Uncertainty Quantification in Machine Learning Models for High-Rate State Estimation

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Background

High-Rate Dynamic Systems

- Systems experiencing dynamic events with amplitudes higher than 100 g_n over a duration of less than 100 ms.

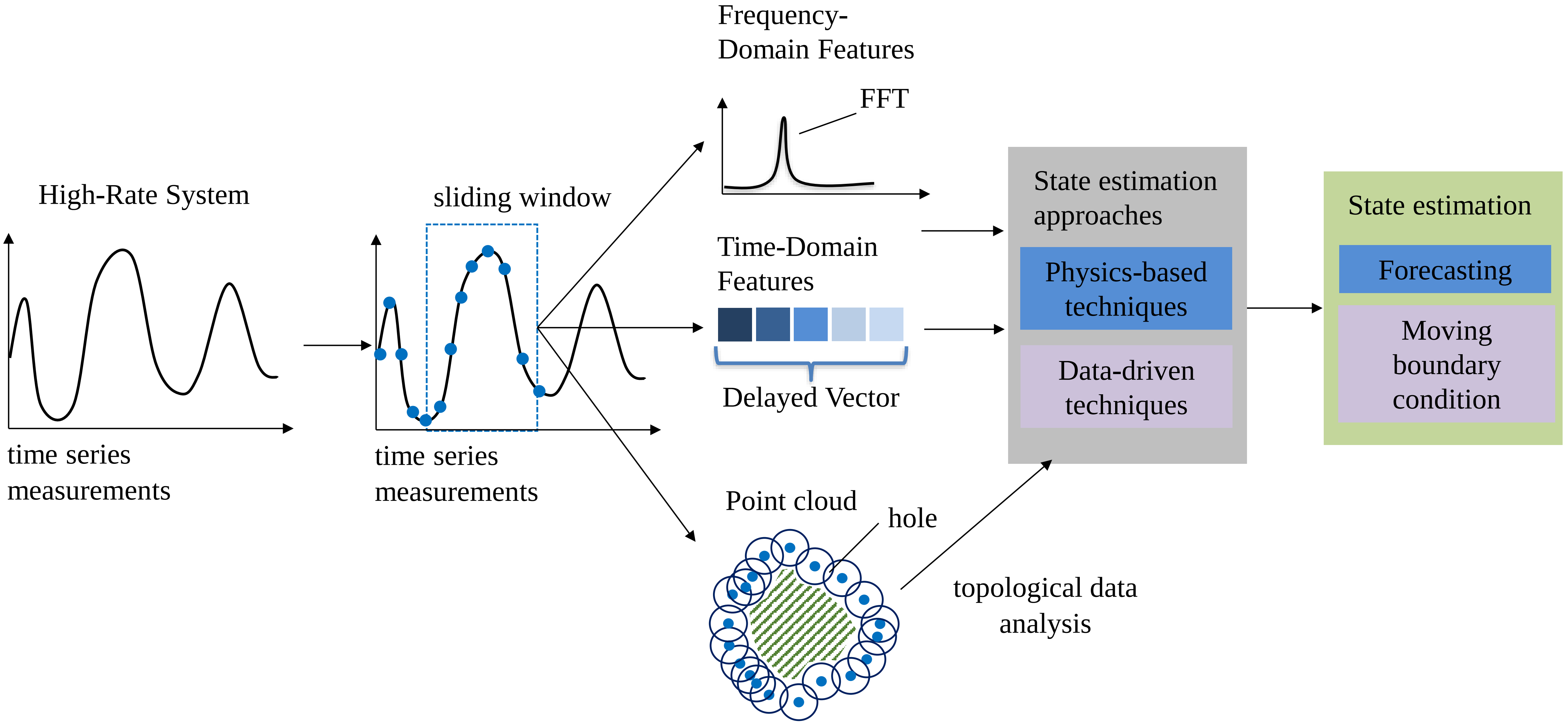
Example

- Active blast mitigation
- Ballistic packages
- Hypersonic vehicle

Characteristics

- Large uncertainties in external loads
- High levels of non-stationarities in the structure and heavy disturbances
- Unmodeled dynamics from changes in system configuration

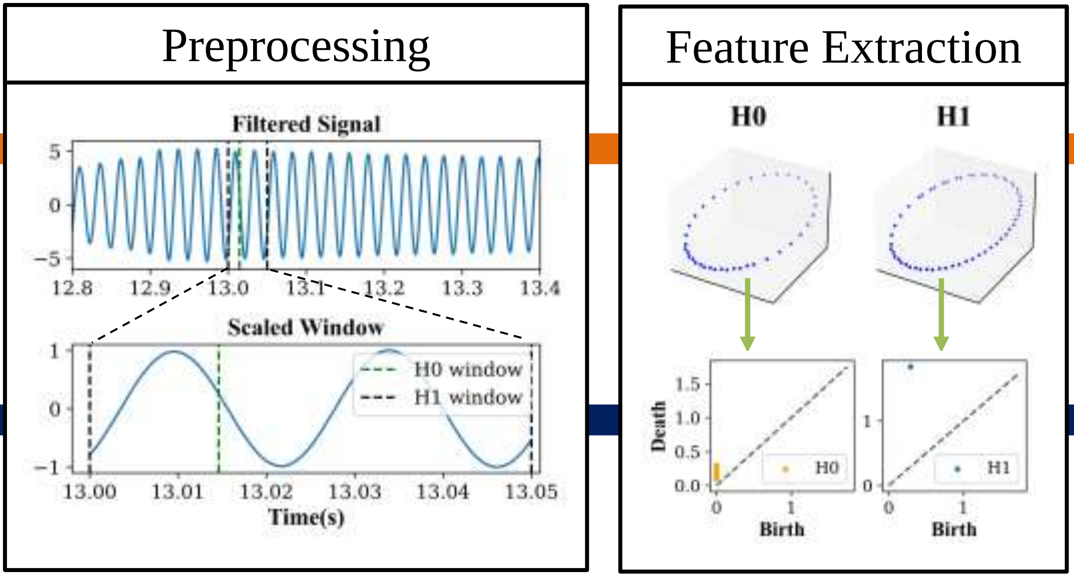
Current State of Art Methods



Uncertainty quantification in state estimation

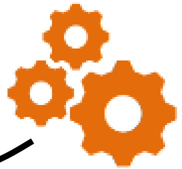
Offline Data

Online Data



Model

Probabilistic ML algorithm



Trained state estimator

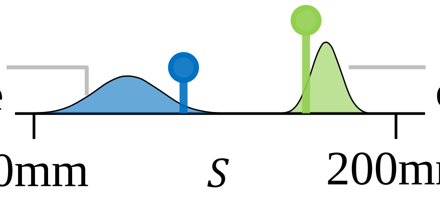
- Gaussian process regression
- Neural network ensemble (NNE)
- Monte Carlo (MC) dropout
- Long short-term memory + NNE
- Long short-term memory + MC dropout

Probabilistic predictions

Compare

Accuracy & uncertainty quantification quality

Low confidence



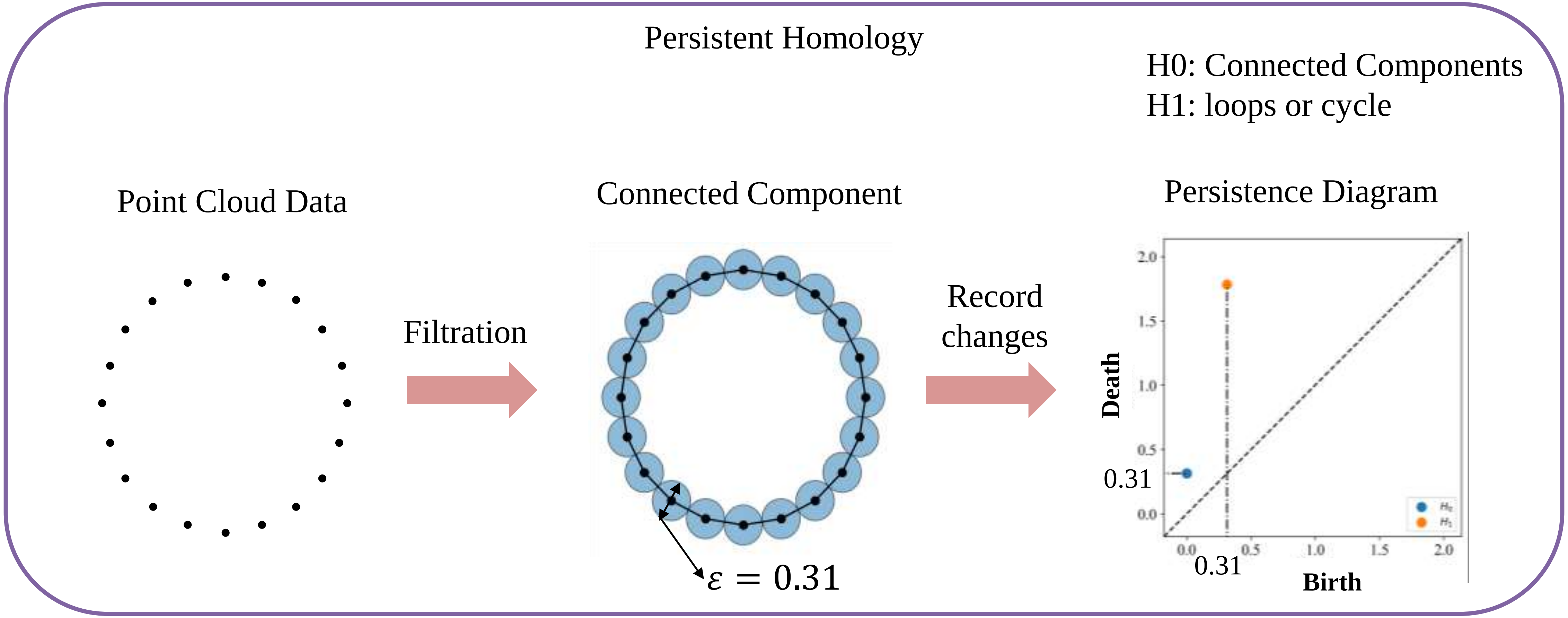
High confidence

Benefits of UQ

- Improve Model Reliability
- Risk-Based Decision Making
- Proxy for Estimation Error

Topological Data Analysis (TDA)

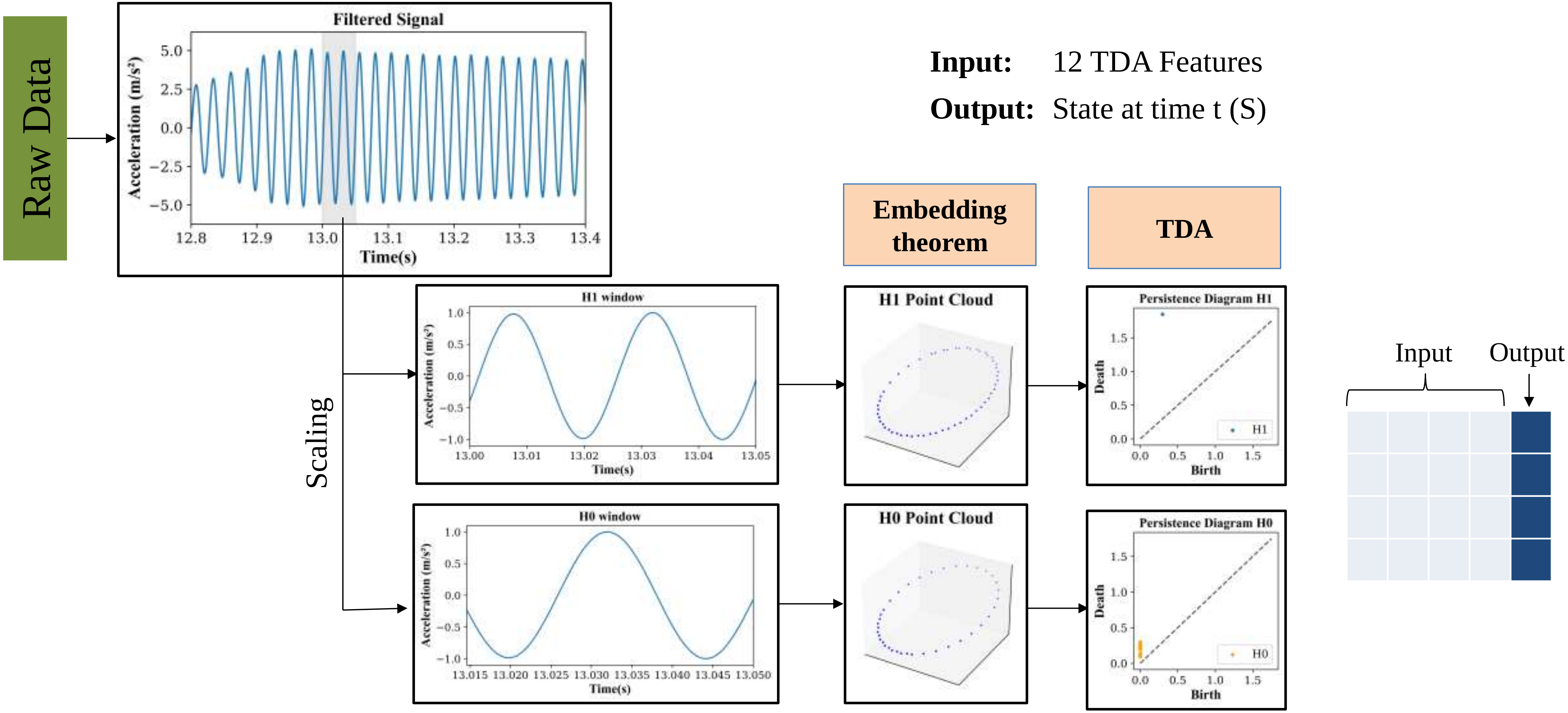
TDA is a method that studies the **shape and structure of data** by identifying patterns and features that persist across multiple scales.



Topological Data Analysis (TDA) feature extraction

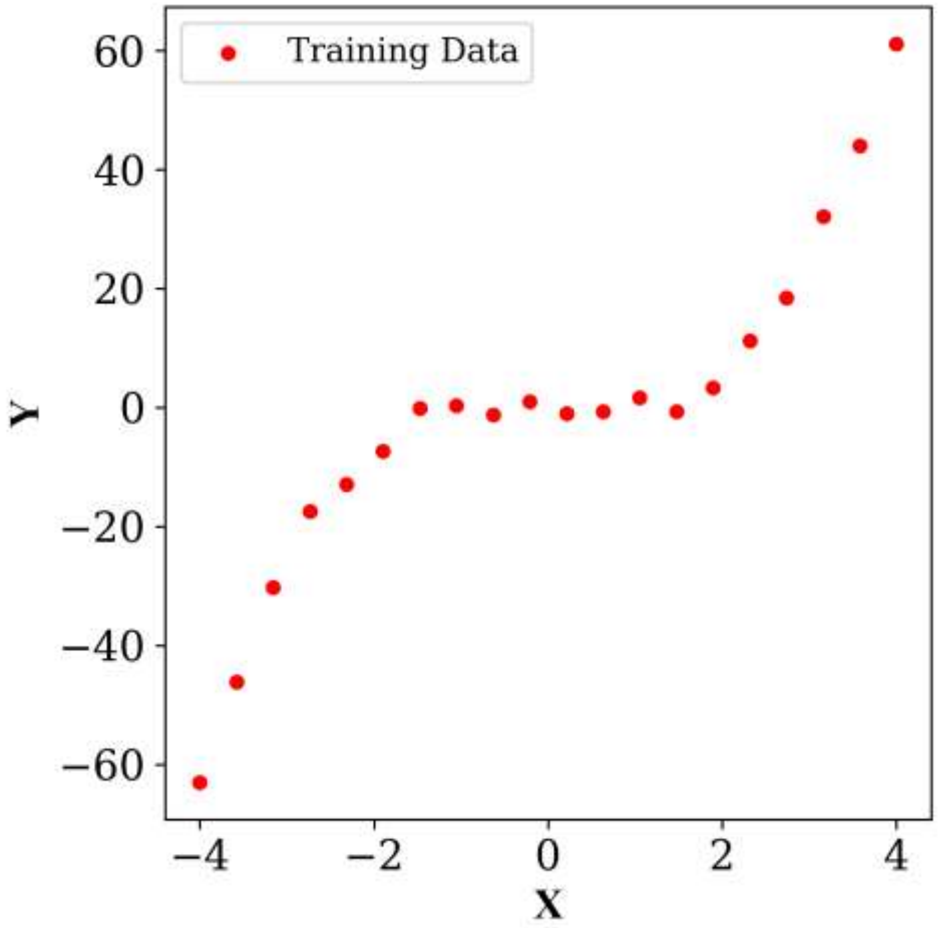
Input: 12 TDA Features

Output: State at time t (S)



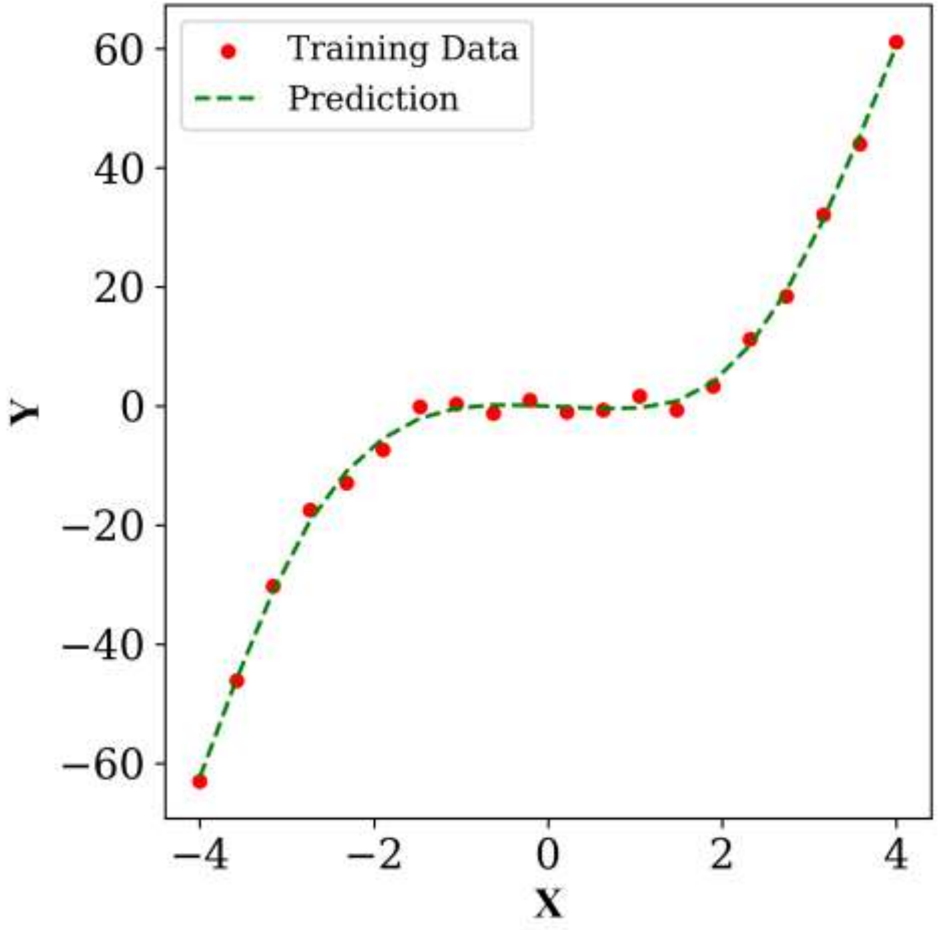
Model Predictions: Basic Point Estimates

Given simple dataset



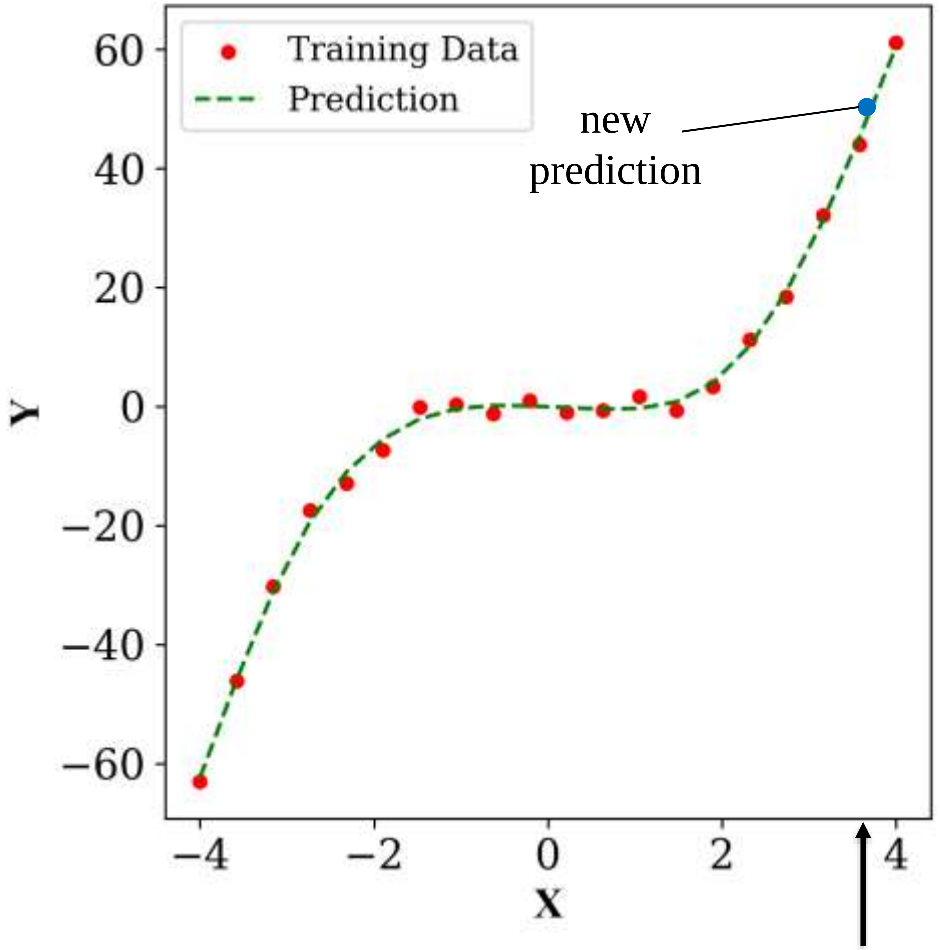
Model Predictions: Basic Point Estimates

Build a model to fit the dataset



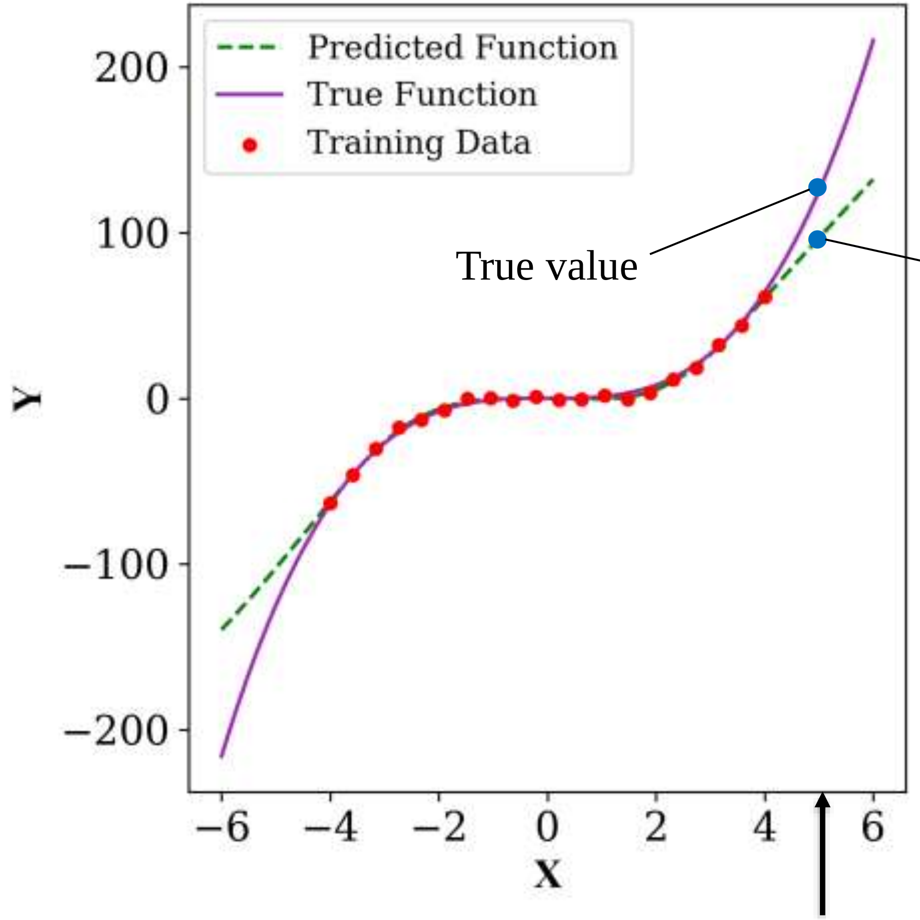
Model Predictions: Basic Point Estimates

Making a new prediction



Point Predictions: Trustworthiness Issues

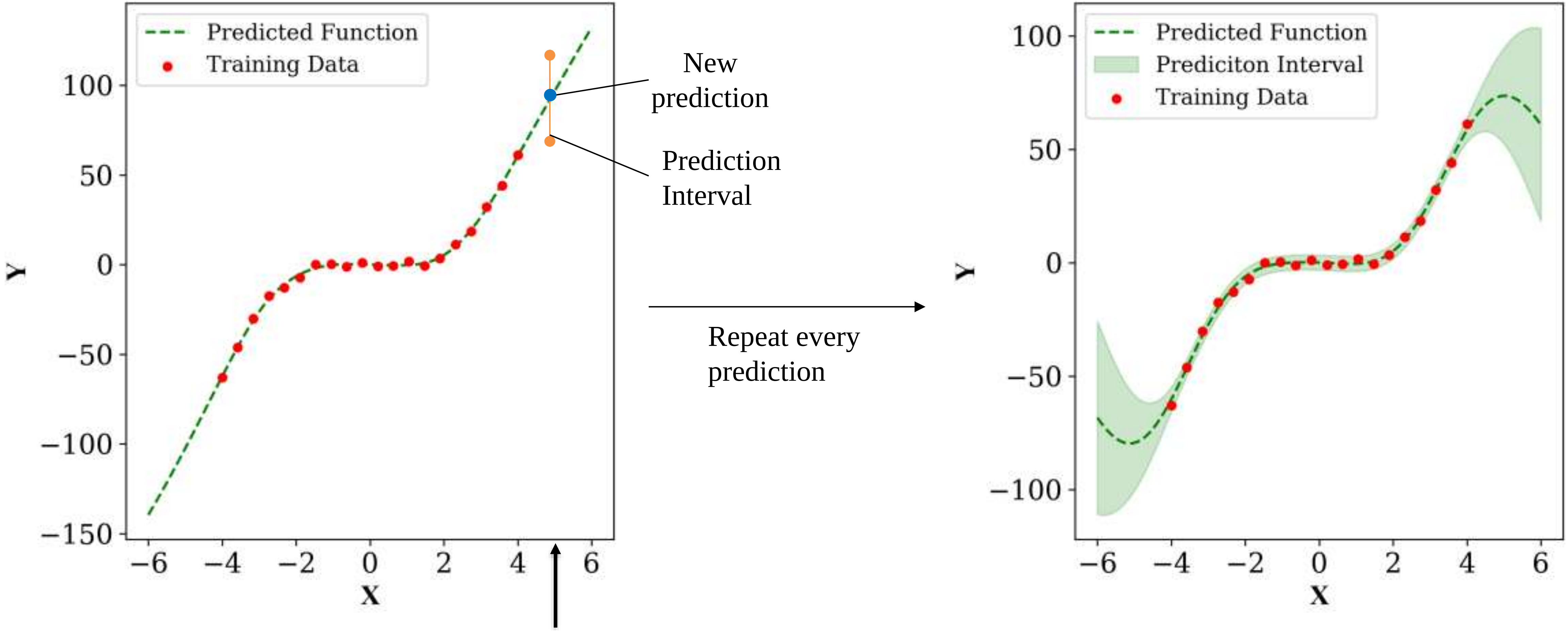
What if we make prediction out of training bound ?



Uncertain Prediction

- Accuracy concerns
- Overconfidence
- Risk management

Enhancing Predictions with Confidence Intervals

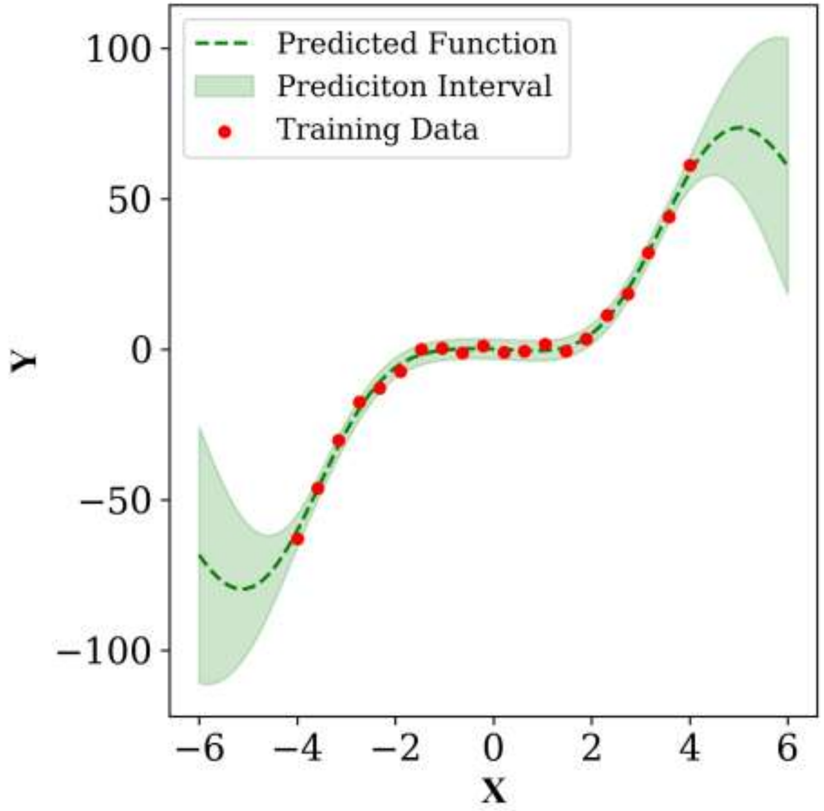


Introducing Uncertainty Quantification (UQ)

Uncertainty Quantification (UQ) is the process of evaluating and managing the uncertainty in model predictions.

UQ models:
- Gaussian process regression
- Monte Carlo (MC) dropout
- Neural network ensemble (NNE)

Models Provide UQ



How UQ is managed

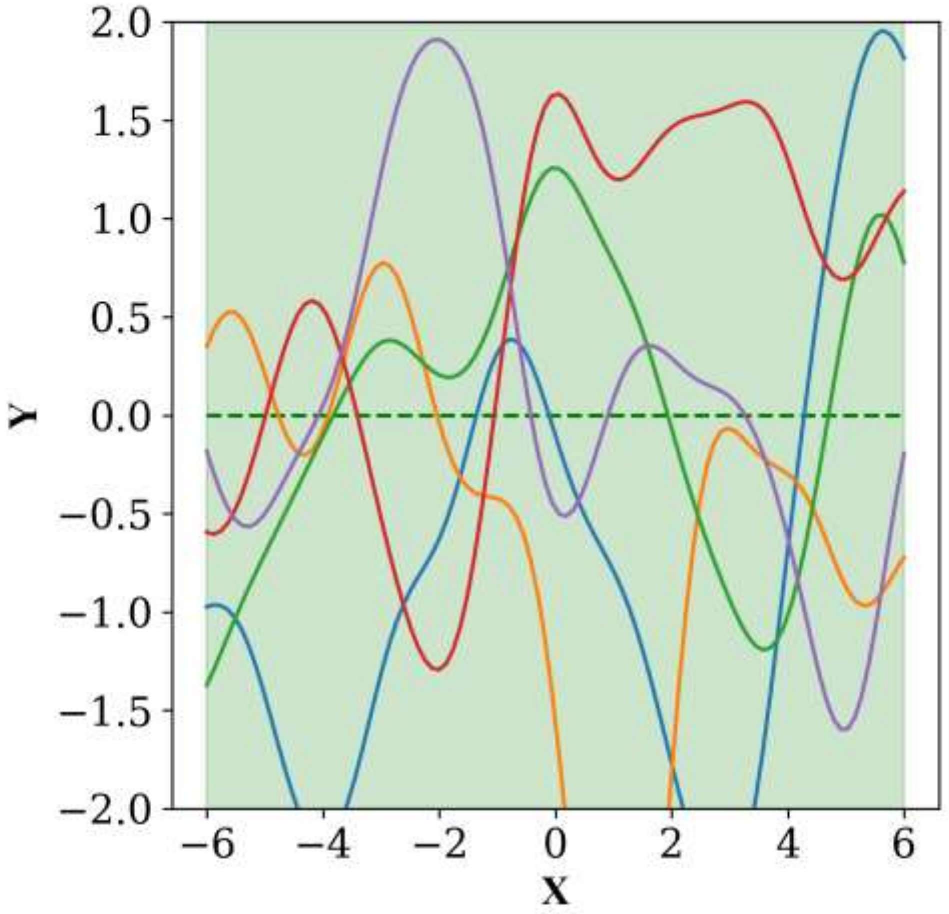
Uncertainty Management

Model Calibration
- Verification
- Validation

Model Prediction
- Identify
- Propagate
- Analysis
- Control

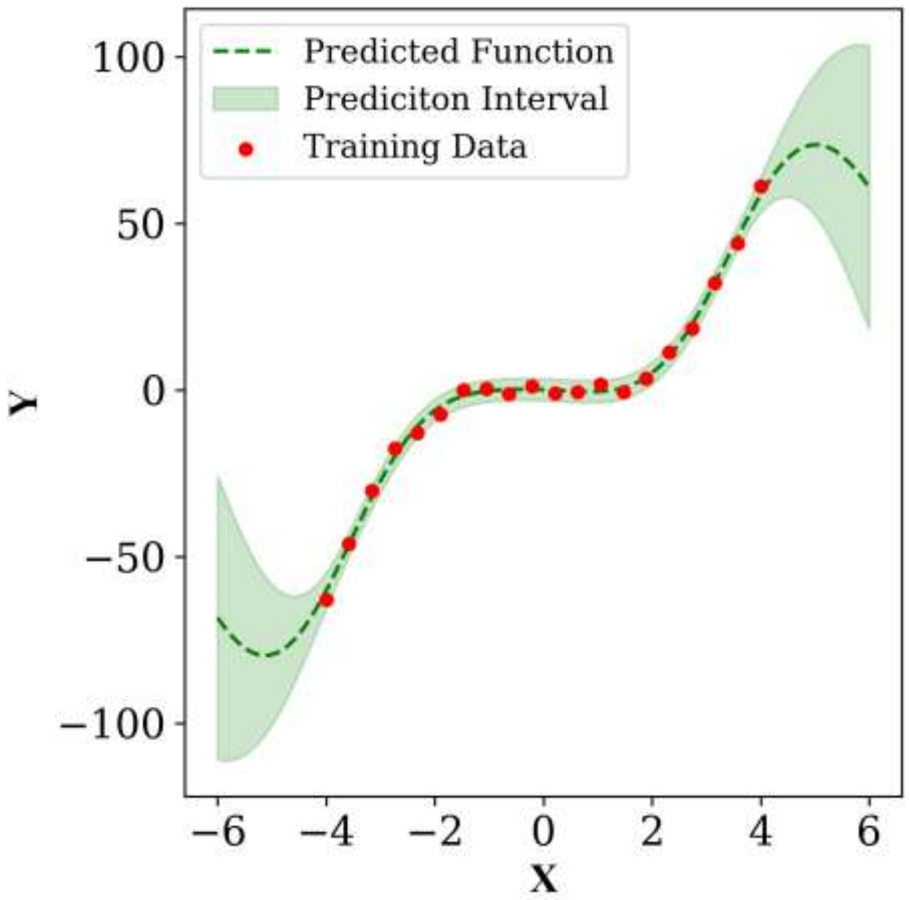
Gaussian Process Regression (GPR)

Prior Distribution (Before seeing data) : $[\mathbf{y}, y_*]^T \sim N(0, \boldsymbol{K})$
$\boldsymbol{K}$: Kernel on input features $\mathbf{x}$

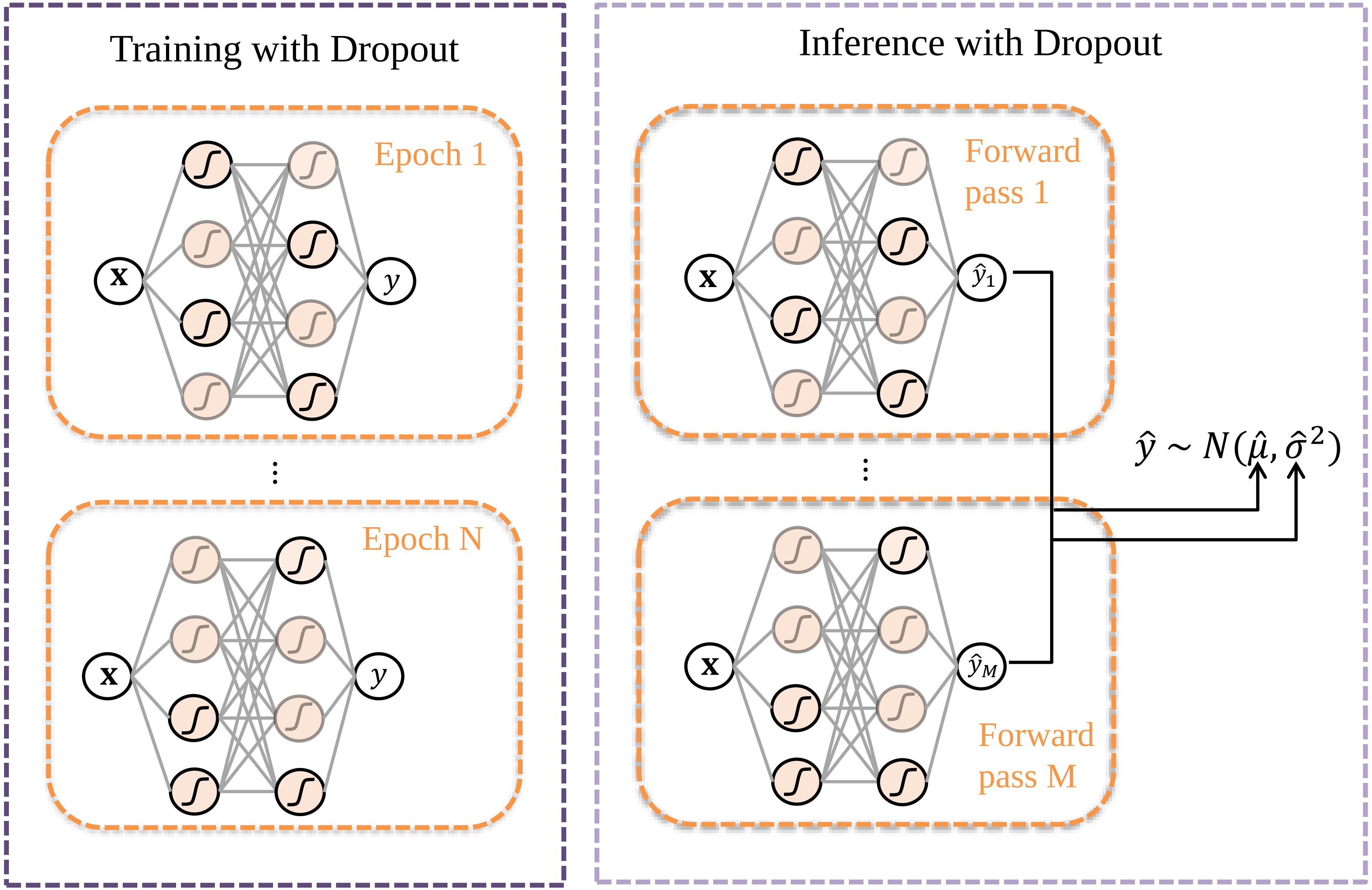


Data observed

Posterior Distribution (After seeing data)
$\hat{y}_* \sim N(\hat{\mu}, \hat{\sigma}^2)$



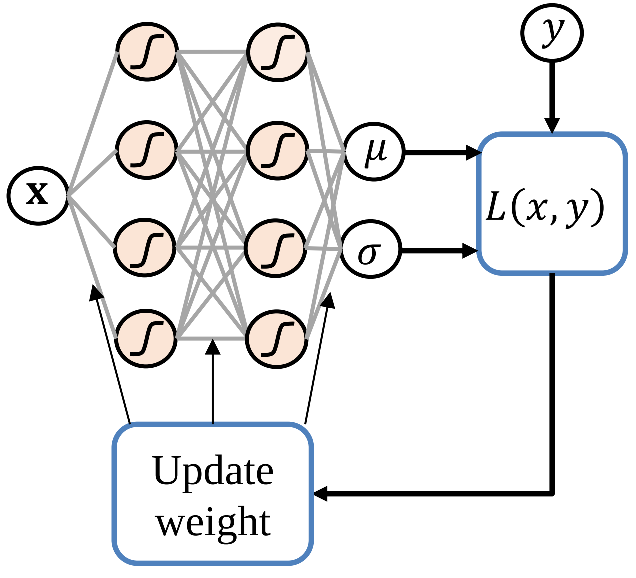
Monte Carlo (MC) Dropout



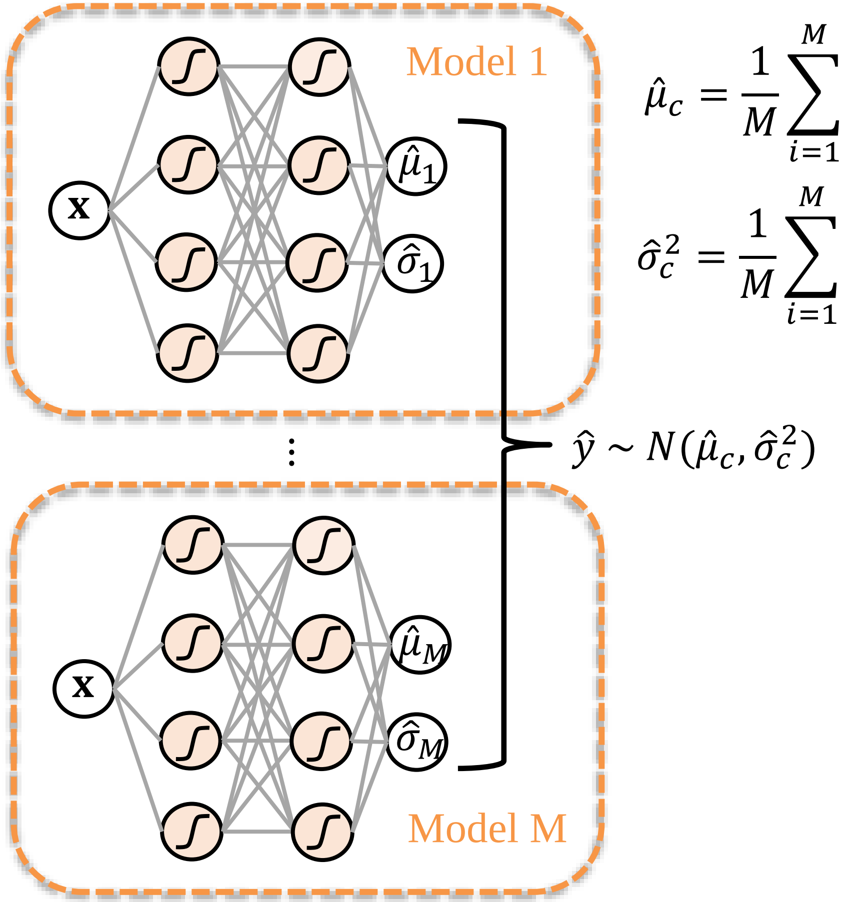
Hyperparameters

Number of forward passes (M)
Number of layer
Number of neurons
Activation function

Neural Network Ensemble (NNE)



$$L(x, y) = \frac{\log(\sigma^2(x))}{2} + \frac{(y - \mu(x))^2}{2\sigma^2(x)}$$



$$\hat{\mu}_c = \frac{1}{M} \sum_{i=1}^{M} \hat{\mu}_i$$

$$\hat{\sigma}_c^2 = \frac{1}{M} \sum_{i=1}^{M} (\sigma_i^2 + \mu_i^2) - \hat{\mu}_c^2$$

Hyperparameters

Ensemble size (M)
Number of layer
Number of neurons
Activation function

Converting LSTM to probabilistic model

Long short-term memory (LSTM):

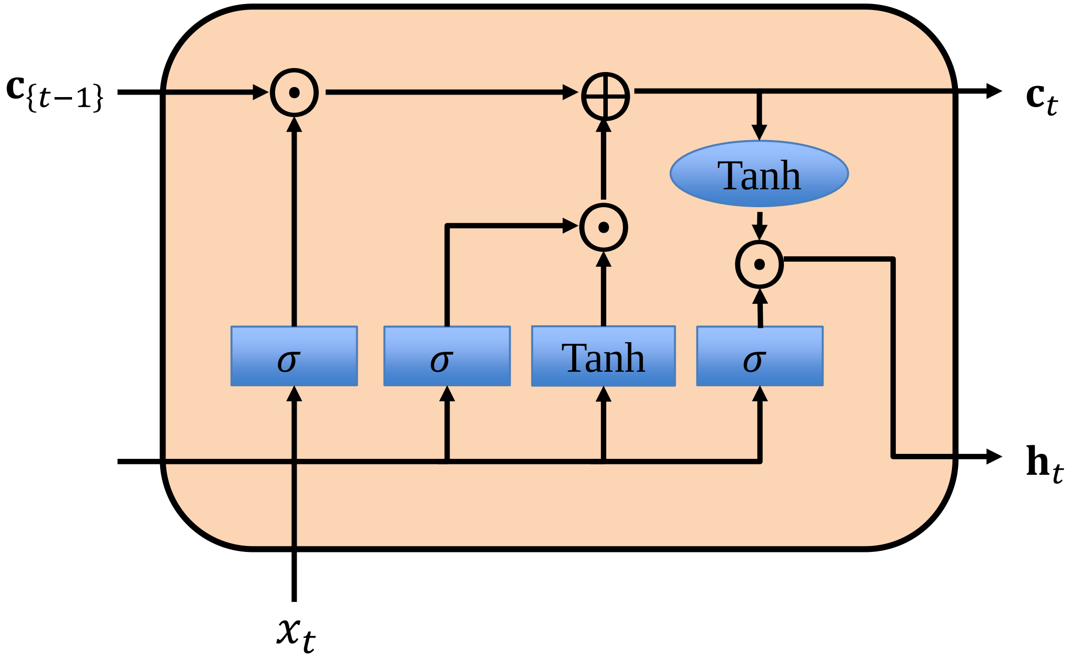
Type of recurrent neural network (RNN), designed to overcome the limitation of traditional RNN in capturing long-term dependencies in sequential data.

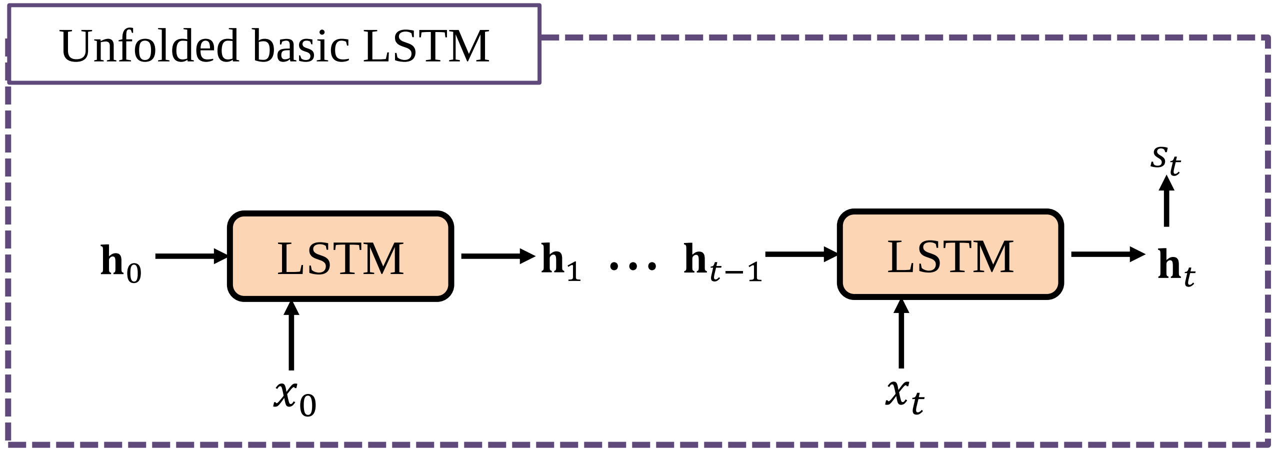
Governing equations:

$$\mathbf{f}_t = \sigma_g(\mathbf{W}_x^f\, x_t + \mathbf{W}_h^f\, \mathbf{h}_{\{t-1\}} + \mathbf{b}_f)$$
$$\mathbf{i}_t = \sigma_g(\mathbf{W}_x^i x_t + \mathbf{W}_h^i \mathbf{h}_{\{t-1\}} + \mathbf{b}_i)$$
$$\mathbf{o}_t = \sigma_g(\mathbf{W}_x x_t + \mathbf{W}_h^o \mathbf{h}_{\{t-1\}} + \mathbf{b}_o)$$
$$\tilde{\mathbf{c}}_t = \sigma_{tanh}(\mathbf{W}_x^{\tilde{c}} x_t + \mathbf{W}_h^{\tilde{c}} \mathbf{h}_{\{t-1\}} + \mathbf{b}_c)$$
$$\mathbf{c}_t = \mathbf{f}_t \circ \mathbf{c}_{\{t-1\}} + \mathbf{i}_t \circ \tilde{\mathbf{c}}_t$$
$$\mathbf{h}_t = \mathbf{o}_t \circ \sigma_h(\mathbf{c}_t)$$

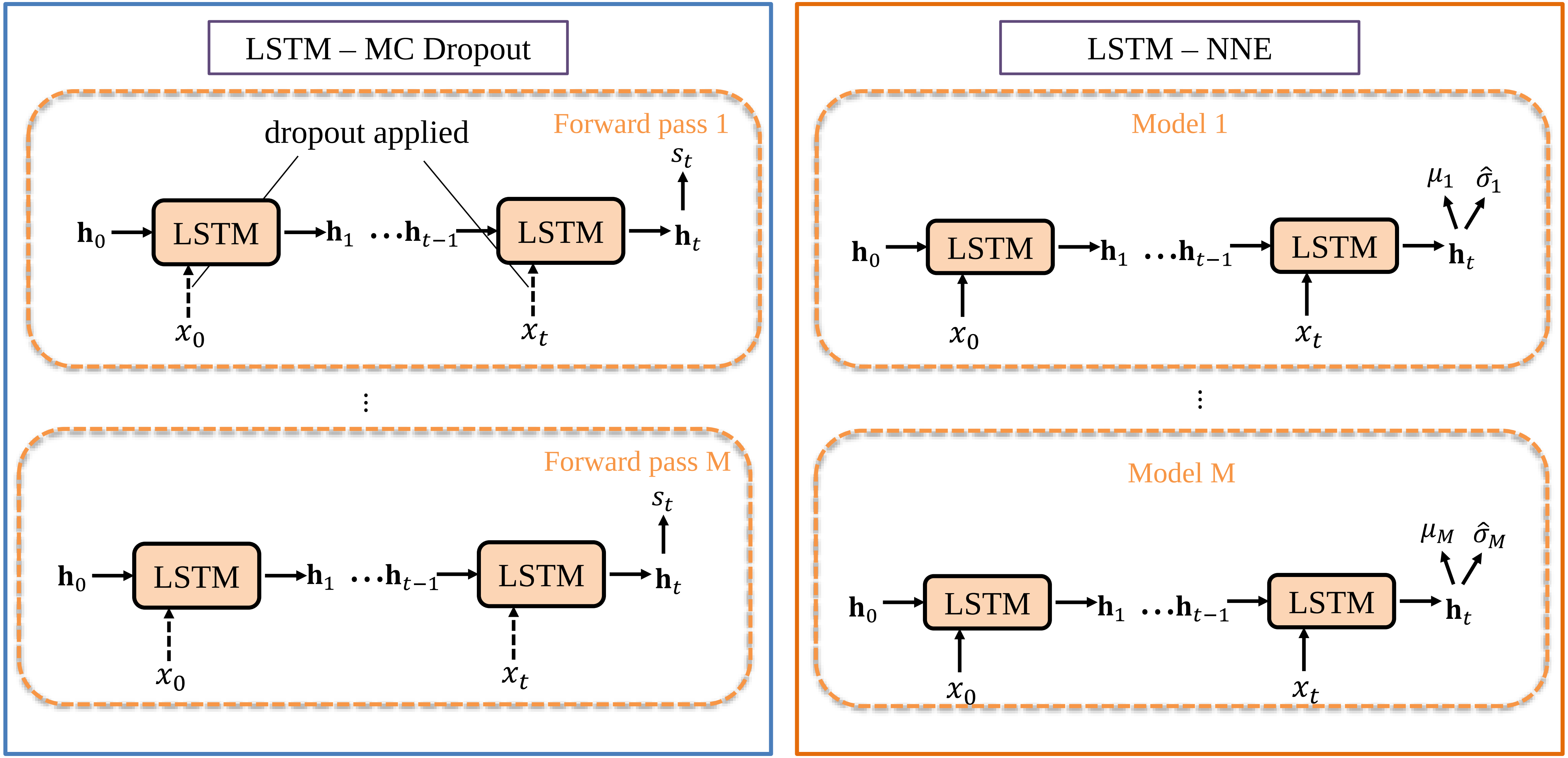
σ_g: sigmoid activation

σ_{tanh}: tanh activation





Converting LSTM to probabilistic model



Case Study 1: Linear Chirp Signal

To model high-rate dynamic systems with well-defined frequency changes and assess the HR-SSEP's performance under controlled conditions. We will use chirp signal:

$$x(t) \; = \cos\left(2\pi\left(\frac{(f_1 - f_0)}{2T}t^2 + f_0 t\right)\right)$$

Parameters:

- f_0 : Initial Frequency
- f_0 : Final Frequency
- T : Total Duration

Training Signal:
10 Hz to 15 Hz

Generate

<u>Feature extraction parameter</u>
H1 window size = 0.13 s
H0 window size = 0.1 s
τ = 0.008 s

Training Signal:
10 Hz to 15 Hz

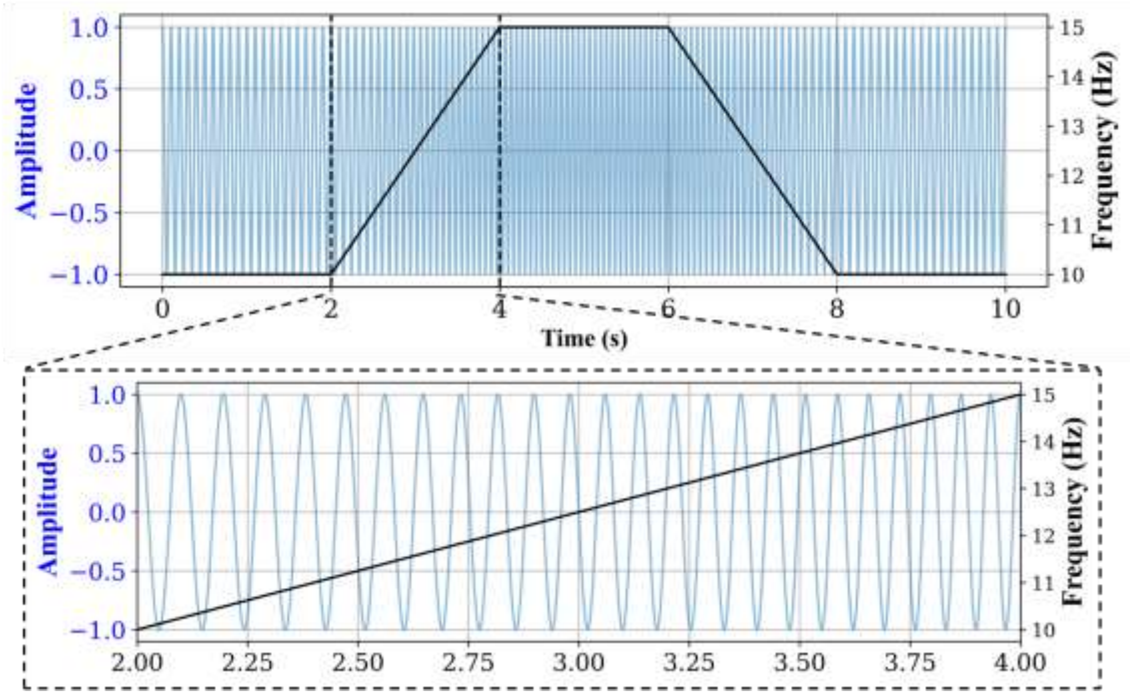
Testing Signal:
5 Hz to 20 Hz

<u>Goal</u>

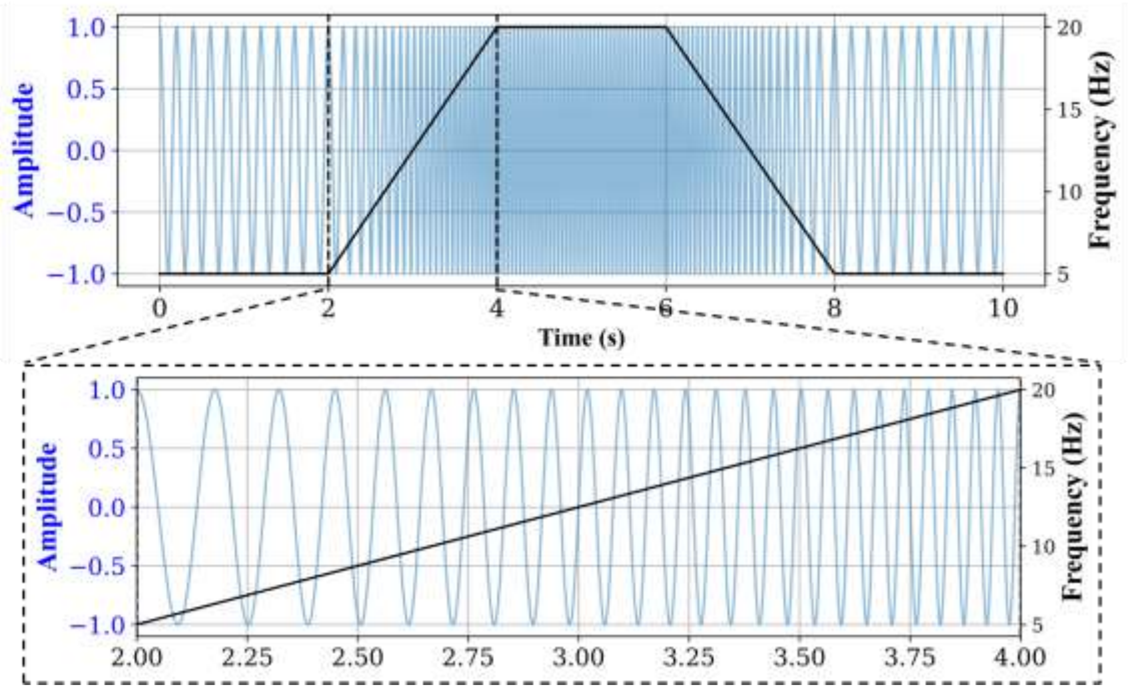
- Evaluate how well the model handles signals outside the trained frequency range.
- Evaluate feature extraction performance when signals are out of the training range.

Case Study 1: Linear Chirp Signal

Training Signal
Frequency Range: 10 Hz to 15 Hz



Testing Signal
Frequency Range: 5 Hz to 20 Hz



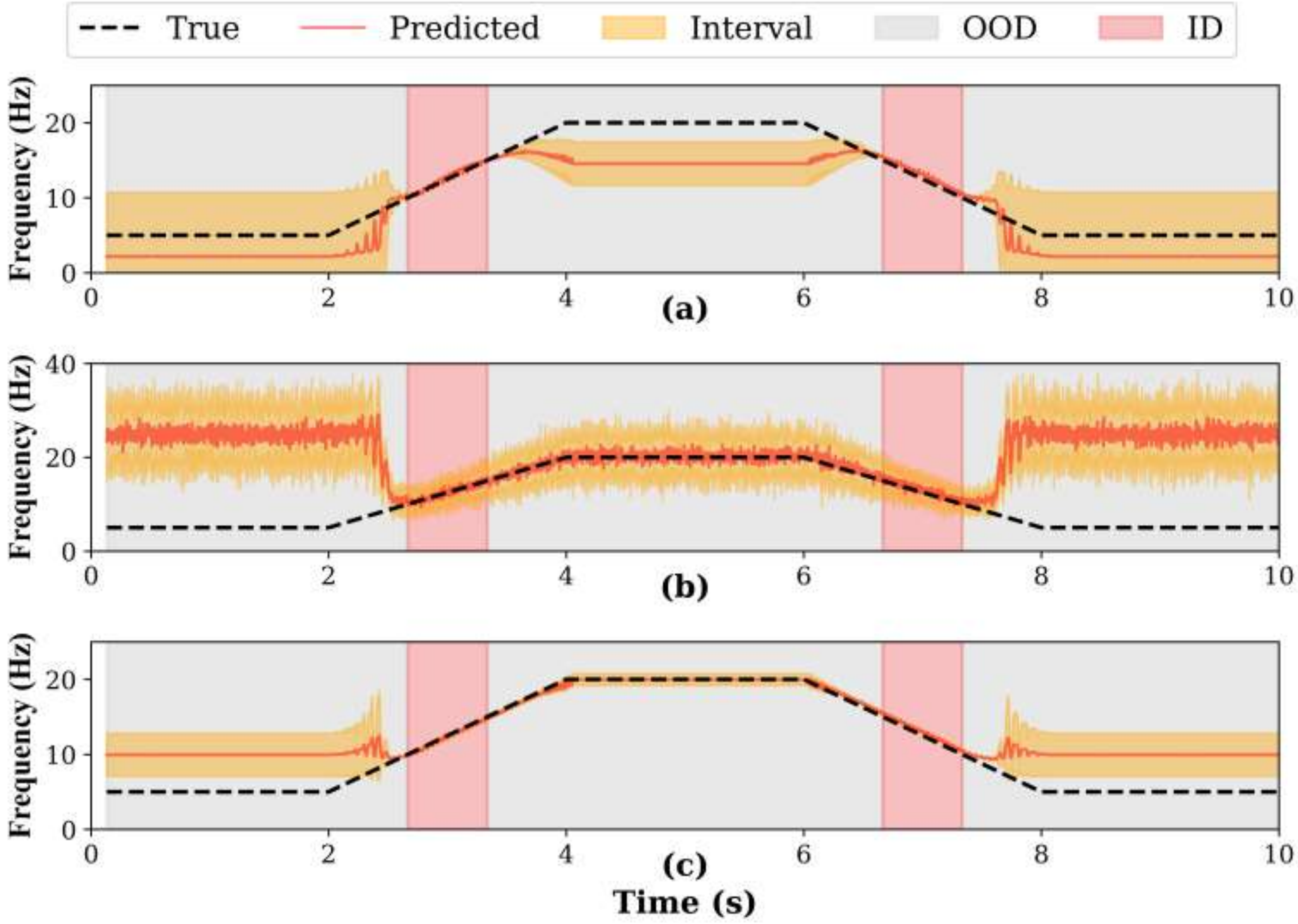
Case Study 1: Linear Chirp Signal

ID: In-Domain
OOD: Out-Of-Domain

Gaussian process regression

Monte Carlo dropout

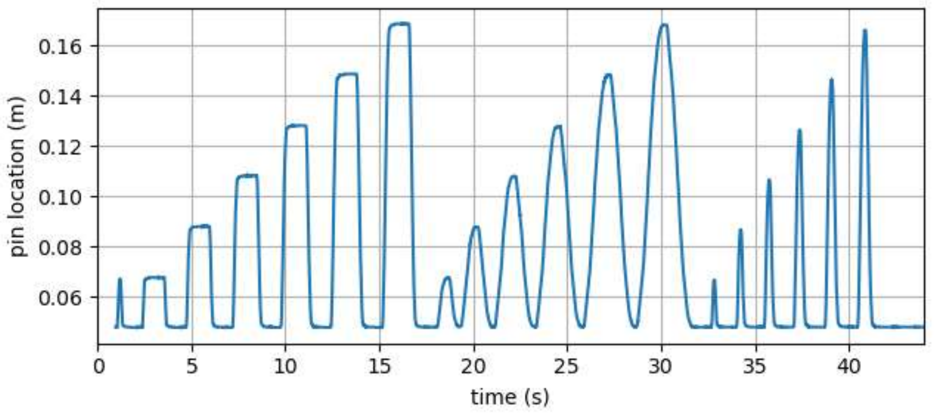
Neural network ensemble



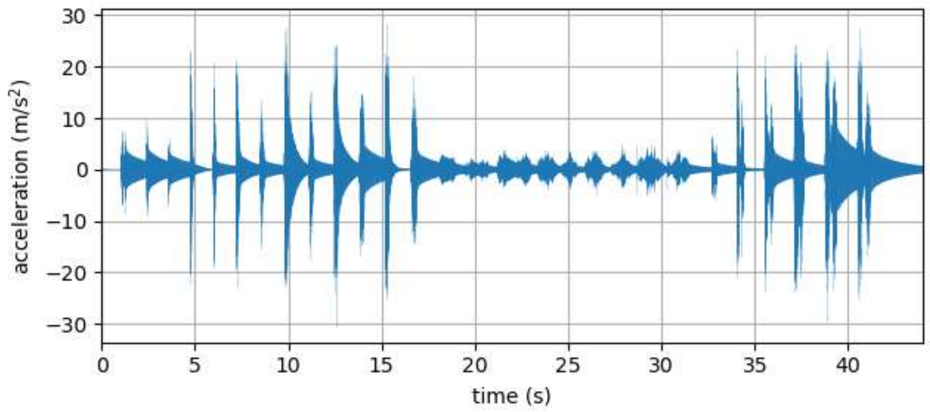
Case Study 2: DROPBEAR Dataset

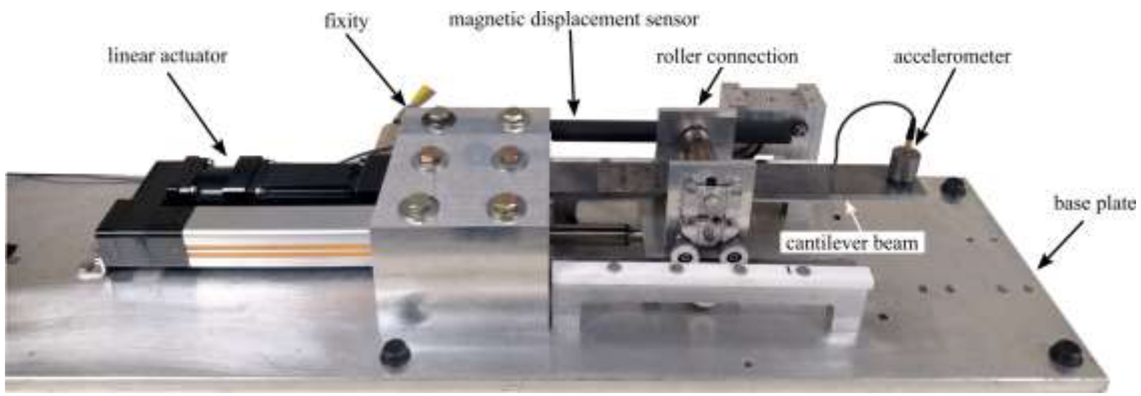
DROPBEAR experimental testbed:
The Dynamic Reproduction of Projectiles in Ballistic Environments for Advanced Research (DROPBEAR) was used to generate the experimental data.

Capabilities:
Reproduce fast boundary condition changes.
Mimic rapid mass changes.
Simulate the rapid dynamics typical of high-rate events.



Generate

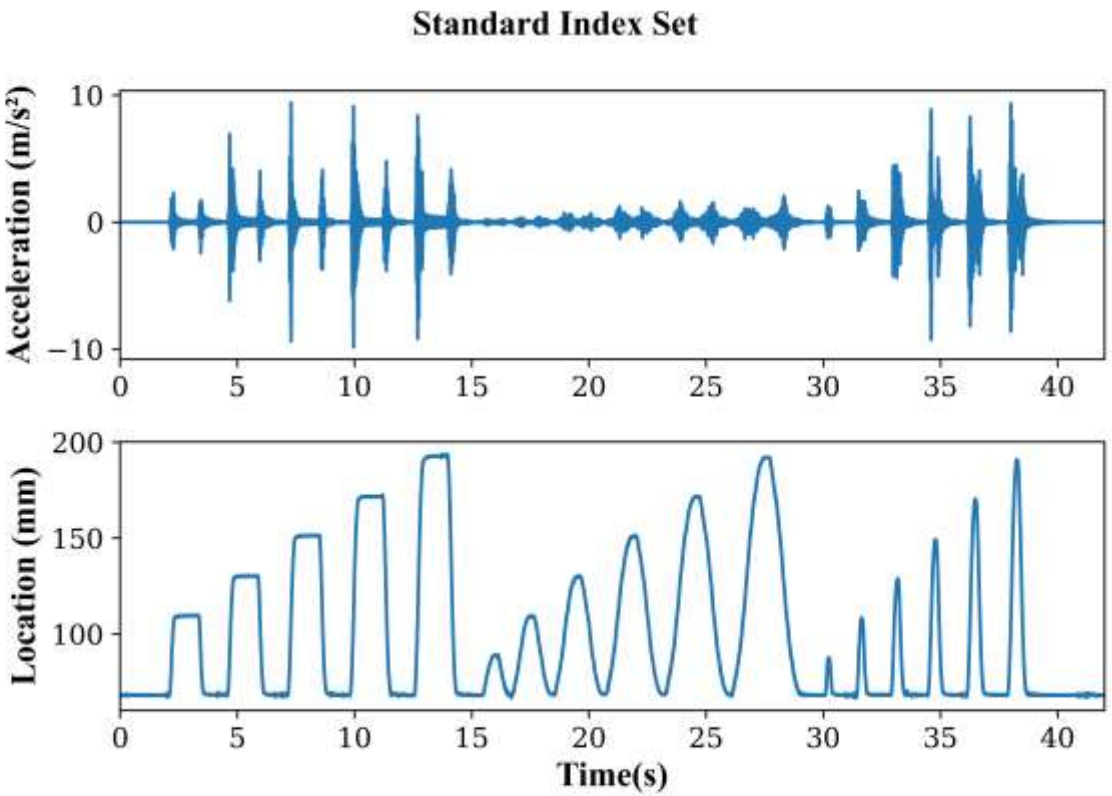


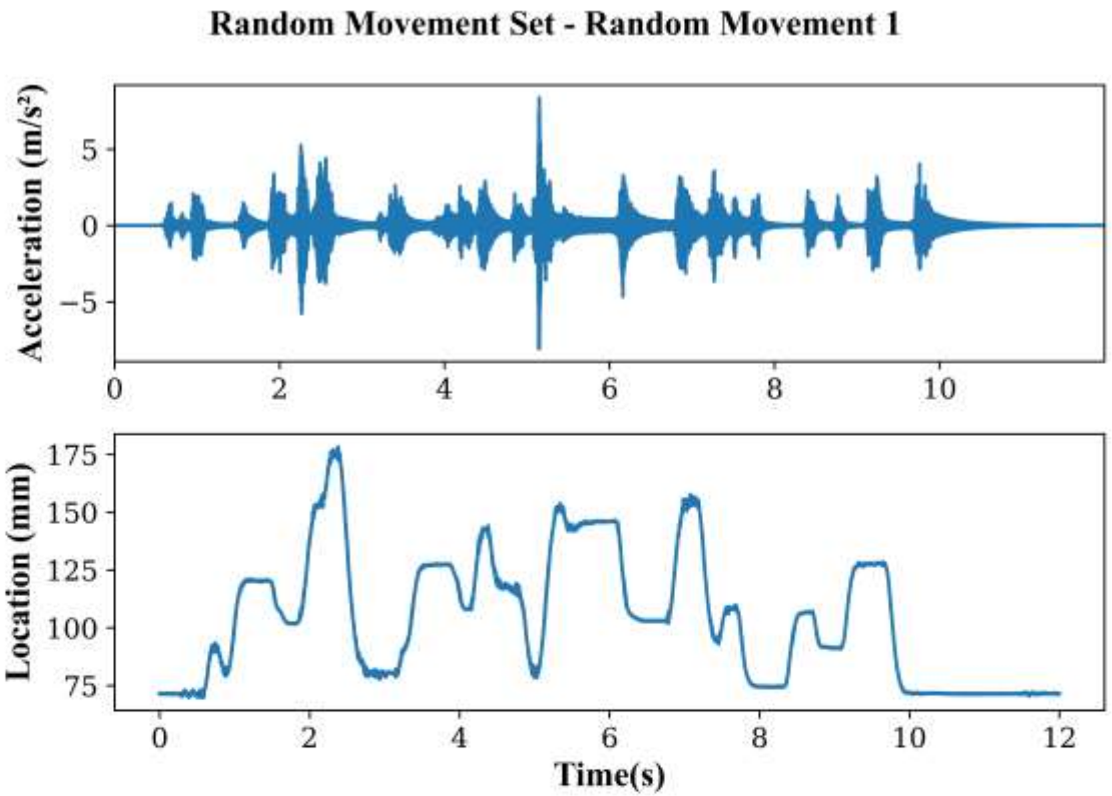


Austin Downey, Jonathan Hong, Jacob Dodson, Michael Carroll, and James Scheppegrell, "Dataset-2-dropbearacceleration-vs-roller-displacement," Dec. 2021. [Online]. Available: https://github.com/High-Rate-SHM-Working-Group/Dataset-2-DROPBEAR-Acceleration-vs-Roller-Displacement

Case Study 2: DROPBEAR Dataset

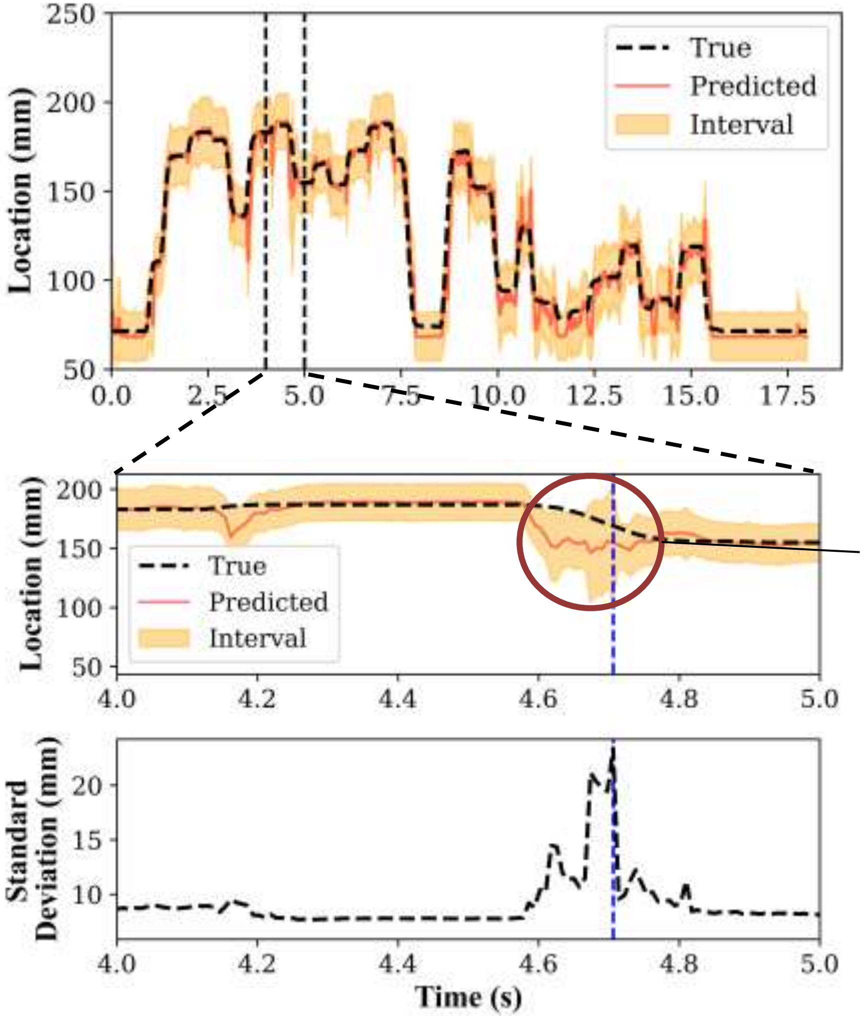
Training and validation	Testing





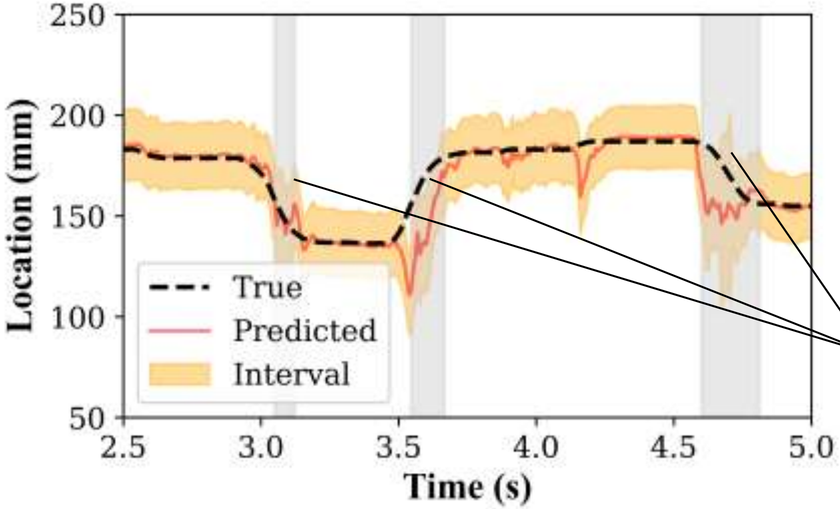
Case Study 2: DROPBEAR Dataset

UQ in cart location prediction



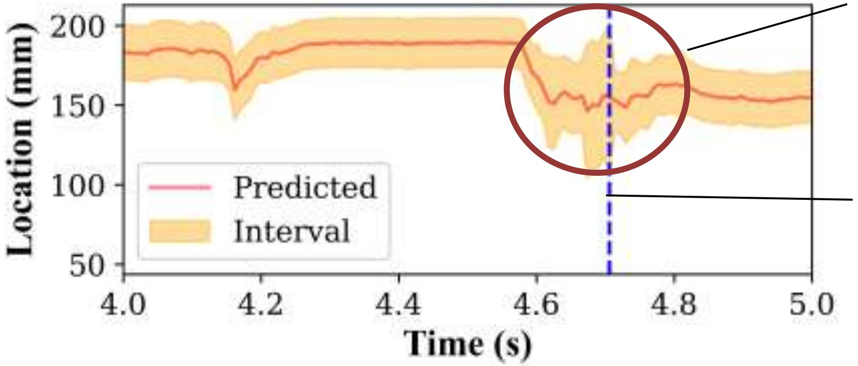
High predictive uncertainty

Use case 1: anomality detection



Sharp changes

Use case 2: proxy for error



High-error region

Uncertainty peak

Summary

Key Outcomes:

- **Advanced State Estimation**: Strong performance with various datasets.
- **Reliable Predictions**: Includes uncertainty quantification for trustworthy results.
- **Real-Time Capability**: Predictions made in under 100 ms.

Future Plans:

- **Expand Metrics**: Add more uncertainty metrics.
- **Broaden Testing**: Apply to diverse high-rate datasets.
- **Enhance Functionality**: Integrate anomaly detection and contextual adaptation.

Acknowledgements

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- Air Force Research Laboratory Munitions Directorate.
- National Science Foundation

Thank You!

Questions?

Backup

Results

Metric Used:
- Mean absolute error (MAE)
- Time response assurance criterion (TRAC)
- Signal-to-noise ratio (SNR_{dB})
- Expected confidence error (ECE)
- Test time: Computation time from pipeline to prediction for each sample.

Model Performance Metrics for random movement 7

Model	**MAE (mm)**	**TRAC**	$\boldsymbol{SNR_{dB}}$	**ECE (%)**	Test time **(ms)**
GPR	6.300	0.991	20.623	16.093	0.665
NN	6.620	0.992	6.460	-	1.283
NNE	6.138	0.993	21.291	18.785	2.963
MC Dropout	8.297	0.991	20.496	5.771	0.523
LSTM	6.280	0.995	6.387	-	1.454
LSTM-MC	6.895	0.996	23.106	26.936	0.454
LSTM-NNE	**4.803**	**0.997**	**24.494**	**15.444**	**2.183**

LSTM-NNE