Data-Driven State Awareness and Health Monitoring for Next Generation Intelligent Systems

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December 17, 2024

Intelligent Structural Systems Lab (ISSL)

- 6 PhD students
- 6 MS/MEng students
- 5 undergraduate students
- 1 visiting Professor

- *Formal verification of stochastic state awareness for aerospace systems*
- *Probabilistic/statistical structural health monitoring (SHM) via vibration-based and acousto-ultrasound methods*
- *Data-driven stochastic identification and online fault diagnosis of eVTOL aircraft*
- *Data-driven modeling and online monitoring of metal additive manufacturing*
- *Integration of physics-based and data-based methods via multi-fidelity modeling*

Introduction

*Future intelligent structural systems will be able to **"feel," "think"** and **"react"!***

Fundamental Challenge: ***State Awareness***

improved performance, control, maintenance, and complete

life-cycle monitoring, management and safety assurance

Drones

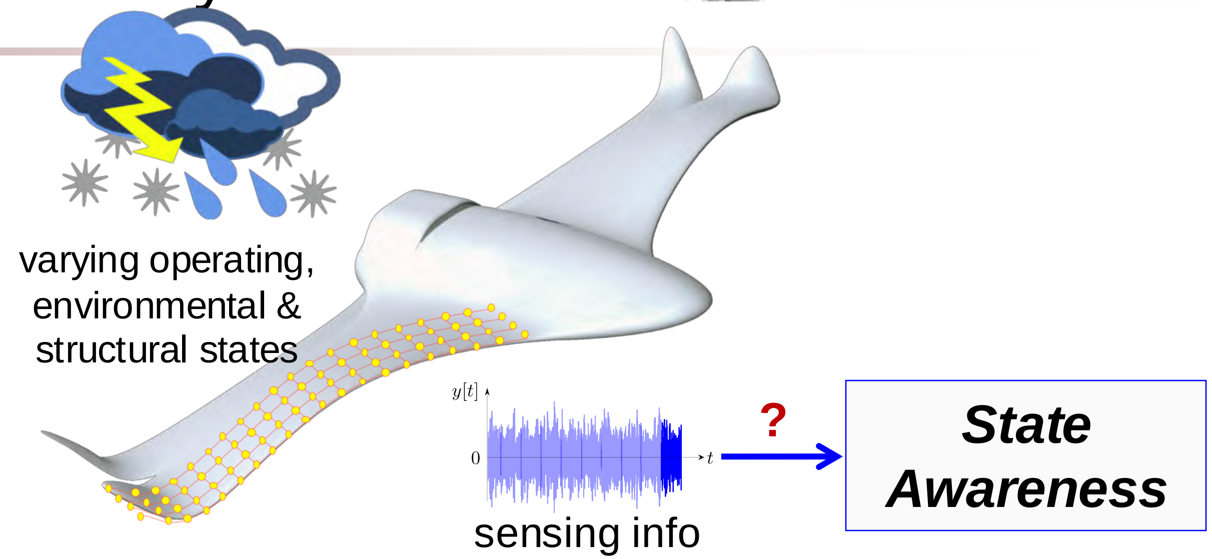
Intelligent Self-Aware Aerospace Systems

State sensing & awareness

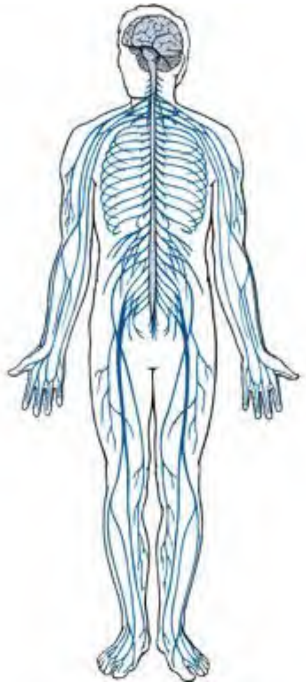
- Sense the environment
- Process the information
- Make optimal decisions
- Improve control and mitigation

State awareness

- Aerodynamic and structural loads
- Aerodynamic phenomena, e.g., *dynamic stall*
- Aeroelastic behavior & structural dynamics
  - ✓ flutter, vibration modes, nat freqs, damp. ratios
- **Flight state (angle of attack, attitude, velocity)**
- **Structural/systems health state (diagnosis/prognosis)**



sensor networks



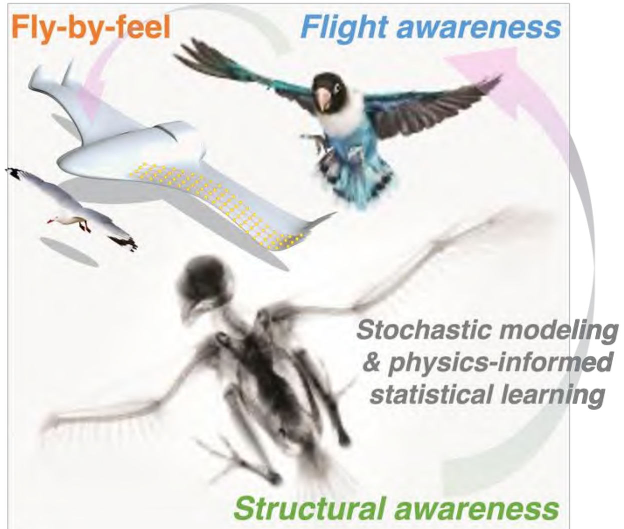
Mission Statement

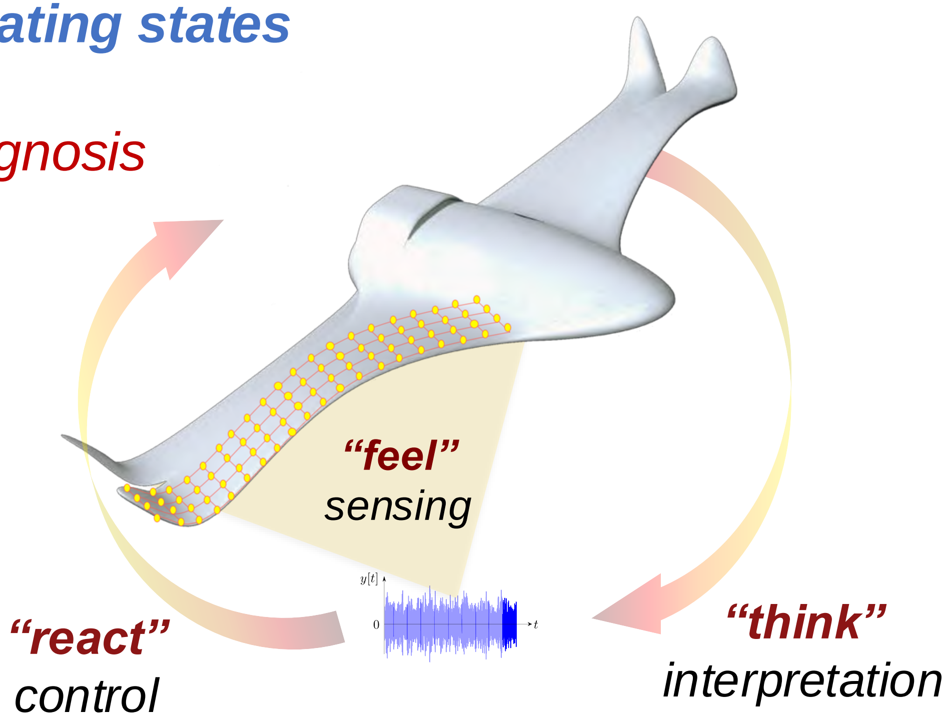
*Development of novel stochastic dynamic data-driven methods that will enable next generation **self-aware** and **self-diagnostic** structural systems that can **“feel,” “think” and “react”***

Main Research Thrusts:

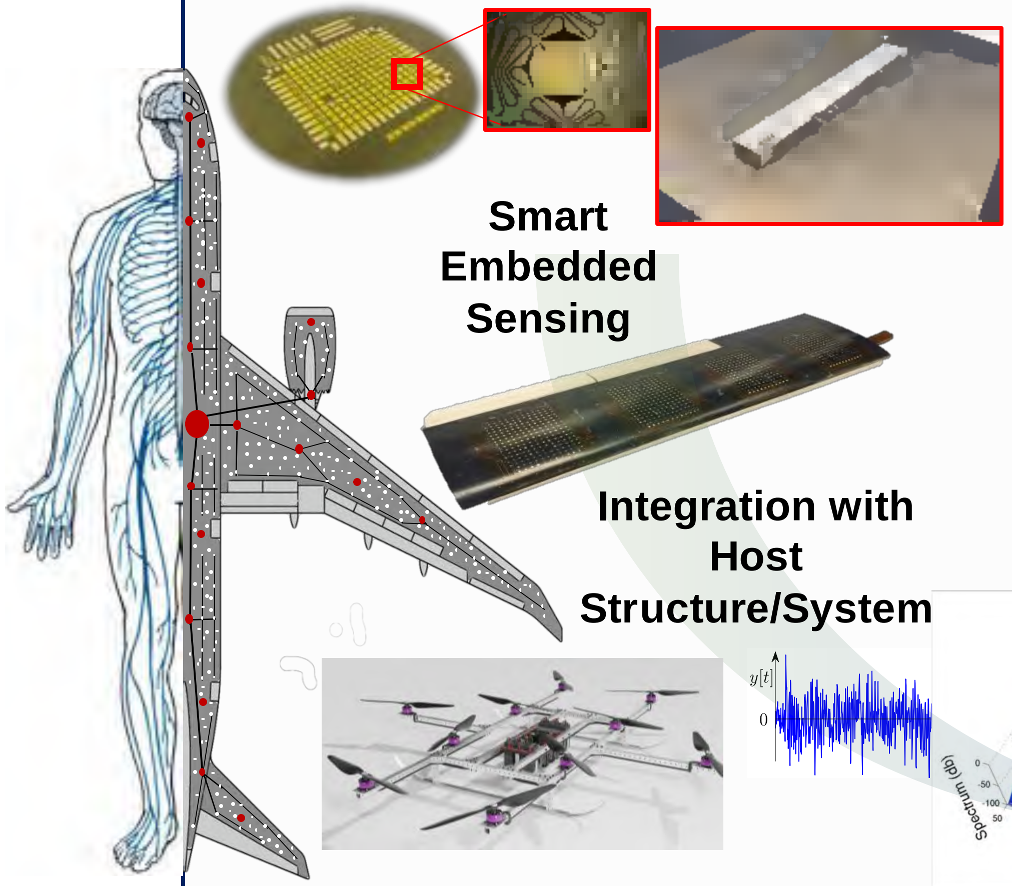
- ***State awareness:*** *monitor the structural state and safety-critical **phenomena and events***
- ***System diagnostics/prognostics:*** *probabilistic **health monitoring, fault detection/identification** within complex dynamic environments under **varying operating states***
- ***Fly-by-feel*** *for next generation intelligent aerial vehicles*
- ***Formal verification*** *of stochastic state awareness and diagnosis*

…“feel,” “think,” and “react”



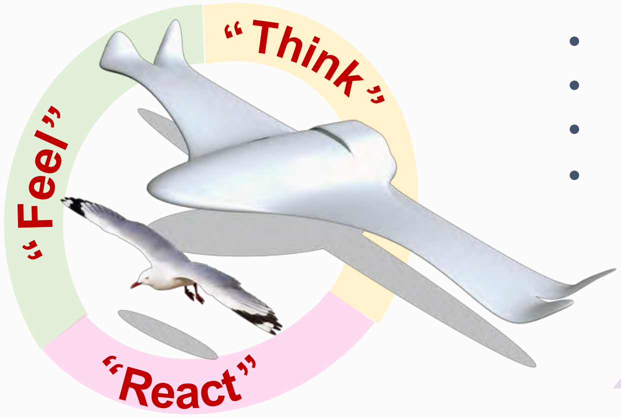


Research Framework



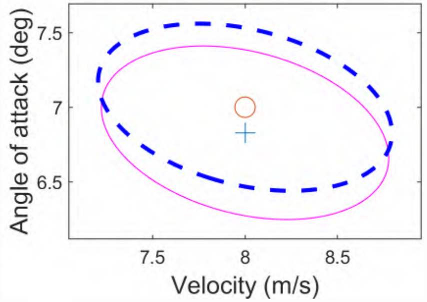
Smart Embedded Sensing

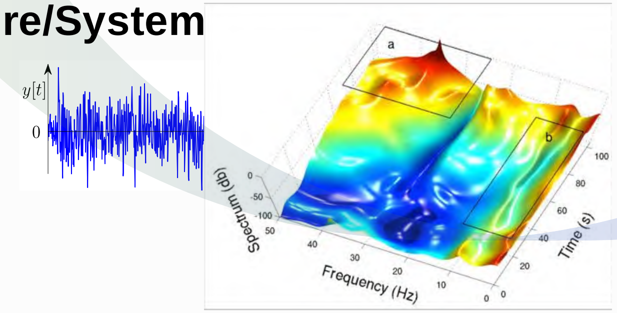
Integration with Host Structure/System

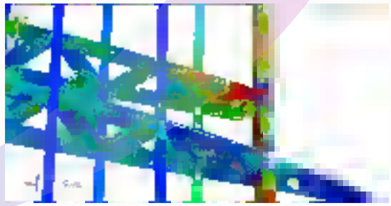


Complete State Awareness

- Real-time **structural** and **flight awareness**
- Real-time **health** monitoring
- **Fly-by-feel**
- Optimized control
- Life-cycle management



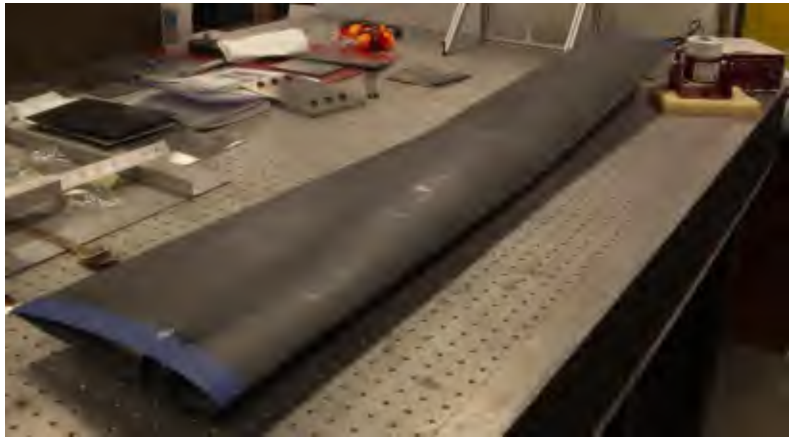


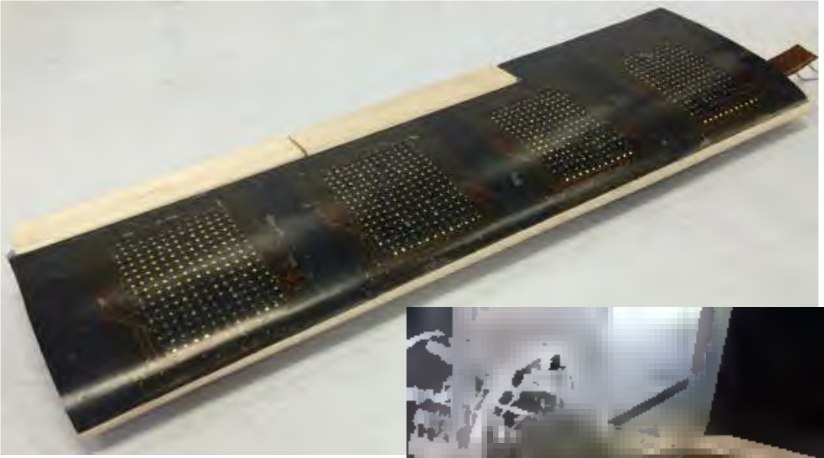


Data-driven stochastic modeling, learning & analysis

Integration with physics-based models for "smart data"

Prototype ISSL Demonstrators





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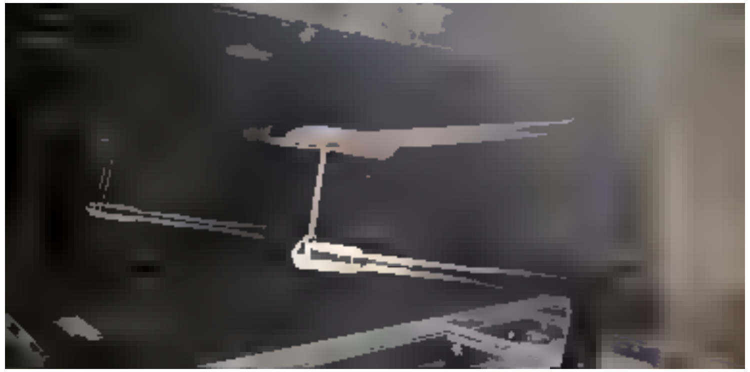
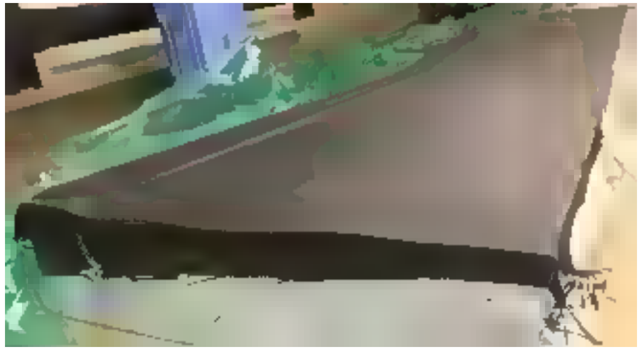
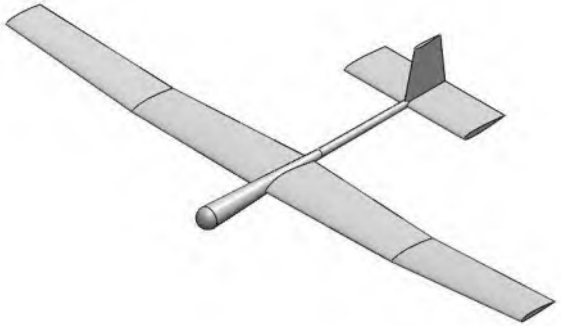
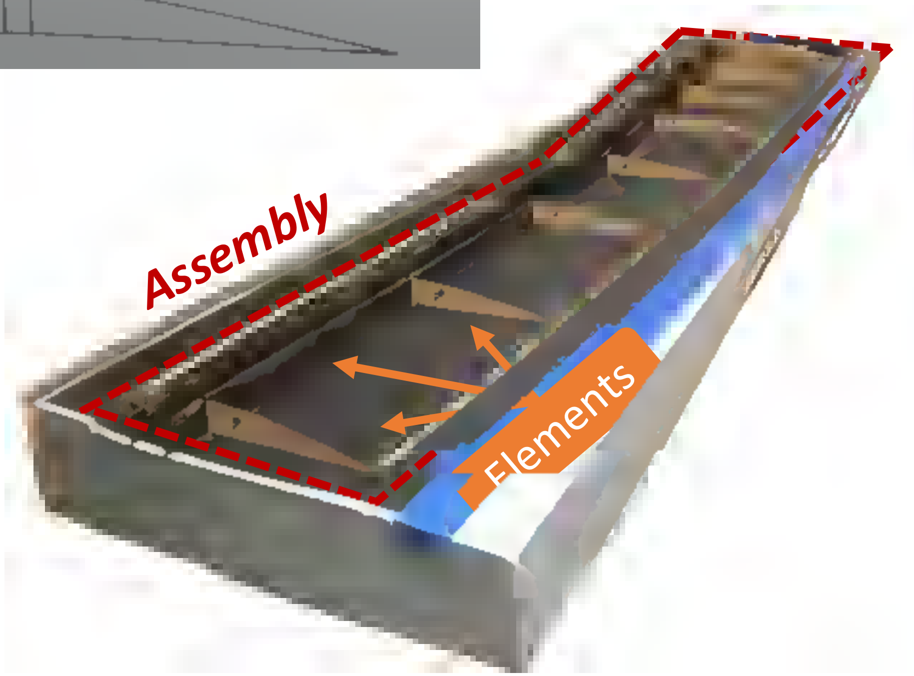




"Albatross"
Intelligent UAV platform

Test Case 1 – Structural Awareness



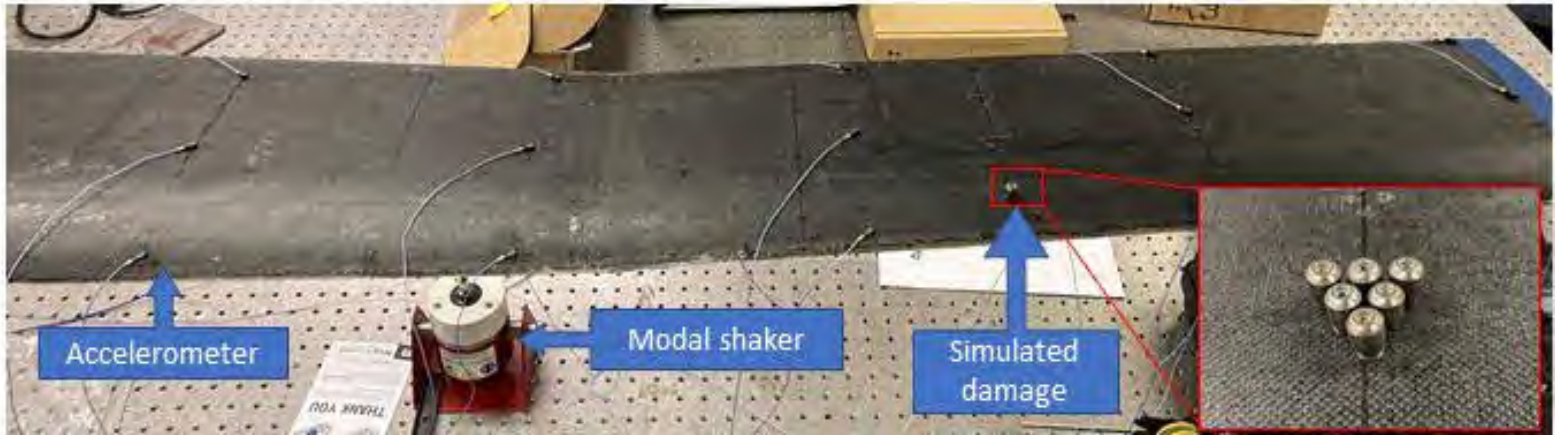


The Composite Wing:

- Half Span: **1.5 m**, chord: **0.3 m**
- Weight: ~**200 g**
- Construction:
  - Spar: CF-Epoxy laminate $(0_{2\times2twill}/0_{UD})\ symmetric$
  - Skin: CF-Epoxy laminate $(0_{2\times2twill})$
  - Rib: Plywood

The Experiments

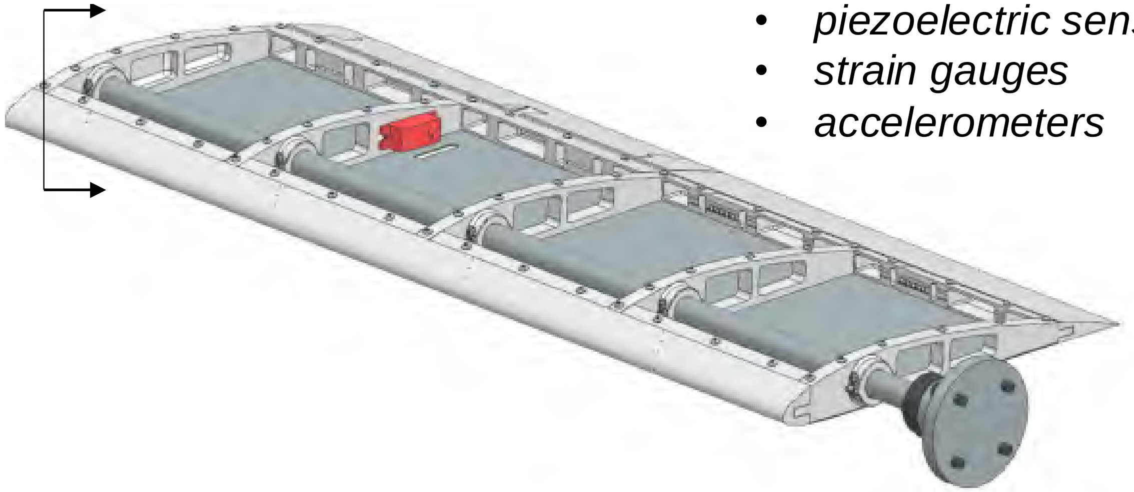
1. *Low-frequency random vibration*
2. *Cantilever fixture*

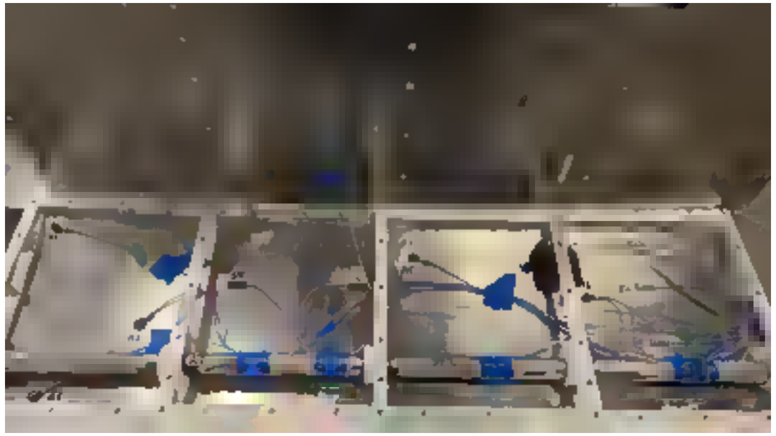


Test Case 2 – Flight Awareness

Modular Wing – Wind Tunnel Experimental Assessment

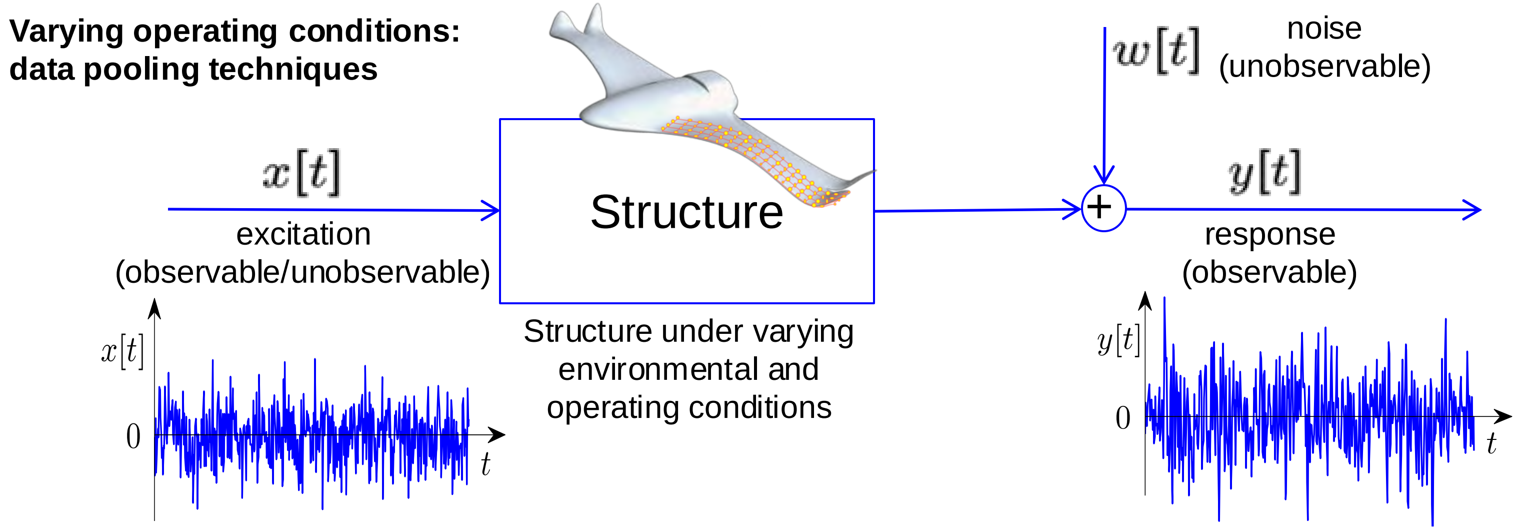
*A total of **48 integrated sensors***
- *piezoelectric sensors*
- *strain gauges*
- *accelerometers*





- **Angle of attack:** [0 – 15] degrees
- **Airspeed:** [8 – 20] m/s
- **Signal length** per data set: 128 s
- **Freq. Bandwidth**: [0.1 – 500] Hz
- **Total number of flight states: 192**

The Identification Approach



Uncertainty: statistical techniques

Differences and comparison with ML models?

*Given the sensing data (noise corrupted signals) identify the **structural dynamics** and determine the actual structural/health state.*

- **System Identification**: build mathematical models from sensing data
- Non-parametric and parametric models
- Stochastic time series models → **Discrete-time difference equations**
- Estimation of dynamics under *uncertainties/noise*

State Awareness – Main Challenge

Main Challenge: Dynamic/Varying States under Uncertainty

Operating State

Loads/moments

Stresses/strains

Speed

Attitude

Health State

System faults

Structural damage

Cascading effects

Propagation

Environment

Wind/gusts

Turbulence

Temperature/humidity

Icing

In addition to

Different missions

Operational usage

Maintenance

Costs

Data-Driven Framework

✓ ***State awareness***

✓ ***Health awareness***

✓ ***Self-diagnosis***

Standard Identification Approach

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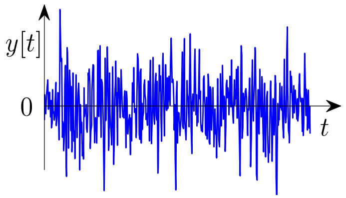
One systems generates all data!

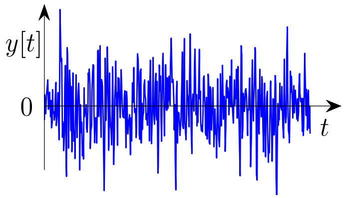
Airspeed: 10 m/s



Identify Model *1*

 # parameters

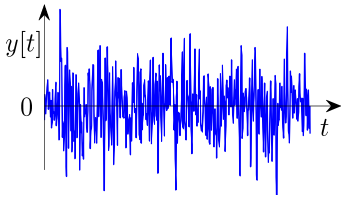
Airspeed: 12 m/s



Identify Model *2*

 # parameters

Airspeed: ... m/s



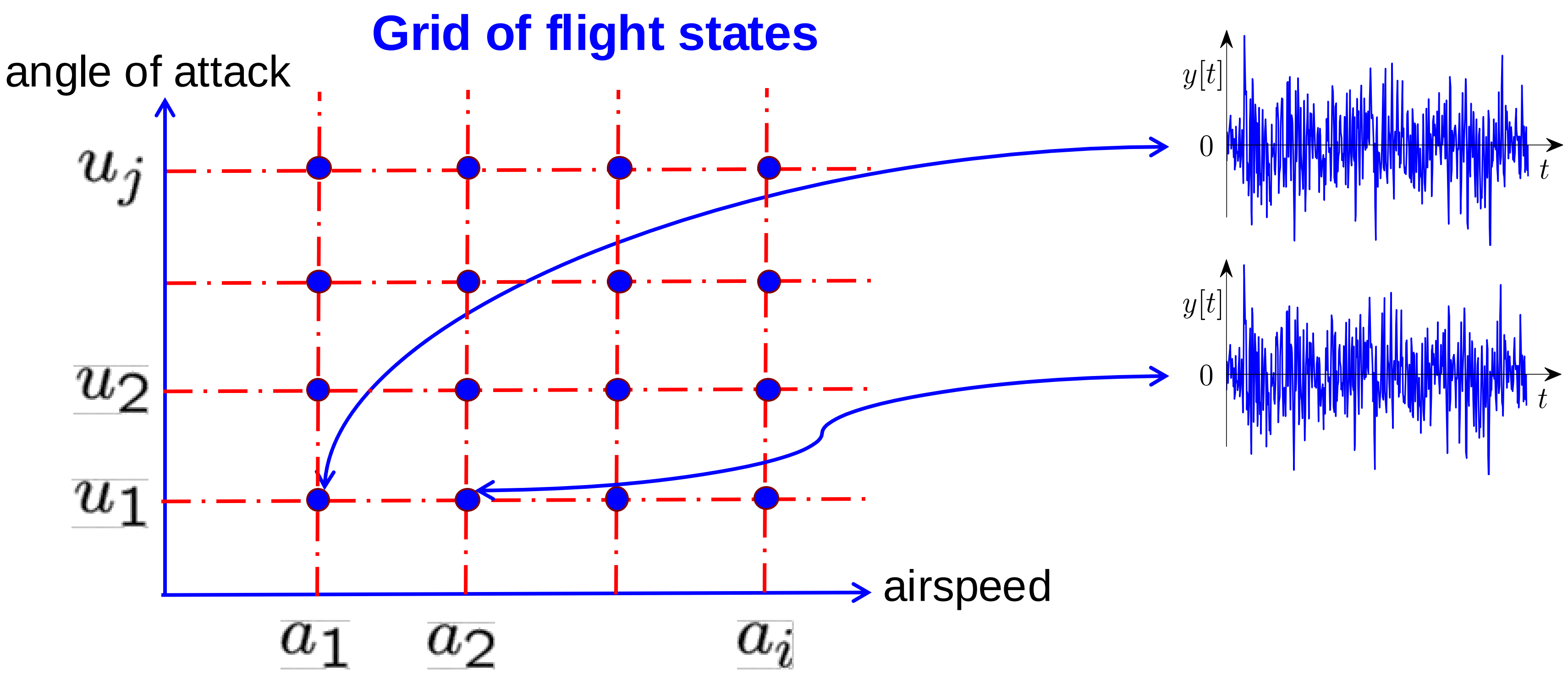
Identify Model *N*

 # parameters

data correlation is ignored!

- One model per data set → break the problem in ***unrelated sub-problems***
- **Total** number of parameters: *(number of models)* x *(number of parameters)*

Multiple Operating States



Data records are collected from sample **flight states**

Operating state vector:

$$\boldsymbol{k} = \begin{bmatrix} a_i \\ u_j \end{bmatrix}$$

airspeed / angle of attack

or

damage size / damage location/size

*The operating state vector **uniquely** defines the system state!*

(Kopsaftopoulos 2012; Kopsaftopoulos et al. 2016, 2018)

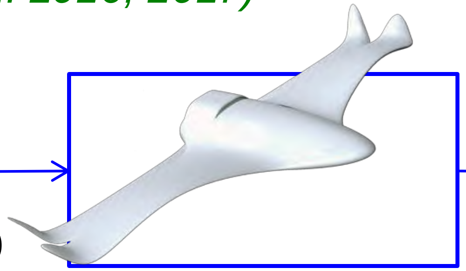
Data Functional Pooling

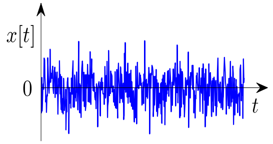
(Kopsaftopoulos 2012; Kopsaftopoulos et al. 2016, 2017)

$f[t]$

aerodynamic forces

(excitation usually unobservable)

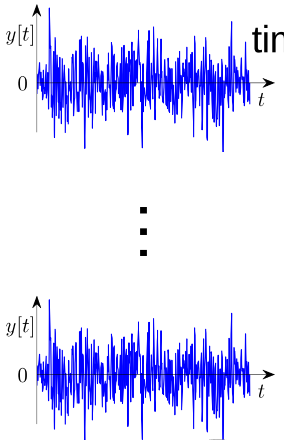




Aeroelastic structure under
varying flight states

$e[t]$ noise (unobservable)

$y[t]$ sensor response (observable)



time series

data under varying flight states ***k***
(cross sections)

Data records collected from sample flight states

functional data pooling

Stochastic "Global" Vector-dependent Functionally Pooled (VFP) Model

$$\boldsymbol{k} = \begin{bmatrix} a_i \\ u_j \end{bmatrix}$$

angle of attack

airspeed

$$y_{\boldsymbol{k}}[t] = \frac{\boldsymbol{B}(B,\boldsymbol{k})}{\boldsymbol{A}(B,\boldsymbol{k})} \cdot f_{\boldsymbol{k}}[t] + \frac{\boldsymbol{C}(B,\boldsymbol{k})}{\boldsymbol{A}(B,\boldsymbol{k})} \cdot w_{\boldsymbol{k}}[t]$$

Model parameters depend functionally on the flight state!

Functionally Pooled (FP) Global Models

Vector-Dependent Functionally Pooled Time Series Model: VFP AutoRegressive Model

sensor signal from flight state ***k***

noise term for flight state ***k***

$$y_{\boldsymbol{k}}[t] = \sum_{i=1}^{na} a_i(\boldsymbol{k}) \cdot y_{\boldsymbol{k}}[t-i] + w_{\boldsymbol{k}}[t]$$

model parameters **function of flight state *k***

$$w_{\boldsymbol{k}}[t] \sim \text{iid}\,\mathcal{N}(0, \sigma_w^2(\boldsymbol{k})) \quad \boldsymbol{k} \in R^2, \qquad E\{w_{k_{i,j}}[t] \cdot w_{k_{m,n}}[t-\tau]\} = \gamma_w[k_{i,j}, k_{m,n}] \cdot \delta[\tau]$$

Model parameters: $a_i(\boldsymbol{k}) \triangleq \sum_{j=1}^{pa} a_{i,j} \cdot G_{d_a(j)}(\boldsymbol{k})$

basis functions

The model parameters **functionally** depend on the flight state vector $\boldsymbol{k}$

Coefficients of projection: $\boldsymbol{\theta} = [a_{1,1}\ a_{1,2}\ \ldots\ a_{na,pa}]^T$ ⟹ *to be estimated from the sensor signals*

(do not depend on the flight state)

$\boldsymbol{k}$	:	flight state vector
$y_{\boldsymbol{k}}[t]$	:	response signals obtained under each flight state
$w_{\boldsymbol{k}}[t]$	:	innovations sequence (noise) signal
$a_{i,j}$	:	AR coefficients of projection
pa	:	AR functional base dimensionality

Stochastic Model Identification

A Functionally Pooled (FP) linear regression framework

The VFP–ARX model may be rewritten in linear regression form:

$$y_{\boldsymbol{k}}[t] = \left[\boldsymbol{\varphi}_{AR}^T[t] \otimes \boldsymbol{g}_{AR}^T(\boldsymbol{k}) \,\vdots\, \boldsymbol{\varphi}_X^T[t] \otimes \boldsymbol{g}_X^T(\boldsymbol{k})\right] \cdot \boldsymbol{\theta} + e_{\boldsymbol{k}}[t] = \boldsymbol{\phi}_{\boldsymbol{k}}^T[t] \cdot \boldsymbol{\theta} + e_{\boldsymbol{k}}[t]$$

with:

$$\begin{aligned}
\boldsymbol{\varphi}_{AR}[t] &\triangleq [-y_{\boldsymbol{k}}[t-1] \;\ldots\; -y_{\boldsymbol{k}}[t-na]]^T \\
\boldsymbol{\varphi}_X[t] &\triangleq [x_{\boldsymbol{k}}[t]\; x_{\boldsymbol{k}}[t-1] \;\ldots\; x_{\boldsymbol{k}}[t-nb]]^T \\
\boldsymbol{g}_{AR}(\boldsymbol{k}) &\triangleq [G_1(\boldsymbol{k}) \;\ldots\; G_{pa}(\boldsymbol{k})]^T_{[pa\times 1]} \\
\boldsymbol{g}_X(\boldsymbol{k}) &\triangleq [G_1(\boldsymbol{k}) \;\ldots\; G_{pb}(\boldsymbol{k})]^T_{[p_b\times 1]}
\end{aligned}$$

Coefficients of projection vector

$$\boldsymbol{\theta} \triangleq [a_{1,1} \;\ldots\; a_{na,pa} \,\vdots\, b_{0,1} \;\ldots\; b_{nb,pb}]^T$$

Having data: $x_{\boldsymbol{k}}[t],\; y_{\boldsymbol{k}}[t] \qquad (t = 1, \ldots, N) \qquad (\boldsymbol{k} \rightarrow \underbrace{k_{1,1}, k_{1,2}, \ldots, k_{M_1,M_2}}_{\text{different operating conditions}})$

the VFP–ARX expression gives:

$$\begin{bmatrix} \boldsymbol{y}[1] \\ \vdots \\ \boldsymbol{y}[N] \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}[1] \\ \vdots \\ \boldsymbol{\Phi}[N] \end{bmatrix} \cdot \boldsymbol{\theta} + \begin{bmatrix} \boldsymbol{e}[1] \\ \vdots \\ \boldsymbol{e}[N] \end{bmatrix} \implies \boxed{\boldsymbol{y} = \boldsymbol{\Phi} \cdot \boldsymbol{\theta} + \boldsymbol{e}}$$

Model Parameter Estimation

Ordinary Least Squares (OLS) criterion

$$J^{\text{OLS}}(\boldsymbol{\theta}, Z^{NM_1M_2}) \triangleq \frac{1}{N}\sum_{t=1}^{N} \boldsymbol{e}^T[t]\boldsymbol{e}[t] = \frac{1}{N}\boldsymbol{e}^T\boldsymbol{e}$$

$$\widehat{\boldsymbol{\theta}}^{\text{OLS}} = [\boldsymbol{\Phi}^T\boldsymbol{\Phi}]^{-1}[\boldsymbol{\Phi}^T\boldsymbol{y}], \quad \widehat{\boldsymbol{\Gamma}}^{\text{OLS}}_{\boldsymbol{w}[t]} = \frac{1}{N}\sum_{t=1}^{N} \boldsymbol{e}[t, \widehat{\boldsymbol{\theta}}^{\text{OLS}}]\boldsymbol{e}^T[t, \widehat{\boldsymbol{\theta}}^{\text{OLS}}]$$

$\widehat{\boldsymbol{\Gamma}}^{\text{OLS}}_{\boldsymbol{w}[t]}$: consistent estimator

Weighted Least Squares (WLS) criterion

$$J^{\text{WLS}}(\boldsymbol{\theta}, Z^{NM_1M_2}) \triangleq \frac{1}{N}\sum_{t=1}^{N} \boldsymbol{e}^T[t]\boldsymbol{\Gamma}^{-1}_{\boldsymbol{w}[t]}\boldsymbol{e}[t] = \frac{1}{N}\boldsymbol{e}^T\boldsymbol{\Gamma}^{-1}_{\boldsymbol{w}}\boldsymbol{e}$$

$$\widehat{\boldsymbol{\theta}}^{\text{WLS}} = [\boldsymbol{\Phi}^T\boldsymbol{\Gamma}^{-1}_{\boldsymbol{w}}\boldsymbol{\Phi}]^{-1}[\boldsymbol{\Phi}^T\boldsymbol{\Gamma}^{-1}_{\boldsymbol{w}}\boldsymbol{y}], \quad \widehat{\boldsymbol{\Gamma}}^{\text{WLS}}_{\boldsymbol{w}[t]} = \frac{1}{N}\sum_{t=1}^{N} \boldsymbol{e}[t, \widehat{\boldsymbol{\theta}}^{\text{WLS}}]\boldsymbol{e}^T[t, \widehat{\boldsymbol{\theta}}^{\text{WLS}}]$$

$\widehat{\boldsymbol{\Gamma}}^{\text{OLS}}_{\boldsymbol{w}[t]}$ used in 1^{st} stage

$\widehat{\boldsymbol{\Gamma}}^{\text{WLS}}_{\boldsymbol{w}[t]}$: consistent estimator

Maximum Likelihood (ML) estimation *(non-linear optimization problem)*

$\widehat{\boldsymbol{\theta}}^{\text{ML}} \triangleq \arg\max_{\boldsymbol{\theta}} L(\boldsymbol{\theta}/\boldsymbol{e}) = \arg\max_{\boldsymbol{\theta}} \ln p(\boldsymbol{e}/\boldsymbol{\theta})$, $L(\cdot)$ the logarithm of the conditional likelihood

$$\widehat{\boldsymbol{\theta}}^{\text{ML}} = \arg\min_{\boldsymbol{\theta}} \ln\det\{\boldsymbol{\Lambda}(\boldsymbol{\theta})\}, \quad \boldsymbol{\Lambda}(\boldsymbol{\theta}) \triangleq \frac{1}{N}\sum_{t=1}^{N} \boldsymbol{e}[t,\boldsymbol{\theta}]\boldsymbol{e}^T[t,\boldsymbol{\theta}], \quad \widehat{\boldsymbol{\Gamma}}_{\boldsymbol{w}[t]} = \frac{1}{N}\sum_{t=1}^{N} \boldsymbol{e}[t, \widehat{\boldsymbol{\theta}}^{\text{ML}}]\boldsymbol{e}^T[t, \widehat{\boldsymbol{\theta}}^{\text{ML}}]$$

Why stochastic time series models?

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Differences and comparison with ML models?

▷ **Consistency**: Probabilistic convergence of the estimator to its "true" ($\boldsymbol{\vartheta}^o$) value:

$$\widehat{\boldsymbol{\vartheta}}_N \xrightarrow{p} \boldsymbol{\vartheta}^o$$

↪ **A physical interpretation:**

"As the number of observations increases more information of the same kind is added".

Theorem 1: *Least Squares estimator consistency*. Let $\boldsymbol{\theta}_o$ be the true projection coefficient vector, $w_{\boldsymbol{k}}[t]$ a white zero mean process with $E\{w_{\boldsymbol{k}}^2[t]\} = \sigma_w^2(\boldsymbol{k})$ for every operating point, and $E\{\boldsymbol{\phi_k}[t]\boldsymbol{\phi_k}^T[t]\}$ a nonsingular matrix. Then:

$$\widehat{\boldsymbol{\vartheta}}_N^{\text{LS}} \xrightarrow{a.s.} \boldsymbol{\vartheta}_o \qquad (N \longrightarrow \infty).$$

with $a.s.$ designating **almost sure** (strong) convergence.

Theorem 2: *Least Squares estimator consistency (projection of innovations std. dev.)*. Let $\boldsymbol{\vartheta}_o$ be the true projection coefficients vector, $w_{\boldsymbol{k}}[t]$ a white zero mean process with the innovations standard deviation bounded $0 < \underline{\sigma_w} \leq \boldsymbol{g}_s^T(\boldsymbol{k})\boldsymbol{s} \leq \overline{\sigma_w} < \infty$ for every operating point, and $E\{\boldsymbol{\varphi_k}[t]\boldsymbol{\varphi_k}^T[t]\}$ a nonsingular matrix. Then,

$$\widehat{\boldsymbol{\vartheta}}_N^{\text{WLS}} \xrightarrow{a.s.} \boldsymbol{\vartheta}_o \qquad (N \longrightarrow \infty).$$

Theorem 3: *Maximum Likelihood estimator consistency*. Let $\bar{\boldsymbol{\theta}}_o = \left[\boldsymbol{\theta}_o^T \ \vdots\ \gamma_w[k_{i,j}, k_{m,n}]\right]$ be the true parameter vector, $w_{\boldsymbol{k}}[t]$ a normally distributed zero mean white process with $E\{w_{\boldsymbol{k}}^2[t]\} = \sigma_w^2(\boldsymbol{k})$ for every operating point, and $E\{\boldsymbol{\phi_k}[t]\boldsymbol{\phi_k}^T[t]\}$ a nonsingular matrix. Then:

$$\widehat{\bar{\boldsymbol{\theta}}}_N^{\text{ML}} \xrightarrow{a.s.} \bar{\boldsymbol{\theta}}_o \qquad (N \longrightarrow \infty).$$

Test Case 1 – Varying Structural States

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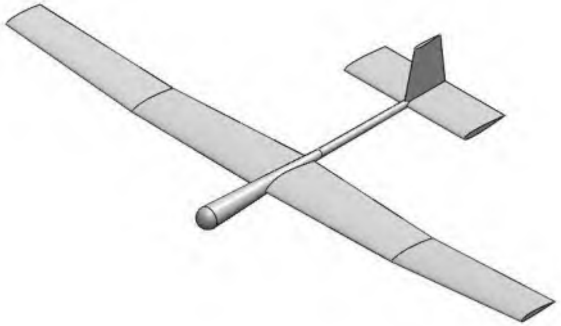
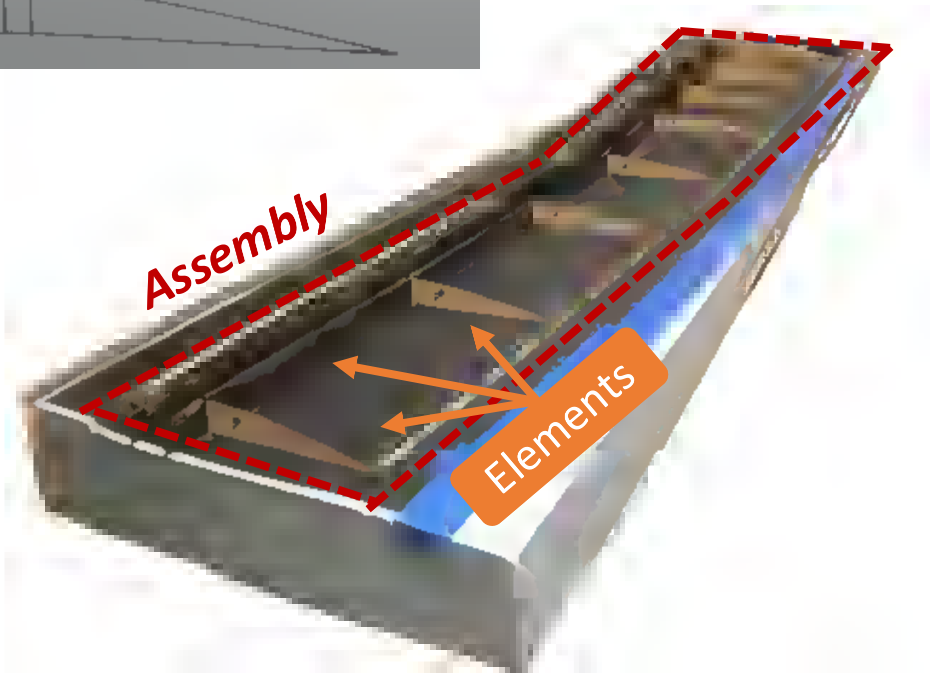


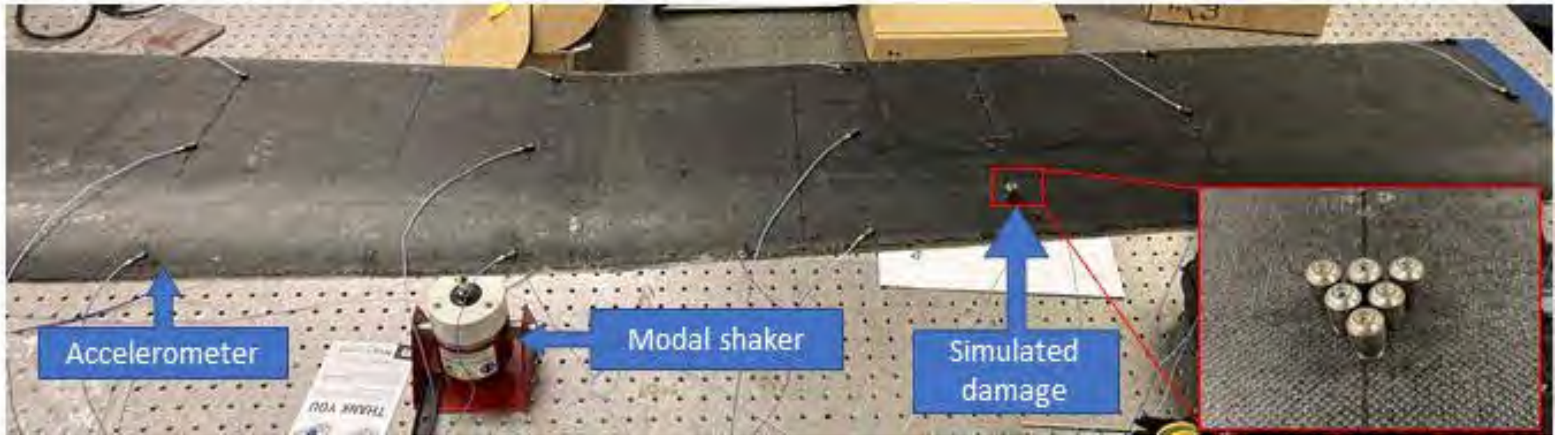


The Composite Wing:

- Half Span: **1.5 m**, chord: **0.3 m**
- Weight: ~**200 g**
- Construction:
  - Spar: CF-Epoxy laminate $(0_{2\times 2twill}/0_{UD})\ symmetric$
  - Skin: CF-Epoxy laminate $(0_{2\times 2twill})$
  - Rib: Plywood

The Experiments

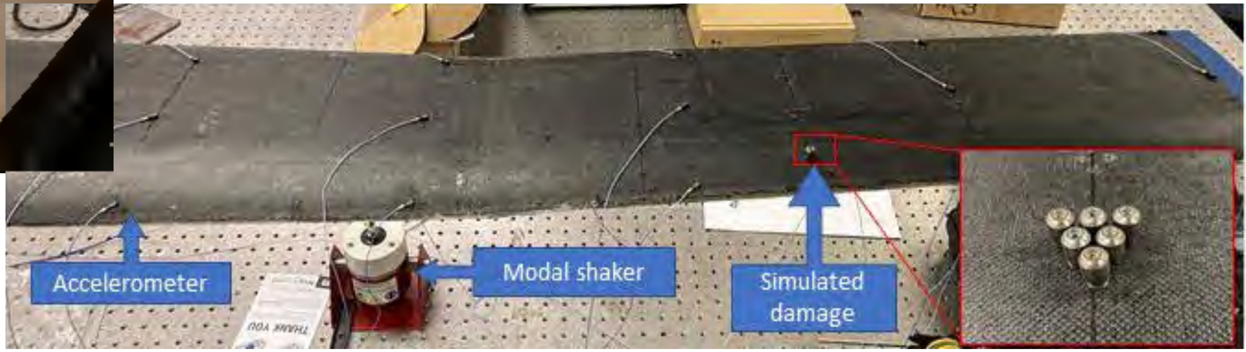
1. *Low-frequency random vibration*
2. *Cantilever fixture*

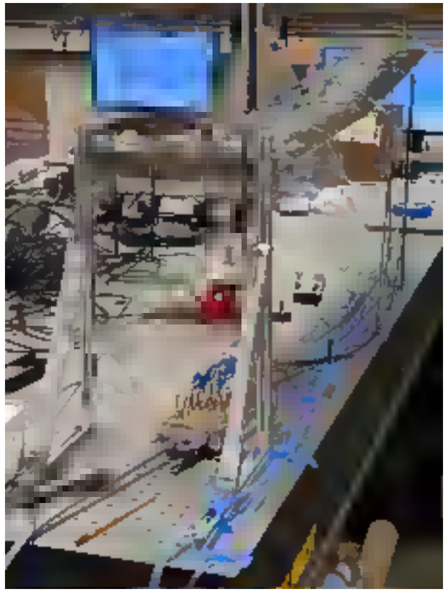


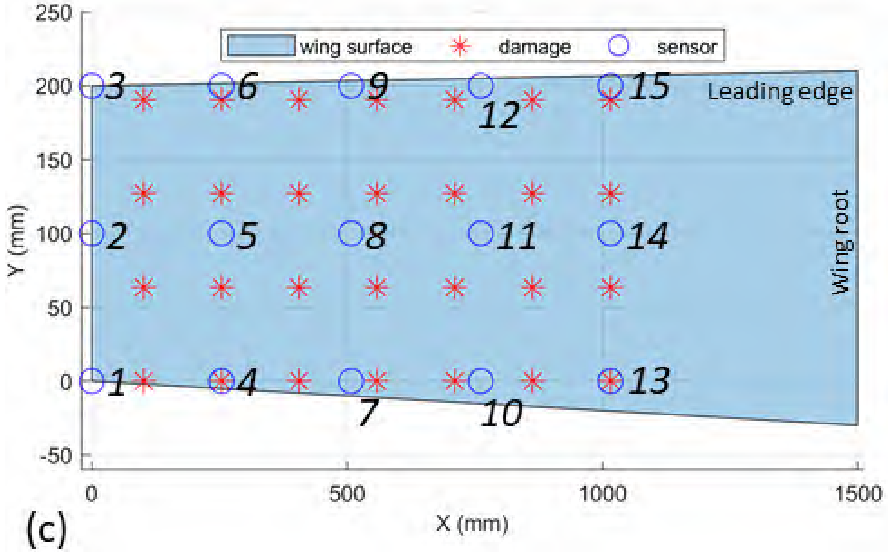
Experimental Setup – Composite Wing

- Sensor types
  - Accelerometers PCB 352C23
  - Glued with Loctite401
- Sensor topology
  - 3x5 grid on wing skin
  - $x_{sensor} \sim \{0, 10, 20, 30, 40\}\ in$
  - $y_{sensor} \sim \{0, 4, 8\}\ in$
- Damage topology
  - **4x7 grid of damage locations**
  - $x_{damage} \sim \{4, 10, 16, 22, 28, 34, 40\}\ in$
  - $y_{damage} \sim \{0, 2.5, 5, 7.5\}\ in$
  - **7 weights to simulate local damage**
  - $w_{damage} \sim \{0, 3, 6, 9, 12, 15, 18\}\ g$
- Data acquisition
  - **Random excitation**
  - Sampling frequency: $F_s = 512\ Hz$
  - Data length: 64 s

Total number of structural states: 168



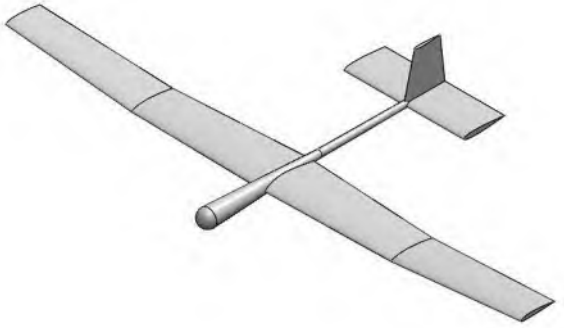




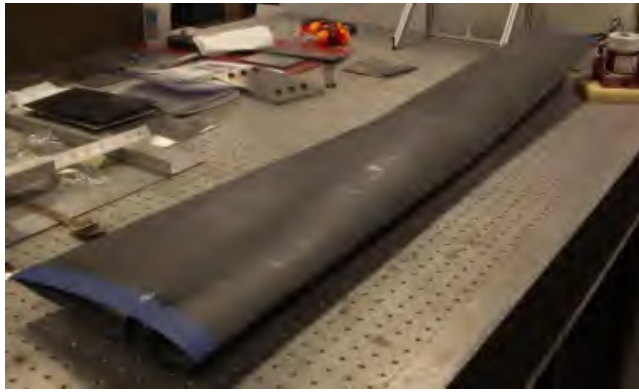
The Main Challenges

A. How to **overcome** the trade-off between detection sensitivity and robustness to uncertainty?

B. How to **detect, localize** and **quantify** damage under uncertainty?



- Varying operational and environmental conditions → system behavior and dynamics continuously change
- Seemingly-identical components across fleet/structure → limited applicability of deterministic techniques



C. How to formulate the **inverse problem** (state estimation)?

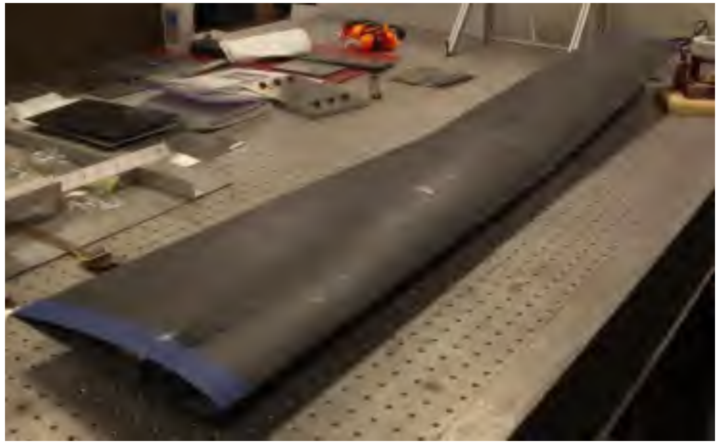
- Model accuracy and predictive capability (**forward problem**) does not necessarily result in "effective" state estimation (**inverse problem**)
- Model **generalization**
- Overfitting vs statistical parsimony vs inverse problem loss function

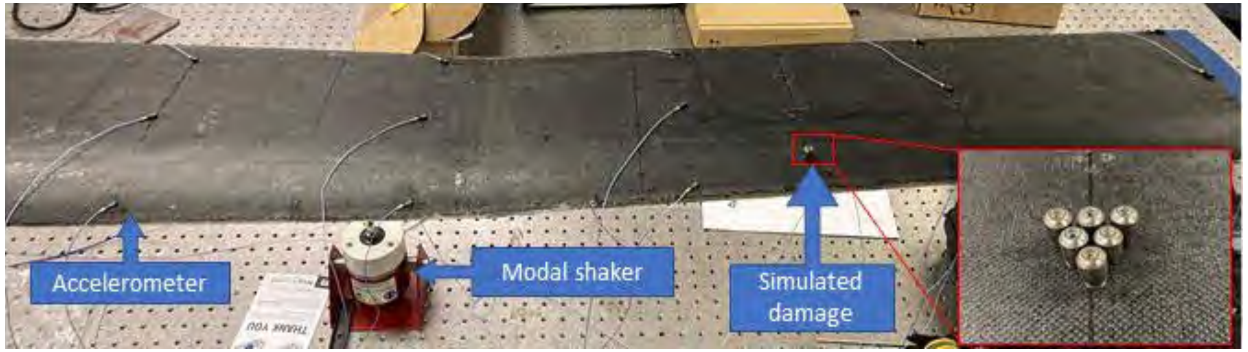
Problem Statement

*Investigate and assess the state estimation (**inverse problem**) performance of stochastic time-series methods for SHM*

Based on dynamic sensing:

- *Identify **accurate** and **robust** data-driven stochastic models under **dynamic operating/environmental/structural states***
- *Detect, localize and quantify **structural damage (inverse problem)***
- ***Assess** model performance and generalization capability*
- ***Explore and assess** model regularization and Bayesian state estimation methods*





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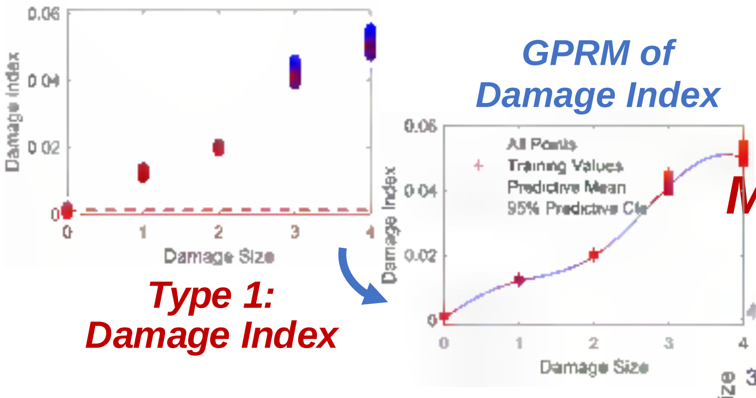
Statistical and Probabilistic Methods for SHM

Proper modeling of uncertainties

↓

***Stochastic** time series and **Gaussian process** regression models*

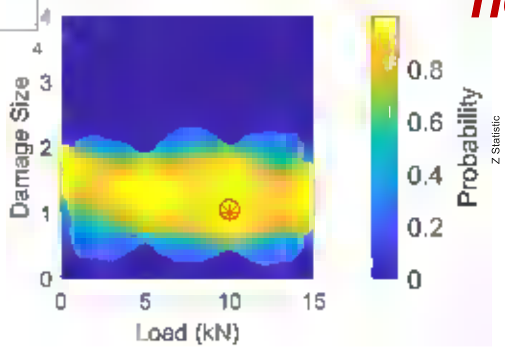
Modelling uncertainty in DIs



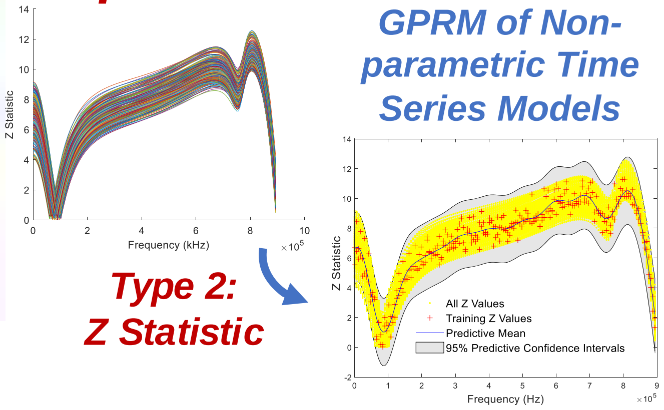
Type 1: Damage Index

GPRM of Damage Index

Modelling loads & damage



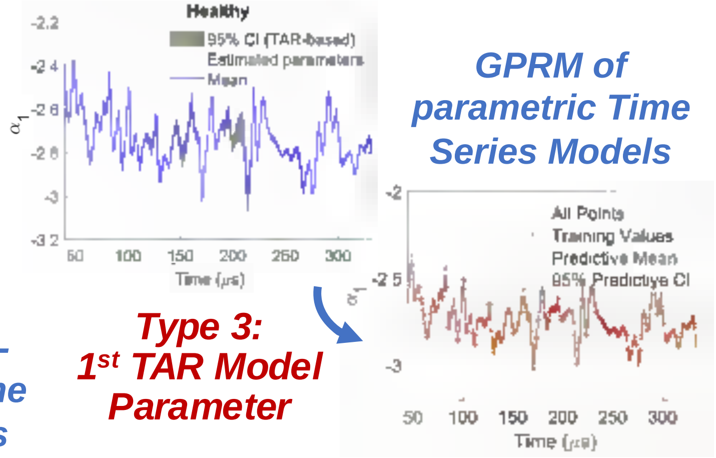
Modelling damage via non-parametric models



Type 2: Z Statistic

GPRM of Non-parametric Time Series Models

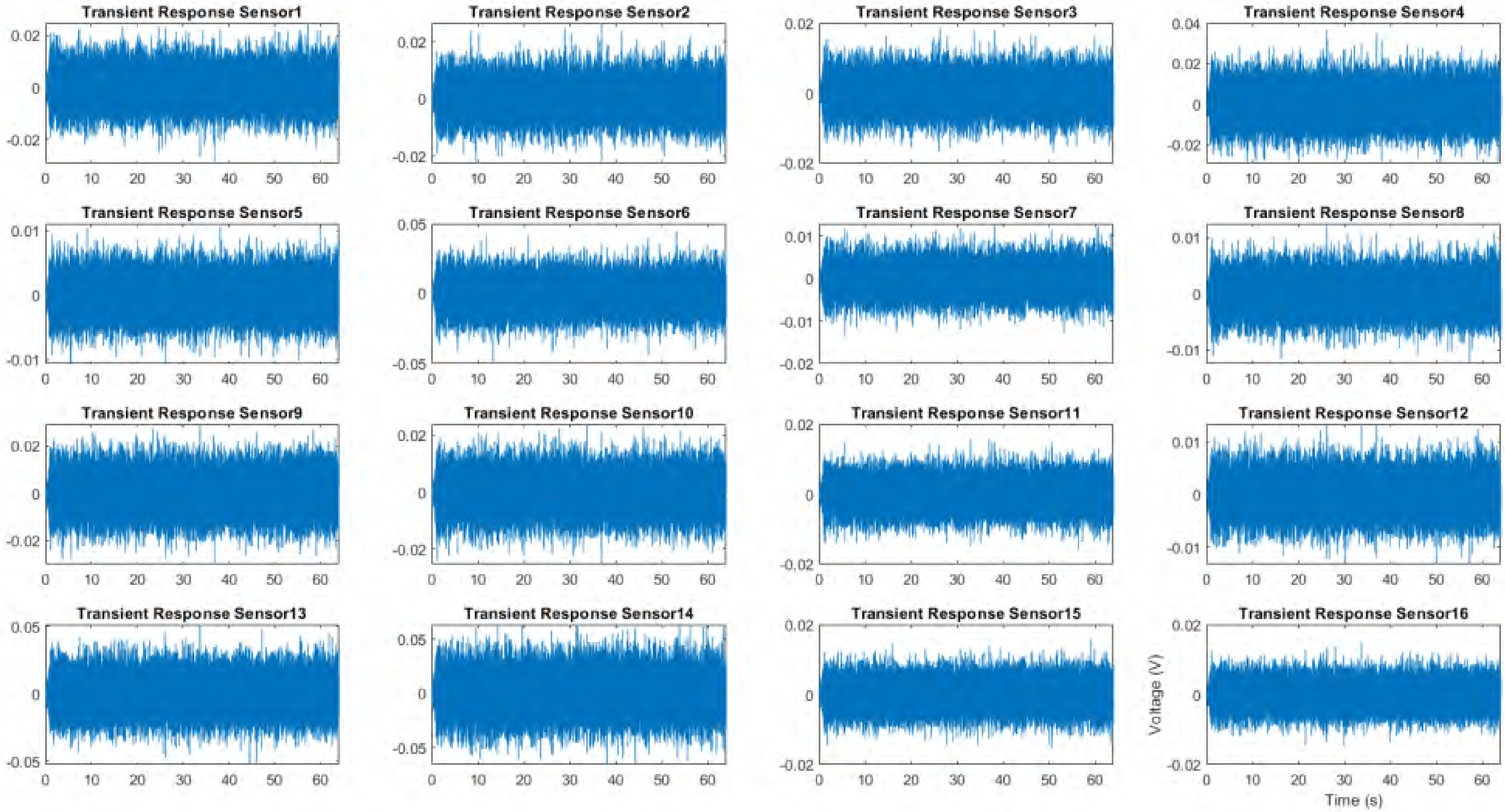
Modelling non-stationarity



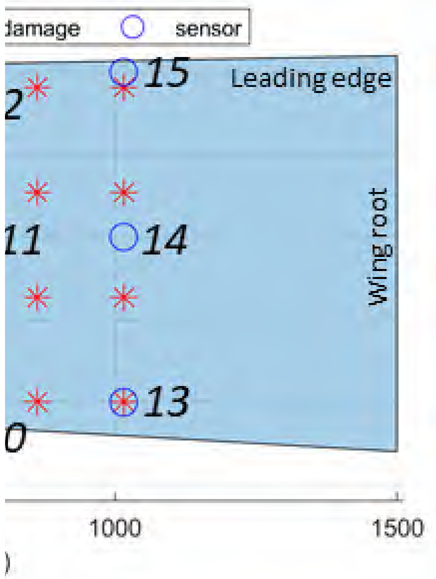
Type 3: 1st TAR Model Parameter

GPRM of parametric Time Series Models

Signals and Non-Parametric Analysis



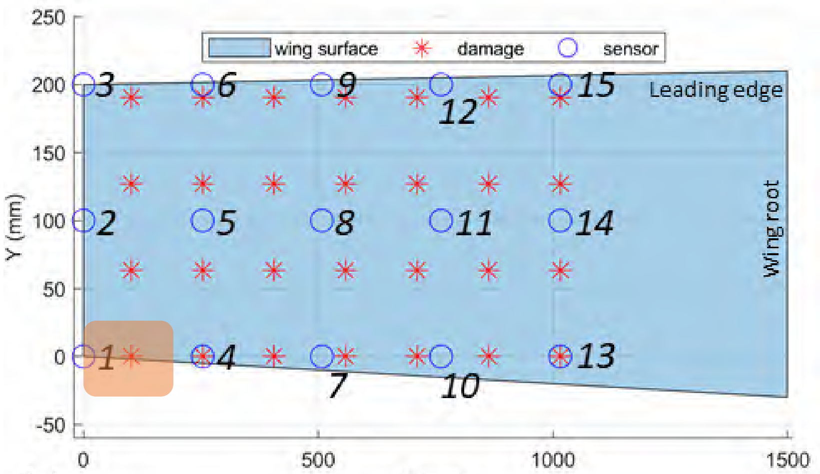
on [x = 4 in, y = 0 in]
or output
#4 d) sensor #8
be observable



The Structural Damage Test Cases

Case I: Damage magnitude estimation on a **single location**

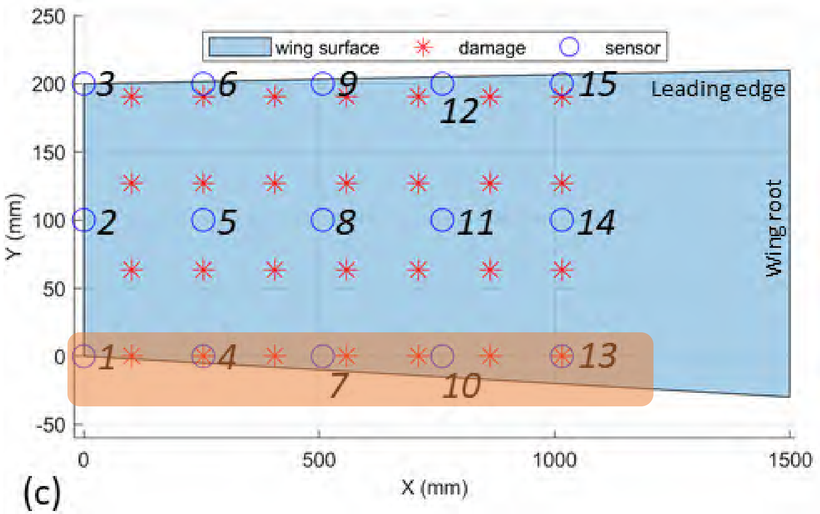
- *Functionally Pooled (FP) ARX model*
  - x_{damage}: $\{4\}$ in
  - y_{damage}: $\{0\}$ in
  - $w_{damage} \in \{0, 3, 6, 9, 12, 15, 18\}$ g
  - **7 structural states**



(c)

Case II: Damage localization and magnitude estimation along ***wing span***

- *Vector-dependent Functionally Pooled (VFP) ARX model*
  - $x_{damage} \in \{4, 10, 16, 22, 28, 34, 40\}$ in
  - y_{damage}: $\{0\}$ in
  - $w_{damage} \in \{0, 3, 6, 9, 12, 15, 18\}$ g
  - **49 structural states**



(c)

State Estimation – The Inverse Problem

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Main idea: *use data from available sensors to estimate **indirectly** the structural state*

current "unknown" signals

"Global" Stochastic Model

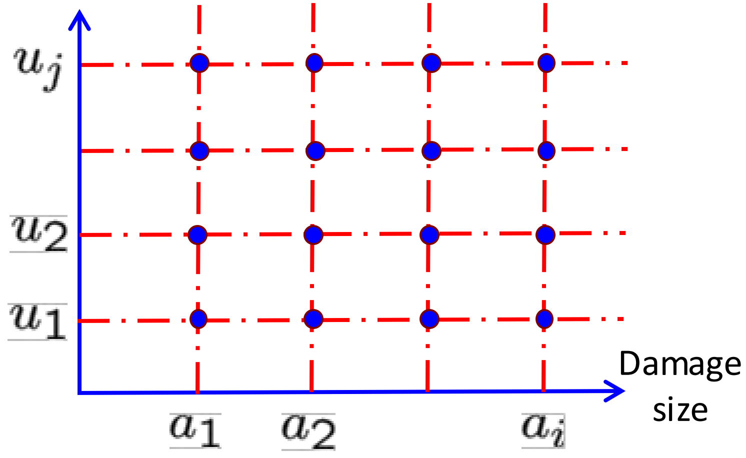
$$y_{\boldsymbol{k}}[t] + \sum_{i=1}^{na} a_i(\boldsymbol{k}) \cdot y_{\boldsymbol{k}}[t-i] = \sum_{i=0}^{nb} b_i(\boldsymbol{k}) \cdot x_{\boldsymbol{k}}[t-i] + w_{\boldsymbol{k}}[t]$$

Model Estimation

Training Phase

Damage location

Grid of structural states



data obtained via simulations, or experiments

Re-parameterized model

(unknown structural state vector)

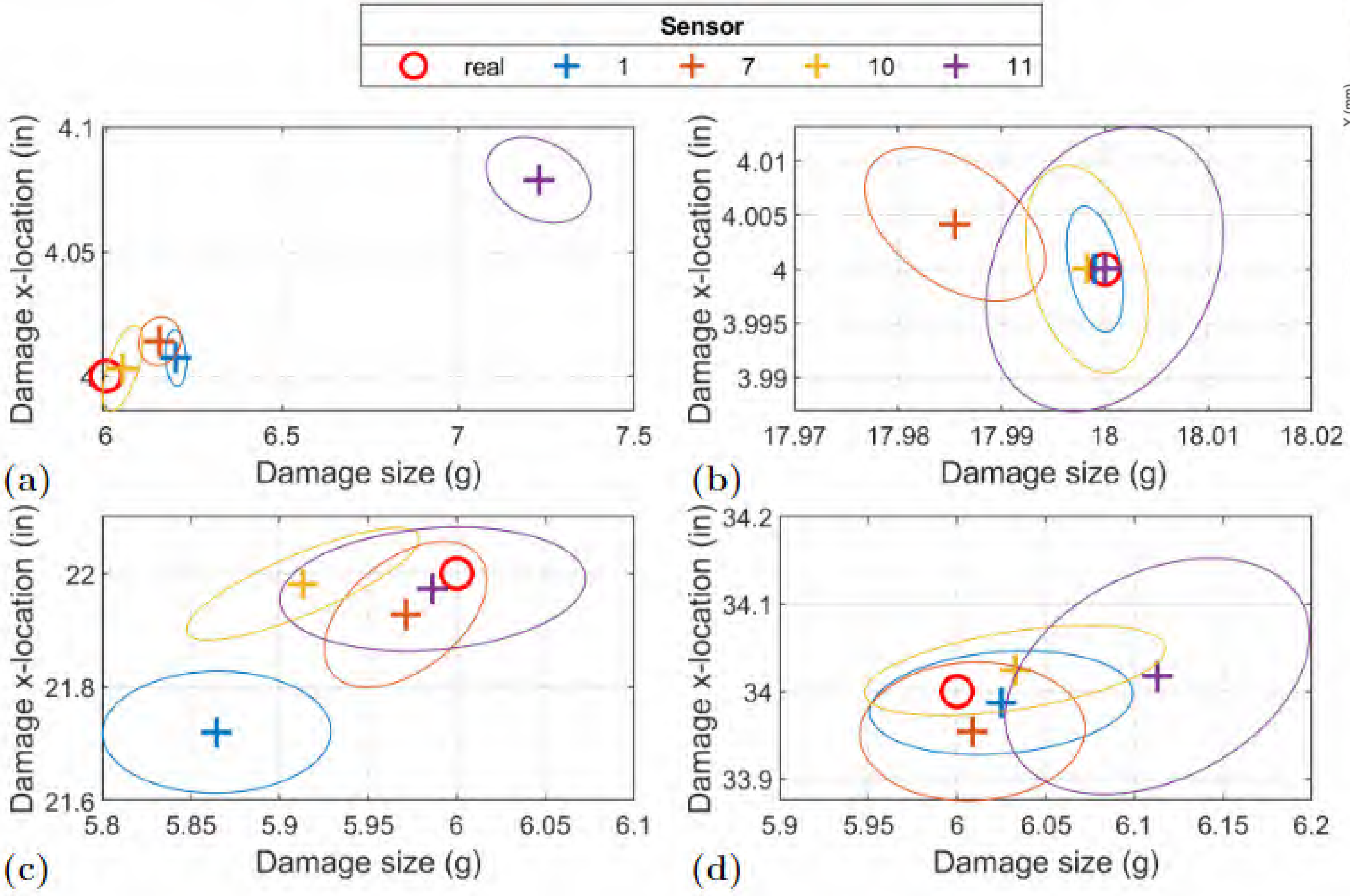
$$\hat{\boldsymbol{k}}^A \triangleq \arg\min_{\boldsymbol{k}} \sum_{t=1}^{N} e_u^2[t] \ , \qquad \sigma_e^2(\hat{\boldsymbol{k}}^A) = \frac{1}{N} \sum_{t=1}^{N} e_u^2[t, \hat{\boldsymbol{k}}^A]$$

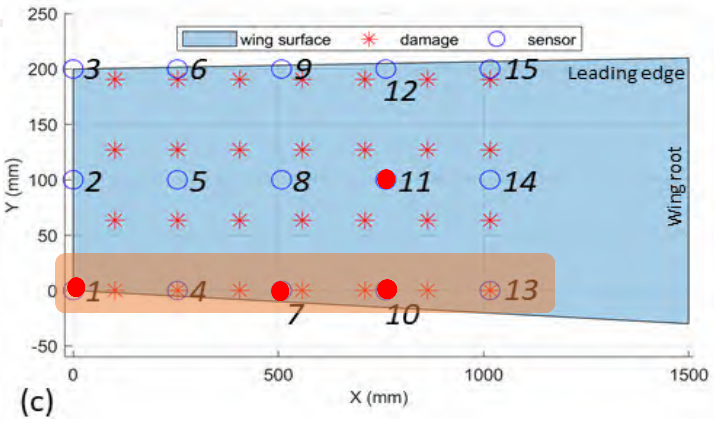
GAs + constrained nonlinear optimization

Inverse estimation

Nonlineasr Optimization Framework

Damage State Estimation





State Estimation – Issues Revealed

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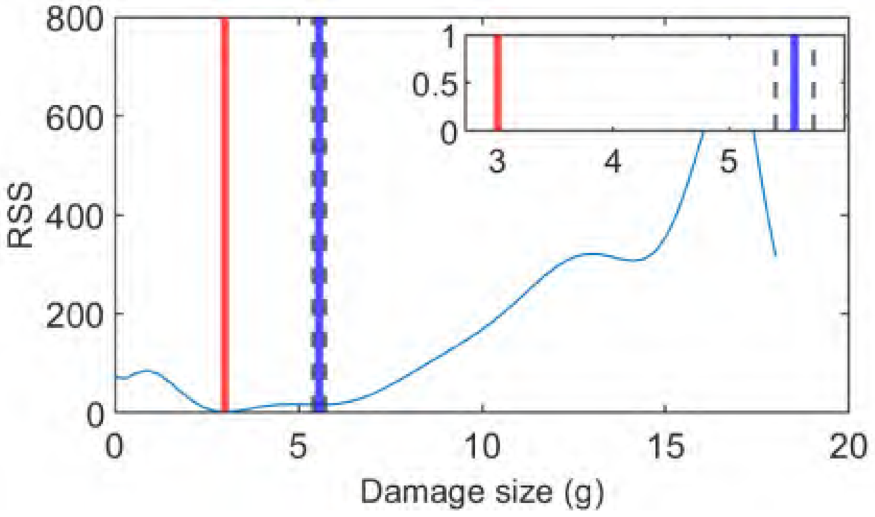
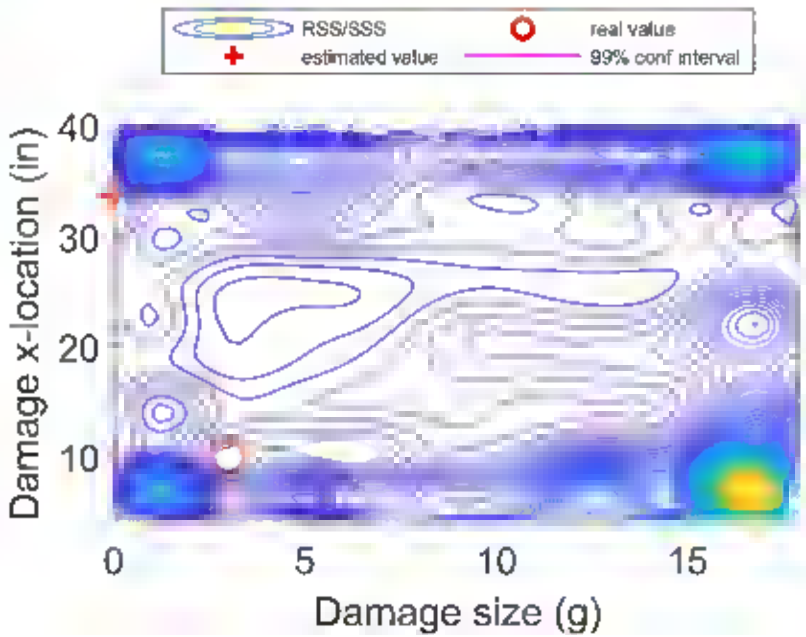
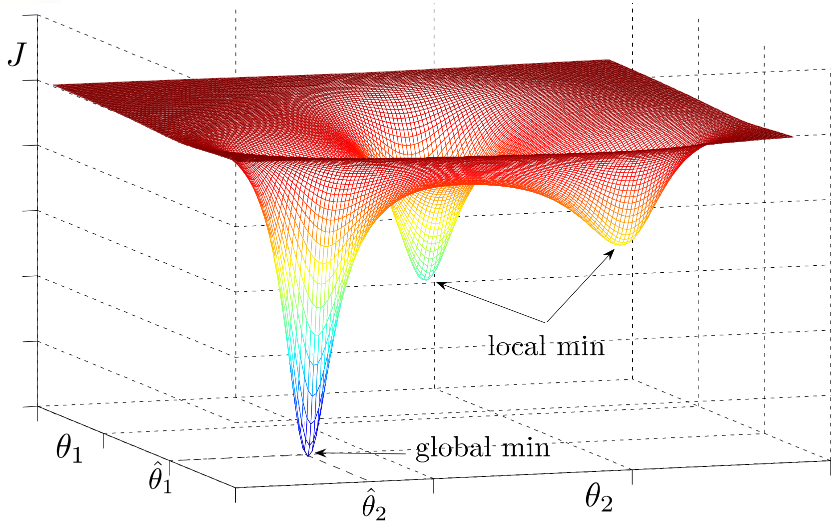






Potential solution: Regularized parameter estimation

Weighted Least Squares (WLS) Estimator

$$\widehat{\boldsymbol{\theta}}^{WLS} = \arg\min_{\boldsymbol{\theta}} \frac{1}{NM_1M_2}\boldsymbol{e}^T\boldsymbol{\Gamma}_{\boldsymbol{e}}^{-1}\boldsymbol{e}$$

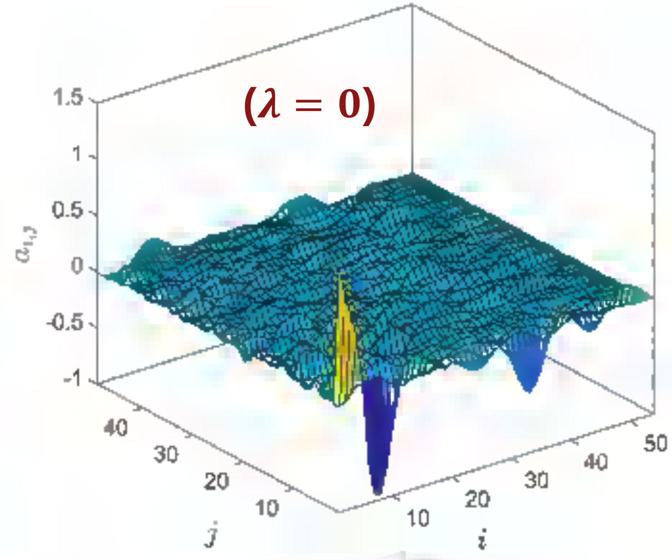
LASSO-WLS Estimator

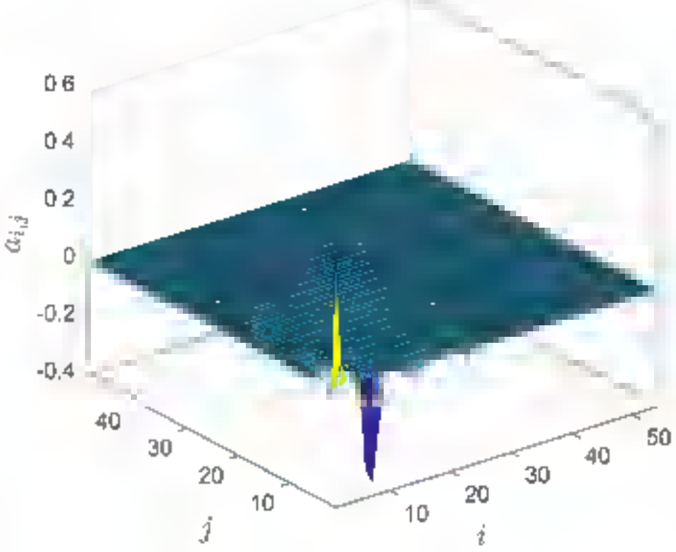
penalty term

$$\widehat{\boldsymbol{\theta}}^{LASSO} = \arg\min_{\boldsymbol{\theta}} \frac{1}{NM_1M_2}\boldsymbol{e}^T\boldsymbol{\Gamma}_{\boldsymbol{e}}^{-1}\boldsymbol{e} + \lambda||\boldsymbol{\theta}||_1$$

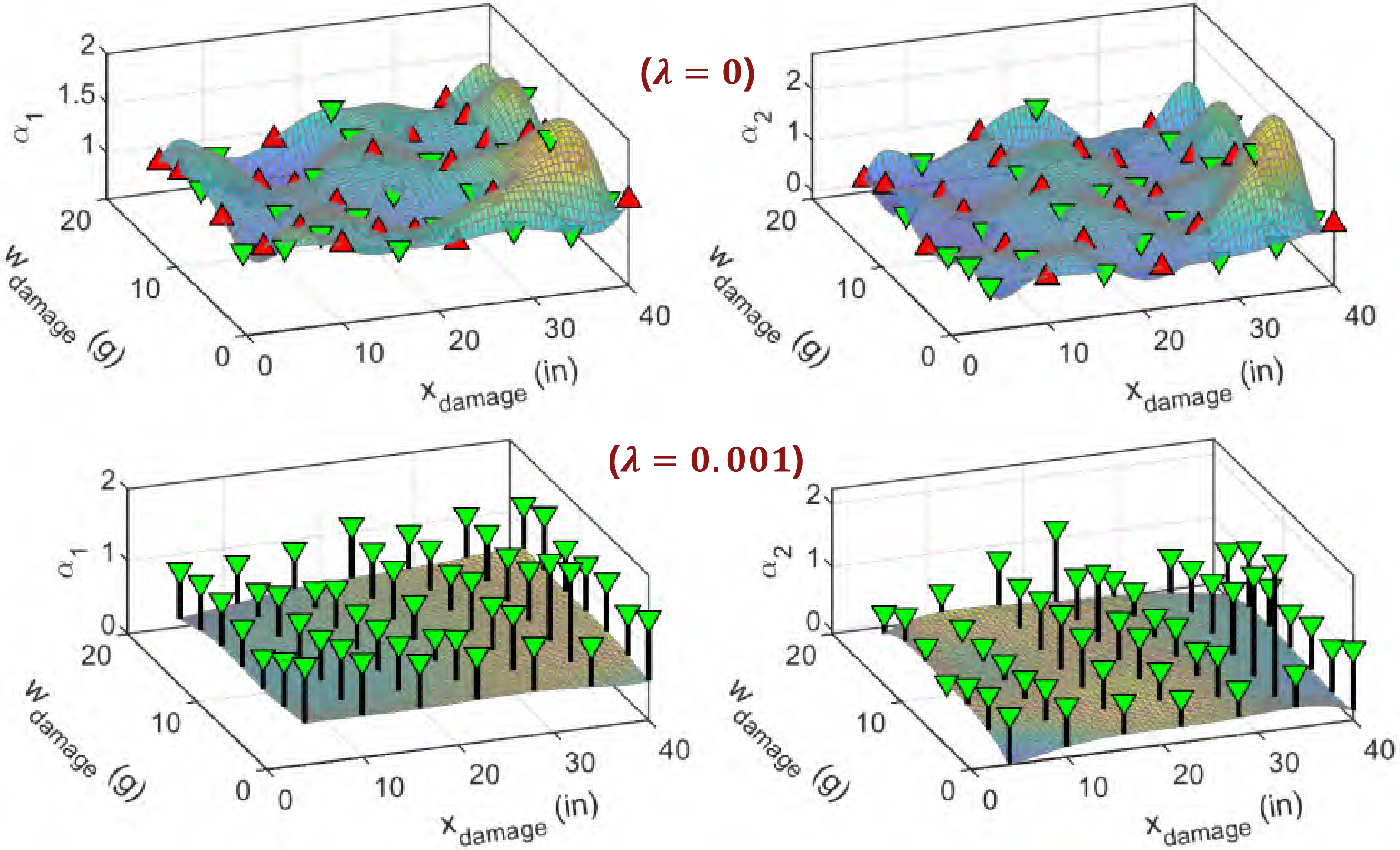
Why Regularization Works?

5341 Coefficients of Projection





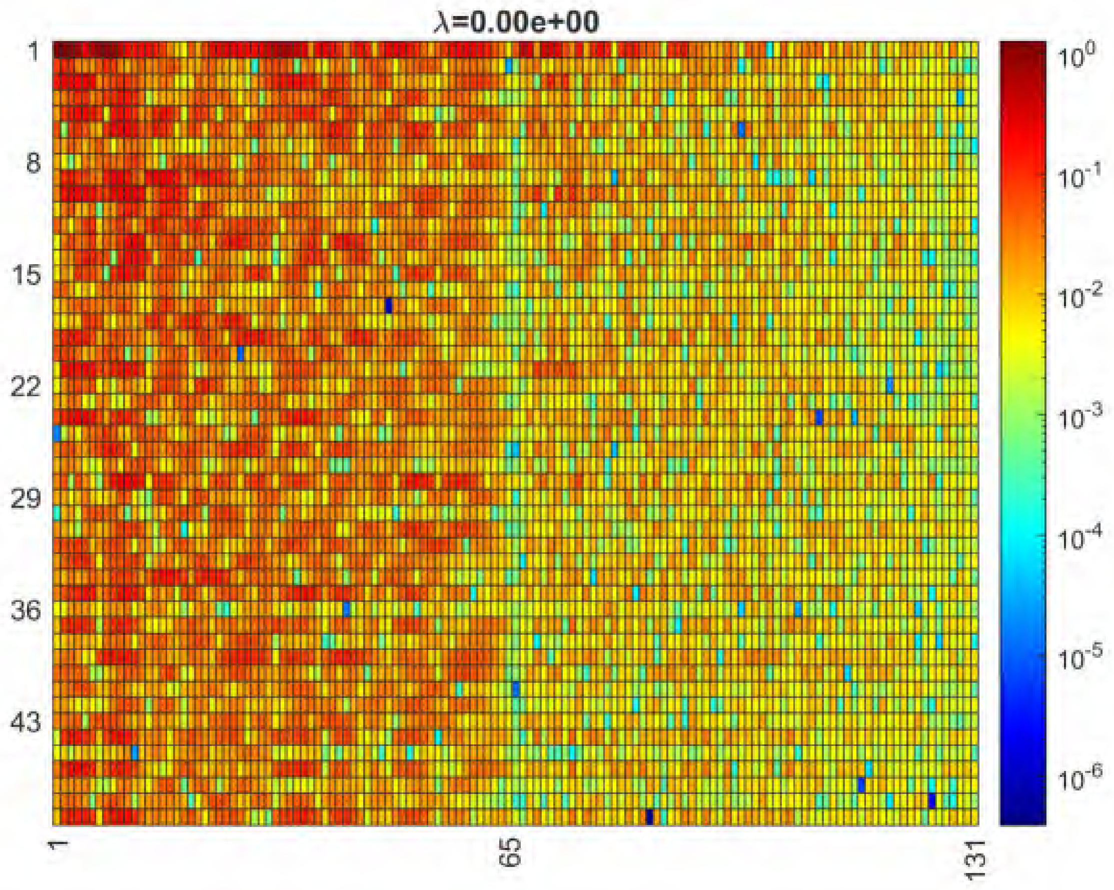
Model parameters vs damage state



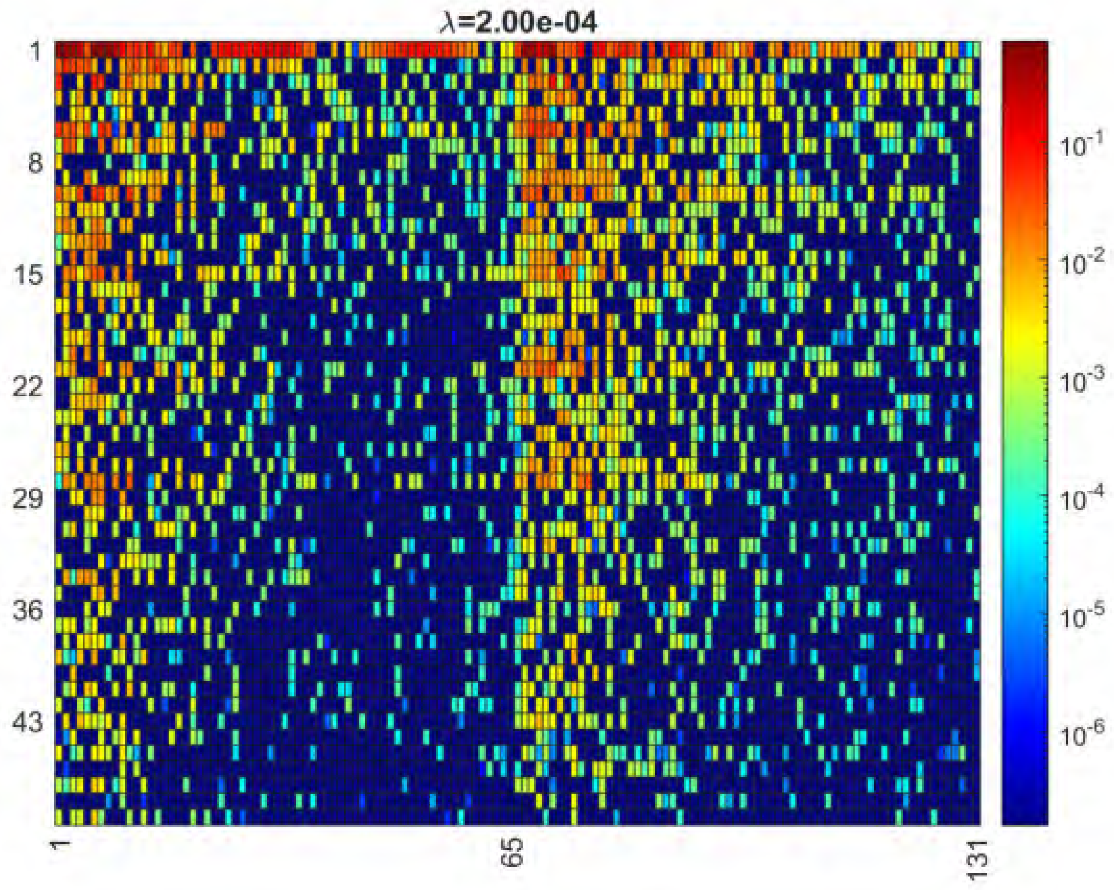
$VFP - ARX(54, 54)_{49}$ **is selected as optimal model**

Model Parameter Sparsity

WLS estimation



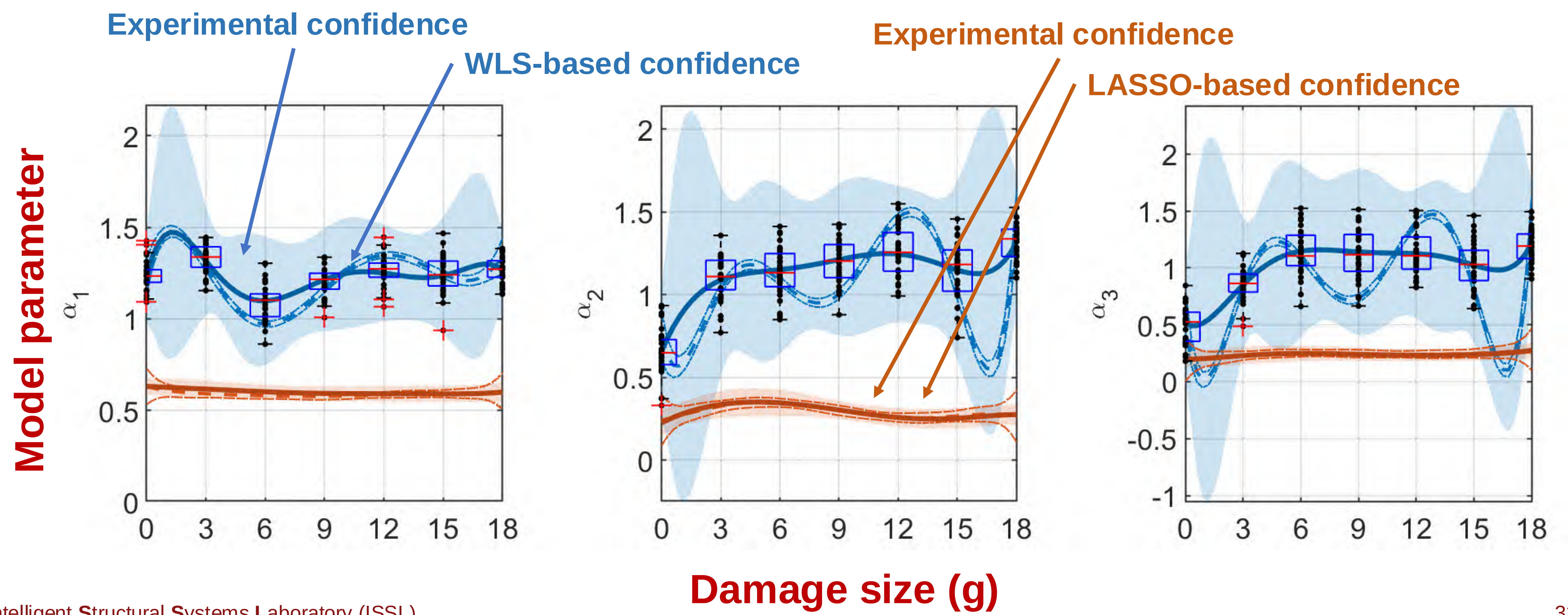
LASSO-WLS estimation



Model Parameter Uncertainty

Shaded blue: WLS model parameters along with 99% confidence intervals

Shaded red: LASSO model parameters along with 99% confidence intervals

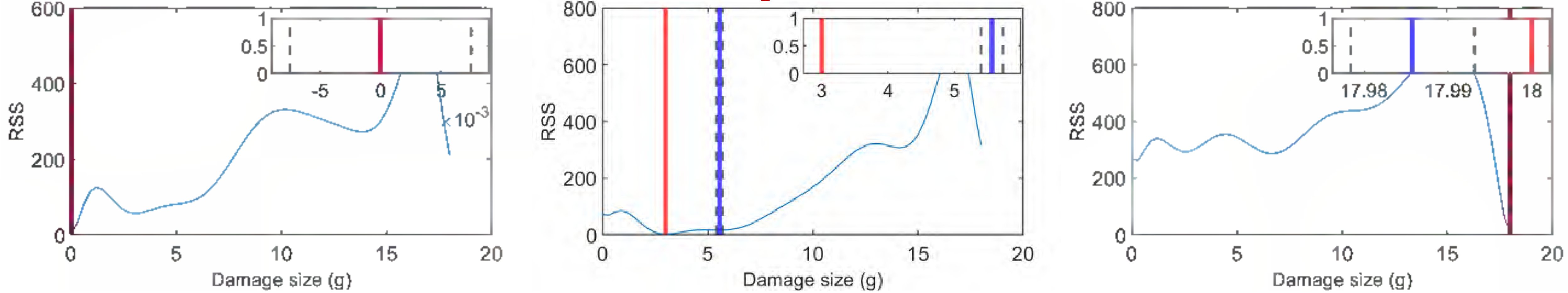


Damage Size Estimation Comparison (1)

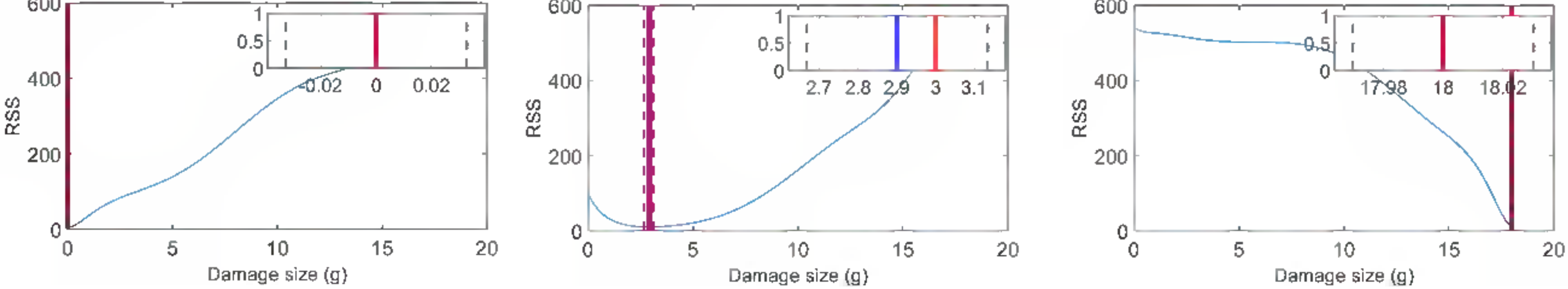
Inverse problem loss function (RSS)

RSS function: '—' 99% CI: '---' true damage: '—' estimated damage: '—'

No Regularization

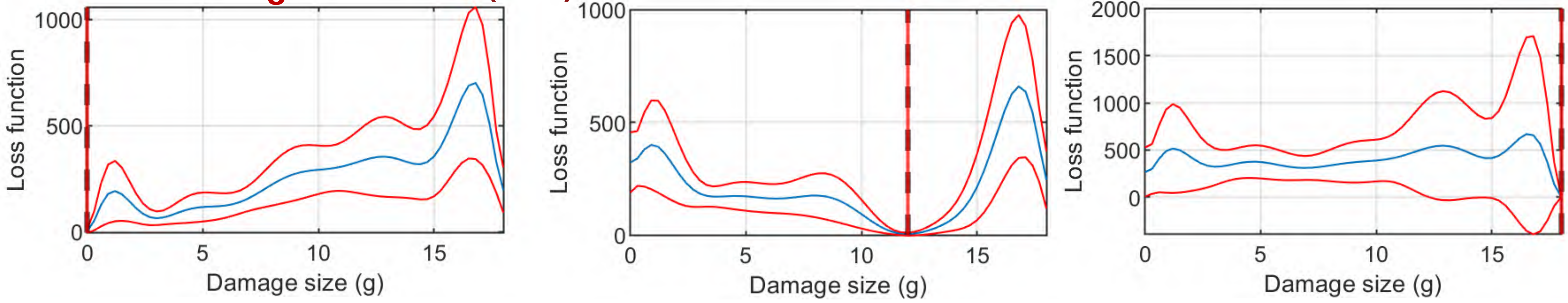


LASSO Regularization

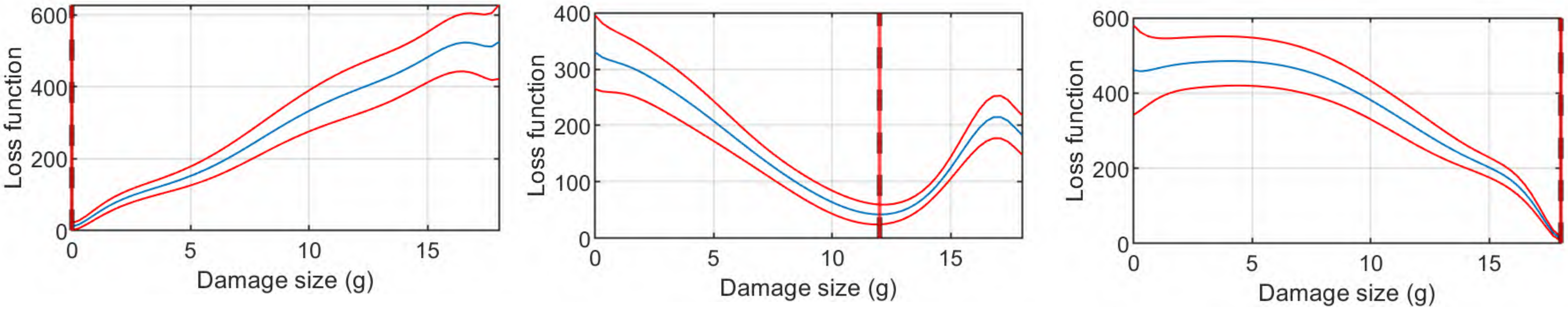


Damage Size Estimation Uncertainty

No Regularization $(\lambda = 0)$



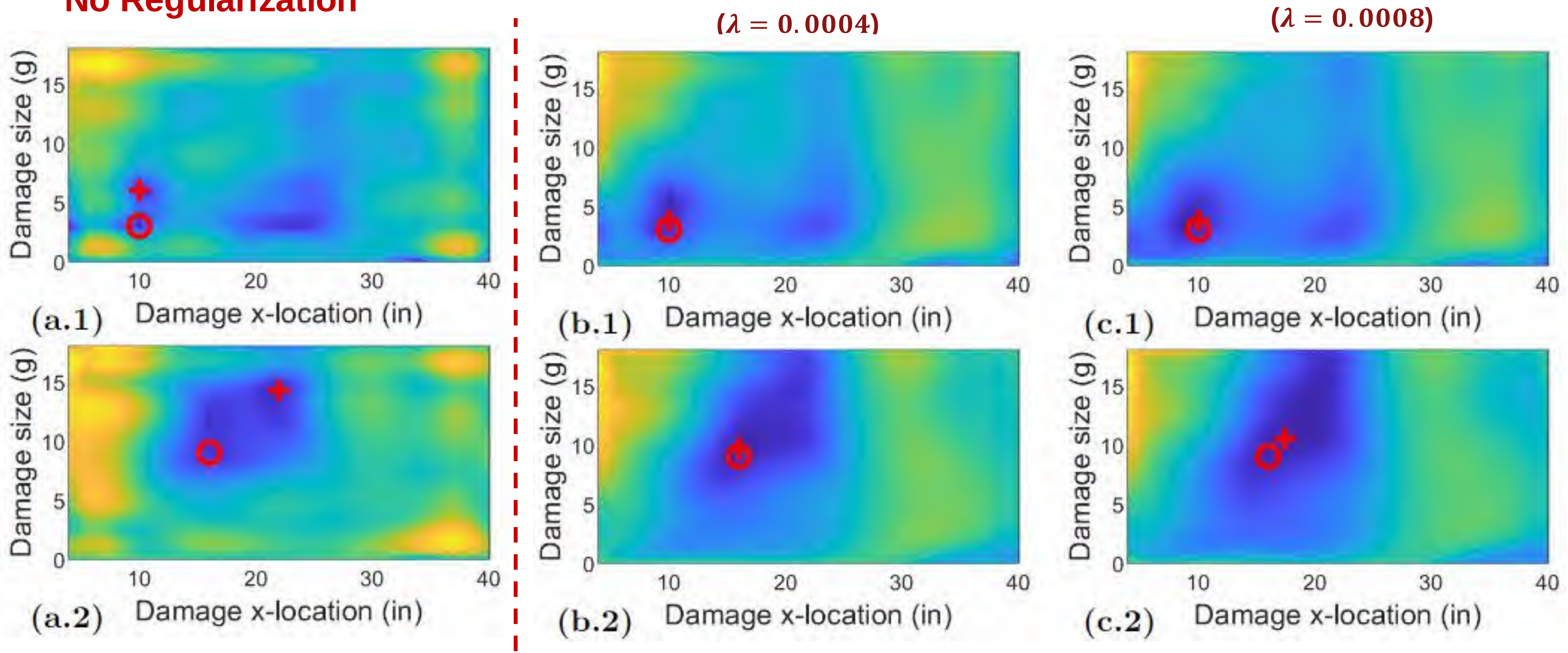
LASSO Regularization $(\lambda = 0.008)$



VFP Model: Model Residual Comparison

No Regularization

LASSO Regularization



Some Issues Remain

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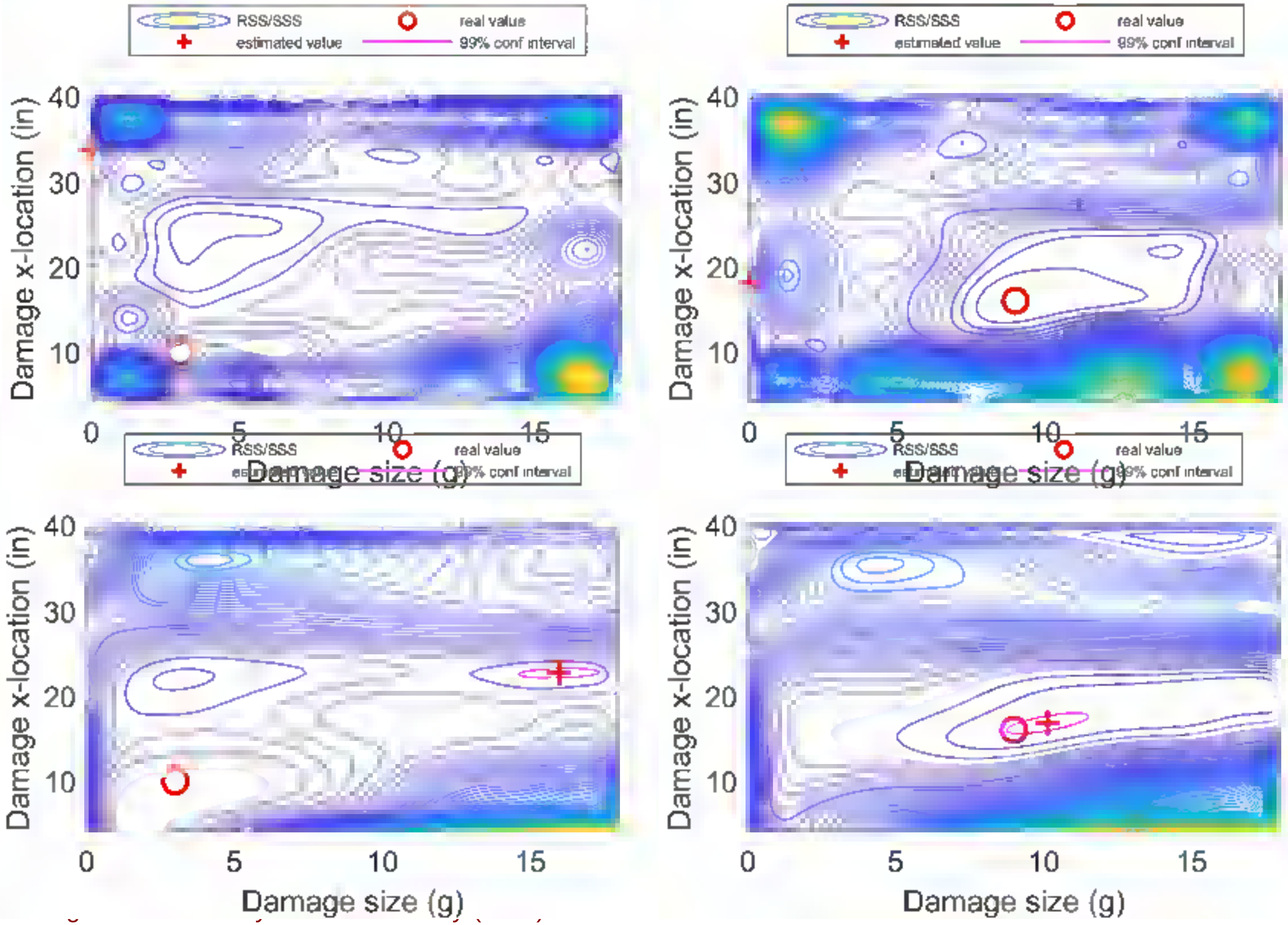


No Regularization

LASSO Regularization

State Estimation Revisited

current signals from “unknown” state

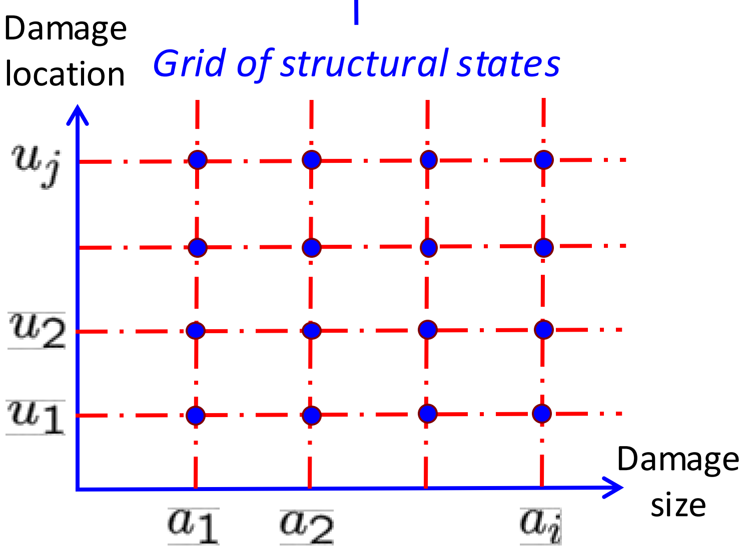
“Global” Stochastic Model

Model Estimation

Training Phase

$$y_{\boldsymbol{k}}[t]+\sum_{i=1}^{na} a_i(\boldsymbol{k})\cdot y_{\boldsymbol{k}}[t-i] = \sum_{i=0}^{nb} b_i(\boldsymbol{k})\cdot x_{\boldsymbol{k}}[t-i]+w_{\boldsymbol{k}}[t]$$

Grid of structural states



data obtained via simulations, or experiments

Re-parameterized model
(unknown structural state vector)

$$\mathcal{L}(\boldsymbol{k},\sigma_e^2;\boldsymbol{y}) = \prod_{t=1}^{N} \frac{1}{\sqrt{(2\pi\sigma_e)^{N_{out}}}} exp(-\frac{\boldsymbol{e}_u^2[t]}{2\sigma_e^2})$$

MCMC sampling

structural state prior

$$\pi(\boldsymbol{k}:\sigma_e^2) = \pi(\boldsymbol{k})\pi(\sigma_e^2)$$

structural state posterior

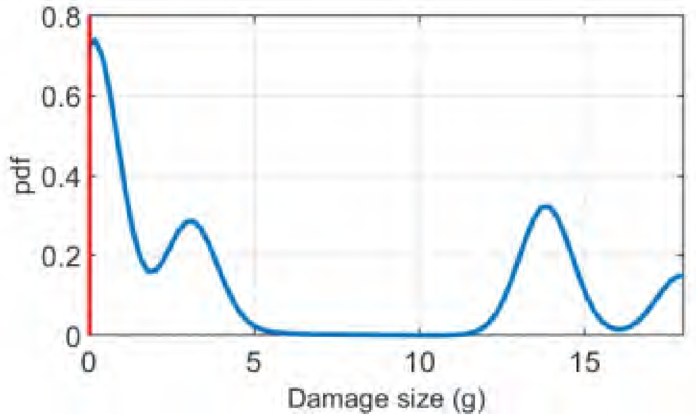
$$\pi(\boldsymbol{k},\sigma_e^2|\boldsymbol{y}) = \frac{1}{Z}\pi(\boldsymbol{k})\pi(\sigma_e^2)\mathcal{L}(\boldsymbol{k},\sigma_e^2;\boldsymbol{y})$$

Convergence?

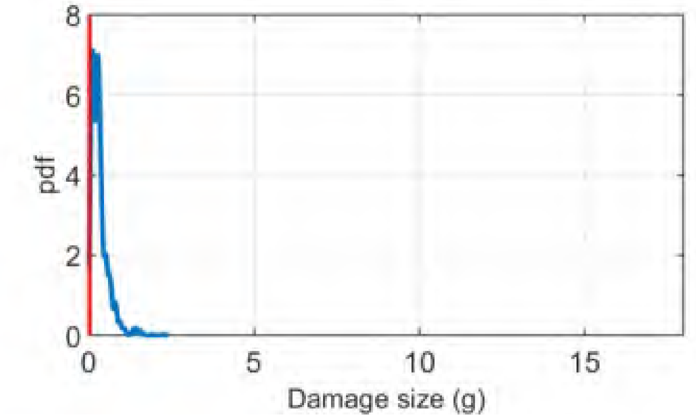
Bayesian Framework

Bayesian Damage Estimation (1)

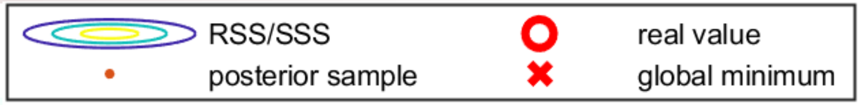
Bayesian MCMC Damage Magnitude Estimation – No Regularization



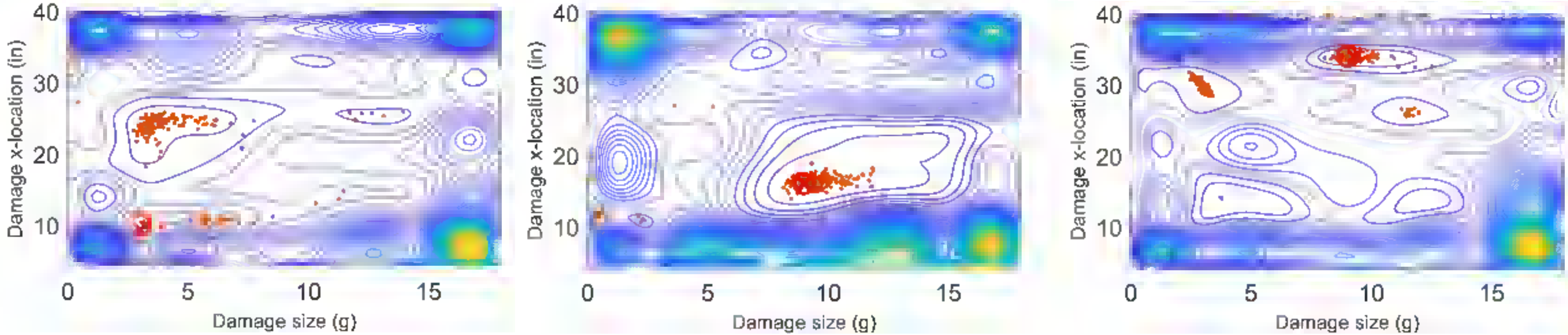
Bayesian MCMC Damage Magnitude Estimation – LASSO Regularization



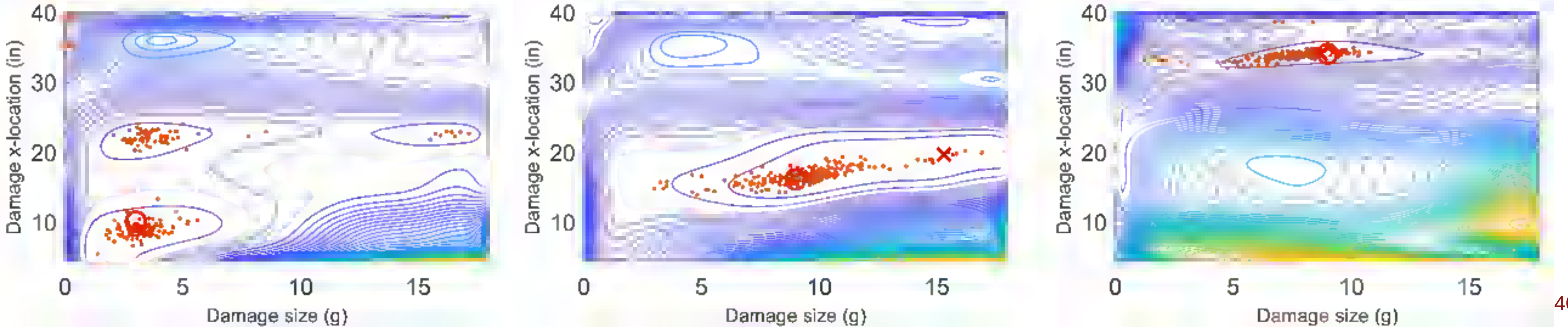
Bayesian Damage Estimation (2)



Bayesian Damage State Estimation – No Regularization

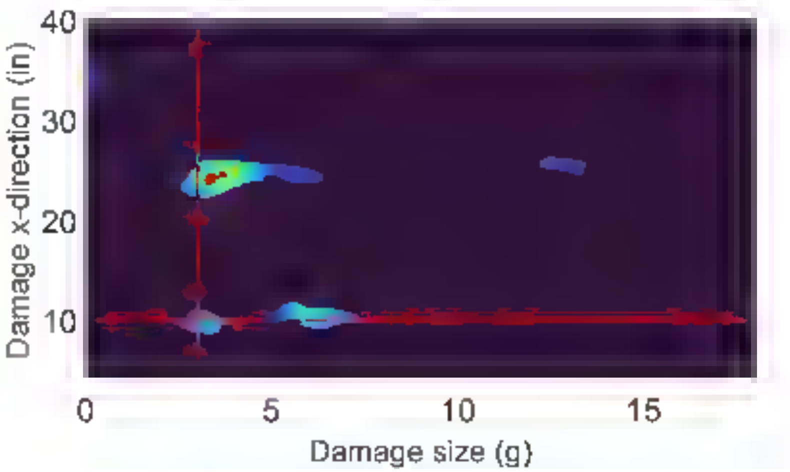


Bayesian Damage Magnitude Estimation – LASSO Regularization

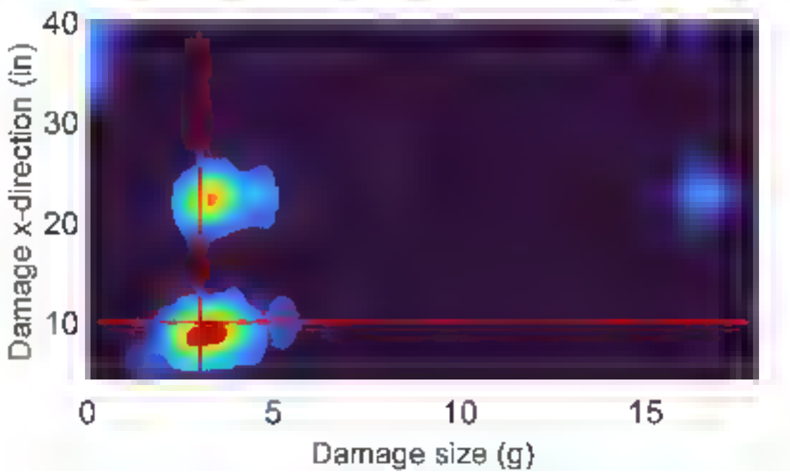


Bayesian Damage Estimation (2)

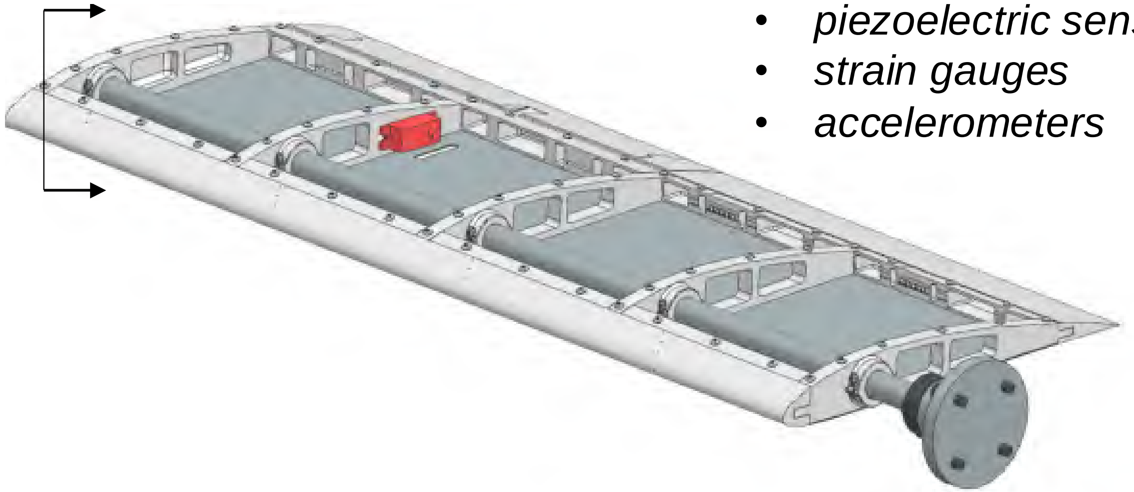
Bayesian Damage State Estimation – No Regularization



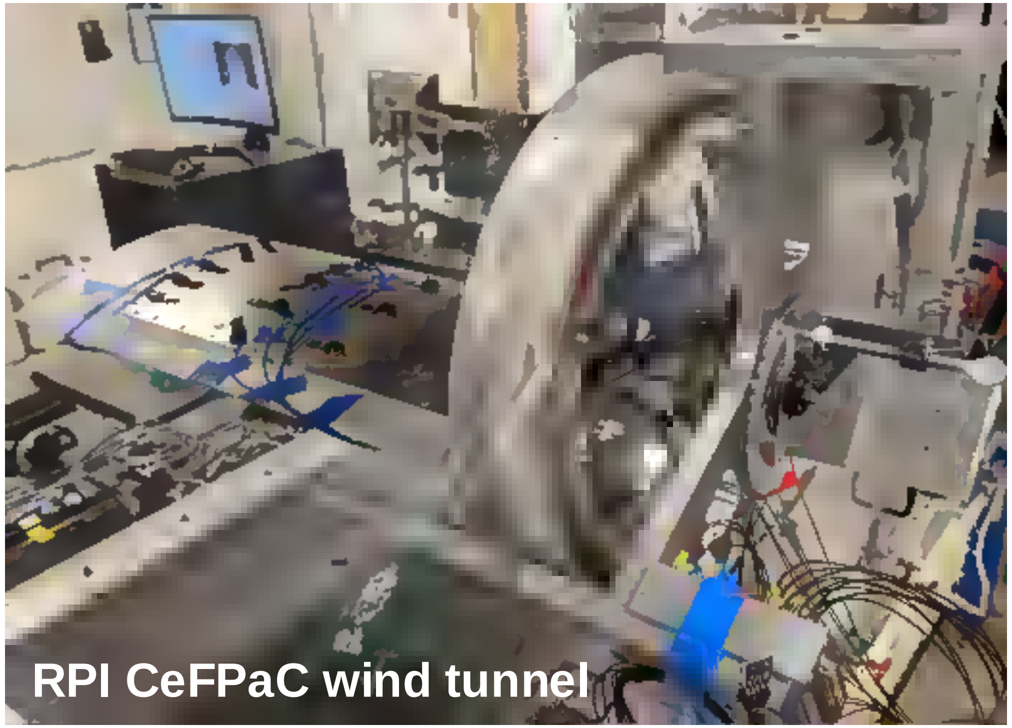
Bayesian Damage Magnitude Estimation – LASSO Regularization

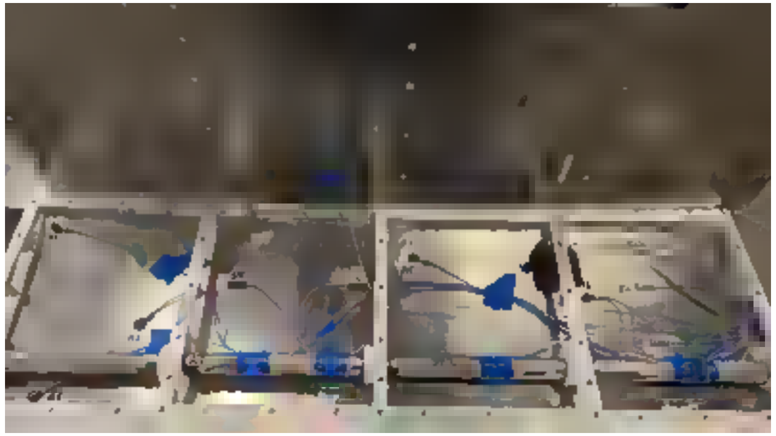


Test Case 2 – Varying Flight States



*A total of **48 integrated sensors***
- *piezoelectric sensors*
- *strain gauges*
- *accelerometers*

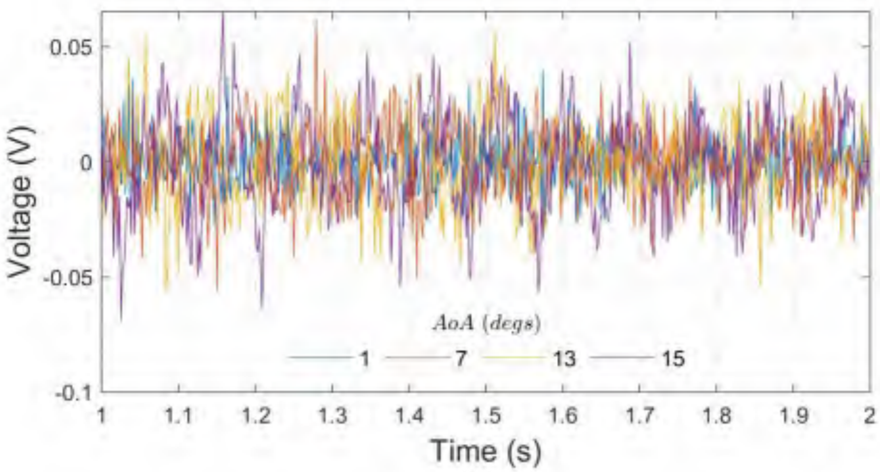




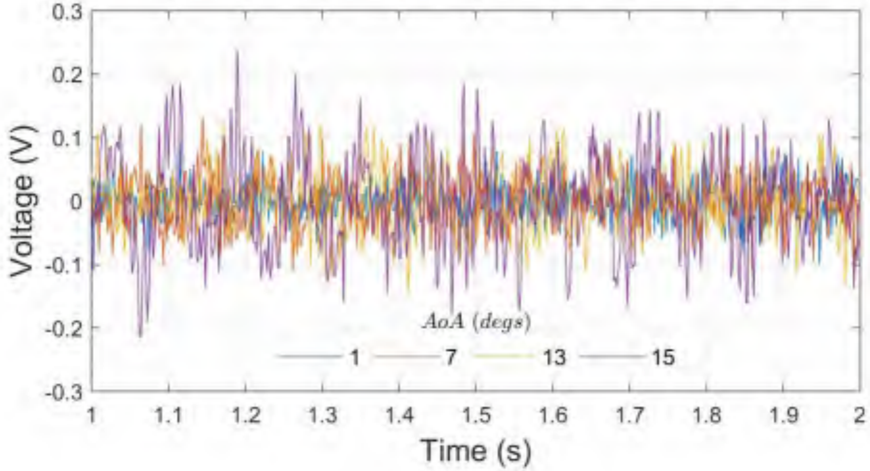
- **Angle of attack:** [0 – 15] degrees
- **Airspeed:** [8 – 20] m/s
- **Signal length** per data set: 128 s
- **Freq. Bandwidth**: [0.1 – 500] Hz
- **Total number of flight states: 192**

The Signals

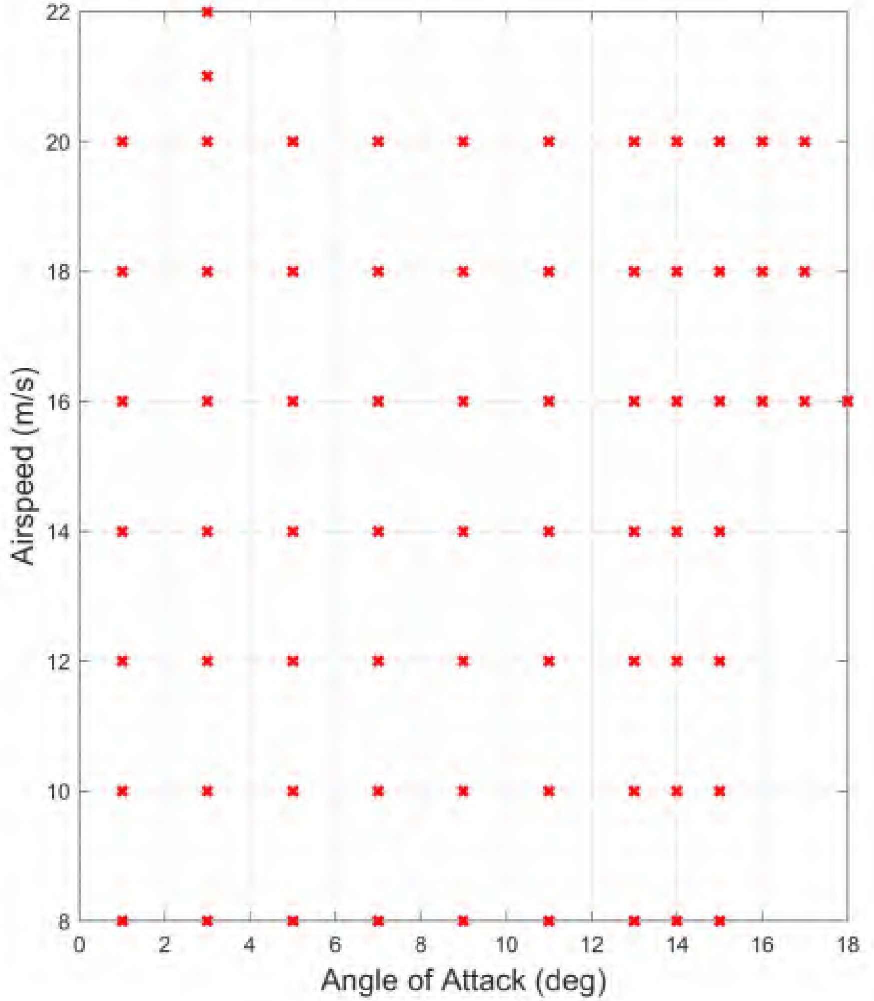
Airflow velocity: 12 m/s



Airflow velocity: 18 m/s



Test envelope

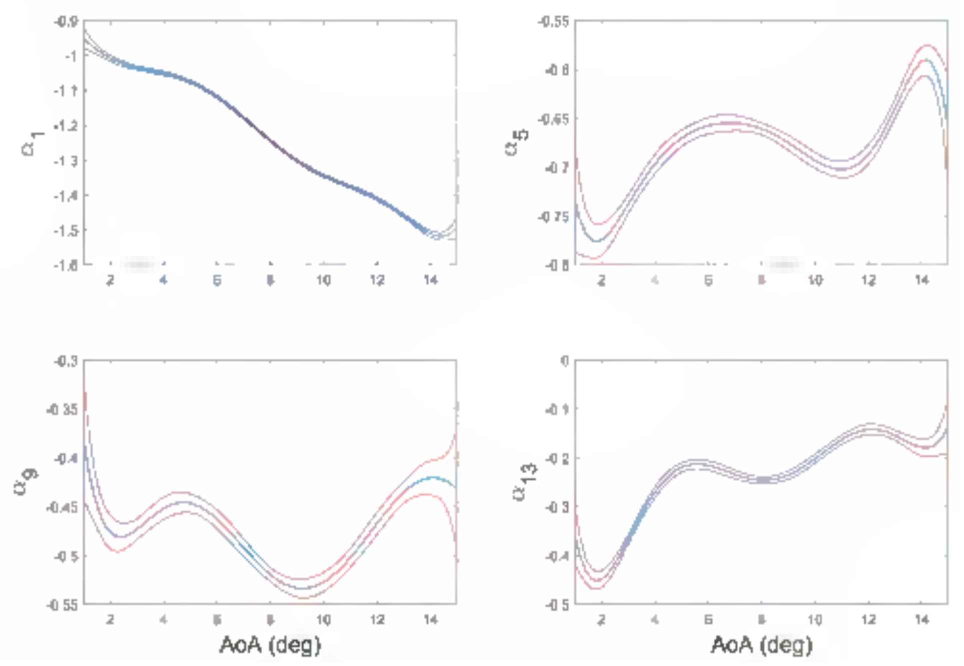


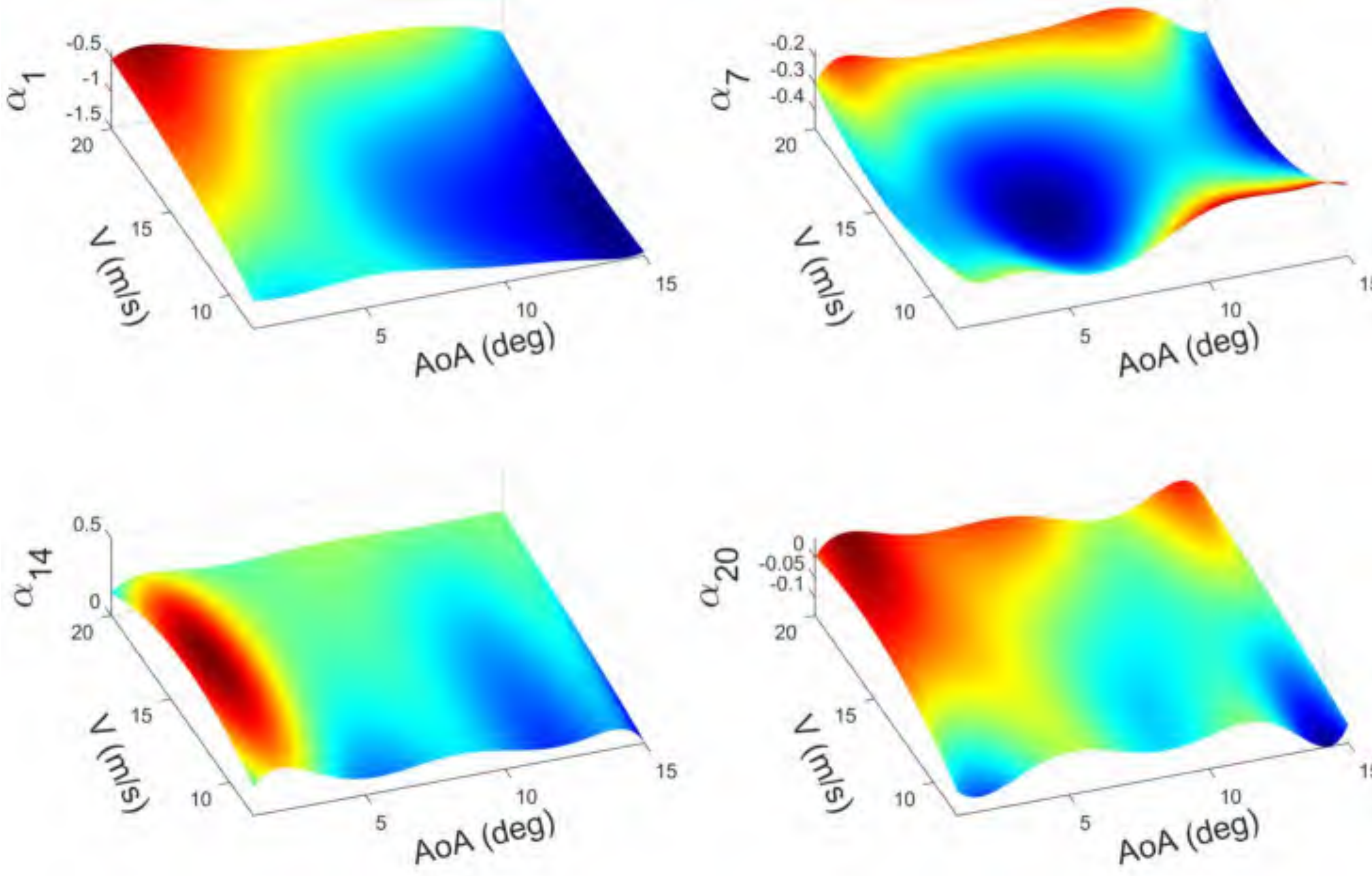
Observation: *as the AoA approaches the critical stall region the* ***amplitude of the signals increases***

The Identified VFP Model

$\boldsymbol{VFPAR(21)_{19}}$ **is selected as optimal model**

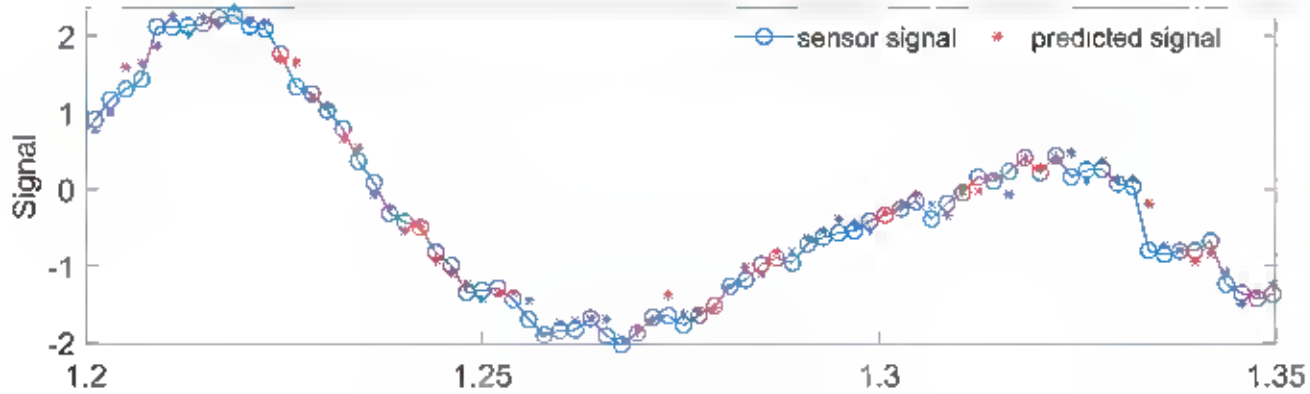
Model parameters vs flight state



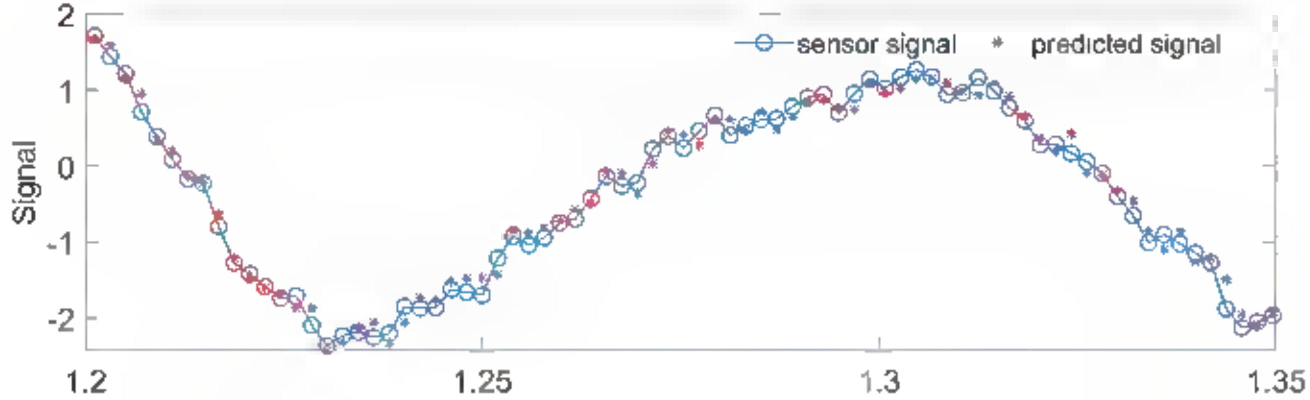


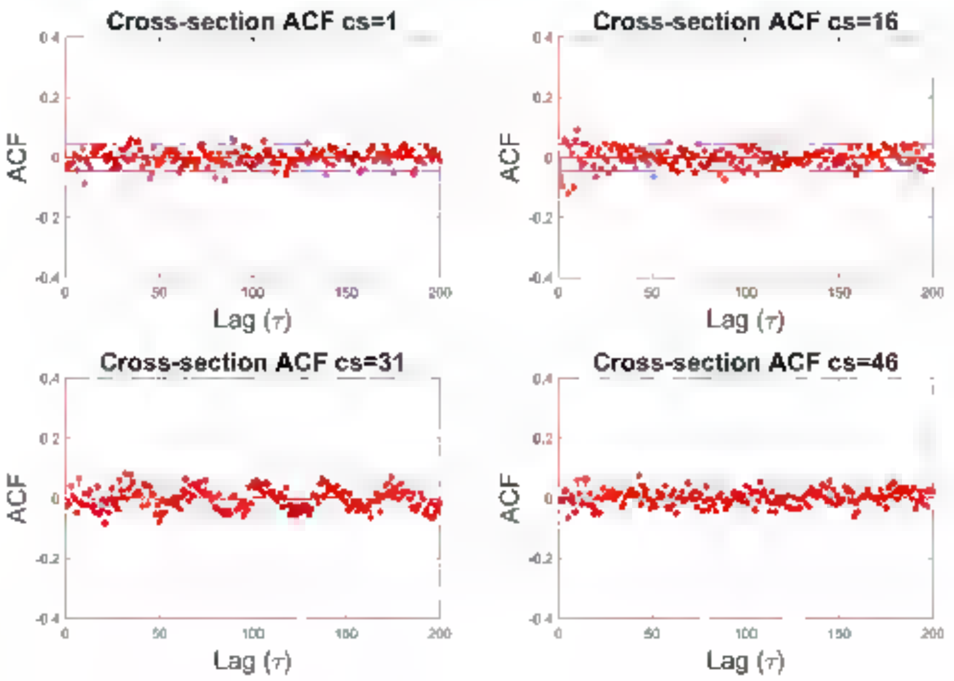
Signal & Prediction

Angle of attack = 1 deg, Airspeed = 8 m/s, cs=1



Angle of attack = 9 deg, Airspeed = 12 m/s, cs=31

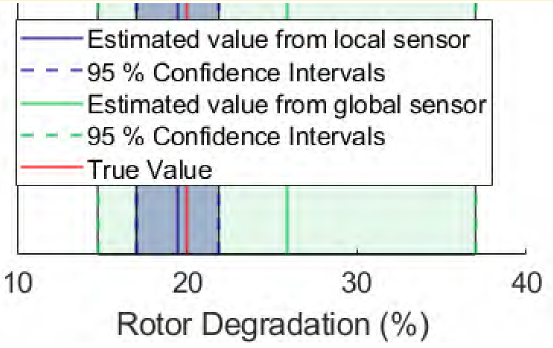




Integration with State Awareness

Data-driven modeling and **state awareness**

Some data will give better accuracy and precision for same diagnostic quantity*



- Account for noise and uncertainty in data from dynamic systems in operation
- Decision made via hypothesis tests at predetermined statistical significance levels

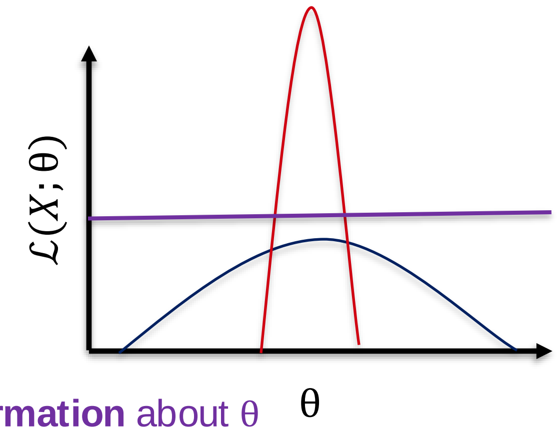
Cramér-Rao Lower bound

- The Cramér-Rao Lower Bound (CRLB) gives a **lower estimate** for the **variance** of an **unbiased estimator**.

$$var(\widehat{\theta}) \geq \frac{1}{I(\theta)}$$

Fisher information

$$I(\theta) = n\mathbf{E}_{\theta}[(\frac{\delta\mathcal{L}(\mathbf{X};\theta)}{\delta\theta})^2]$$



CRLB formulations for FP time series models:

1. CRLB wrt **model parameter vector**
2. CRLB wrt **state estimation vector**

- The data gives **no information** about θ
- The data gives **some information** regarding which value of θ will maximize the likelihood of observing this set of data.
- This data has **high information** about the **precise value of *θ***

Fisher information measures the expected amount of information given by a random variable (X) for a parameter(θ) of interest

*Dutta, A., McKay, M., Kopsaftopoulos, F., and Gandhi, F., “Unified Statistical Framework for Rotor Fault Diagnosis on a Hexacopter via Functionally Pooled Stochastic Models,” Vertical Flight Society 77th Annual Forum, Virtual, May 2021.

CRLB w.r.t VFP-AR state vector ($\boldsymbol{k}$)

Model parameter estimator

$$\widehat{\boldsymbol{k}} = \arg\min_{\boldsymbol{k}\in\mathbf{R}^m} \sum_{i=1}^{N} e_u^T[t,\boldsymbol{k}]e_u[t,\boldsymbol{k}] \ , \quad \sigma_u^2(\widehat{\boldsymbol{k}}) = \frac{1}{N}\sum_{t=1}^{N} e_u[t,\widehat{\boldsymbol{k}}]e_u^T[t,\widehat{\boldsymbol{k}}]$$

Log likelihood

$$\ln\mathcal{L}(\hat{\boldsymbol{k}},\sigma_u^2(\hat{\boldsymbol{k}})) = -\frac{N}{2}ln(2\pi) - \frac{N}{2}ln(\sigma_u^2) - \frac{1}{2}\sum_{t=1}^{N}\frac{e_u^T(\hat{\boldsymbol{k}}),t)e_u(\hat{\boldsymbol{k}}),t)}{\sigma_u^2(\hat{\boldsymbol{k}})}$$

CRLB formulation

$$\boldsymbol{\Sigma}_{CRLB} = \left[\mathbf{E}\left[\left(\frac{\delta\ln\mathcal{L}(\boldsymbol{k},\sigma_u^2)}{\delta\boldsymbol{k}}\right)\left(\frac{\delta\ln\mathcal{L}(\boldsymbol{k},\sigma_u^2)}{\delta\boldsymbol{k}}\right)^T\right]\right]^{-1}$$

e_u : model residual

$\sigma_u^2(\hat{k})$: model residual covariance

$$\frac{\delta\ln\mathcal{L}(\mathbf{k},\sigma_u^2)}{\delta\mathbf{k}} = 0 + 0 - \frac{1}{2}\sum_{t=1}^{N}\frac{\delta\mathbf{e}(\mathbf{k},t)}{\delta\mathbf{k}}\cdot 2\cdot\boldsymbol{\Sigma}^{-1}\mathbf{e}^T(\mathbf{k},t)$$

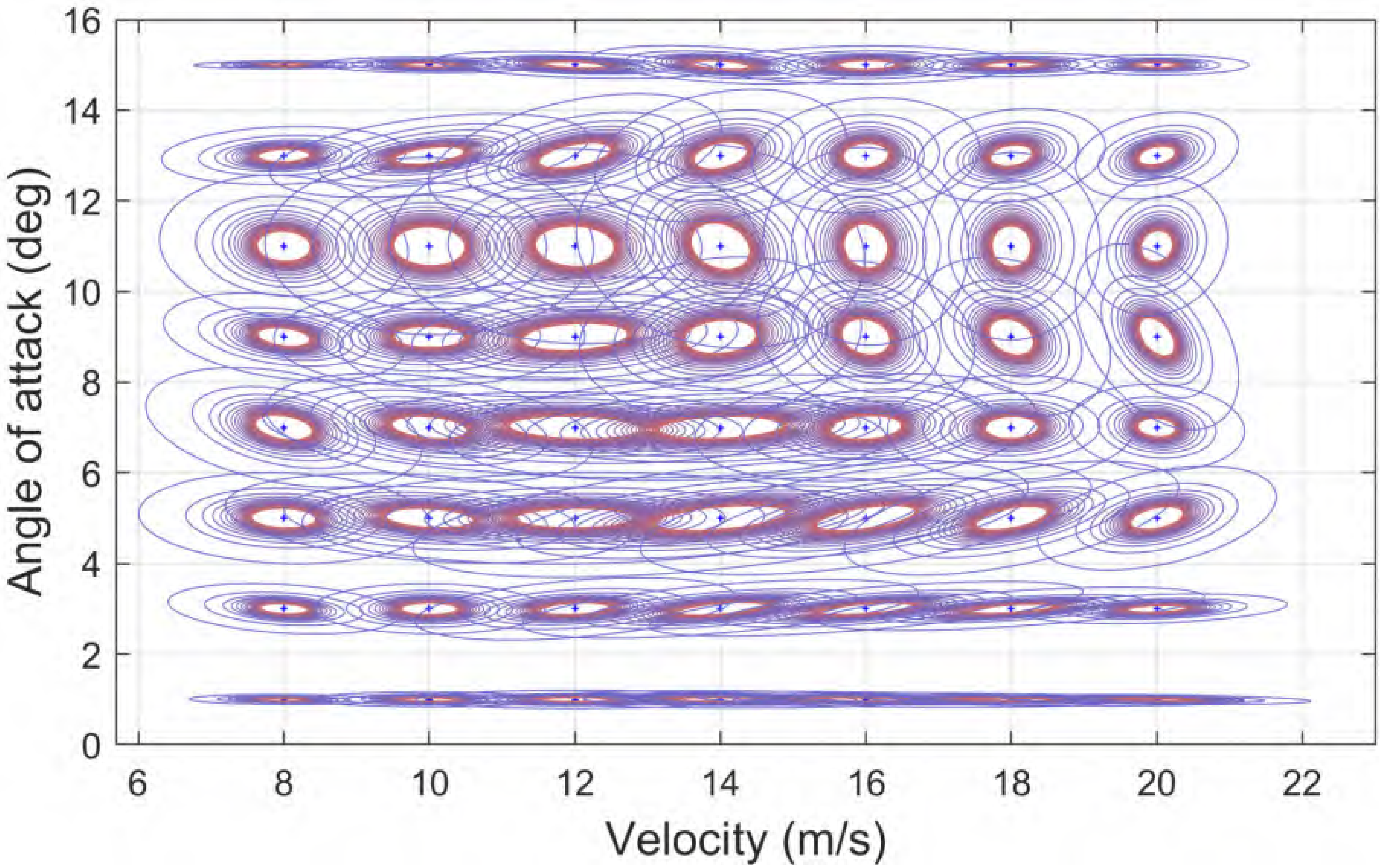
functional subspace

data

$$\boldsymbol{\Sigma}_{CRLB} = \sigma_u^2(\hat{\boldsymbol{k}})\left[\sum_{t=1}^{N}\varepsilon(\boldsymbol{k},t)\varepsilon(\boldsymbol{k},t)^T\right]^{-1} \ , \quad \varepsilon(\boldsymbol{k},t) = \frac{\delta e_u^T(\boldsymbol{k},t)}{\delta\boldsymbol{k}} = 0 - \boldsymbol{\theta}^T\cdot\psi_u[t]\otimes\frac{\delta\boldsymbol{G}(\boldsymbol{k})}{\delta\boldsymbol{k}}$$

model structure

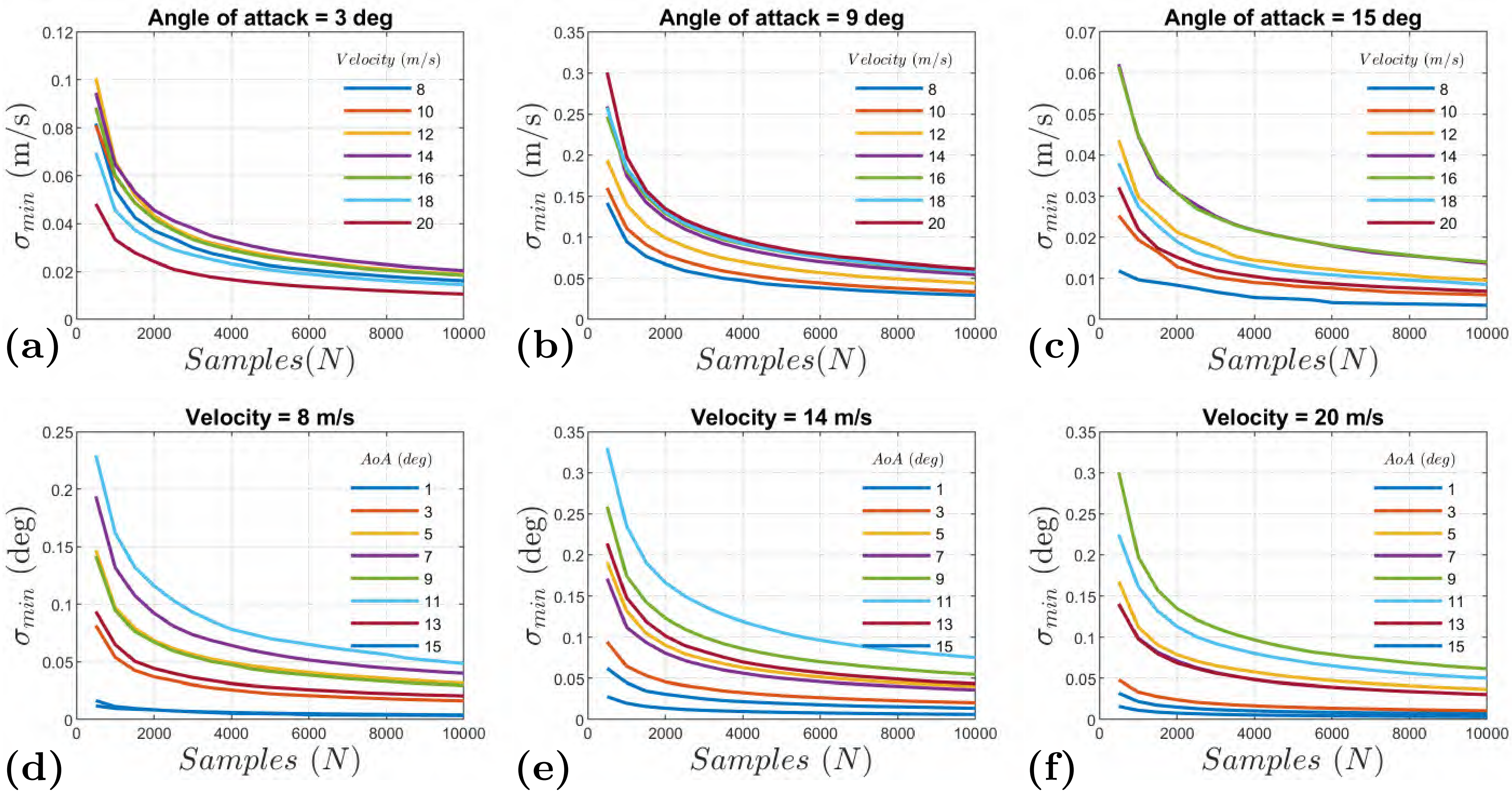
State Estimation CRLB



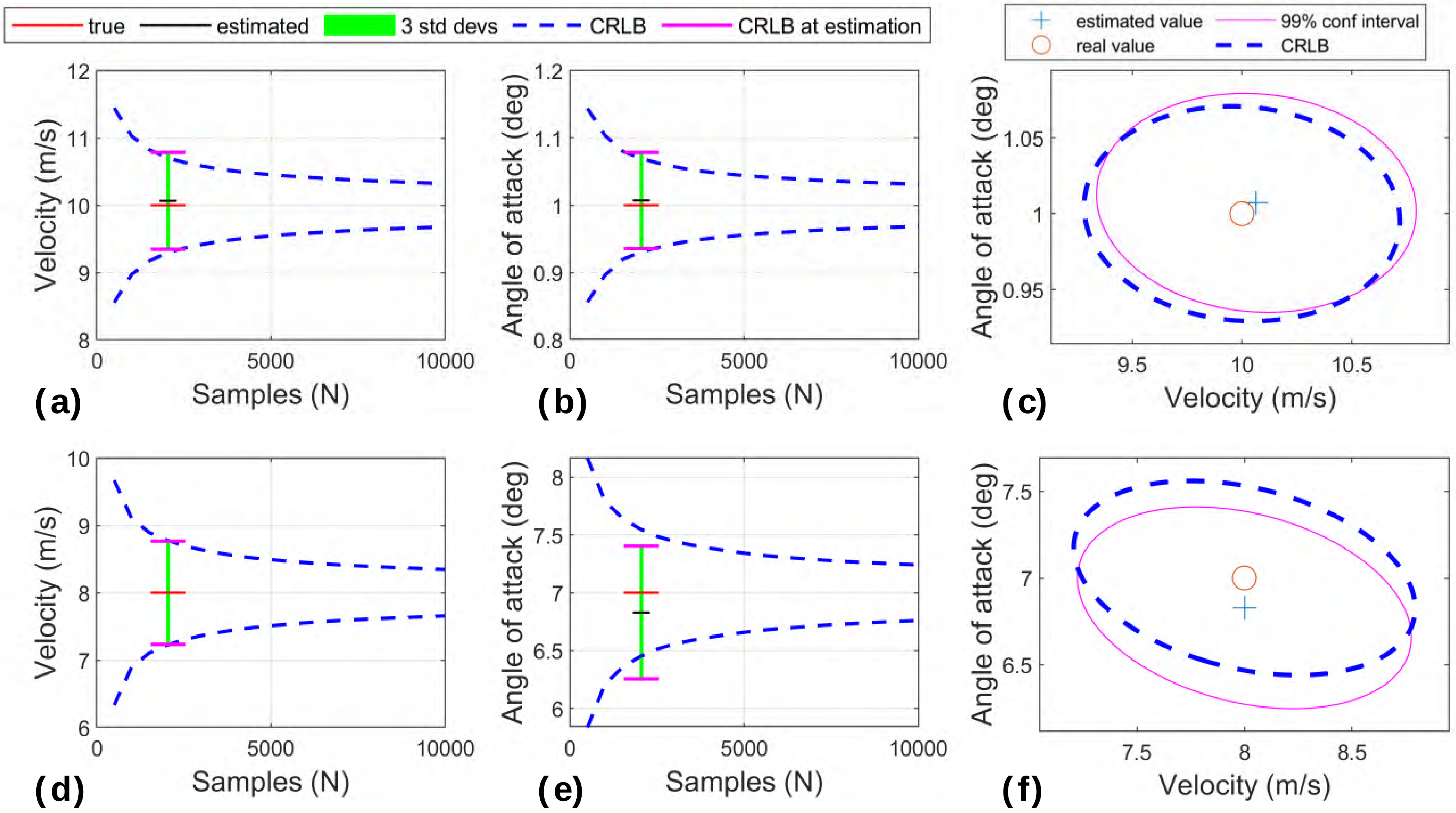
Blue ellipsoids: 500 samples

Red ellipsoids: 10,000 samples

CRLB State Estimation Assessment (1)

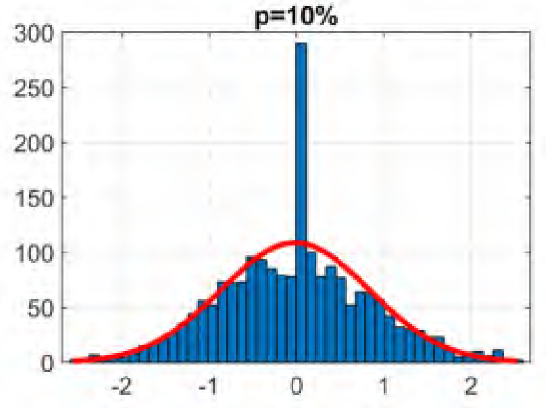


CRLB State Estimation Assessment (2)



Simulating Data Corruption

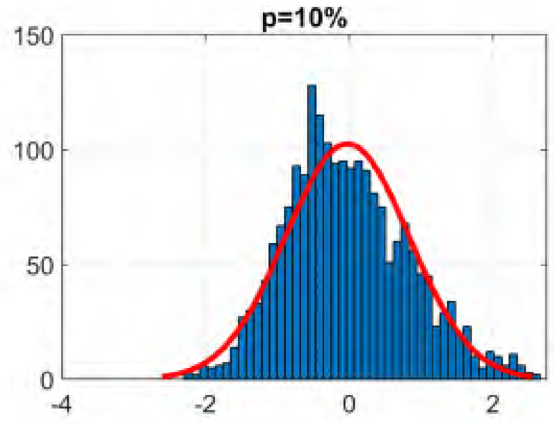
10% data is set to zero



Sensor error:

Zero padding

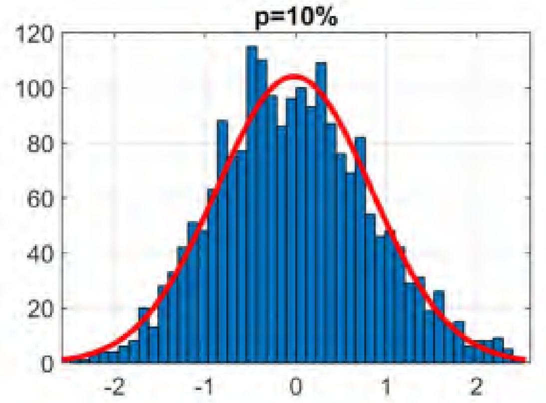
$y_{corr}(t) = y(t)(1 + sign\big(y(t)\big) \times p)$



Sensor error:

Skewed data value

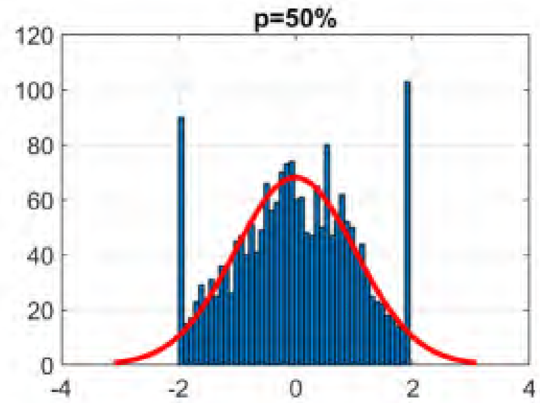
$y_{corr}(t) = y(t) + X, X \sim Normal(0, p^2 Var(y))$



Sensor error:

Additional noise

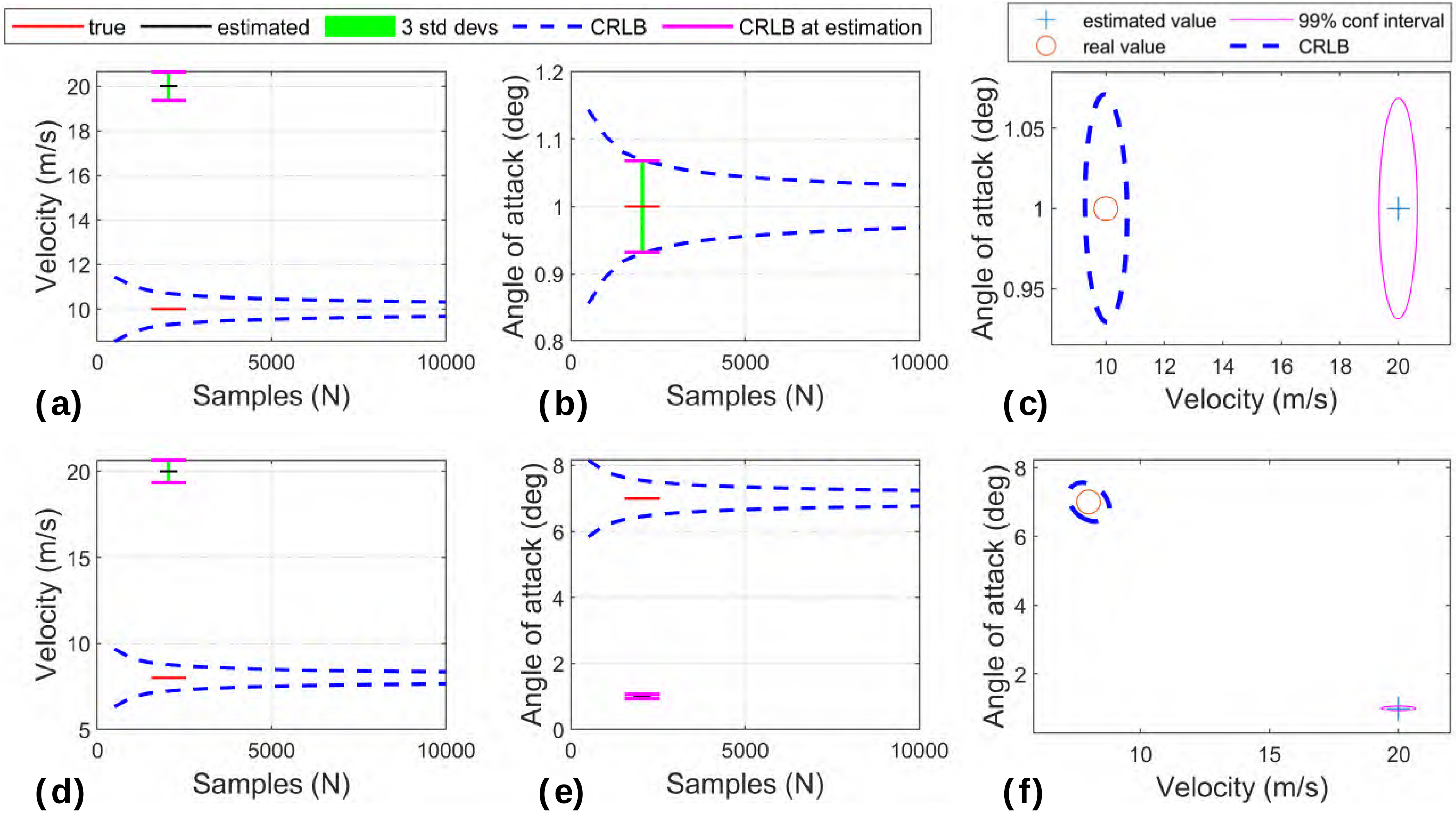
$y_{corr}(t) = \min(y(t), p \times \max(y(t)))$



Sensor error:

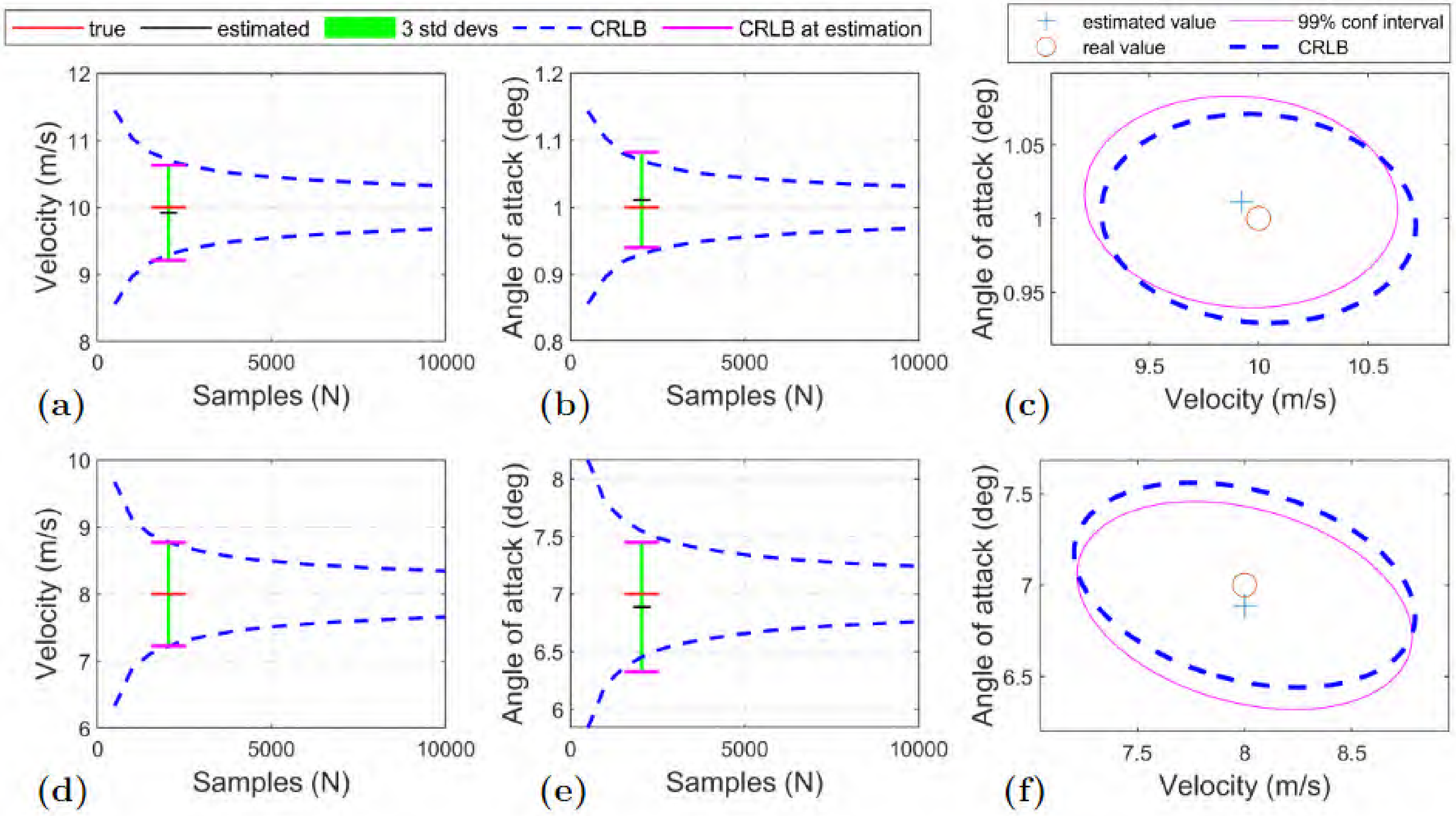
Data clipping

Sensor error: Zero padding



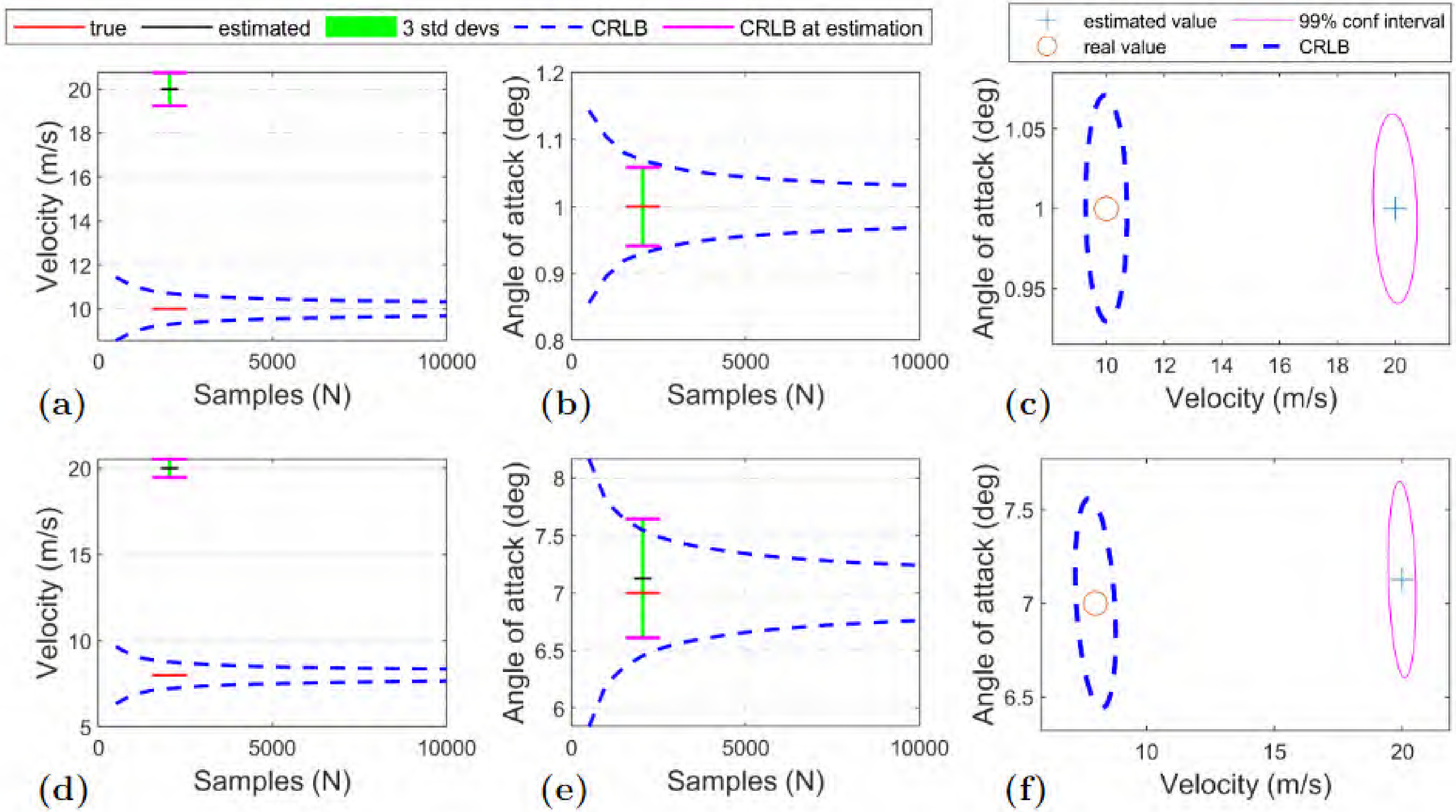
CRLB State Estimation Assessment (4)

Sensor error: Skewed data



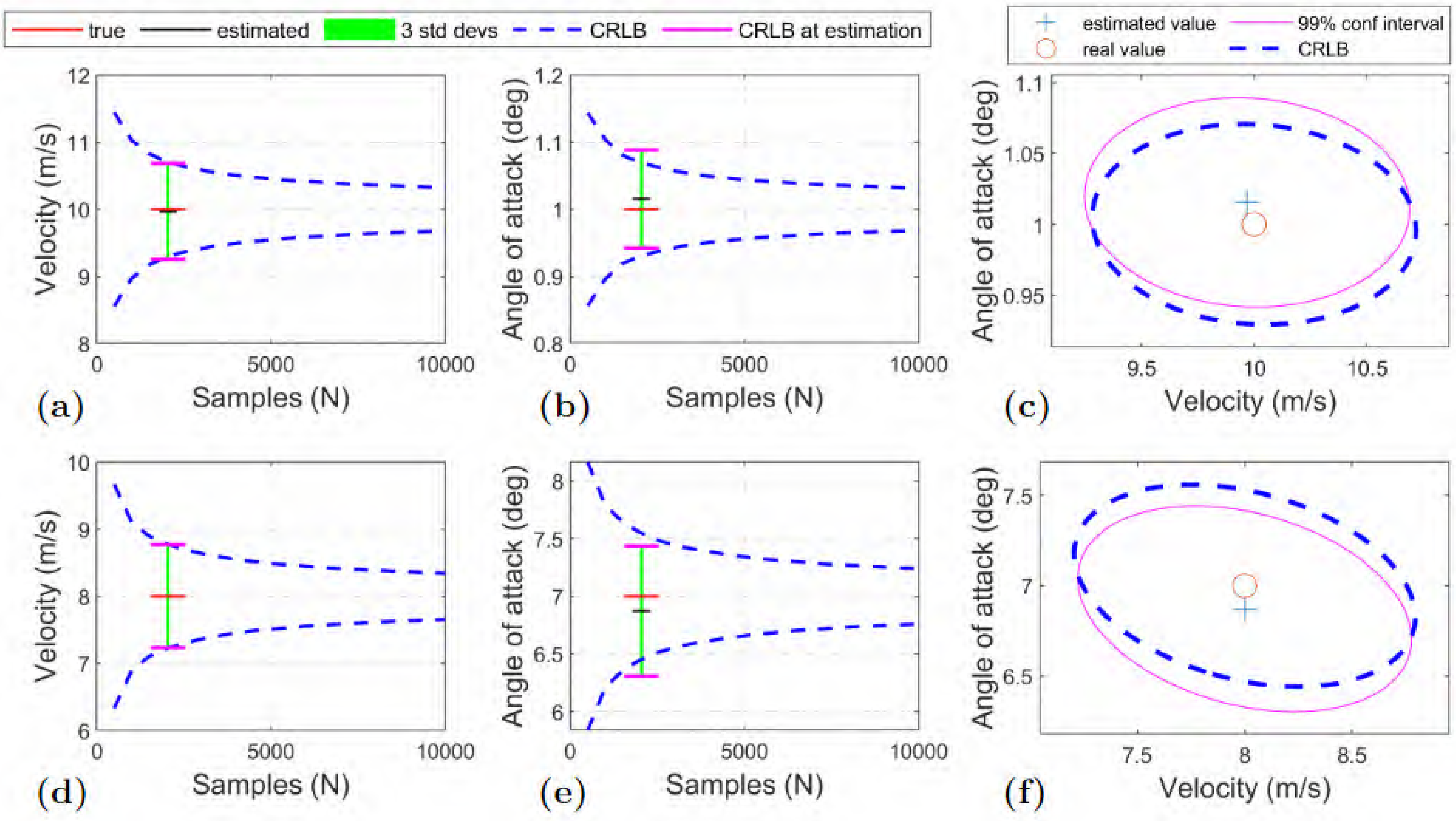
CRLB State Estimation Assessment (5)

Sensor error: Addition noise



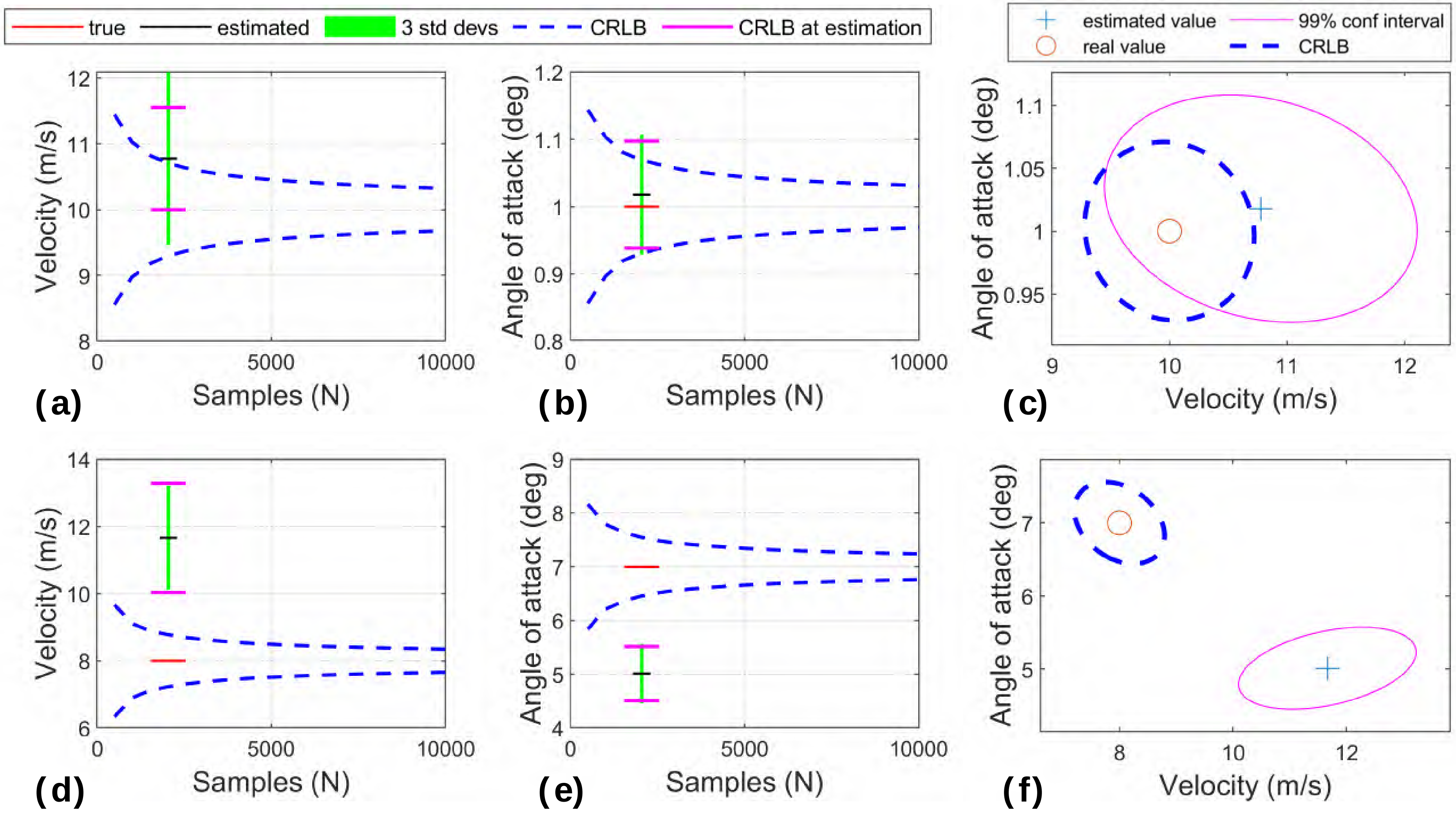
CRLB State Estimation Assessment (6)

Sensor error: Data Clipping



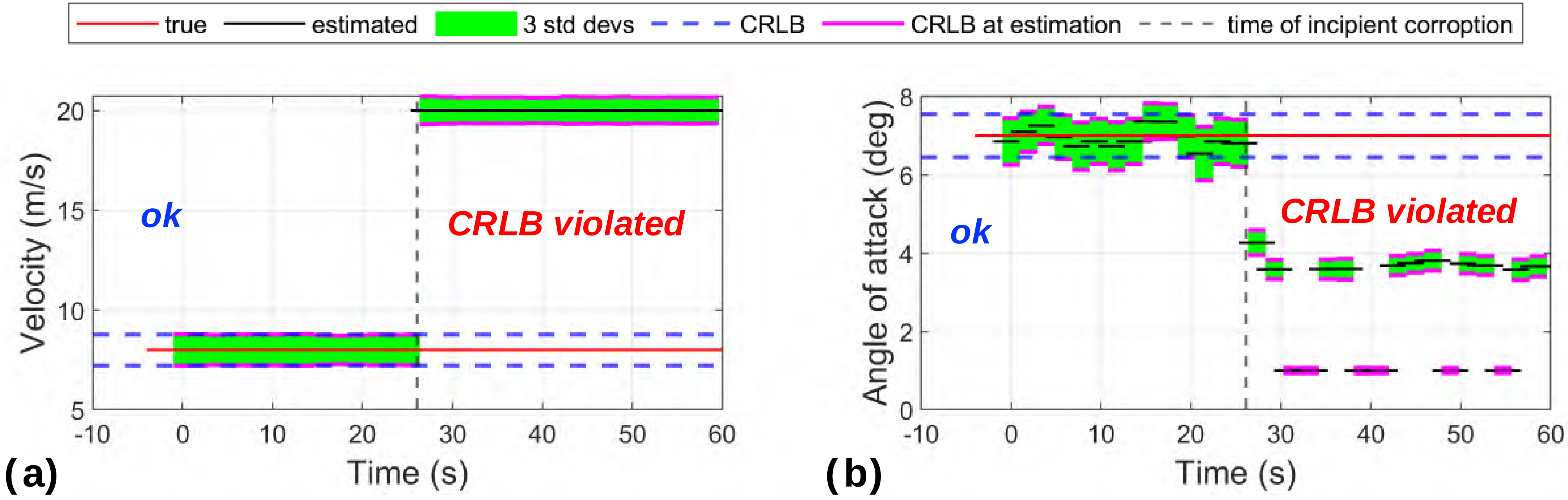
CRLB State Estimation Assessment (7)

Suboptimal model selection: reduced functional subspace (9 basis)



State Estimation Runtime Monitoring

10% samples corrupted by zero padding at 26 s

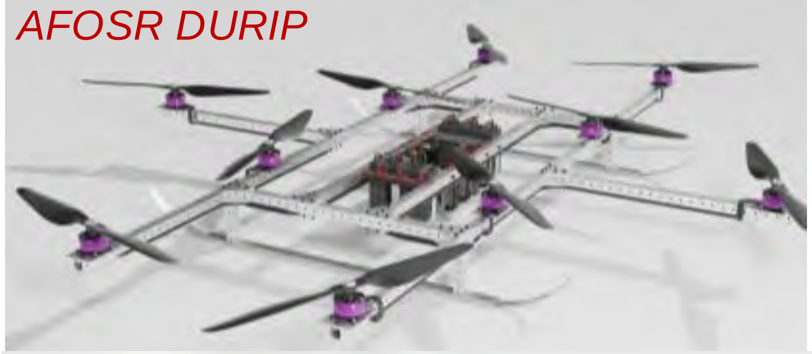


Concluding Remarks & Next Steps

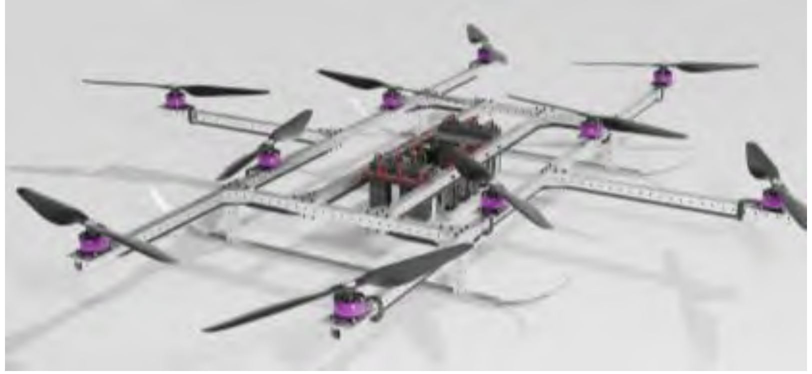
- **State awareness** for **intelligent** systems:
  - ✓ multi-modal distributed **sensing**
  - ✓ combination of stochastic identification and machine learning approaches
- Formal verification of **stochastic state awareness** algorithms

Current & Future Steps

- **Continuous model adaptation and learning**
- **Stochastic properties for formal verification**
- **Modular verification & safety envelopes**
- **Evaluation & assessment**
- **Prototype self-aware UAV demos & flight tests**



"Albatross"
Intelligent UAV platform



Thank you!