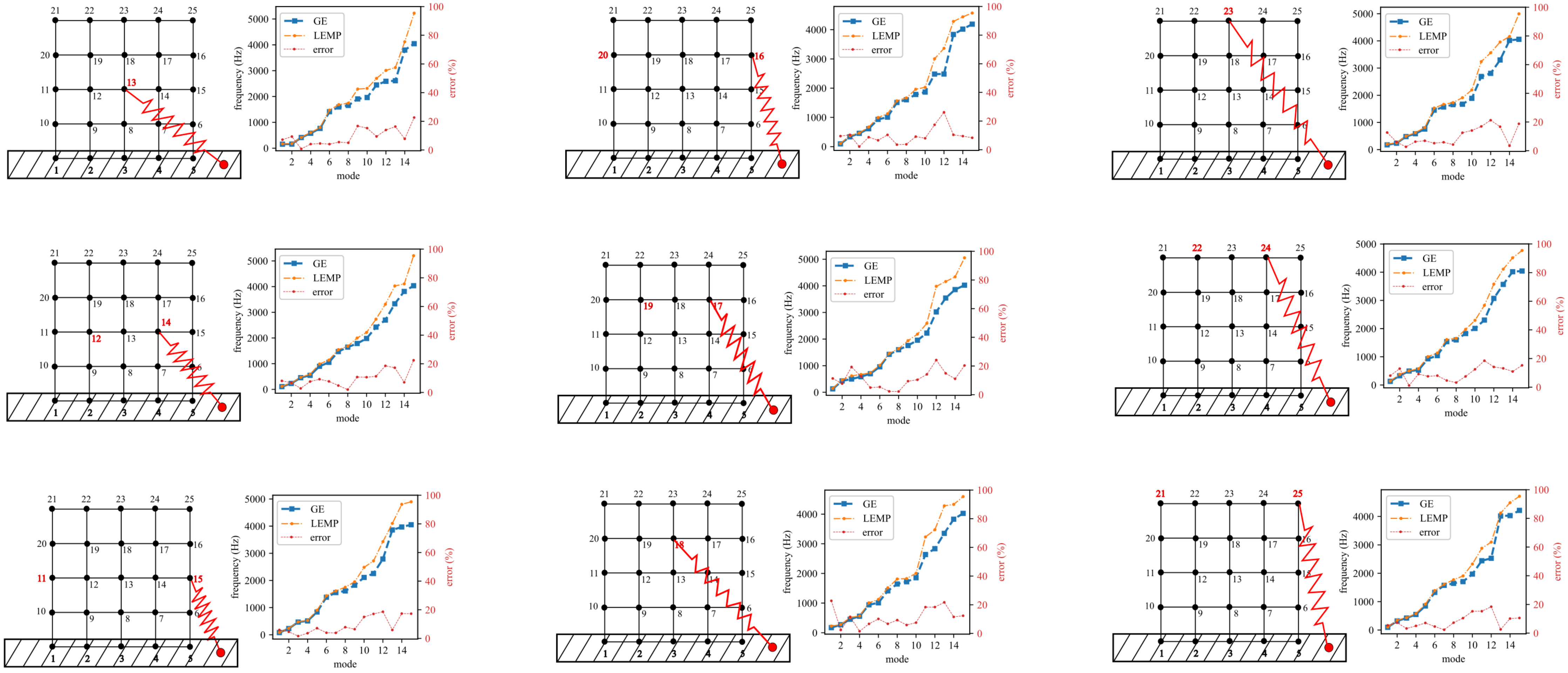
fix nodes 1,2,3,4 and 5

Local change introduction



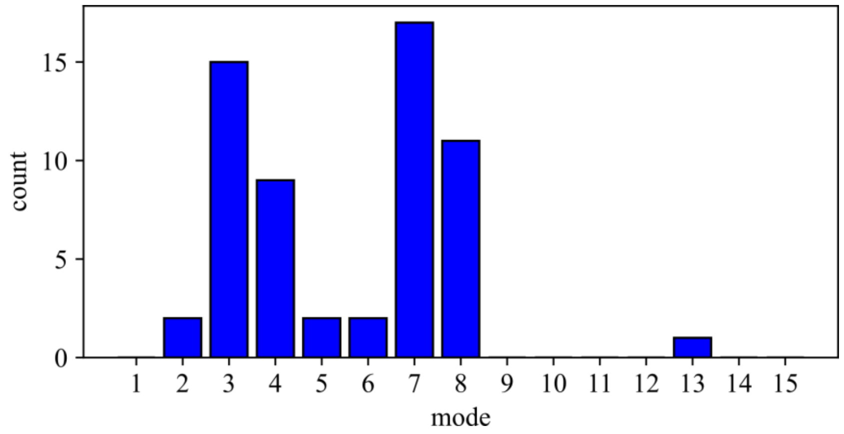


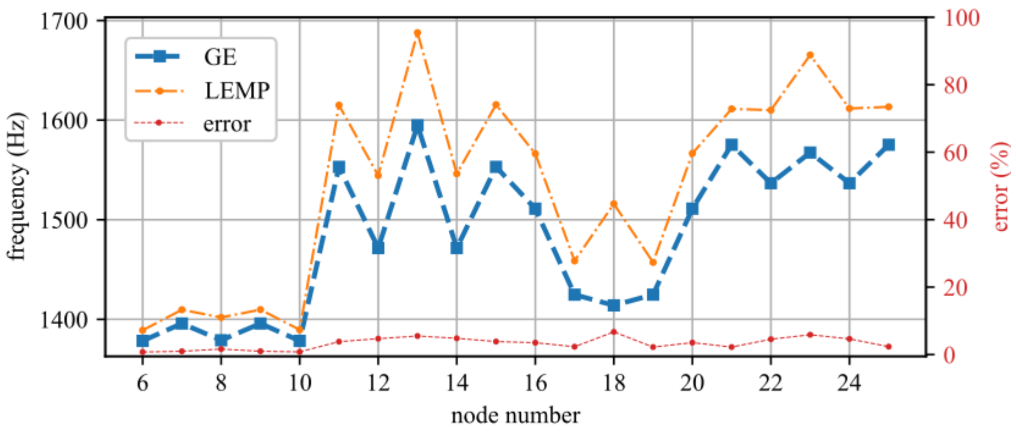
Local change introduction



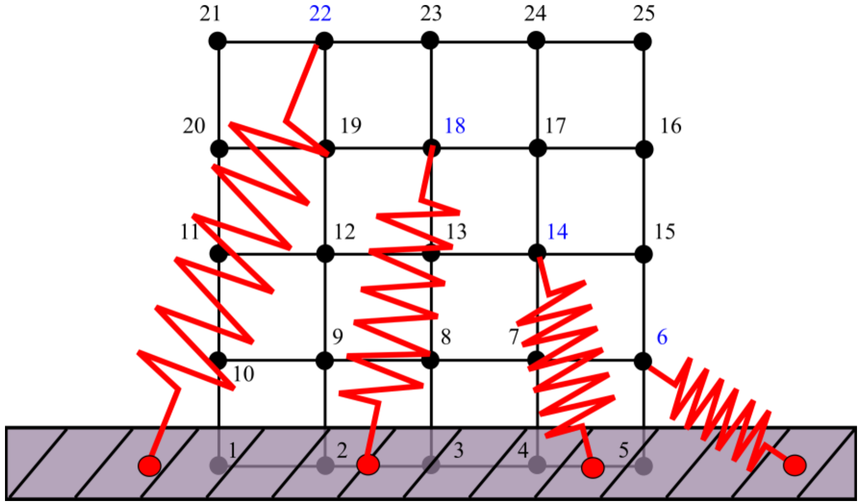
Error at nodes

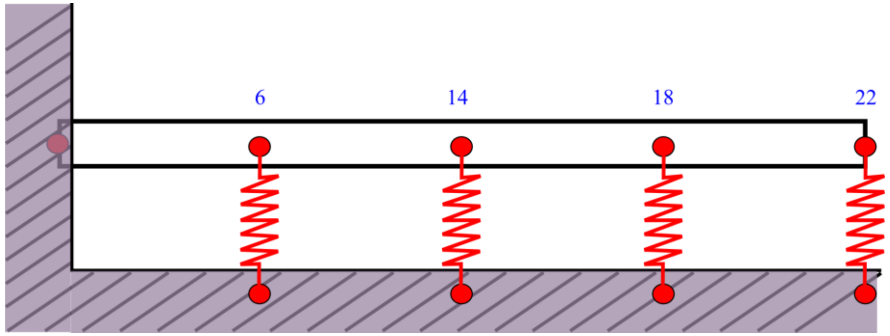
mode	% error at nodes					% error at nodes					% error at nodes					% error at nodes					count
mode	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	count
1	2.532117	1.987089	2.686049	2.039476	2.574719	5.890645	7.965409	7.192803	8.101189	5.912551	9.497408	11.3315	22.76656	11.47762	9.488718	5.442432	8.088518	12.67039	8.049198	5.424116	0
2	1.748493	2.466974	2.074156	2.063405	1.719738	4.619095	7.172105	9.368445	6.82218	4.638922	10.38528	7.715702	2.41712	7.677894	10.38767	7.161676	12.92952	6.074223	12.94797	7.218211	2
3	0.953186	1.316698	0.813959	1.117853	0.946491	1.622843	3.017837	0.802334	2.8354	1.606298	2.176486	19.31973	11.59083	18.66725	2.193322	3.166835	1.183541	2.701019	1.199321	3.237814	15
4	0.521762	1.485406	2.826987	1.614323	0.552273	3.596503	7.637173	4.136615	7.63092	3.629259	8.734441	12.74896	1.519571	12.62904	8.707256	5.018165	9.199682	6.276045	9.173027	5.037879	9
5	4.700076	6.679389	3.521589	6.013716	4.744664	7.170588	9.990671	4.552547	9.260694	7.141813	6.561462	4.950766	6.674359	5.247573	6.607372	7.077355	7.587955	6.927779	7.587955	7.083466	2
6	5.323287	6.451122	4.052768	6.503801	5.323287	3.910083	7.563697	4.103226	7.73393	3.9568	10.35219	5.378282	10.129	5.434294	10.38407	4.592651	8.067761	5.231131	8.051535	4.599511	2
7	0.766019	0.979711	1.636351	0.986735	0.801727	3.834675	4.717042	5.510132	4.827965	3.864438	3.524868	2.327415	6.722739	2.226873	3.543342	2.234841	4.55528	5.883271	4.655953	2.35604	17
8	0.615614	3.333333	3.5367	3.298704	0.708041	7.865297	2.171119	5.033873	1.990343	7.854778	3.76996	2.219439	9.296427	2.279138	3.821924	7.067435	3.274497	4.274147	3.215992	7.256163	11
9	7.734375	3.905734	5.084374	3.62077	7.997004	6.362704	11.00658	16.736	10.7485	6.453861	9.132671	9.428189	5.873433	9.418856	9.002907	10.38084	7.378596	12.53273	7.524065	10.44199	0
10	7.12605	3.78518	2.77063	3.562164	7.038741	15.00322	10.56295	15.25906	10.64363	14.99295	7.980973	10.4069	7.539742	10.28782	8.098754	15.11541	12.33968	14.06895	12.4735	15.21763	0
11	5.682823	10.78792	7.697923	10.6093	5.700386	17.08992	11.37399	9.440886	11.26693	17.09602	17.397	14.18785	18.35887	14.21432	17.44371	15.27005	18.39393	16.90804	18.48926	15.27005	0
12	16.41868	13.98043	16.83436	14.0326	16.50432	18.69787	18.48873	13.96	18.58958	18.63622	26.12778	24.16503	18.38648	24.10023	26.1124	18.29072	14.16208	21.24383	14.14289	18.4094	0
13	10.00126	16.19612	18.27292	16.28644	9.983127	5.86878	17.09173	16.32266	17.25441	5.889439	10.39051	14.89921	21.69847	15.00948	10.35073	2.5435	13.18581	16.70431	13.16257	2.571872	1
14	12.15216	7.737914	5.404227	7.856141	12.17155	17.33568	7.093032	7.79415	7.11567	17.34773	9.475202	11.05434	11.52126	11.05639	9.475202	10.13109	10.91125	3.439656	10.95864	10.1351	0
15	12.0695	12.4493	12.30422	12.37868	12.09239	17.03949	22.45205	22.63504	22.4595	17.26478	8.317759	20.35461	12.34558	20.46007	8.32779	10.62872	15.2532	18.76341	15.25852	10.55855	0





Multiple state change timing with LEMP





Single state change time on 25 node plate

GE	LEMP	speed
9.01 ms	0.43 ms	20 x

Four state change time on 25 node plate

GE	LEMP	speed
36.04 ms	1.62 ms	22 x

Contents

Conclusion

- Developed a real-time structural model updating framework using the Local Eigenvalue Modification Procedure (LEMP) tailored for high-rate dynamic environments.

- Achieved millisecond to microsecond-level latency in structural state estimation, significantly outperforming traditional general eigenvalue-based approach.

- Validated LEMP on both numerical simulations and experimental testbeds (DROPBEAR), demonstrating robust performance under changing boundary conditions and structural modifications.

- Extended LEMP from 1D beam systems to 2D plate structures, showcasing its versatility across domains and scales.

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THANK YOU!

Questions or Comments?

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