MICROSECOND MODEL UPDATING FOR 2D STRUCTURAL SYSTEMS USING THE LOCAL EIGENVALUE MODIFICATION PROCEDURE (LEMP)

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Contents

High-rate Dynamics in the Real-world

Civil Structures Exposed to blast

Samali, B., et al., *Review of the basics of state of the art of blast loading. Asian Journal of Civil Engineering.* (2018).

Automotive safety systems against Collision

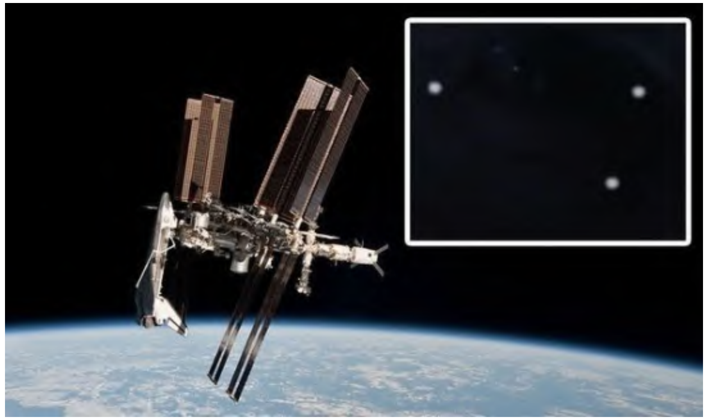
airbag deployment

Space shuttle and Aerial Vehicles Prone to In-Flight Anomalies

Hypersonic vehicles

Ballistic packages

Debris approaching space shuttle

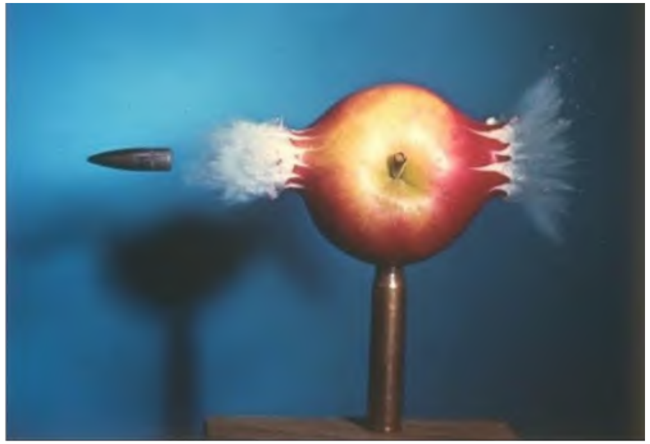


Lightning strikes on aircraft

Fighter jets

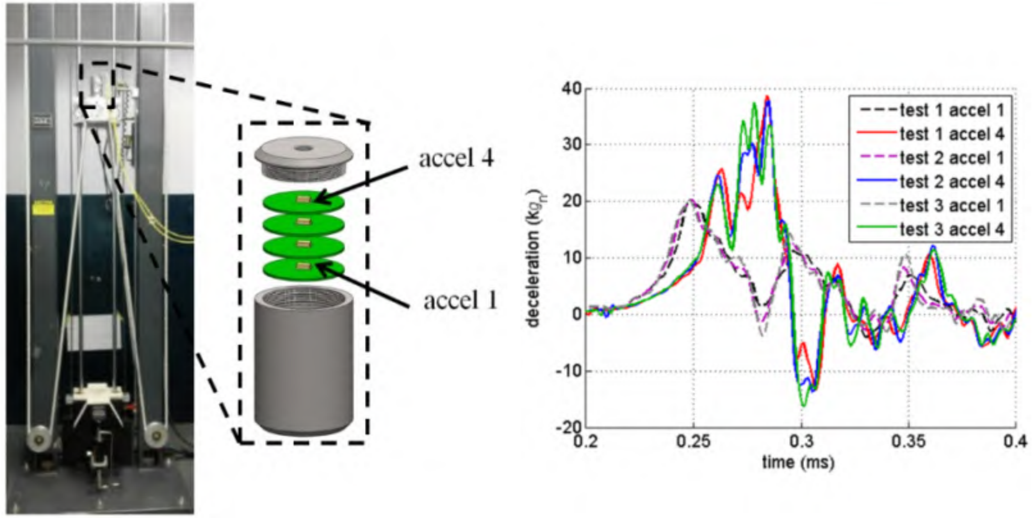
Description of High-rate Dynamics

High-rate (<100ms)



High-amplitude (acceleration > 100 g)

The deceleration event in drop tower tests typically lasts for 0.5ms



- Large uncertainties in the external loads.
- High levels of nonstationarity and heavy disturbance.
- Generations of unmodeled dynamics from changes in mechanical configuration.

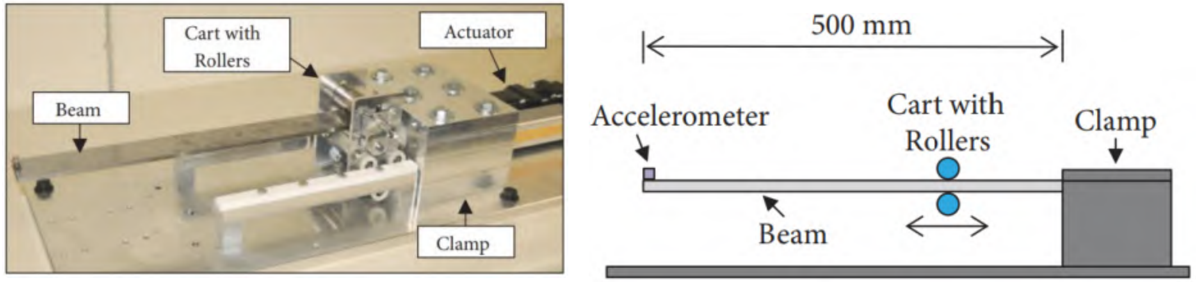
GitHub

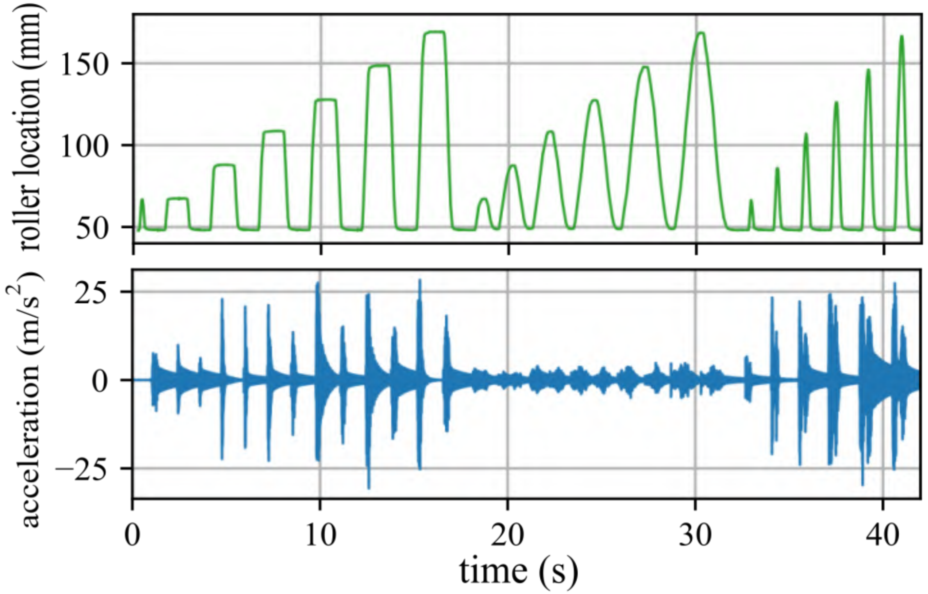
Hong, J. et al,. *Introduction to state estimation of high-rate system dynamics*. Sensors, 18(2):217, Jan 2018

Reproducing High-rate Dynamics in Laboratory

DROPBEAR experimental testbed:

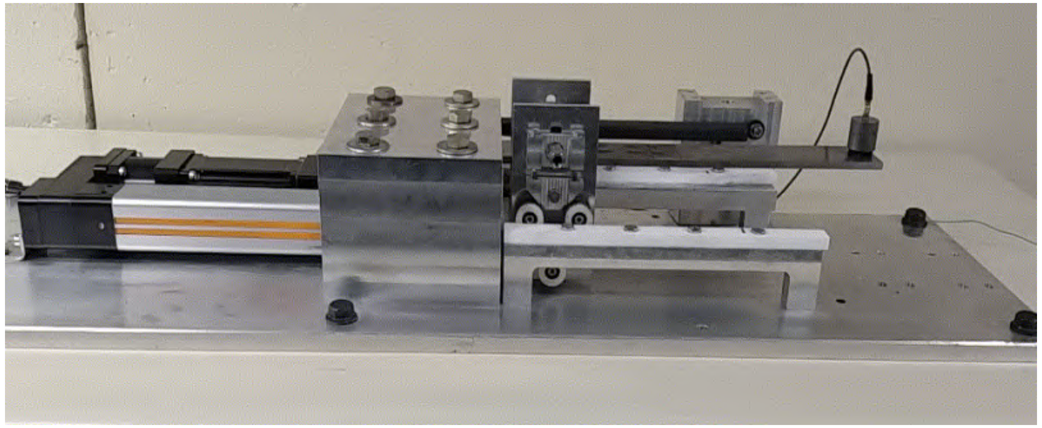
- The Dynamic Reproduction of Projectiles in Ballistic Environments for Advanced Research (DROPBEAR) was used to generate the experimental data in this work.
- Cantilever beam with a controllable roller to alter the state.
- Acceleration and pin location are recorded.
- Dataset available on GitHub at: https://github.com/High-Rate-SHM-Working-Group/Dataset-2-DROPBEAR-Acceleration-vs-Roller-Displacement

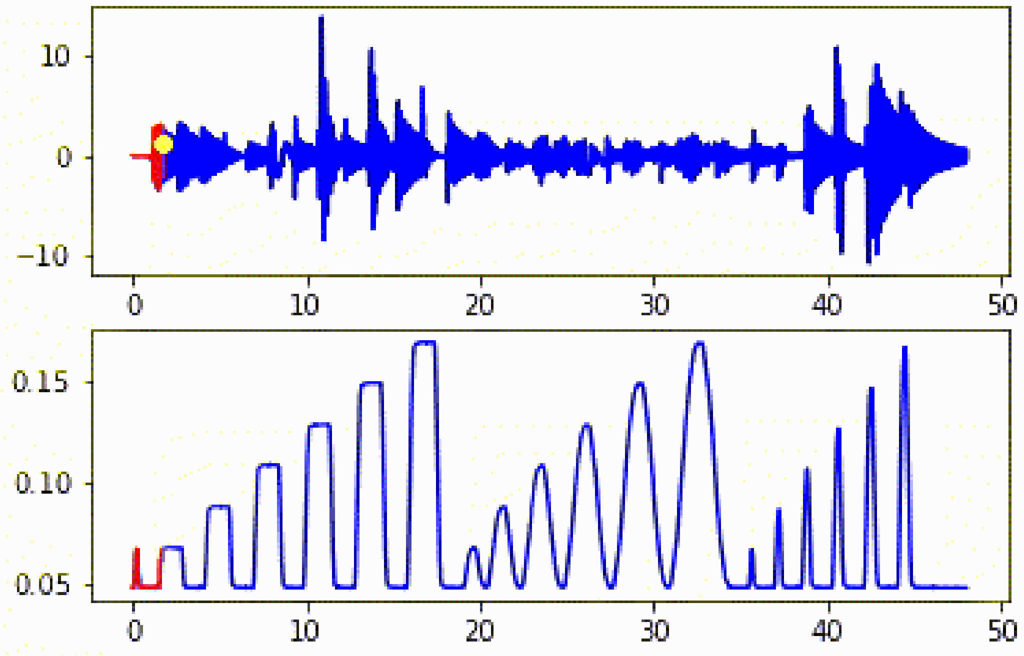




Joyce, B., Dodson, J., Laflamme, S., & Hong, J. *An experimental test bed for developing high-rate structural health monitoring methods*. Shock and Vibration, 2018.

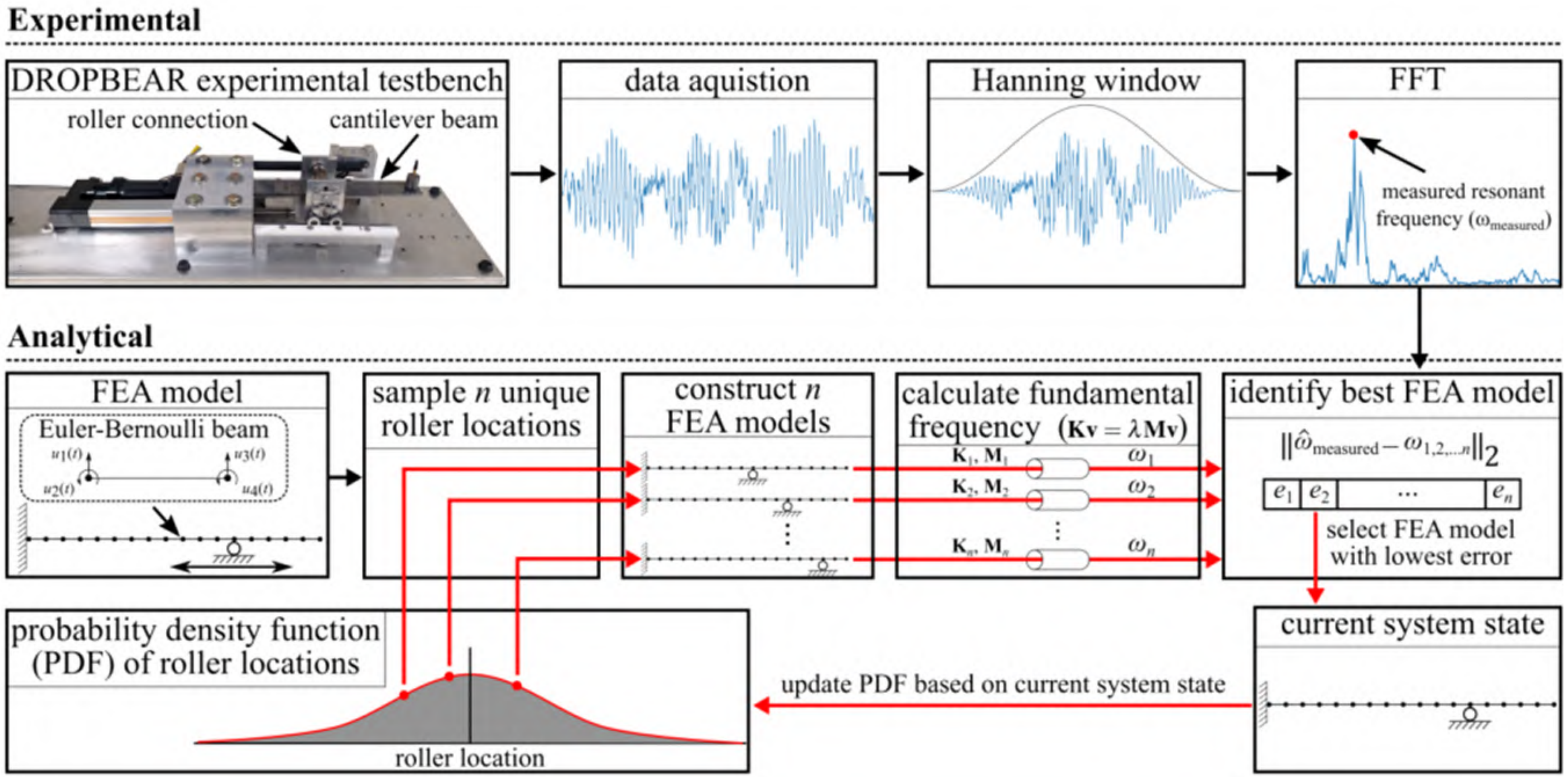
Experimental System used for Validation





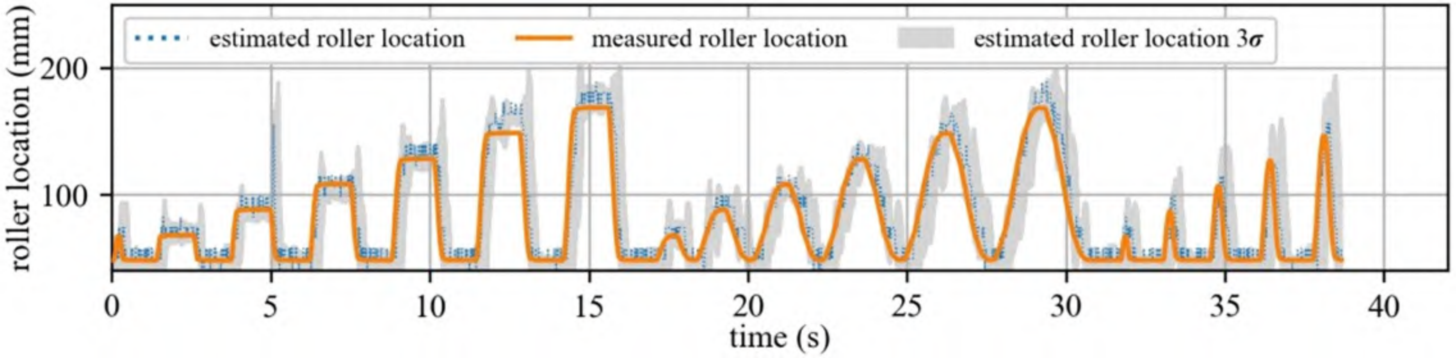
Joyce, B., Dodson, J., Laflamme, S., & Hong, J. *An experimental test bed for developing high-rate structural health monitoring methods*. Shock and Vibration, 2018.

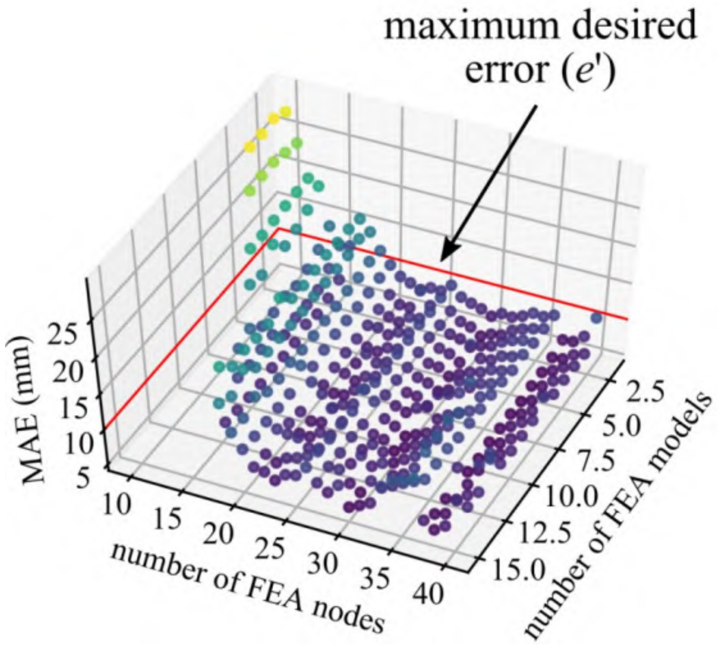
Real-time FEA model updating (1D)

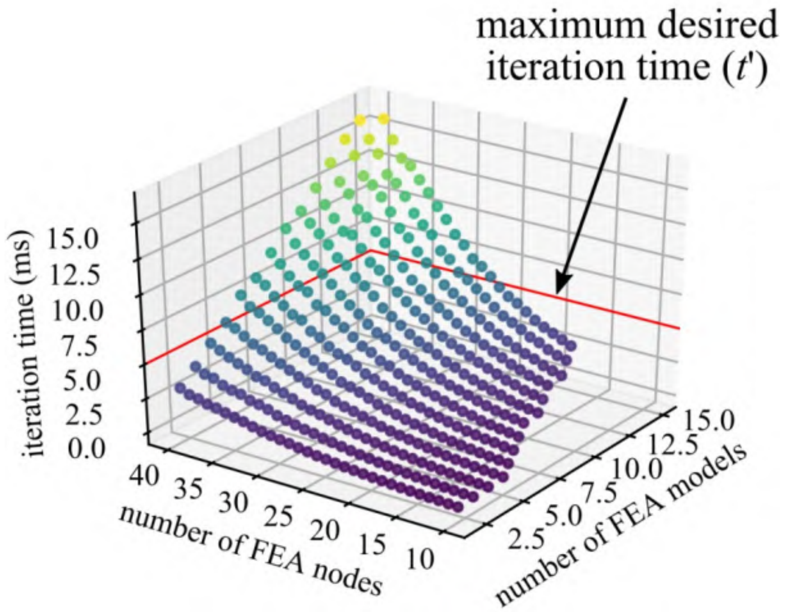


Downey A., et al,. "Millisecond Model Updating for Structures Experiencing Unmodeled High-Rate Dynamic Events" *Mechanical Systems and Signal Processing* ***138***, 2020

Real-time FEA model updating Results





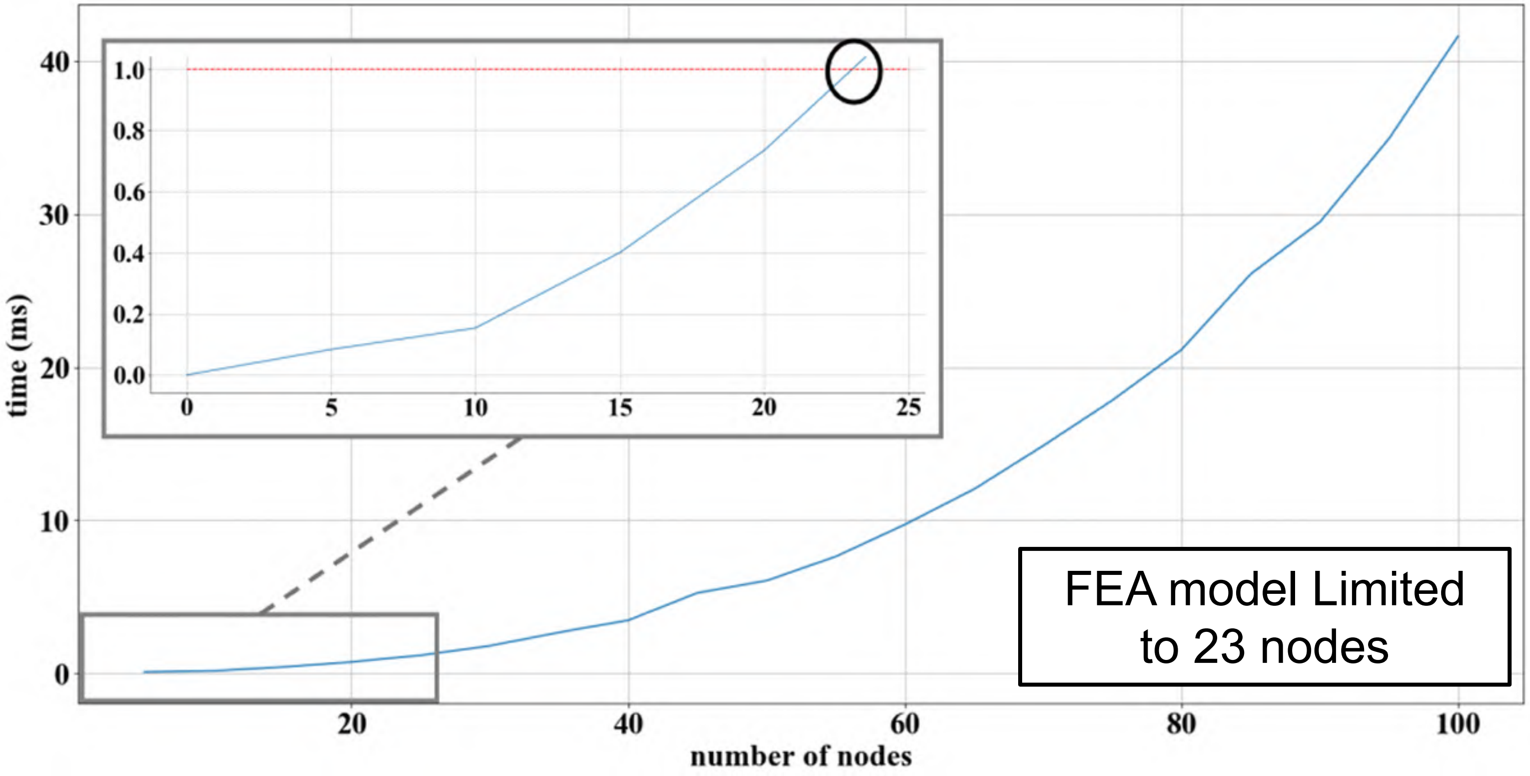


Downey A., et al,. "Millisecond Model Updating for Structures Experiencing Unmodeled High-Rate Dynamic Events" *Mechanical Systems and Signal Processing* ***138***, 2020

FEA Computation speed for the DROPBEAR

General Eigenvalue solutions accurately estimates the state of the DROPBEAR

Solving for system's frequencies accounted for 90% of algorithm iteration time

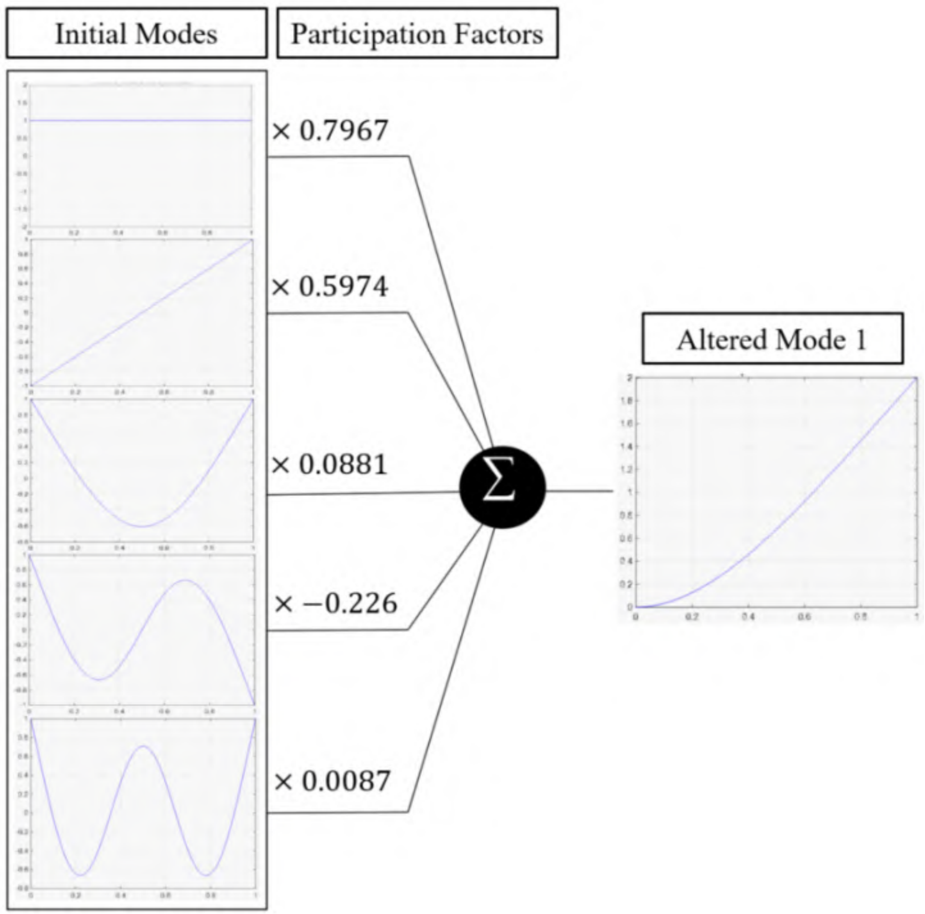


FEA model Limited to 23 nodes

Carroll, M., Downey, A., Dodson, J., Hong, J. and Scheppegrell, J., "Analysis of Computation Speeds of Eigenvalue Solutions for High-Rate Structural Health Monitoring."

Local Eigenvalue Modification Procedure (LEMP)

- Developed by Wesseinburger in 1968
- Identifies physical changes to the system such as mass, stiffness or damping using changes such as frequencies or mode shapes
- Model the altered state as a mixture of the initial state and changes made to the initial state
- Reduces the GE equation to a set of second-order equations

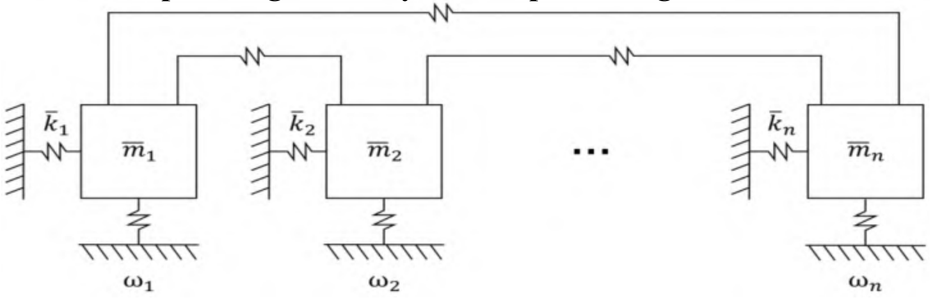


Avitabile, P., "*Twenty Years of Structural Dynamic Modification- A Review*," *Sound and Vibration,* pp. 14-25. 2003
Drnek, C. R., "*Local eigenvalue modification procedure for real-time model updating of structures experiencing high-rate dynamic events*," (2020).

Local Eigenvalue Modification Procedure (LEMP)

n independent single DOF systems representing the initial state

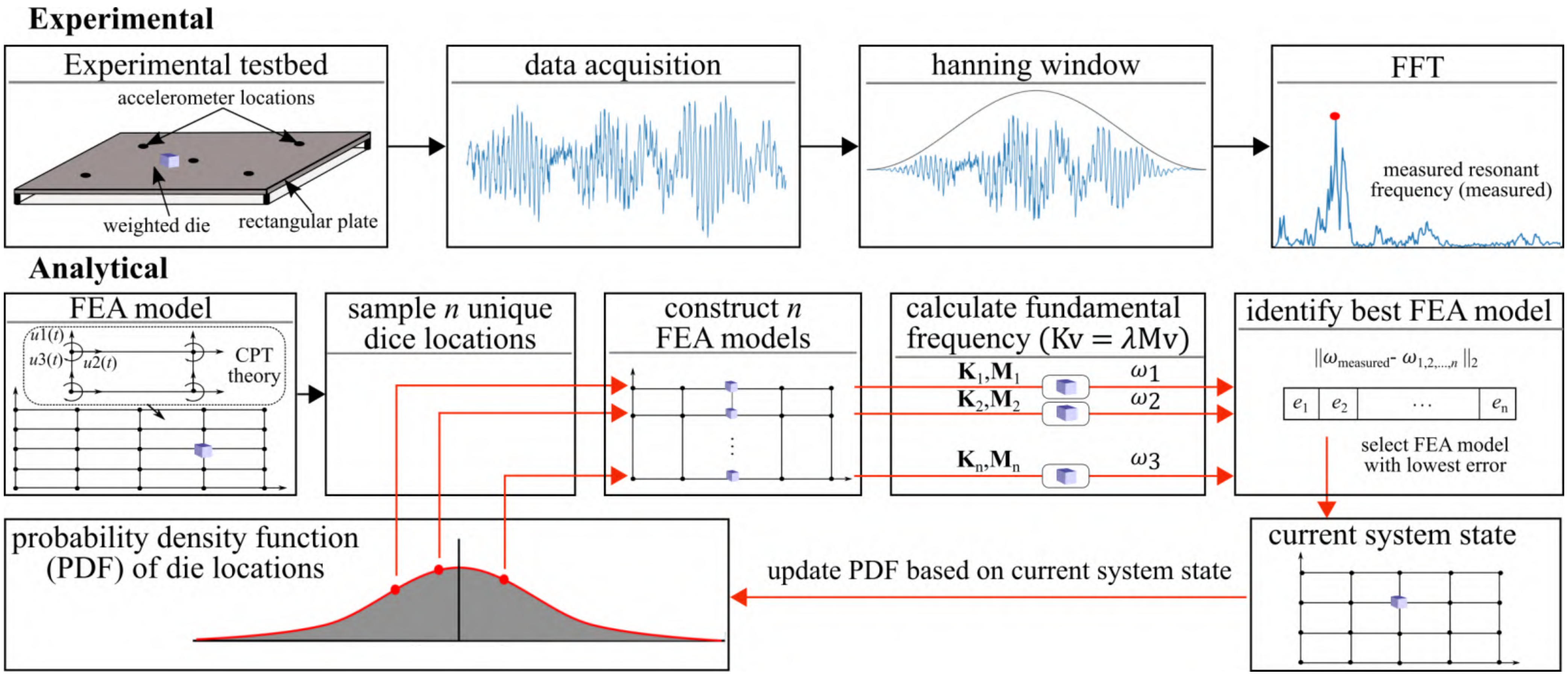
Coupled single DOF systems representing the altered state



	Initial State	Modification	Altered State	
Physical Space	$[\mathbf{M}_1], [\mathbf{K}_1]$	$[\Delta\mathbf{M}_{12}], [\Delta\mathbf{K}_{12}]$	$[\mathbf{M}_2], [\mathbf{K}_2]$	*'n'* Physical DOF
Modal Transformation	$\{x\} = [U_1]\{p_1\}$	$\frac{-1}{\alpha} = \sum_{r=1}^{m} \frac{v_r^2}{\omega_r^2 - \Omega_r^2}$ **Solved using Divide and Conquer method**	$\{x\} = [U_2]\{p_2\}$	*m<<n*
Modal Space	$[\omega_1^2], [U_1]$	$\{p_1\} = [U_{12}]\{p_2\}$	$[\Omega_2^2], [U_2]$	*'m'* Modal DOF

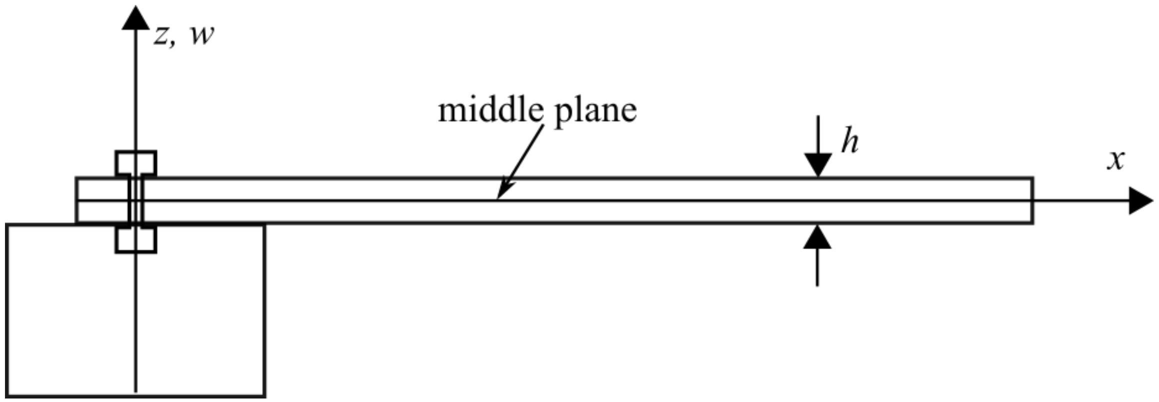
Avitabile, P., "*Twenty Years of Structural Dynamic Modification- A Review*," *Sound and Vibration*, pp. 14-25. 2003
Drnek, C. R., "*Local eigenvalue modification procedure for real-time model updating of structures experiencing high-rate dynamic events*," (2020).

Current methodology

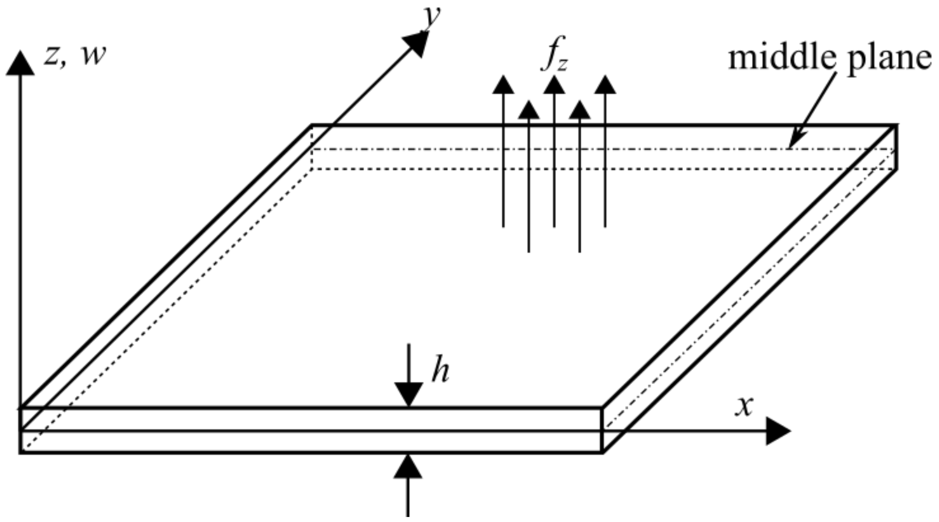


1D vs 2D Structural systems

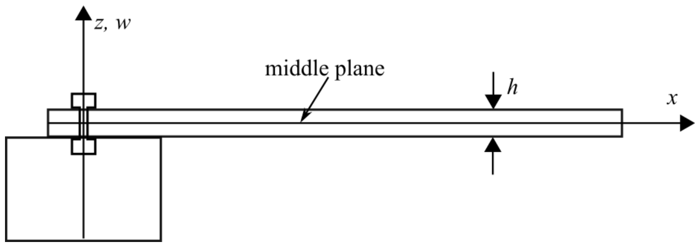
1D

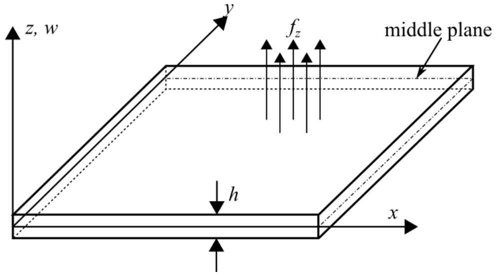


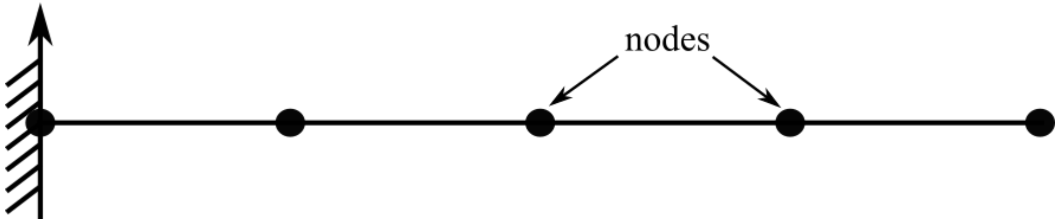
2D

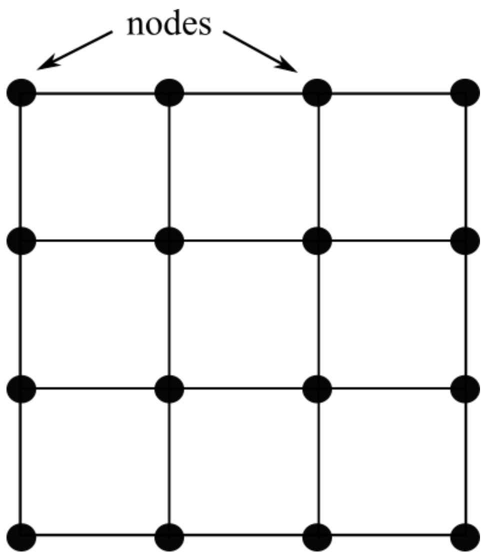


1D vs 2D Node construction



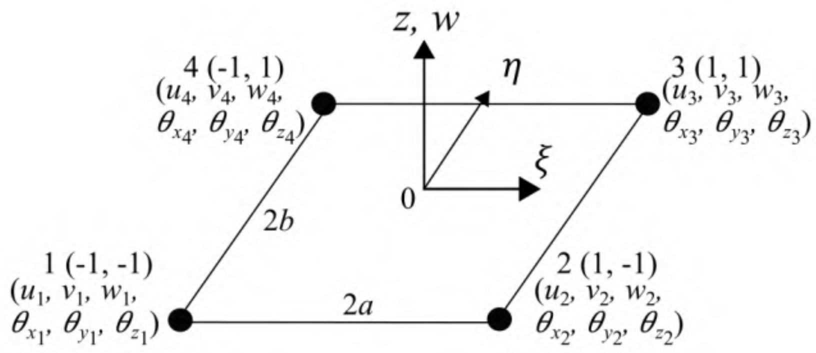






2D Model Formulation

Shell element



Three translational displacements in the *x*, *y*, and *z* directions, and three rotational deformations with respect to the *x*, *y*, and *z* axes.

$$\mathbf{d}_e = \begin{Bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \mathbf{d}_3 \\ \mathbf{d}_4 \end{Bmatrix} \begin{matrix} \text{node 1} \\ \text{node 2} \\ \text{node 3} \\ \text{node 4} \end{matrix}$$

where **d***i* (*i*=1, 2, 3, 4) are the displacement vector at node *i*:

$$\mathbf{d}_i = \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_{xi} \\ \theta_{yi} \\ \theta_{zi} \end{Bmatrix} \begin{matrix} \text{displacement in } x \text{ direction} \\ \text{displacement in } y \text{ direction} \\ \text{displacement in } z \text{ direction} \\ \text{rotation about } x\text{-axis} \\ \text{rotation about } y\text{-}axis \\ \text{rotation about } z\text{-axis} \end{matrix}$$

2D Model Formulation

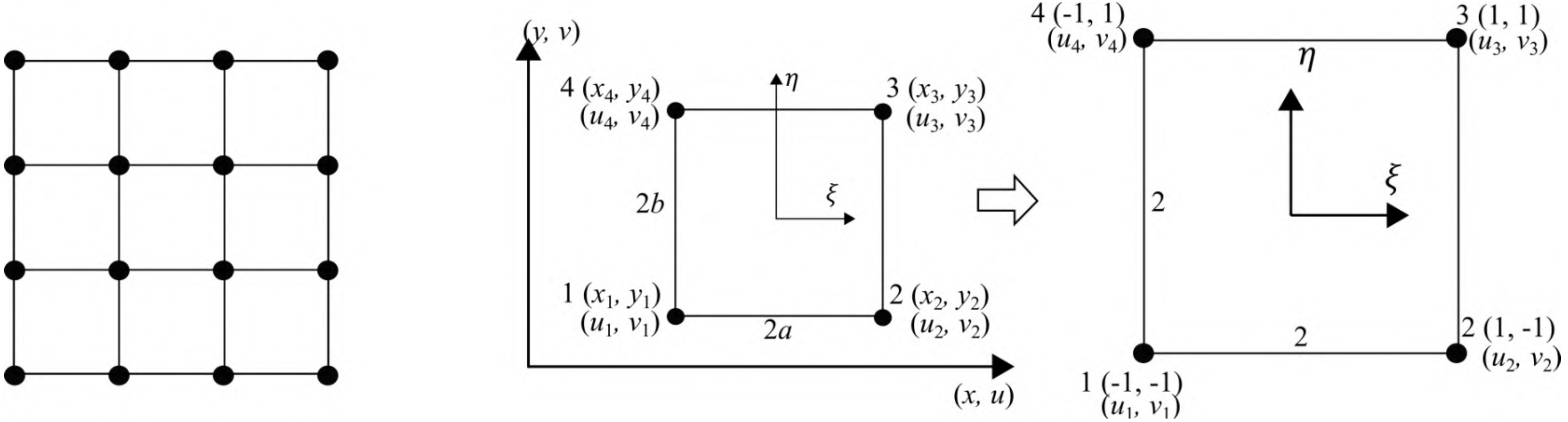


Figure 1. Shell element formation and its coordinate system where; (a) represents the nodal construction on the element; (b) shows the coordinate system of a 2D solid element with 2 DOFs; (c) shows the transformation of the coordinate system with dimension; (d) depicts a plate structure, and; (e) is the shell coordinate system that combines the 2D solid element and plate structure.

2D Model Formulation

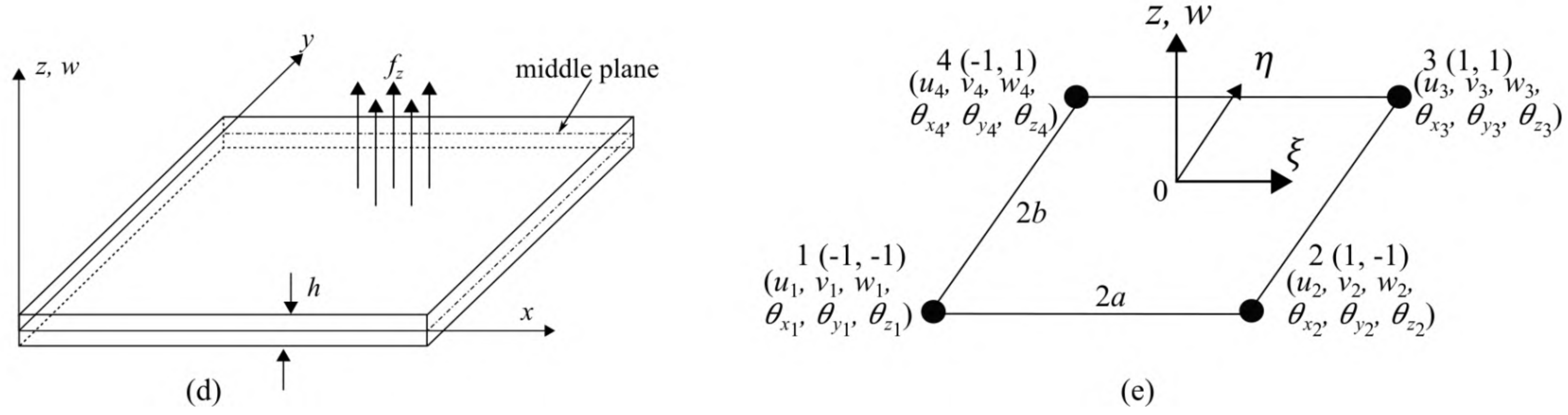


Figure 1. Shell element formation and its coordinate system where; (a) represents the nodal construction on the element; (b) shows the coordinate system of a 2D solid element with 2 DOFs; (c) shows the transformation of the coordinate system with dimension; (d) depicts a plate structure, and; (e) is the shell coordinate system that combines the 2D solid element and plate structure.

2D Model Formulation

Modeling steps

1. Construction of shape functions matrix **N** that satisfies Eqs. 1

2. Formulation of the strain matrix for 2D element **B**, Eq. 3 and 2D plate, **BI** and **BO** shown in Eqs. 4 and 5.

3. Calculation of **ke** and **me** using shape functions **N** and strain matrix in step 2 to obtain Eqs. 5 and 6.

2D Model Formulation

1. Construction of shape functions matrix **N** that satisfies Eqs. 1

2D element

$$\mathbf{N}_e = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \tag{1}$$

2D plate

$$\mathbf{N}_p = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 \end{bmatrix} \tag{2}$$

<u>Subscript</u>

e – 2D element
p – 2D plate

2D Model Formulation

2. Formulation of the strain matrix for 2D element **B**, Eq. 3 and 2D plate, **B**I and **BO** shown in Eqs. 4 and 5.

2D element

$$\mathbf{B} = \mathbf{LN} = \frac{1}{4}\begin{bmatrix} -\frac{1-\eta}{a} & 0 & \frac{1-\eta}{a} & 0 & \frac{1+\eta}{a} & 0 & -\frac{1+\eta}{a} & 0 \\ 0 & -\frac{1-\xi}{b} & 0 & -\frac{1+\xi}{b} & 0 & \frac{1+\xi}{b} & 0 & \frac{1-\xi}{b} \\ -\frac{1-\xi}{b} & -\frac{1-\eta}{a} & -\frac{1+\xi}{b} & \frac{1-\eta}{a} & \frac{1+\xi}{b} & \frac{1+\eta}{a} & \frac{1-\xi}{b} & -\frac{1+\eta}{a} \end{bmatrix} \quad (3)$$

2D plate

$$\mathbf{B}^{\mathrm{I}} = \begin{bmatrix} \mathbf{B}_1^{\mathrm{I}} & \mathbf{B}_2^{\mathrm{I}} & \mathbf{B}_3^{\mathrm{I}} & \mathbf{B}_4^{\mathrm{I}} \end{bmatrix}, \qquad \mathbf{B}_{\mathrm{j}}^{\mathrm{I}} = \begin{bmatrix} 0 & 0 & -\partial N_j/\partial x \\ 0 & \partial N_j/\partial x & 0 \\ 0 & \partial N_j/\partial y & -\partial N_j \partial y \end{bmatrix} \quad (4)$$

$$\mathbf{B}^{\mathrm{O}} = \begin{bmatrix} \mathbf{B}_1^{\mathrm{O}} & \mathbf{B}_2^{\mathrm{O}} & \mathbf{B}_3^{0} & \mathbf{B}_4^{0} \end{bmatrix}, \qquad \mathbf{B}_{\mathrm{j}}^{\mathrm{O}} = \begin{bmatrix} \partial N_j/\partial x & 0 & N_j \\ \partial N_j/\partial y & -N_j & 0 \end{bmatrix} \quad (5)$$

2D Model Formulation

3. Calculation of **ke** and **me** using shape functions **N** and strain matrix in step 2 to obtain Eqs. 5 and 6.

mass matrix

$$\mathbf{m}_\mathrm{e} = \int_A h\rho\mathbf{N}^\mathrm{T}\mathbf{N}\mathrm{d}\mathrm{A}, \qquad \mathbf{m}_\mathrm{p} = \int_{A_p} \mathbf{N}^\mathrm{T}\mathbf{I}\mathbf{N}\mathrm{d}\mathrm{A} \quad (6)$$

$$\mathbf{I} = \begin{bmatrix} \rho h & 0 & 0 \\ 0 & \rho h^3/12 & 0 \\ 0 & 0 & \rho h^3/12 \end{bmatrix}$$

stiffness matrix

$$\mathbf{k}_\mathrm{e} = \int_A h\mathbf{B}^\mathrm{T}\mathbf{c}\mathbf{B}\mathrm{d}\mathrm{A}, \qquad \mathbf{k}_\mathrm{p} = \int_{A_p} \frac{h^3}{12}\left[\mathbf{B}^\mathrm{I}\right]^\mathrm{T}\mathbf{c}\mathbf{B}^\mathrm{I}dA + \int_{A_p} \kappa h\left[\mathbf{B}^\mathrm{O}\right]^\mathrm{T}\mathbf{c}_s\mathbf{B}^\mathrm{O}dA \quad (7)$$

2D Model Formulation

The stiffness matrix for a 2D solid, rectangular element is used to account for the membrane effects of the element, which corresponds to DOFs of *u* and *v*.

$$\mathbf{k}_e = \begin{array}{c c} \begin{array}{cccc} \text{node 1} & \text{node 2} & \text{node 3} & \text{node 4} \end{array} & \\ \left[\begin{array}{ccc ccc ccc ccc} \mathbf{k}^m_{11} & \mathbf{0} & 0 & \mathbf{k}^m_{12} & \mathbf{0} & 0 & \mathbf{k}^m_{13} & \mathbf{0} & 0 & \mathbf{k}^m_{14} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{k}^b_{11} & 0 & \mathbf{0} & \mathbf{k}^b_{12} & 0 & \mathbf{0} & \mathbf{k}^b_{13} & 0 & \mathbf{0} & \mathbf{k}^b_{14} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{k}^m_{21} & \mathbf{0} & 0 & \mathbf{k}^m_{22} & \mathbf{0} & 0 & \mathbf{k}^m_{23} & \mathbf{0} & 0 & \mathbf{k}^m_{24} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{k}^b_{21} & 0 & \mathbf{0} & \mathbf{k}^b_{23} & 0 & \mathbf{0} & \mathbf{k}^b_{23} & 0 & \mathbf{0} & \mathbf{k}^b_{24} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{k}^m_{31} & \mathbf{0} & 0 & \mathbf{k}^m_{32} & \mathbf{0} & 0 & \mathbf{k}^m_{33} & \mathbf{0} & 0 & \mathbf{k}^m_{34} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{k}^b_{31} & 0 & \mathbf{0} & \mathbf{k}^b_{33} & 0 & \mathbf{0} & \mathbf{k}^b_{33} & 0 & \mathbf{0} & \mathbf{k}^b_{34} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{k}^m_{41} & \mathbf{0} & 0 & \mathbf{k}^m_{44} & \mathbf{0} & 0 & \mathbf{k}^m_{43} & \mathbf{0} & 0 & \mathbf{k}^m_{44} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{k}^b_{41} & 0 & \mathbf{0} & \mathbf{k}^b_{43} & 0 & \mathbf{0} & \mathbf{k}^b_{43} & 0 & \mathbf{0} & \mathbf{k}^b_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] & \begin{array}{c} \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{node 1} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{node 2} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{node 3} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{node 4} \end{array} \end{array}$$

The stiffness matrix for a rectangular plate element is used for the bending effects, corresponding to DOFs of *w* and ϑx, ϑy.

$$\mathbf{k}_e^m = \begin{array}{cc} \begin{array}{cccc} \text{node 1} & \text{node 2} & \text{node 3} & \text{node 4} \end{array} & \\ \begin{bmatrix} \mathbf{k}^m_{11} & \mathbf{k}^m_{12} & \mathbf{k}^m_{13} & \mathbf{k}^m_{14} \\ \mathbf{k}^m_{21} & \mathbf{k}^m_{22} & \mathbf{k}^m_{23} & \mathbf{k}^m_{24} \\ \mathbf{k}^m_{31} & \mathbf{k}^m_{32} & \mathbf{k}^m_{33} & \mathbf{k}^m_{34} \\ \mathbf{k}^m_{41} & \mathbf{k}^m_{42} & \mathbf{k}^m_{43} & \mathbf{k}^m_{44} \end{bmatrix} & \begin{array}{c} \text{node 1} \\ \text{node 2} \\ \text{node 3} \\ \text{node 4} \end{array} \end{array}$$

$$\mathbf{k}_e^b = \begin{array}{cc} \begin{array}{cccc} \text{node 1} & \text{node 2} & \text{node 3} & \text{node 4} \end{array} & \\ \begin{bmatrix} \mathbf{k}^b_{11} & \mathbf{k}^b_{12} & \mathbf{k}^b_{13} & \mathbf{k}^b_{14} \\ \mathbf{k}^b_{21} & \mathbf{k}^b_{22} & \mathbf{k}^b_{23} & \mathbf{k}^b_{24} \\ \mathbf{k}^b_{31} & \mathbf{k}^b_{32} & \mathbf{k}^b_{33} & \mathbf{k}^b_{34} \\ \mathbf{k}^b_{41} & \mathbf{k}^b_{42} & \mathbf{k}^b_{43} & \mathbf{k}^b_{44} \end{bmatrix} & \begin{array}{c} \text{node 1} \\ \text{node 2} \\ \text{node 3} \\ \text{node 4} \end{array} \end{array}$$

2D Model Formulation

The mass matrix for the 2D solid element is used for the membrane effects, corresponding to DOFs of u and v.

$$
\mathbf{m}_e^m = \begin{array}{c} \begin{array}{cccc} \text{node1} & \text{node2} & \text{node3} & \text{node4} \end{array} \\ \begin{bmatrix} \mathbf{m}_{11}^m & \mathbf{m}_{12}^m & \mathbf{m}_{13}^m & \mathbf{m}_{14}^m \\ \mathbf{m}_{21}^m & \mathbf{m}_{22}^m & \mathbf{m}_{23}^m & \mathbf{m}_{24}^m \\ \mathbf{m}_{31}^m & \mathbf{m}_{32}^m & \mathbf{m}_{33}^m & \mathbf{m}_{34}^m \\ \mathbf{m}_{41}^m & \mathbf{m}_{42}^m & \mathbf{m}_{43}^m & \mathbf{m}_{44}^m \end{bmatrix} \end{array} \begin{array}{l} \\ \text{node1} \\ \text{node2} \\ \text{node3} \\ \text{node4} \end{array}
$$

$$
\mathbf{k}_e = \begin{array}{c} \begin{array}{cccc} \text{node 1} & \text{node 2} & \text{node 3} & \text{node 4} \end{array} \\ \left[\begin{array}{cccccccccccc} \mathbf{m}_{11}^m & \mathbf{0} & 0 & \mathbf{m}_{12}^m & \mathbf{0} & 0 & \mathbf{m}_{13}^m & \mathbf{0} & 0 & \mathbf{m}_{14}^m & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{m}_{11}^b & 0 & \mathbf{0} & \mathbf{m}_{12}^b & 0 & \mathbf{0} & \mathbf{m}_{13}^b & 0 & \mathbf{0} & \mathbf{m}_{14}^b & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{m}_{21}^m & \mathbf{0} & 0 & \mathbf{m}_{22}^m & \mathbf{0} & 0 & \mathbf{m}_{23}^m & \mathbf{0} & 0 & \mathbf{m}_{24}^m & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{m}_{21}^b & 0 & \mathbf{0} & \mathbf{m}_{23}^b & 0 & \mathbf{0} & \mathbf{m}_{23}^b & 0 & \mathbf{0} & \mathbf{m}_{24}^b & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{m}_{31}^m & \mathbf{0} & 0 & \mathbf{m}_{32}^m & \mathbf{0} & 0 & \mathbf{m}_{33}^m & \mathbf{0} & 0 & \mathbf{m}_{34}^m & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{m}_{31}^b & 0 & \mathbf{0} & \mathbf{m}_{33}^b & 0 & \mathbf{0} & \mathbf{m}_{33}^b & 0 & \mathbf{0} & \mathbf{m}_{34}^b & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{m}_{41}^m & \mathbf{0} & 0 & \mathbf{m}_{44}^m & \mathbf{0} & 0 & \mathbf{m}_{43}^m & \mathbf{0} & 0 & \mathbf{m}_{44}^m & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{m}_{41}^b & 0 & \mathbf{0} & \mathbf{m}_{43}^b & 0 & \mathbf{0} & \mathbf{m}_{43}^b & 0 & \mathbf{0} & \mathbf{m}_{44}^b & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{l} \\ \\ \text{node 1} \\ \\ \\ \text{node 2} \\ \\ \\ \text{node 3} \\ \\ \\ \text{node 4} \\ \\ \end{array}
$$

The mass matrix for a rectangular plate element is used for the bending effects, corresponding to DOFs of w and ϑx, ϑy.

$$
\mathbf{m}_e^b = \begin{array}{c} \begin{array}{cccc} \text{node1} & \text{node2} & \text{node3} & \text{node4} \end{array} \\ \begin{bmatrix} \mathbf{m}_{11}^b & \mathbf{m}_{12}^b & \mathbf{m}_{13}^b & \mathbf{m}_{14}^b \\ \mathbf{m}_{21}^b & \mathbf{m}_{22}^b & \mathbf{m}_{23}^b & \mathbf{m}_{24}^b \\ \mathbf{m}_{31}^b & \mathbf{m}_{32}^b & \mathbf{m}_{33}^b & \mathbf{m}_{34}^b \\ \mathbf{m}_{41}^b & \mathbf{m}_{42}^b & \mathbf{m}_{43}^b & \mathbf{m}_{44}^b \end{bmatrix} \end{array} \begin{array}{l} \\ \text{node1} \\ \text{node2} \\ \text{node3} \\ \text{node4} \end{array}
$$

2D Model Formulation

Elements in the global coordinate system

$$\mathbf{K}_e = \mathbf{T}^T \mathbf{k}_e \mathbf{T}$$

$$\mathbf{M}_e = \mathbf{T}^T \mathbf{m}_e \mathbf{T}$$

$$\mathbf{F}_e = \mathbf{T}^T \mathbf{f}_e$$

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_3 \end{bmatrix}_{24\times 24}$$

T is the transformation matrix

$$\mathbf{T}_3 = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix}_{3\times 3}$$

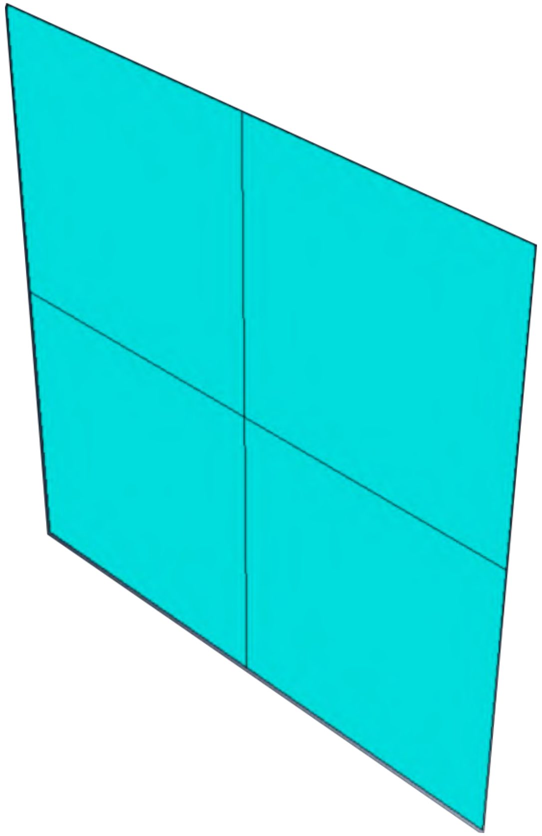
where lk, mk and nk (k=x, y, z) are direction cosines

2D Plate properties

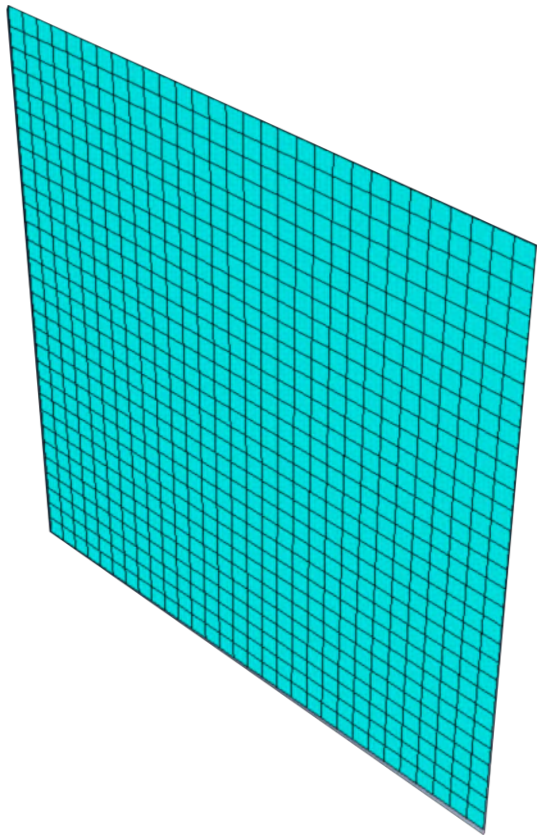
Type	Poisson’s ration	Young’s modulus	density	length	width	thickness
Steel	0.3	200e9	7700 kg/m3	0.3 m	0.3 m	0.006 m

Abaqus modeling

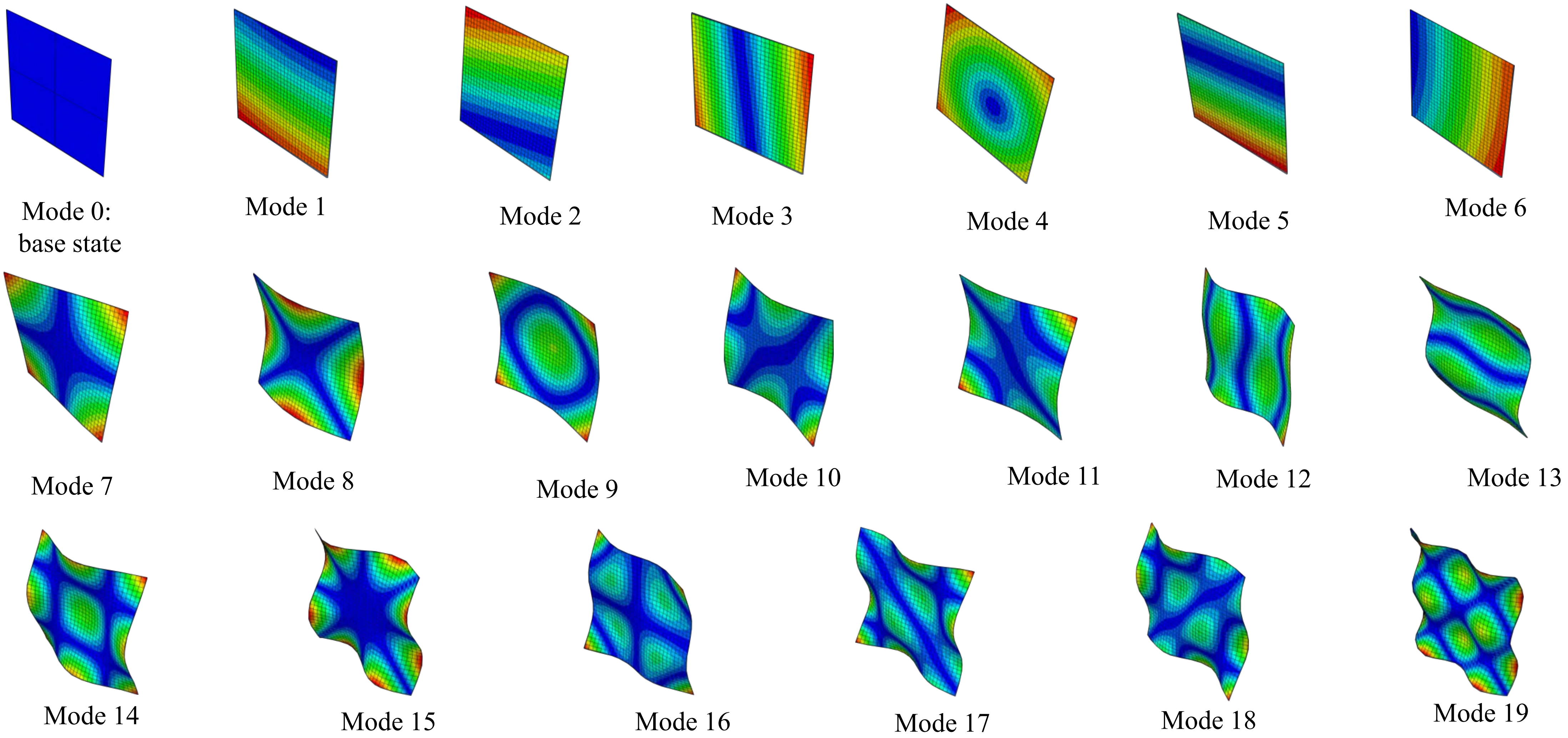
4 elements



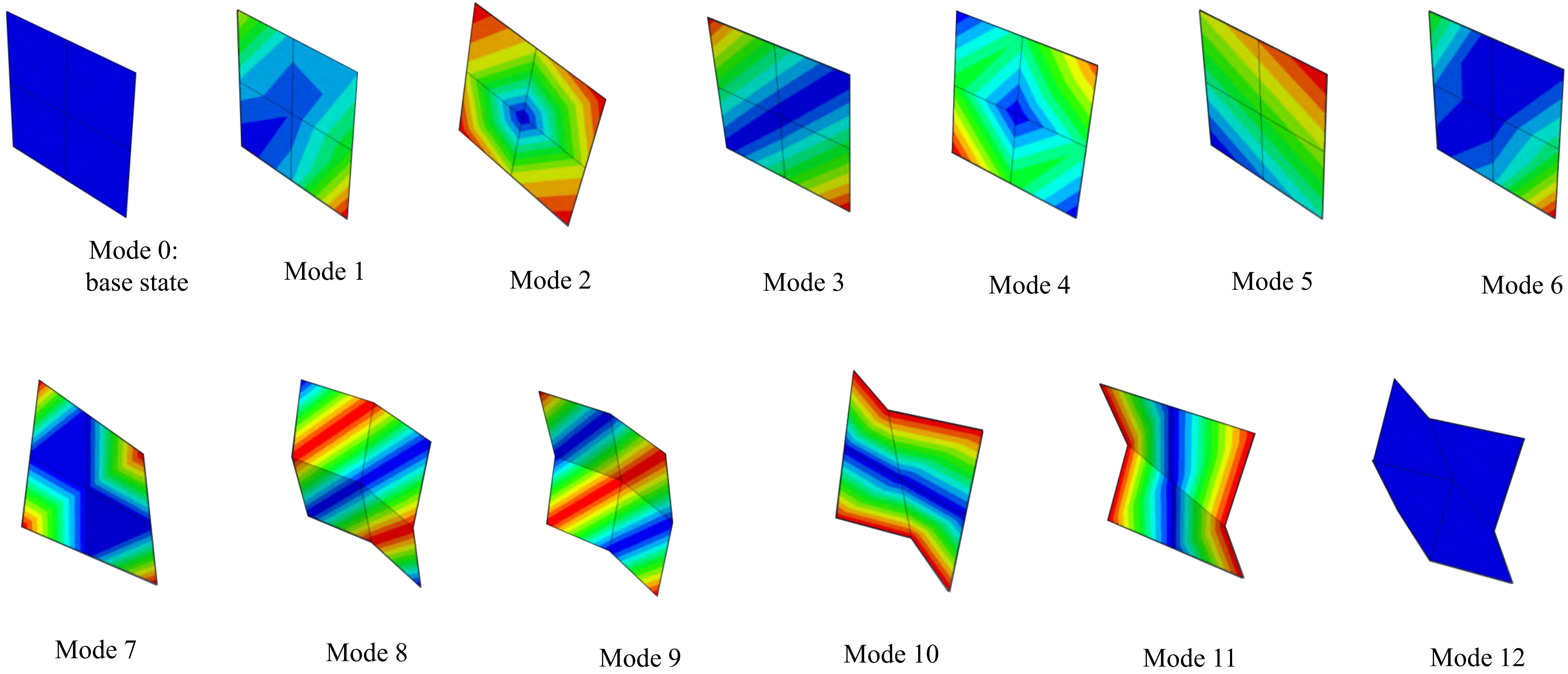
900 elements



Abaqus modeling



Abaqus modeling



Mode Frequency

4 elements

Step/Frame

Step Name	Description
Step-1	

Frame

Index	Description
0	Increment 0: Base State
1	Mode 1: Value = -3.19909E-07 Freq = 0.0000 (cycles/time)
2	Mode 2: Value = -2.69152E-07 Freq = 0.0000 (cycles/time)
3	Mode 3: Value = -1.24332E-07 Freq = 0.0000 (cycles/time)
4	Mode 4: Value = -8.33534E-08 Freq = 0.0000 (cycles/time)
5	Mode 5: Value = -4.33065E-08 Freq = 0.0000 (cycles/time)
6	Mode 6: Value = -3.72529E-09 Freq = 0.0000 (cycles/time)
7	Mode 7: Value = 2.12713E+06 Freq = 232.12 (cycles/time)
8	Mode 8: Value = 5.66377E+06 Freq = 378.77 (cycles/time)
9	Mode 9: Value = 1.05068E+07 Freq = 515.89 (cycles/time)
10	Mode 10: Value = 1.41477E+07 Freq = 598.64 (cycles/time)
11	Mode 11: Value = 1.41477E+07 Freq = 598.64 (cycles/time)
12	Mode 12: Value = 3.52346E+07 Freq = 944.72 (cycles/time)

900 elements

Step/Frame

Step Name	Description
Step-1	

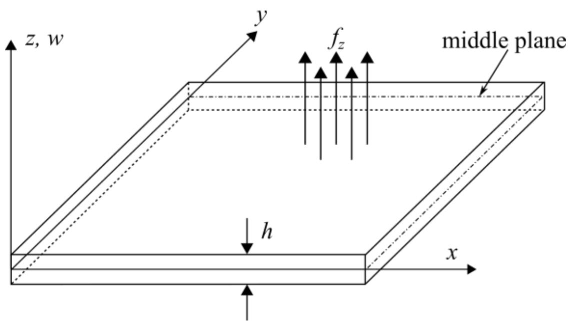
Frame

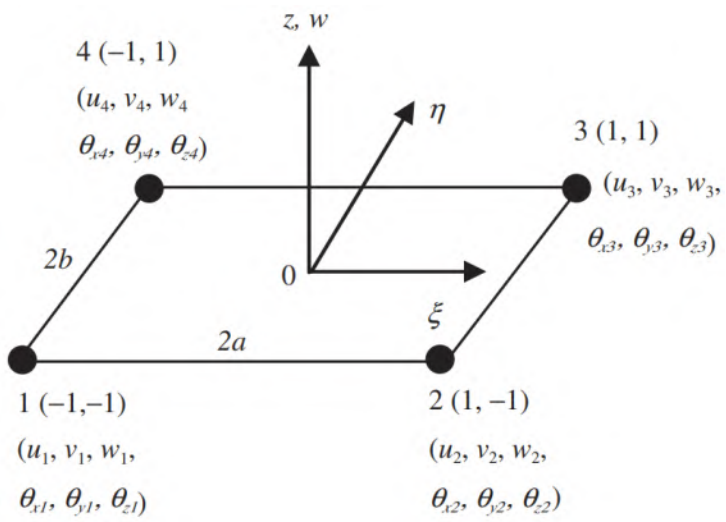
Index	Description
0	Increment 0: Base State
1	Mode 1: Value = 2.11708E-06 Freq = 2.31573E-04 (cycles/time)
2	Mode 2: Value = 3.40977E-06 Freq = 2.93888E-04 (cycles/time)
3	Mode 3: Value = 5.05996E-06 Freq = 3.58009E-04 (cycles/time)
4	Mode 4: Value = 6.18608E-06 Freq = 3.95847E-04 (cycles/time)
5	Mode 5: Value = 7.60294E-06 Freq = 4.38845E-04 (cycles/time)
6	Mode 6: Value = 1.44800E-05 Freq = 6.05625E-04 (cycles/time)
7	Mode 7: Value = 1.89263E+06 Freq = 218.95 (cycles/time)
8	Mode 8: Value = 4.05830E+06 Freq = 320.62 (cycles/time)
9	Mode 9: Value = 6.23002E+06 Freq = 397.25 (cycles/time)
10	Mode 10: Value = 1.26330E+07 Freq = 565.68 (cycles/time)
11	Mode 11: Value = 1.26330E+07 Freq = 565.68 (cycles/time)
12	Mode 12: Value = 3.95886E+07 Freq = 1001.4 (cycles/time)
13	Mode 13: Value = 3.95886E+07 Freq = 1001.4 (cycles/time)
14	Mode 14: Value = 4.20637E+07 Freq = 1032.2 (cycles/time)
15	Mode 15: Value = 5.01417E+07 Freq = 1127.0 (cycles/time)
16	Mode 16: Value = 6.26389E+07 Freq = 1259.6 (cycles/time)
17	Mode 17: Value = 1.15204E+08 Freq = 1708.3 (cycles/time)
18	Mode 18: Value = 1.15204E+08 Freq = 1708.3 (cycles/time)
19	Mode 19: Value = 1.46137E+08 Freq = 1924.0 (cycles/time)

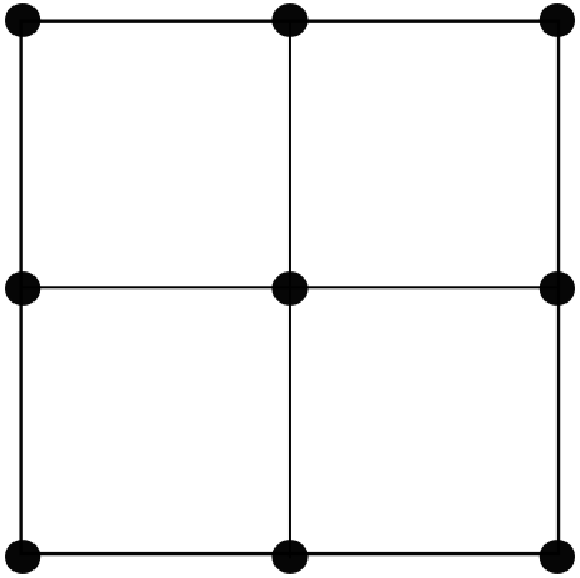
Base state estimation with GE

Mode	Abaqus	Generalized Eigenvalue	Error (abs)
7	232.12	232.027	0.0093
8	378.77	379.044	0.274
9	515.89	515.983	0.0093
10	598.64	598.768	0.128
11	598.64	598.768	0.128
12	944.72	945.03	0.31

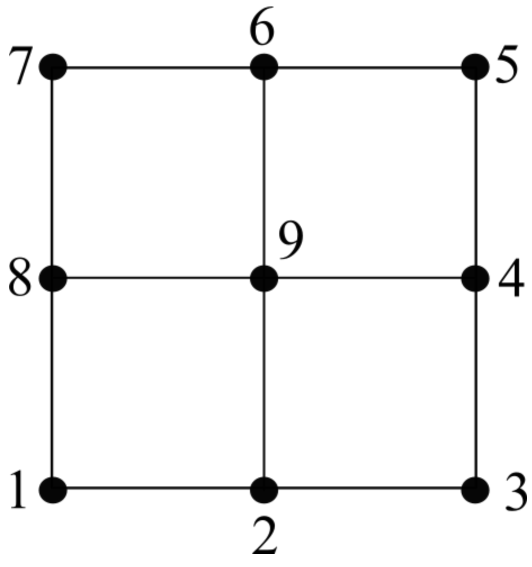
Single state change with GE and LEMP

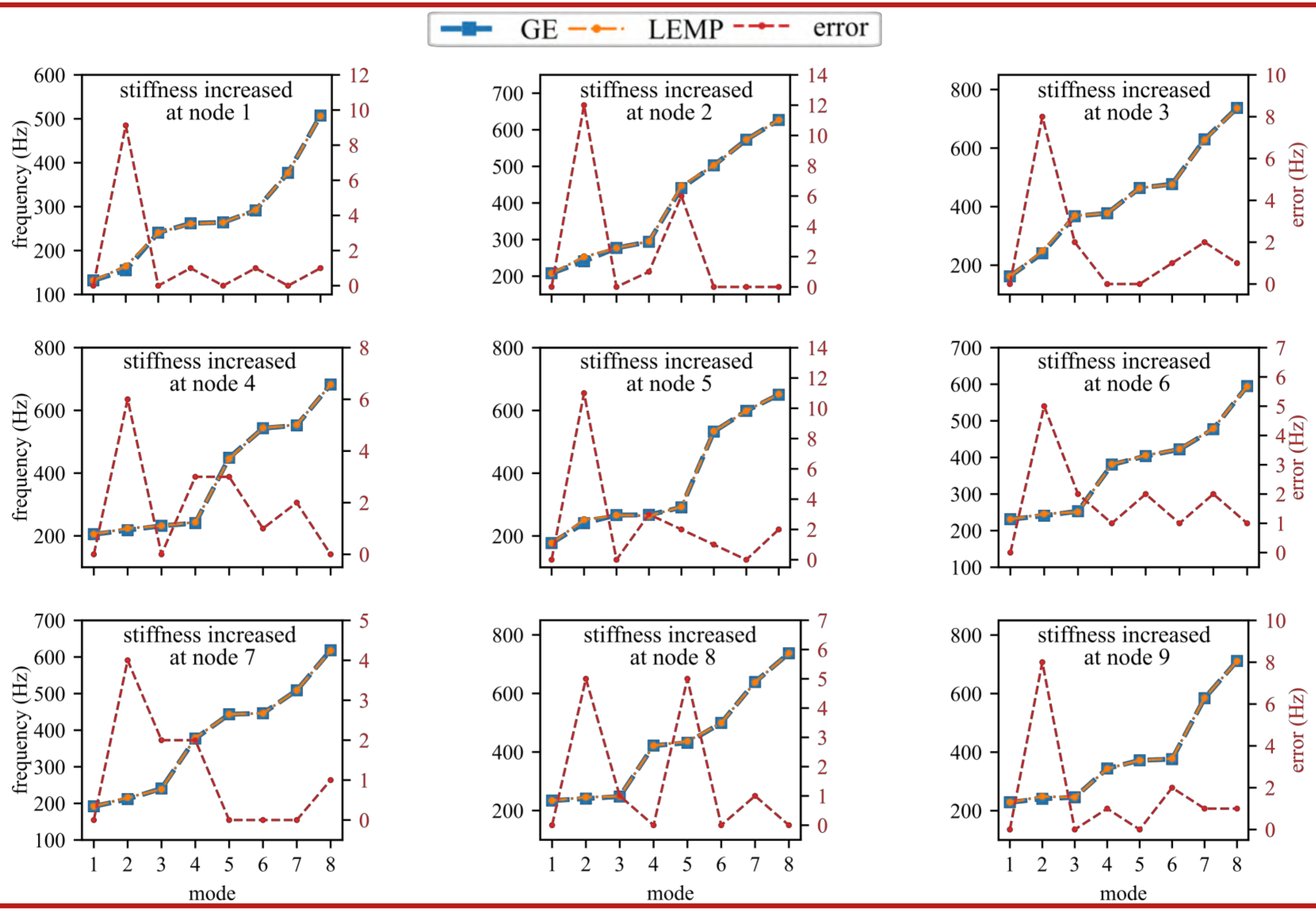






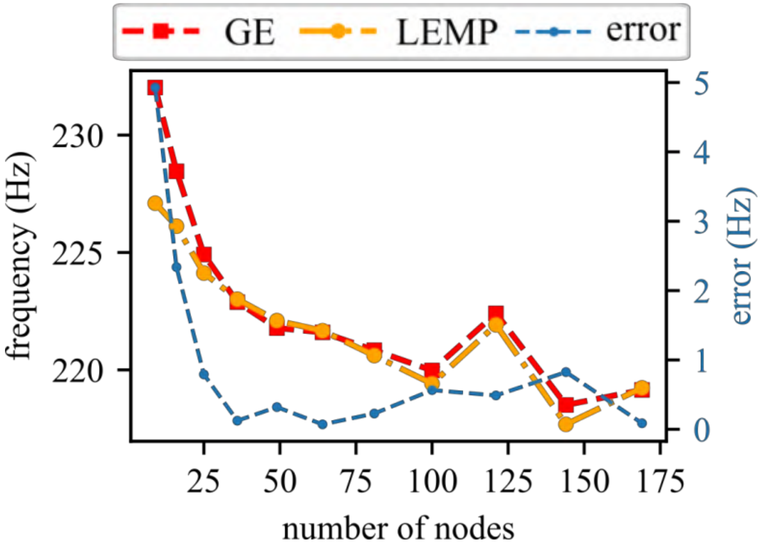
Single state change with GE and LEMP

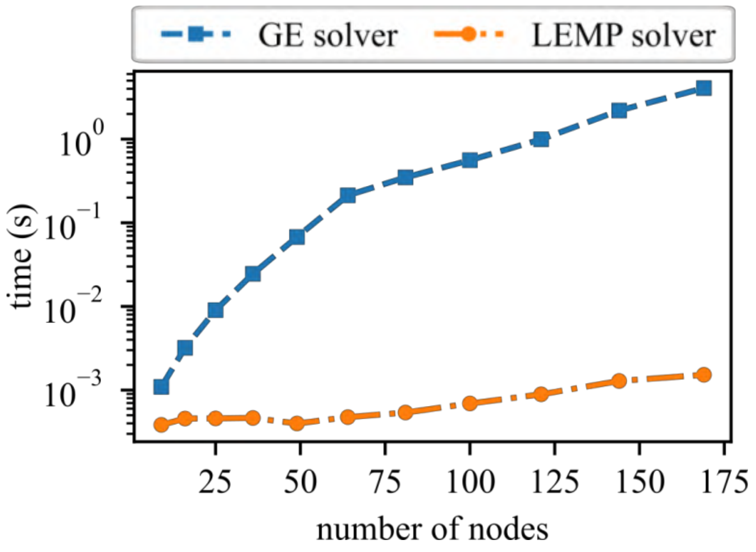




Single state change with GE and LEMP

Model update time





Single state change with GE and LEMP

Estimation Timing for GE and LEMP

single change calculated using:				generalized eigenvalue		LEMP		
no. of nodes	no. of element	DOF	matrix size	freq (Hz)	time GE (s)	freq (Hz)	time LEMP (s)	error (Hz)
9	4	54	54 x 54	232.027	0.001093	227.099	0.000384	4.928
16	9	96	96 x 96	228.458	0.00320	226.120	0.000456	2.338
25	16	150	150 x 150	224.914	0.009031	224.123	0.000458	0.791
36	25	216	216 x 216	222.886	0.024529	223.01	0.000464	-0.124
49	36	294	294 x 294	221.78	0.067579	222.1	0.000399	-0.32
64	49	384	384 x 384	221.599	0.212773	221.67	0.000475	-0.071
81	64	486	486 x 486	220.837	0.348656	220.610	0.000539	0.227
100	81	600	600 x 600	219.975	0.559744	219.41	0.000691	0.565
121	100	726	726 x 726	222.409	0.994675	221.919	0.000890	0.488
144	121	864	864 x 864	218.505	2.197694	217.68	0.001285	0.825
169	144	1014	1014 x 1014	219.147	4.075451	219.234	0.001523	-0.087

Conclusion

- The LEMP algorithm can be useful for faster solving of system equation for 2D structures.
- LEMP accuracy compared to the Generalized Eigenvalue process is good.
- Alternative 2D model construction should be used before employing LEMP algorithm to solve the system equation.

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