# Model Reduction/Expansion with Applications for Linear and Nonlinear Dynamic Response and Related Topics

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University of Massachusetts Lowell

# Half a Century

## How Some Old Stuff Can Still Be Useful



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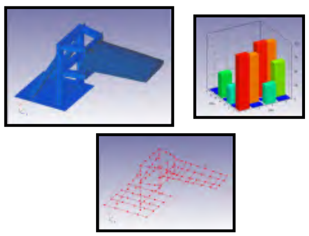
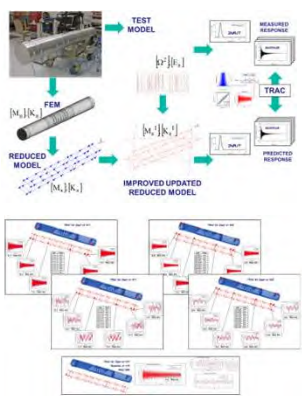
# Structural Dynamics Thrust Areas – Peter Avitabile

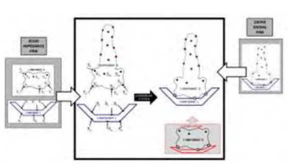
## Test/Analysis Correlation Updating



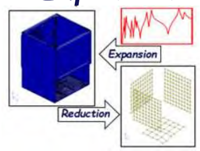
### MDM Reduced Order Updated Response Predictions




### Field to Laboratory Fixture Neutralization

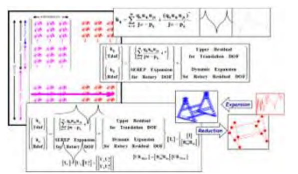


## Model Reduction Expansion

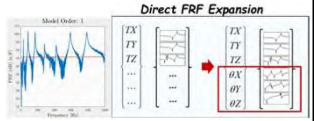



### VIKING/SEREP Self-Smoothing

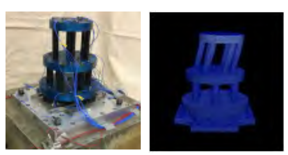
### FRF Smoothing/Expansion



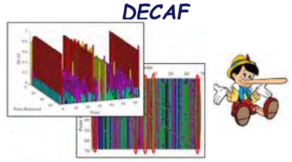
### Direct FRF Expansion



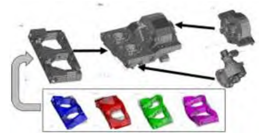
### SEREP SMOOTHING

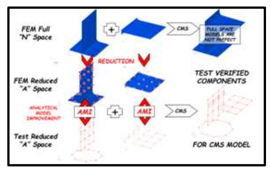


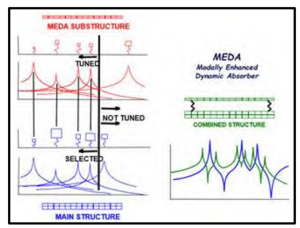
### DECAF

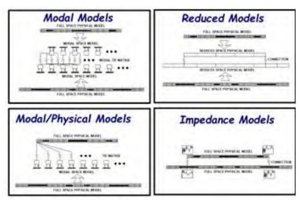


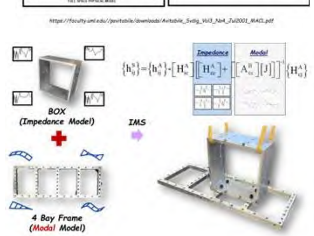
## System Modeling





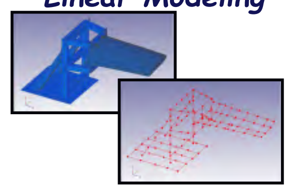



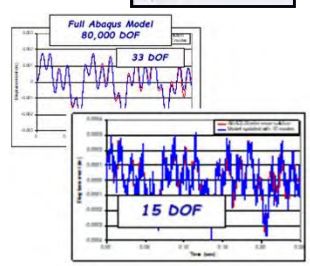



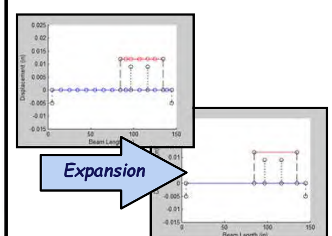

## Linear & Nonlinear Response Modeling

### Linear Modeling

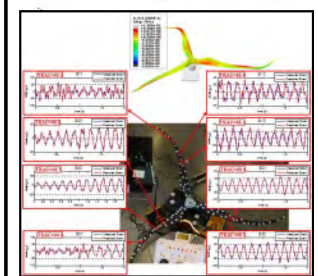


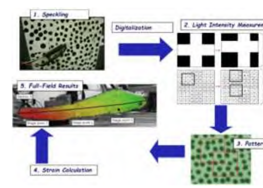


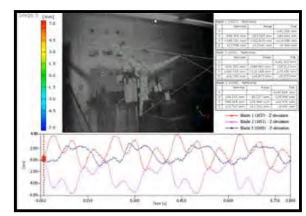

### Nonlinear Modeling

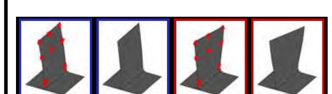



## Full Field Optical Techniques

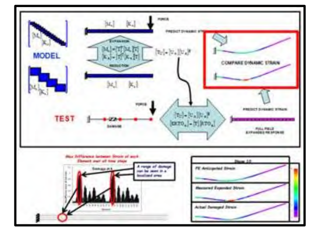


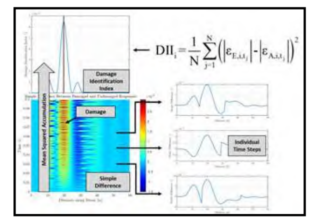


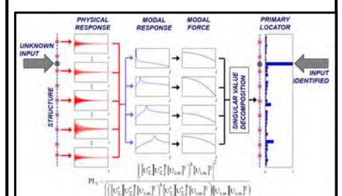


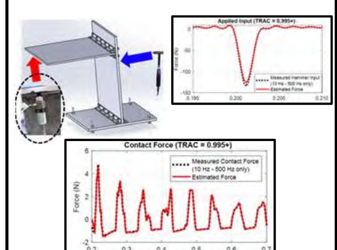
## Damage Identification

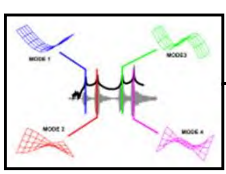




### Force Reconstruction



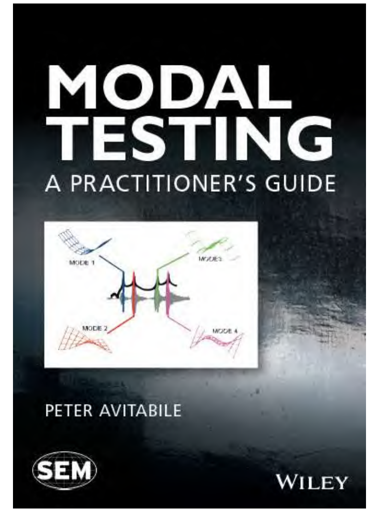



# Dr. Peter Avitabile, PE

*Professor Emeritus, Mechanical Engineering*
*Research Professor, Michigan Technological U*
*Co-Director, Structural Dynamics*
*and Acoustic Systems Laboratory*

*President 2016/2017 & Vice President 2014*
*for Society for Experimental Mechanics*
*Associate Editor – Handbook of*
*Experimental Structural Mechanics*

*B.S.M.E., Manhattan College*
*M.S.M.E., University of Rhode Island*
*D. Eng., University of Massachusetts Lowell*
*Professional Engineer, Rhode Island*




***Pete has five decades of experience in design and analysis using FEM and experimental techniques. His main area of research is structural dynamics specializing in the areas of modeling, testing and correlation of analytical and experimental models along with advanced applications for developing structural dynamic models. Pete has contributed over 300 technical papers in the area as well as his "Modal Space" article series in the Experimental Techniques magazine published by the Society for Experimental Mechanics for 17 years. He is the 2004 recipient of the prestigious SEM DeMichele Award and elected SEM Fellow in 2020. He is recognized worldwide as an expert in structural dynamic modeling applications. He often provides consulting services for a wide variety industries in these specialty areas of expertise.***

# Structural Dynamics and Acoustic Systems Laboratory

FEMtools
FEMAP
LMS
MSC Software
ANSYS
MATLAB
lms
SDRC

***Modeling Tools***

***Testing Equipment***

Vibrant Technology, Inc.
STAR
LMS CADA-X
LMS Test.Lab
MTS
SPECTRAL DYNAMICS

***Experimental Tools***

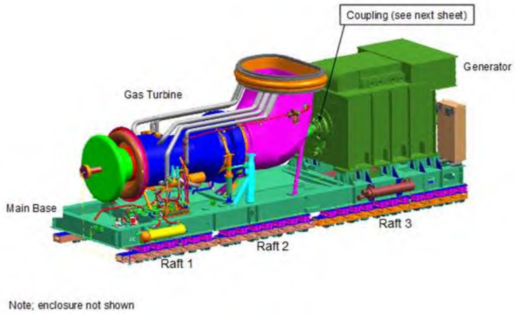
***The Structural Dynamics and Acoustic Systems Laboratory is one of the best equipped labs in the country to do the work that the lab does.***

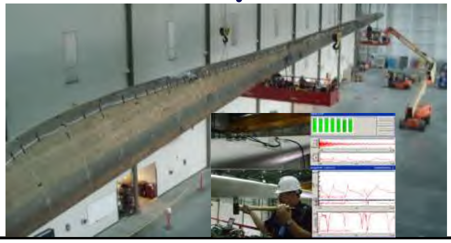
***In addition to research efforts, the SDASL has been involved in many industry related structural dynamic tests due to their capability***

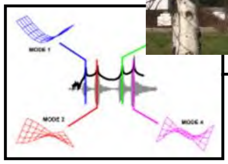
***- Performed detailed modal tests for several 50m+ wind turbine blades***

***- DDG 1000 Propulsion System for new Navy destroyer***

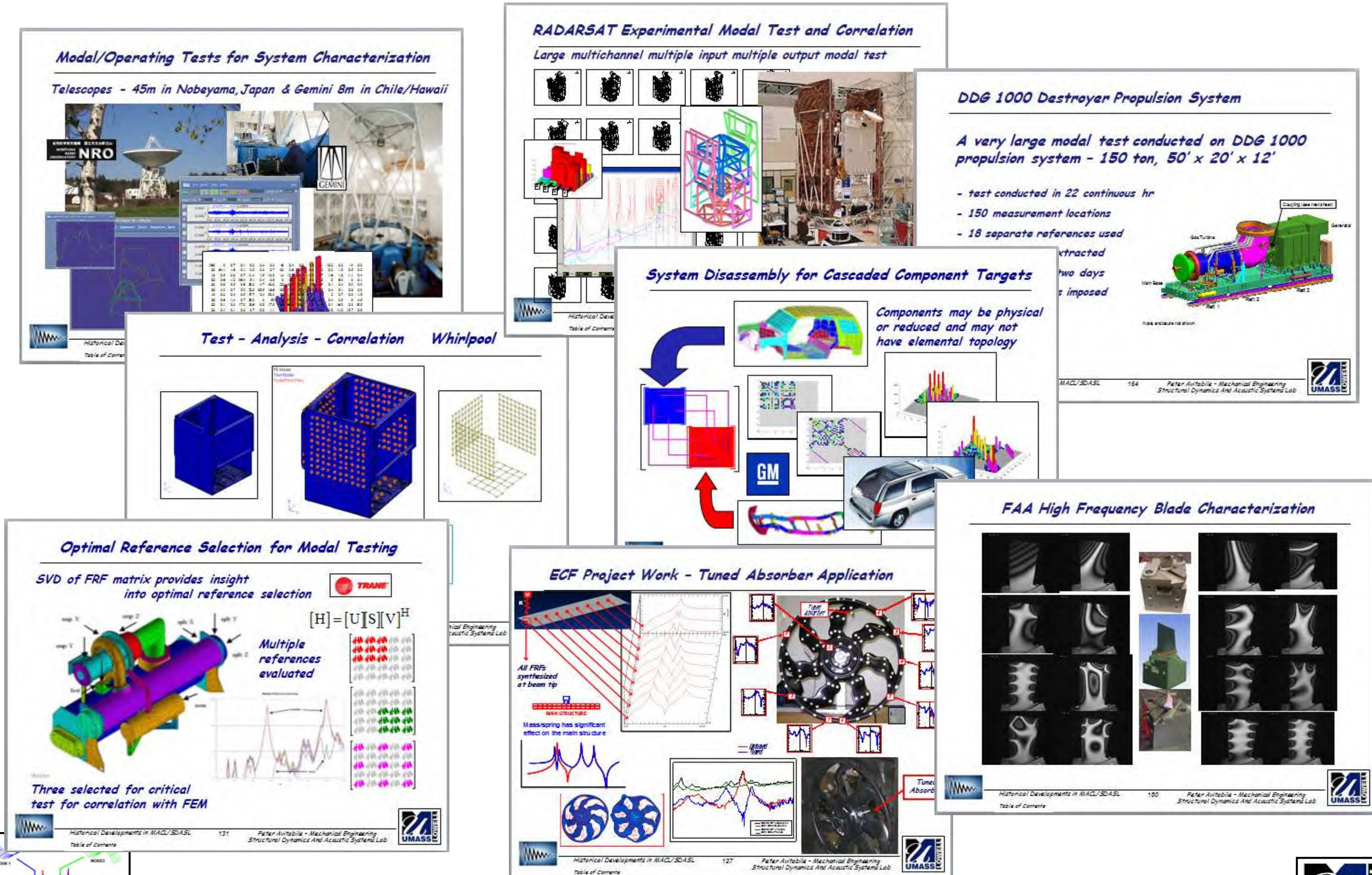
***- Optical & radio telescopes (Nobeyama, Gemini, Haystack)***

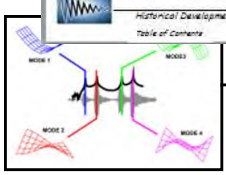




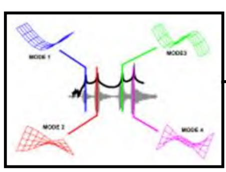
# Peter Avitabile Work Spans Many Industries and Applications



# Where is UMASS Lowell ???



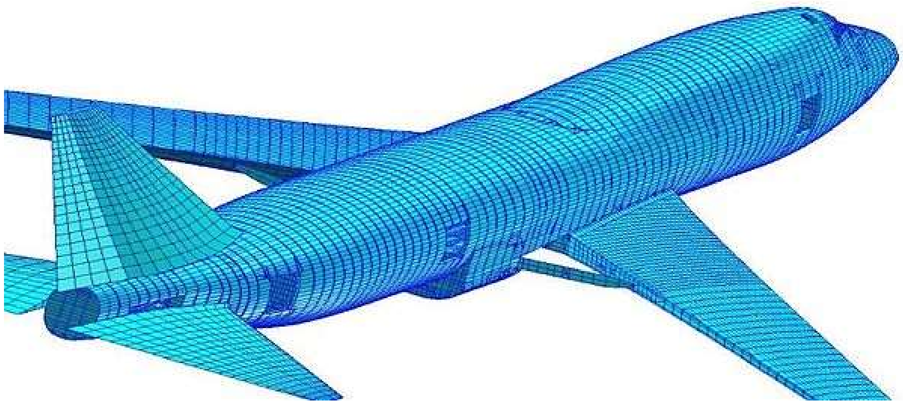


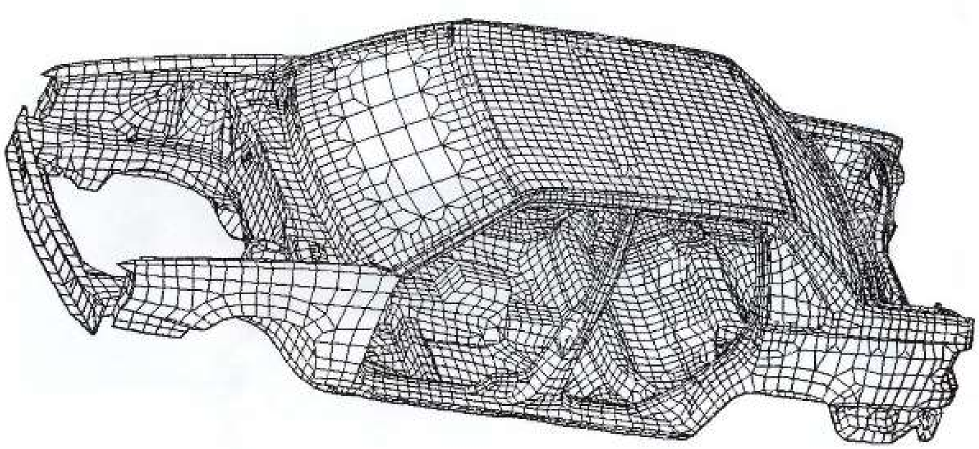
**Some general topics:**

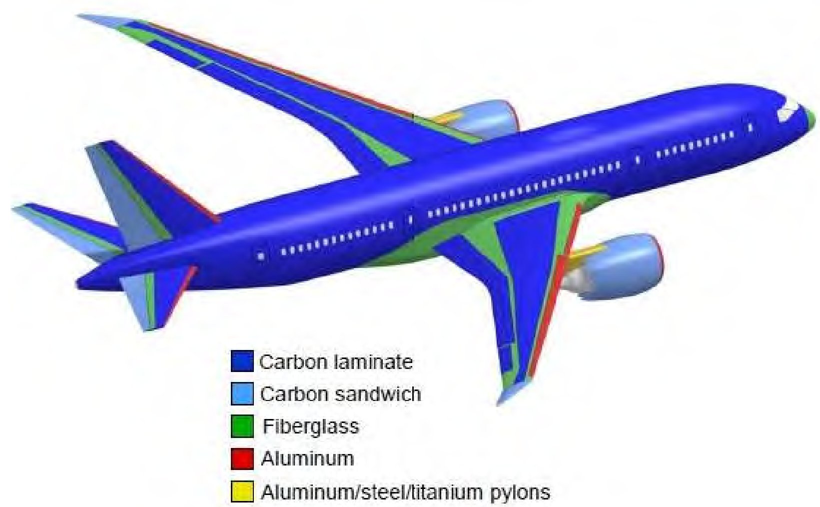
- Model Reduction and Expansion
- Correlation/updating of structural dynamic models
- Structural dynamic modification and system modeling
- Highly reduced order analytical models for efficient response
- Incorporating measured experimental data (from limited data) for full field dynamic response/strain
- Utilizing linear modeling techniques to efficiently address nonlinear response applications
- Damage detection utilizing dynamic response and dynamic strain from real-time operating data
- Force Reconstruction – Beyond Measured Points

# Finite Element Modeling

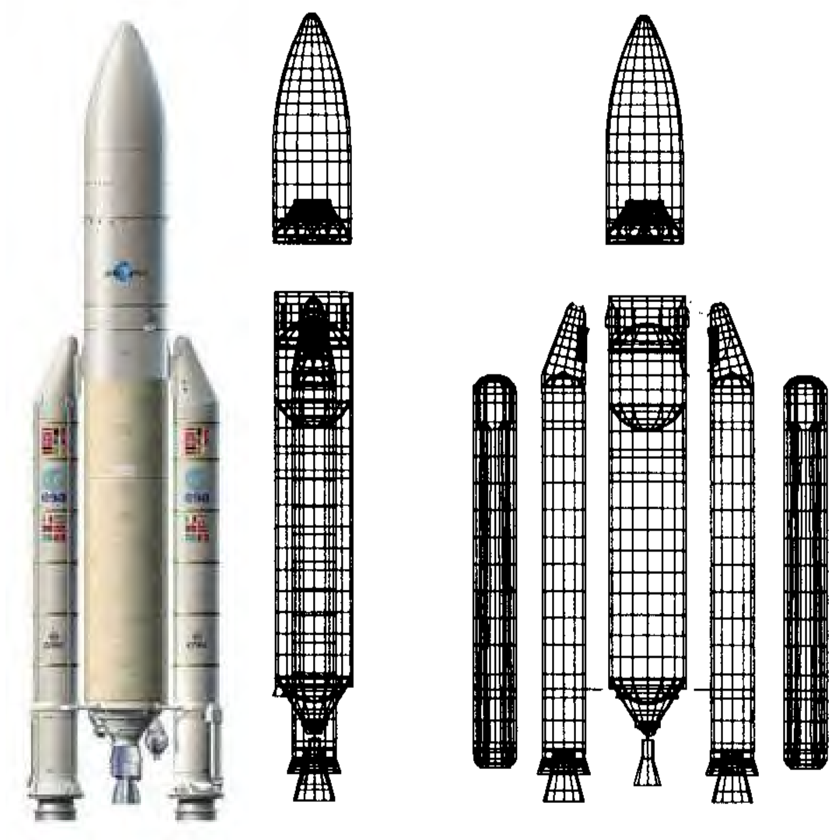
# Obsessively Large Finite Element Models Dominate the Landscape Today (only because we think we need them to be this large)

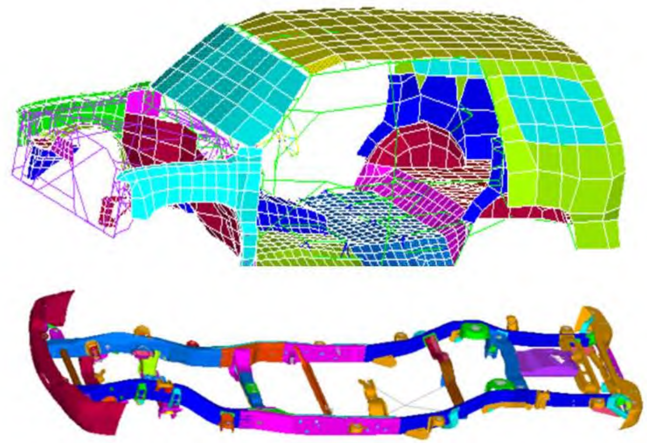


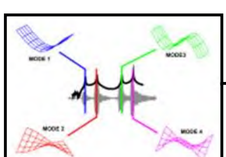




# Models so large that reduced order models necessary







# Larger Models vs Smaller Accurate Models

- Large models are often difficult to interpret
- Especially true when multiple perturbations are evaluated

- Often time smaller reduced modes are needed
- Models must be accurate for any response studies

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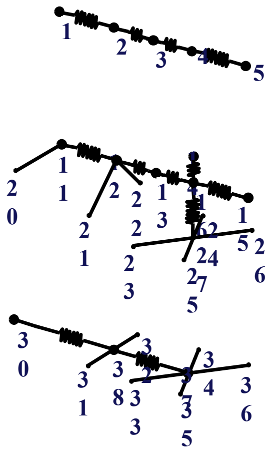
Sometimes it seems like we are going back to old approaches


# TECHNOLOGY IMPROVEMENTS IN MODELING

*(or were the old techniques adequate?)*

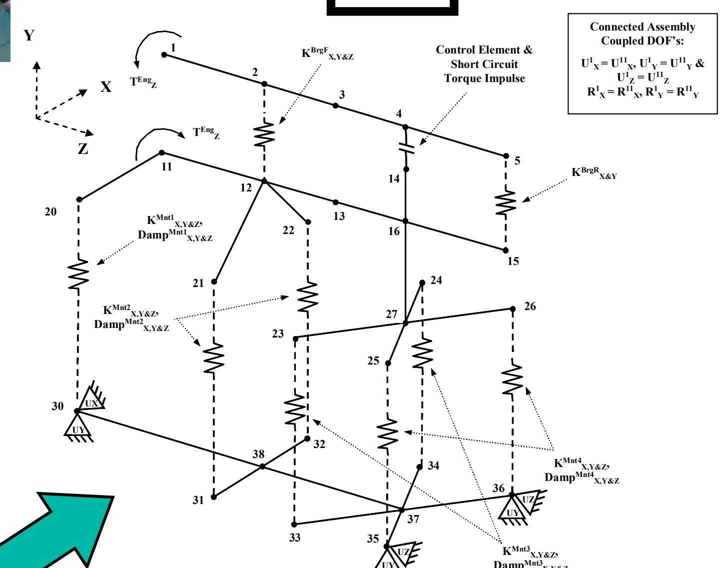
**1**

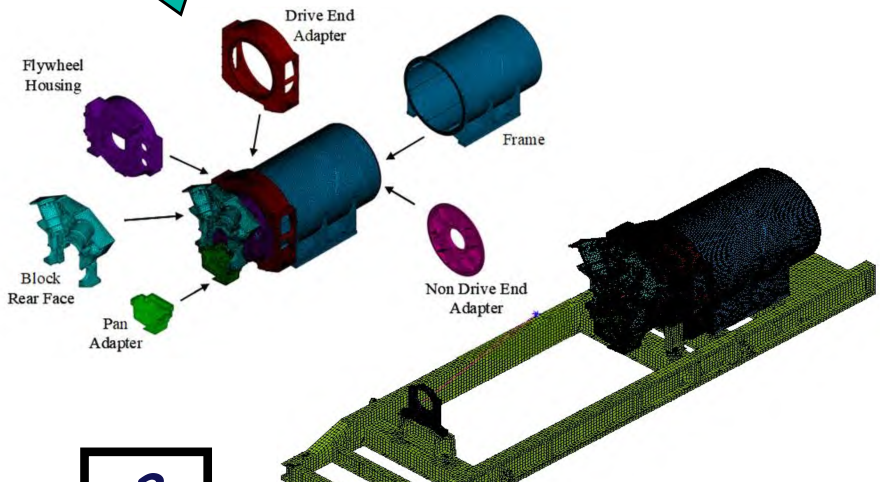
*Prehistoric models were simple due to lack of computational ability*



*Simpler models are developed from detailed models to gain better understanding of effects observed from studies made*

**3**

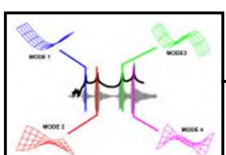



**2**

*Improved computers and software allow development of sophisticated models*


## BACK TO THE FUTURE

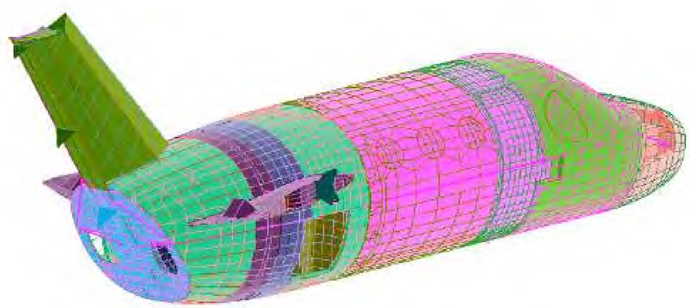


# TECHNOLOGY IMPROVEMENTS IN MODELING

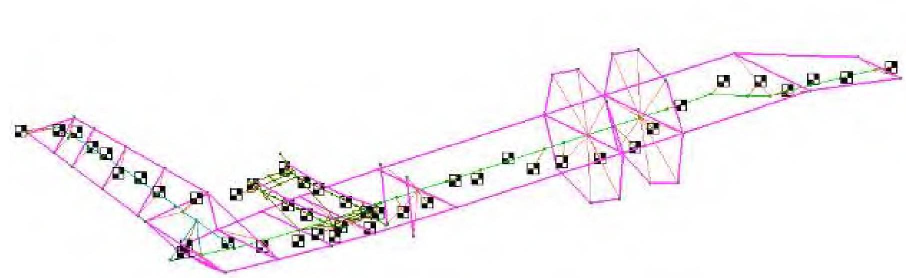
*(or were the old techniques adequate?)*

**1** Early models were simple due to lack of computational ability

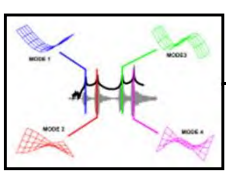
**2** Improved computers and software allow development of sophisticated models



**3** Computational restraints force use of simpler models



## BACK TO THE FUTURE

# Larger Models vs Smaller Accurate Models

- Large models are often difficult to interpret
- Especially true when multiple perturbations are evaluated

- Often time smaller reduced modes are needed
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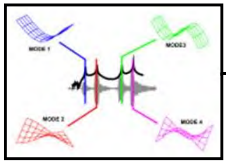
Sometimes it seems like we are going back to old approaches


BTTF

# Model Reduction

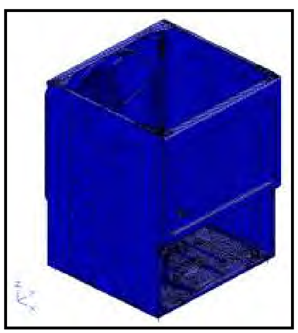


# Model Reduction – An Important Piece

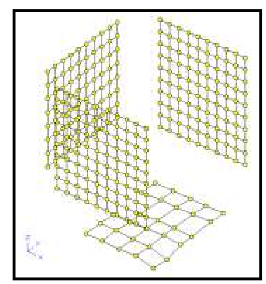
*Dynamic reduction means :*


*reducing a given dynamic finite element model*
*to one with fewer degrees of freedom*
*while maintaining the dynamic characteristics of the system.*

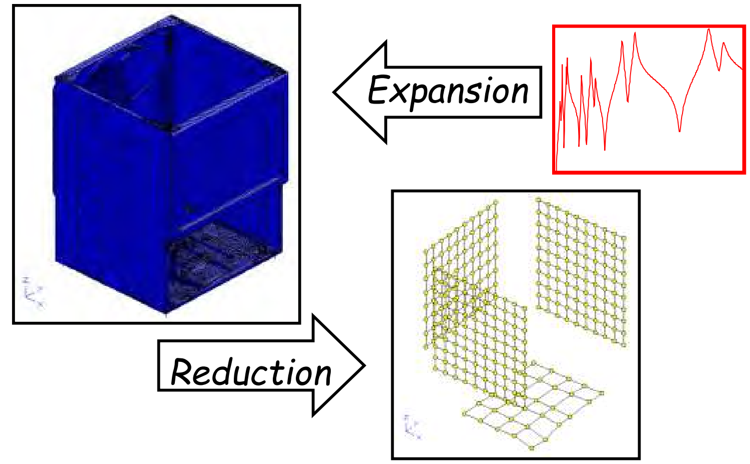
*The transformation T will take on various forms*
*depending on the transformation technique utilized*



$$\{x_n\} = \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = [T]\{x_a\}$$



# Model Reduction and Expansion

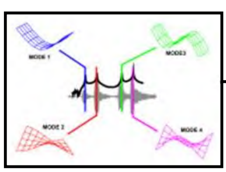



- *Model reduction - capture system dynamics at a subset of points*
- *Model expansion – obtain full field information from limited points*
- *The general transformation between the 'n' full space DOF and 'a' subset of DOF is*

$$\{x_n\} = \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = [T]\{x_a\}$$

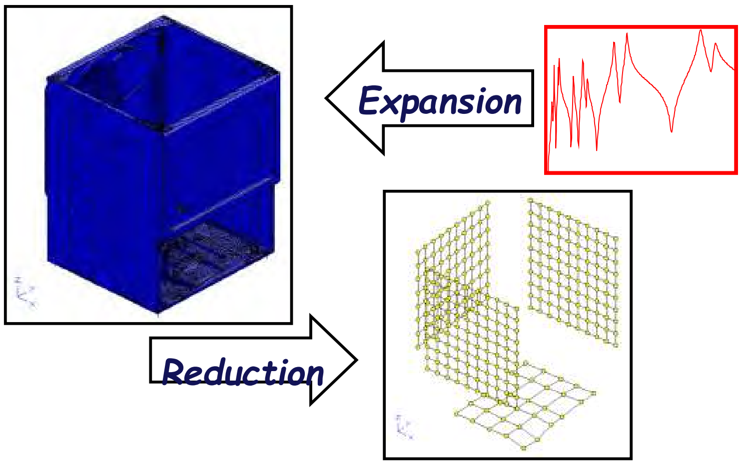
- *And the mass and stiffness matrices are written as*

$$[M_a] = [T]^T [M_n][T] \qquad [K_a] = [T]^T [K_n][T]$$



# Model Reduction and Expansion

***Reduction/Expansion are critical to many structural Dynamic modeling activities – correlation, modification, updating, …***




$$\{x_n\} = \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = [T]\{x_a\}$$

$$[M_a] = [T]^T [M_n] [T]$$

$$[K_a] = [T]^T [K_n] [T]$$

## Guyan Condensation

$$[T_s] = \begin{bmatrix} [I] \\ [t_s] \end{bmatrix} = \begin{bmatrix} [I] \\ -[K_{dd}]^{-1}[K_{da}] \end{bmatrix}$$

## Improved Reduced System

$$[T_i] = \begin{bmatrix} [I] \\ [t_s] \end{bmatrix} + [t_i]$$

$$[t_s] = -[K_{dd}]^{-1}[K_{da}]$$

$$[t_i] = \begin{bmatrix} [0] & [0] \\ [0] & [K_{dd}^{-1}] \end{bmatrix} [M_n][T_s][M_a]^{-1}[K_a]$$

## Dynamic Reduction

$$[D_n] = [[K_n] + f[M_n]]$$

$$[T_f] = \begin{bmatrix} [I] \\ [t_f] \end{bmatrix} = \begin{bmatrix} [I] \\ -[D_{dd}]^{-1}[D_{da}] \end{bmatrix}$$

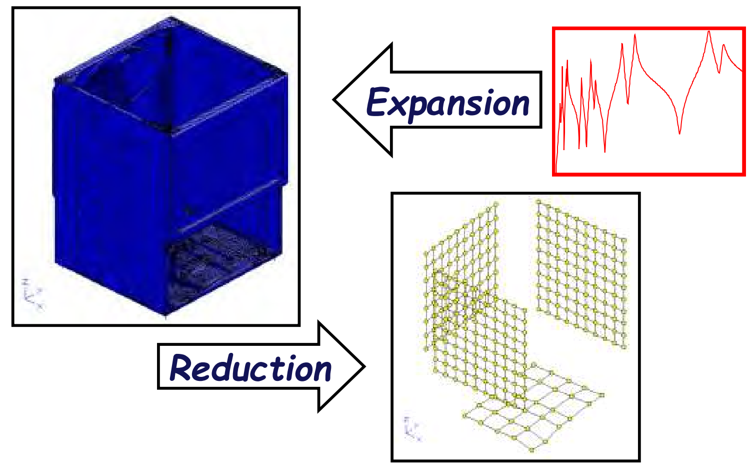
# Model Reduction and Expansion

## System Equivalent Reduction Expansion Process

## SEREP is a significant breakthrough in reduction/expansion

$$\begin{Bmatrix}\{x_a\}\\ \{x_d\}\end{Bmatrix} = \{x_n\} = [U_n]\{p\} = \begin{bmatrix}[U_a]\\ [U_d]\end{bmatrix}\{p\}$$

$$\begin{Bmatrix}\{x_a\}\\ \{x_d\}\end{Bmatrix} = \begin{bmatrix}[U_a]\\ [U_d]\end{bmatrix}[U_a]^g\{x_a\}$$



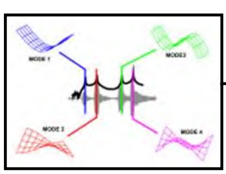

## Reduced matrices saw tremendous computational savings

$$[M_a^S] = [T_U]^T[M_n][T_U] = [U_a^g]^T[U_a^g] \qquad [K_a^S] = [T_U]^T[K_n][T_U] = [U_a^g]^T[\Omega^2][U_a^g]$$

## and breakthrough changes in correlation techniques

$$[U_a]^T[M_a][E_a] \equiv [U_n]^T[M_n][E_n] \equiv [U_a]^g[E_a]$$

https://faculty.uml.edu//pavitabile/downloads/IMAC7_SEREP.pdf



# *Model Reduction and Expansion – SEREP*

- ***The System Equivalent Reduction Expansion Process (SEREP) transformation is***

$$[T_U] = [U_n][U_a]^g$$

- ***The eigenvectors & eigenvalues are preserved exactly using this technique***

- ***The mass and stiffness are written as***

$$[M_a] = [T_U]^T [M_n][T_U] = \left[U_a{}^g\right]^T \left[U_a{}^g\right]$$

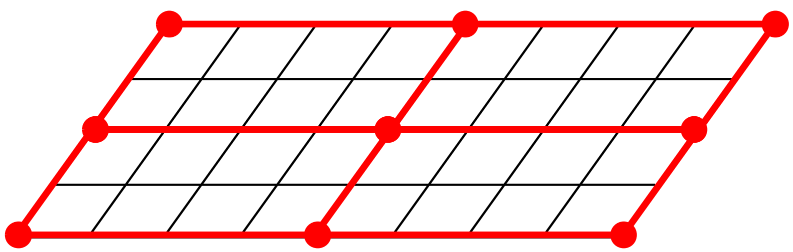
$$[K_a] = [T_U]^T [K_n][T_U] = \left[U_a{}^g\right]^T \left[\Omega^2\right]\left[U_a{}^g\right]$$

*https://faculty.uml.edu//pavitabile/downloads/IMAC7_SEREP.pdf*

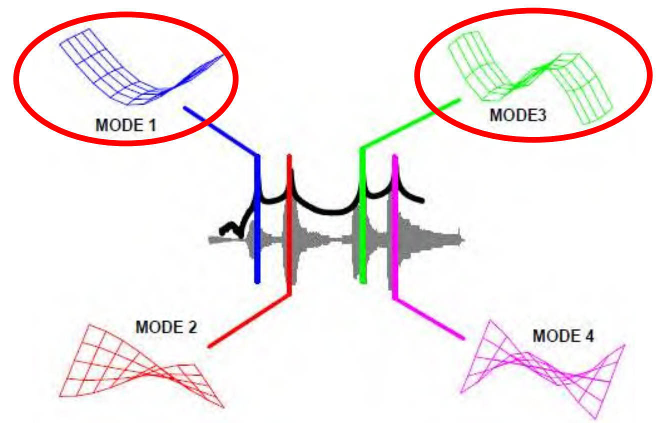
# Efficient Time Response Calculation Techniques

- *Equivalent Reduced Model Technique (ERMT)*
- *Modal Modification Response Technique (MMRT)*

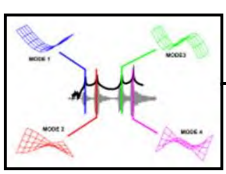
*ERMT - Physical approach*



*MMRT - Modal approach*



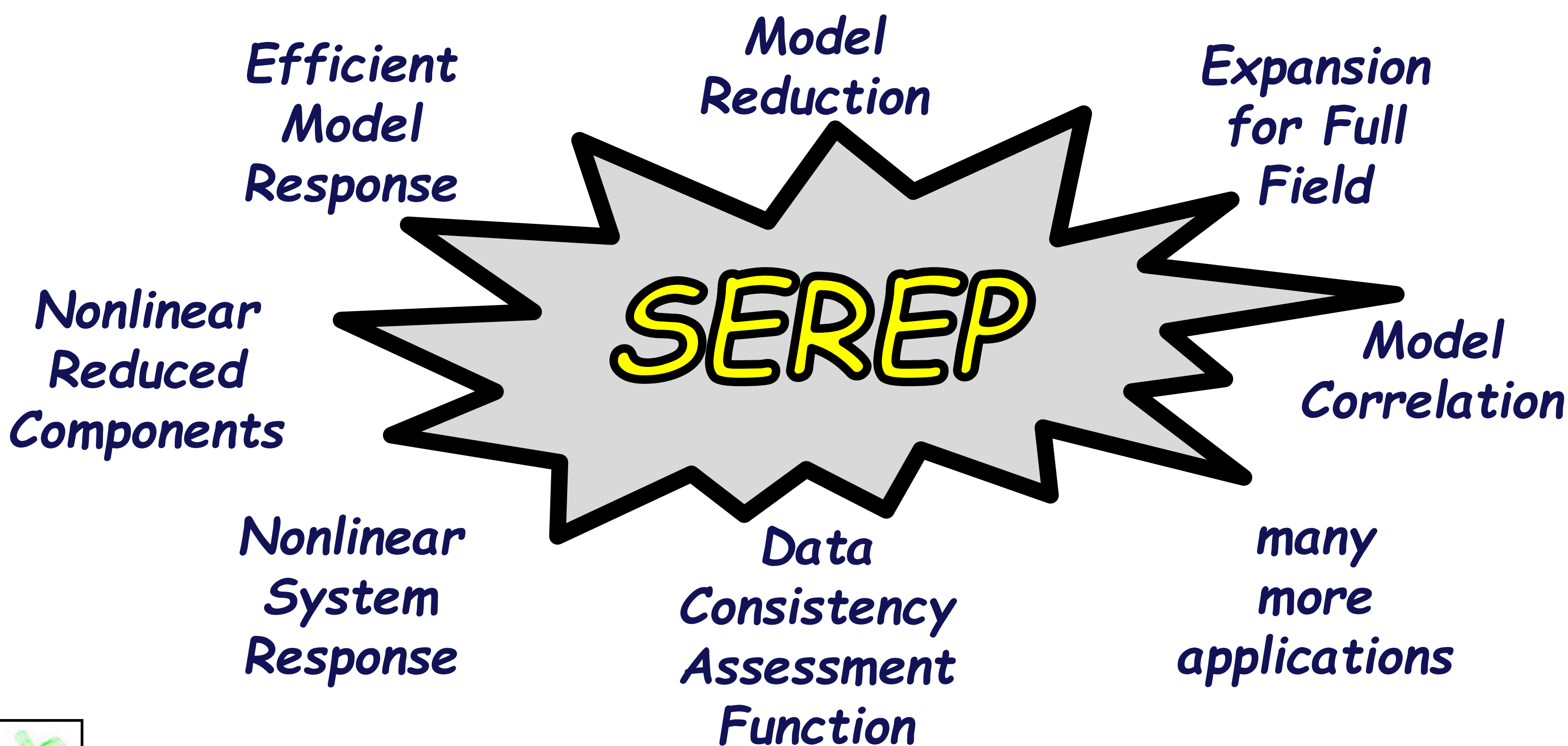

- *Numerical integration performed at each time step for the specific configuration*
  - *Newmark time integration*
  - *Wilsion-Penzien damping formulation*



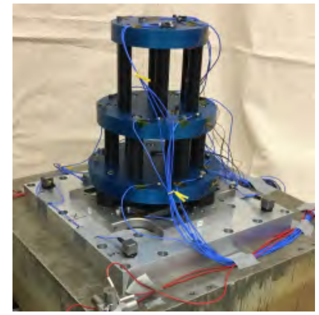
# Theory – Model Reduction/Expansion

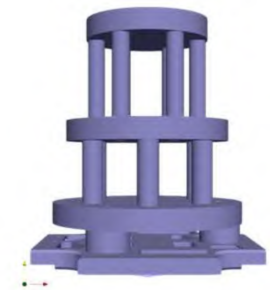
Reduction/Expansion are critical to many structural dynamic modeling activities – correlation, modification, updating, …

The System Equivalent Reduction Expansion Process (SEREP) has been widely used in many applications

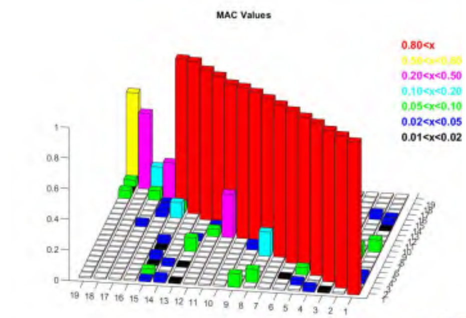


# Model Correlation – SEREP-POC-CORTHOG



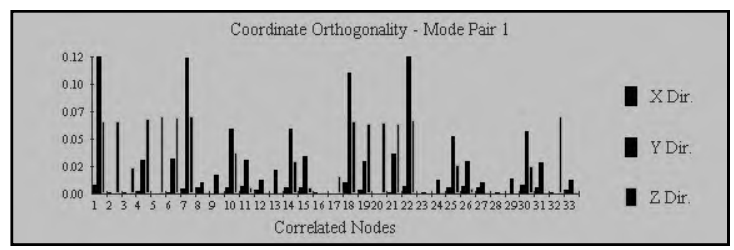


SEREP is widely used and has shown significant accuracy in many applications.




Pseudo Orthogonality Check allows for efficient mass orthogonality check without need for mass or stiffness matrices.

SEREP



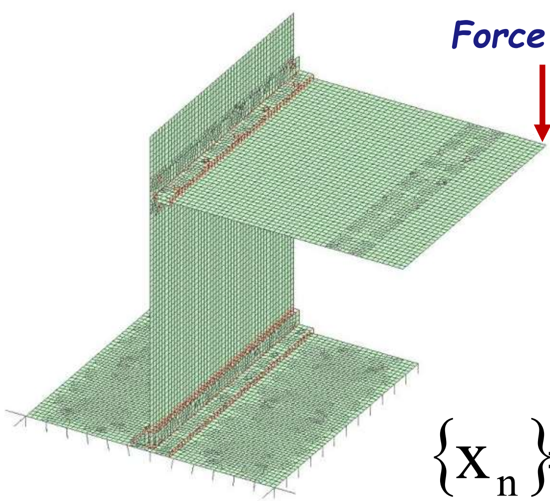

CORTHOG is a mass scaled degree of freedom check for comparison to the FEA and indicates where errors exist.

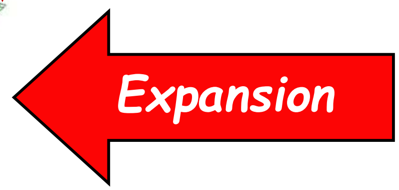
# Model Expansion

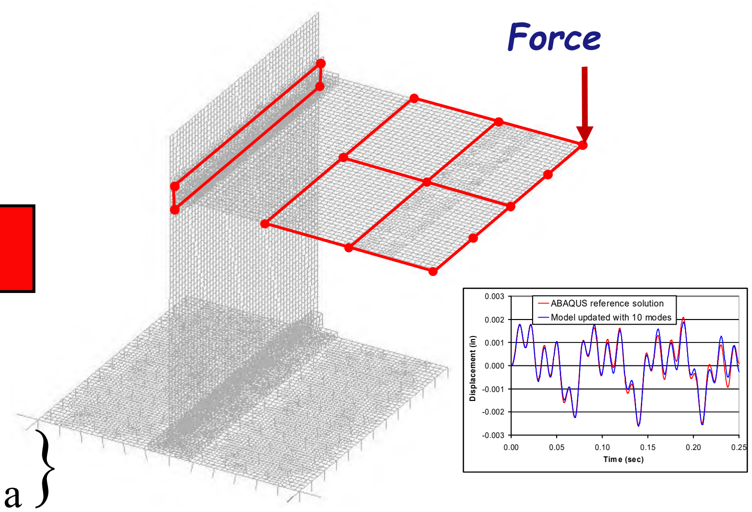
**Expansion as Important as Reduction**

# Expansion Enables Additional Possibilities

***Using force or displacement at attachment points, expansion processes can be applied to identify full model response (dynamic stress/strain)***



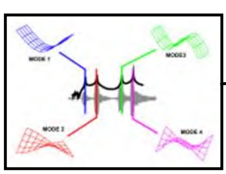




$$\{x_n\} = \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = [T]\{x_a\}$$

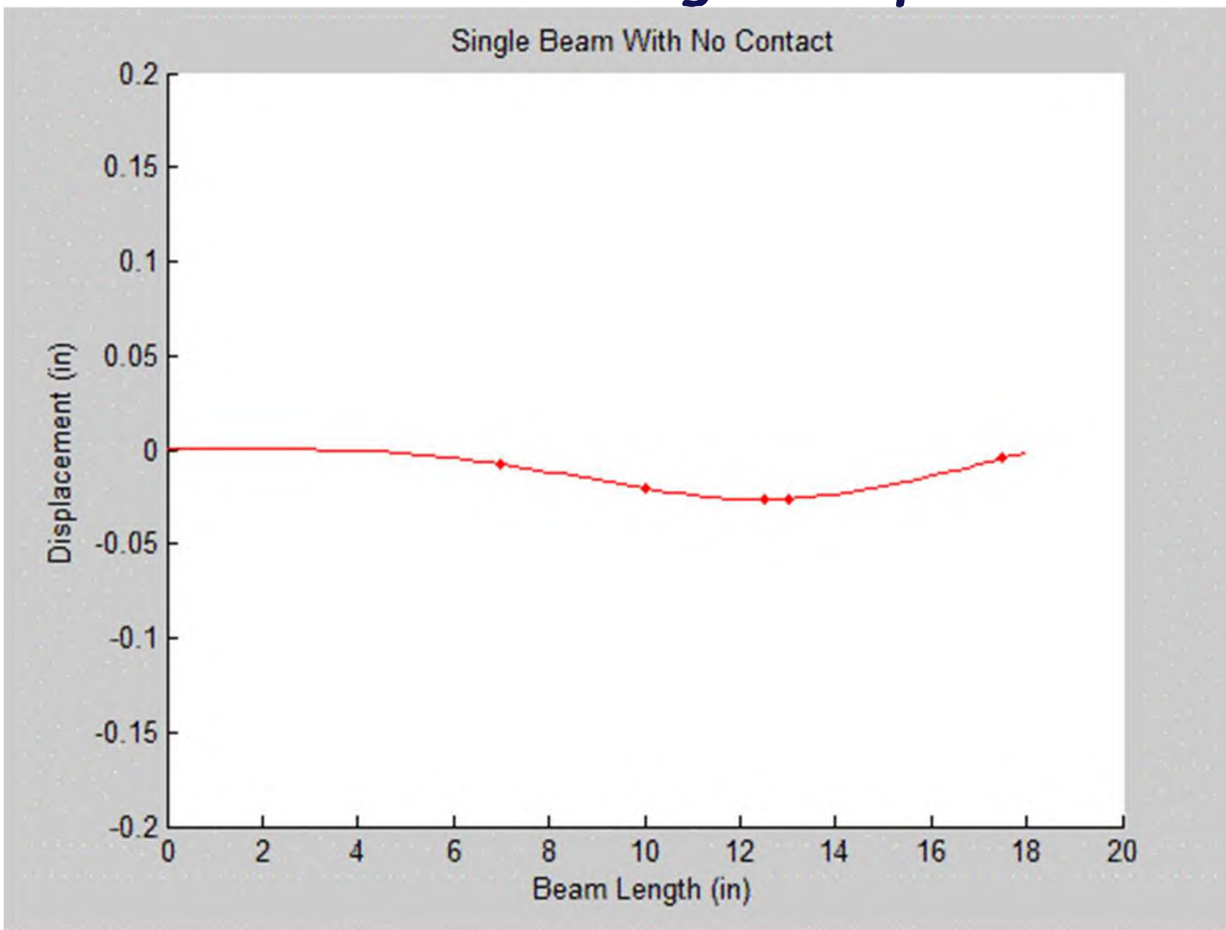
***Formerly only used for linear components***

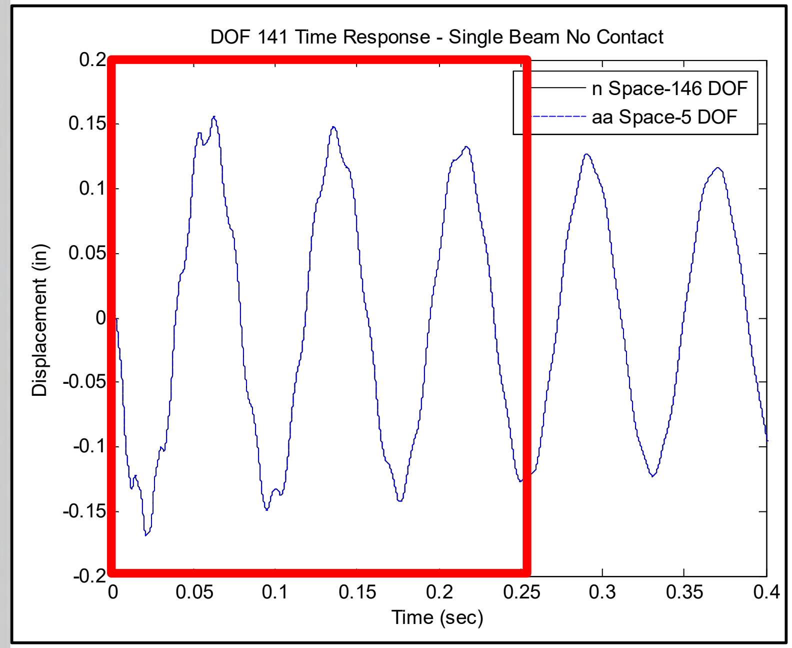
***(but extensions to systems and nonlinear possible)***



# Full Field Response From Limited Set of Data

***The full field displacement obtained from limited sets of data either for single component or for the nonlinear system***

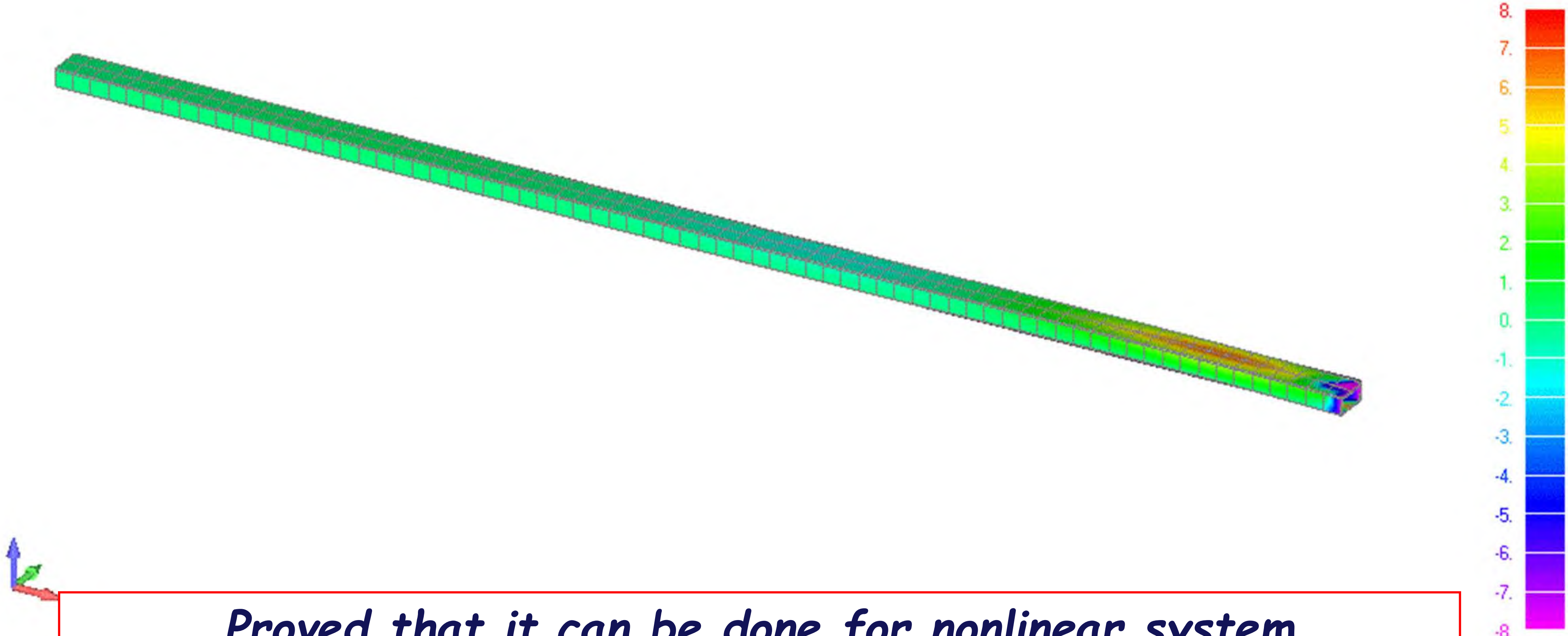



ANIMATION

# Expansion Allows Full Field from Limited Data

**Eventual Aim:**
**Expansion of Nonlinear Displacements**
**to Compute Dynamic Stress-Strain**

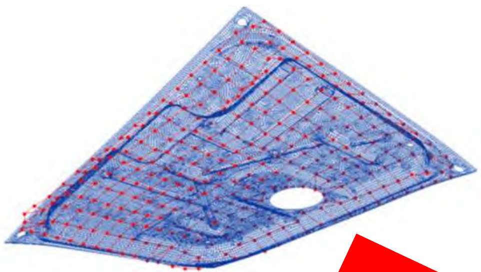



**Proved that it can be done for nonlinear system**

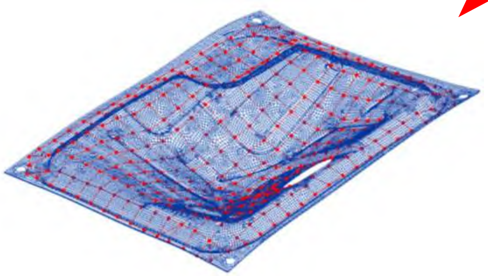
ANIMATION

# Expansion Real Time Operating Data

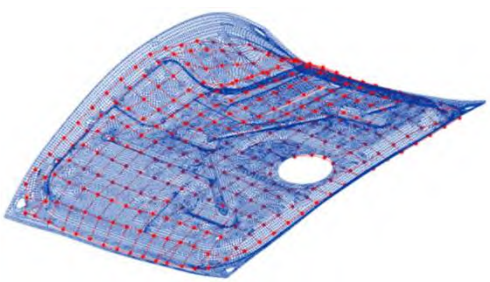
MODE SHAPES MATTER



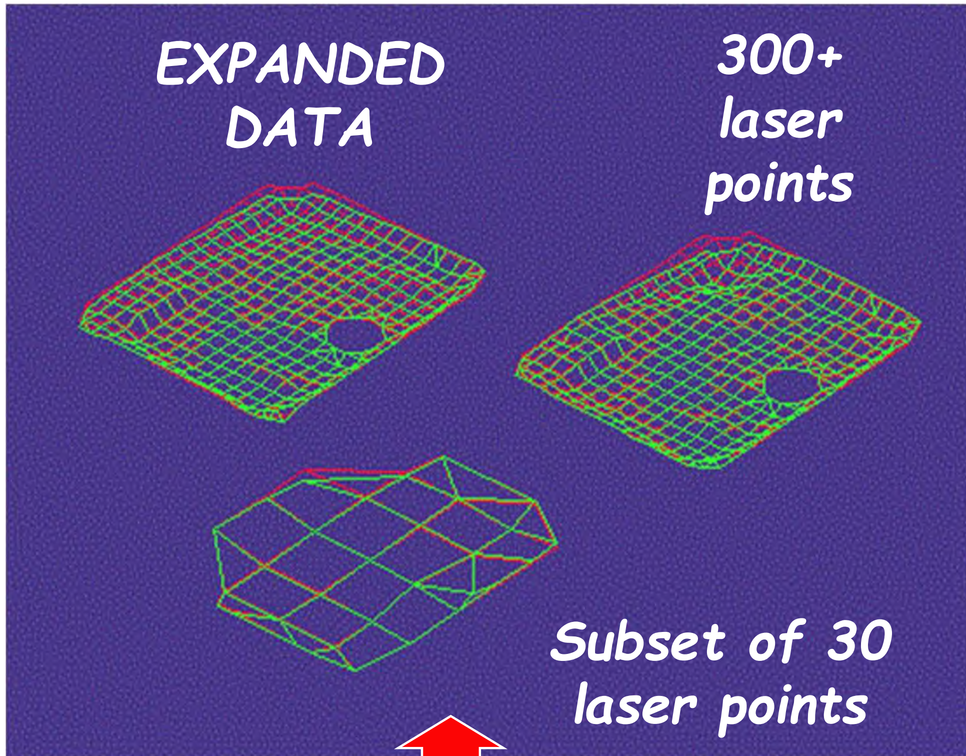
Mode 1

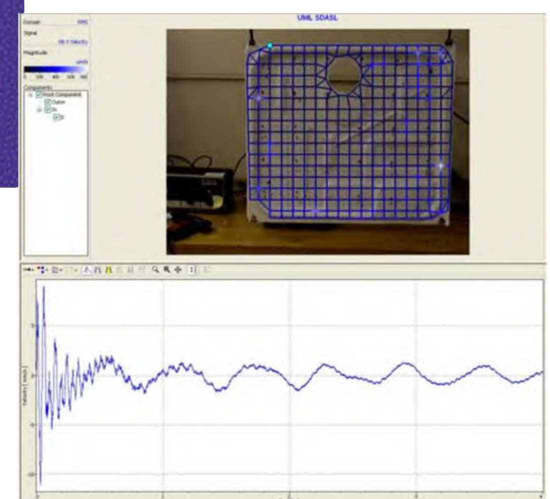


Mode 2



Mode 3



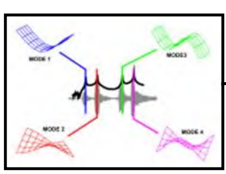
Polytec Scanning Vibrometer

'a' set
Limited Measurement Points

$$[T_U] = [E_n][E_a]^g$$

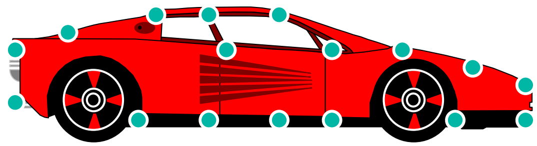
$$[RTO_n] = [T][RTO_a]$$

ANIMATION

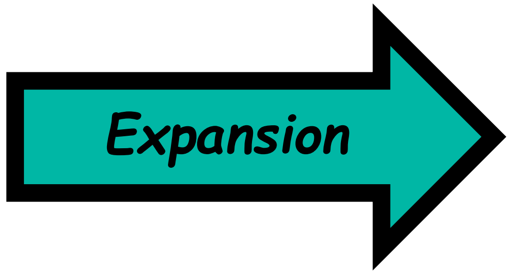


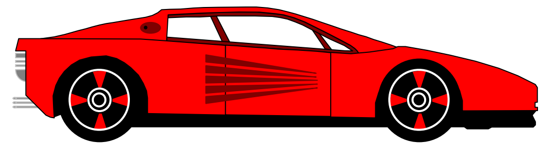
# Expansion – Very Important Piece

## Dynamic expansion means :



$X_A$ = active set of dof’s

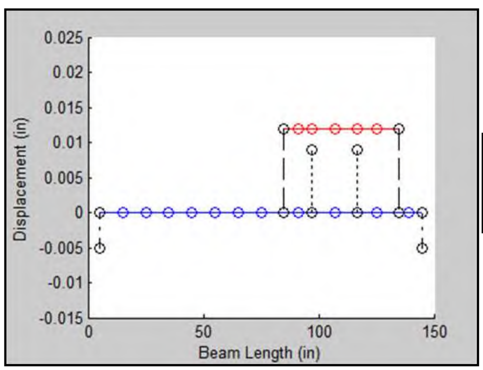


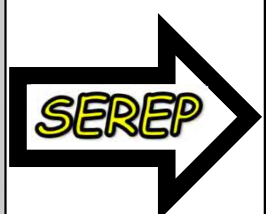



$X_F$ = full set of dof’s

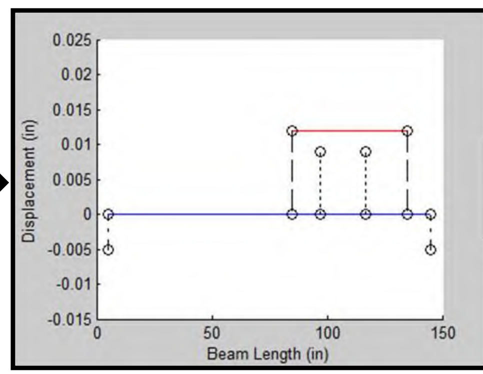
***Starting with a sparse set of degrees of freedom and “expanding” to a much larger set of degrees of freedom (possibly intermediate space or full field space or FEA dofs)***

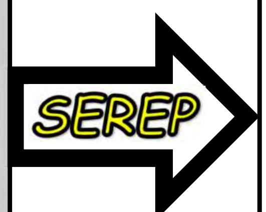
*Response at a-space*



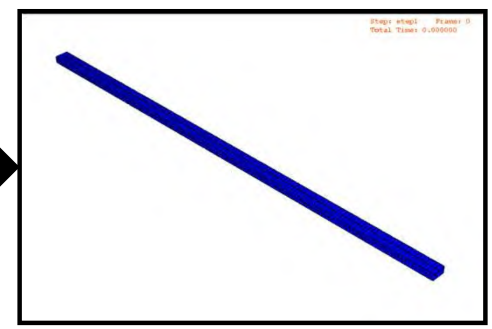




*Expanded Response at N-space*



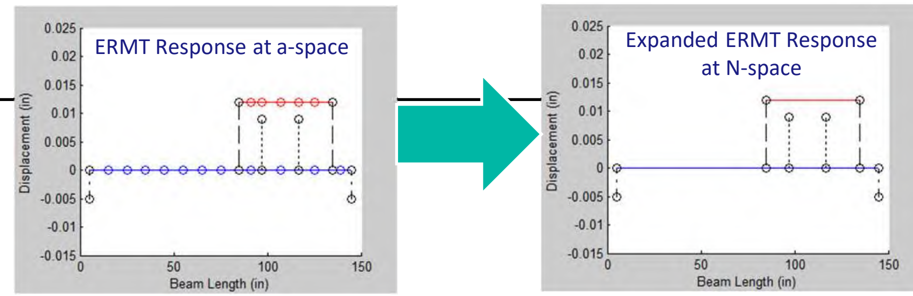

*Component Expanded Response Full Field*



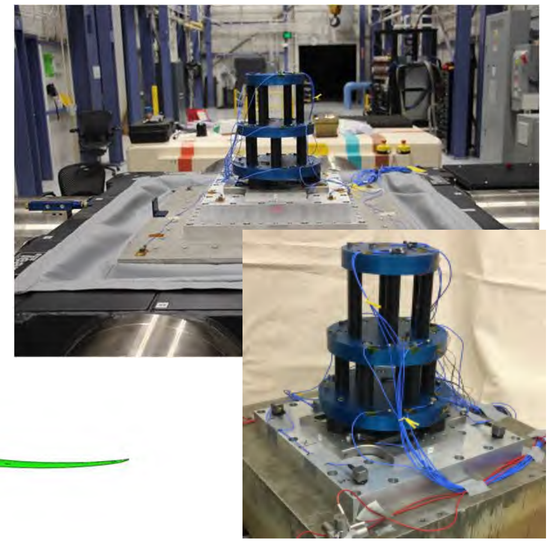
*https://faculty.uml.edu//pavitabile/downloads/IMAC7_SEREP.pdf*

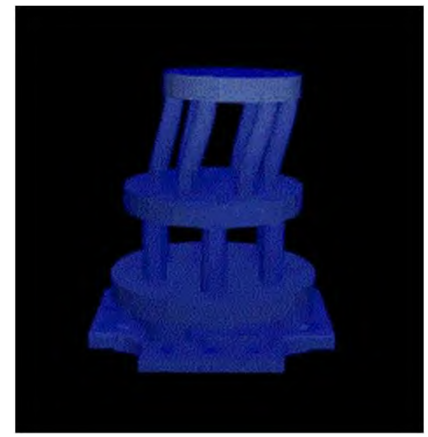
ANIMATION

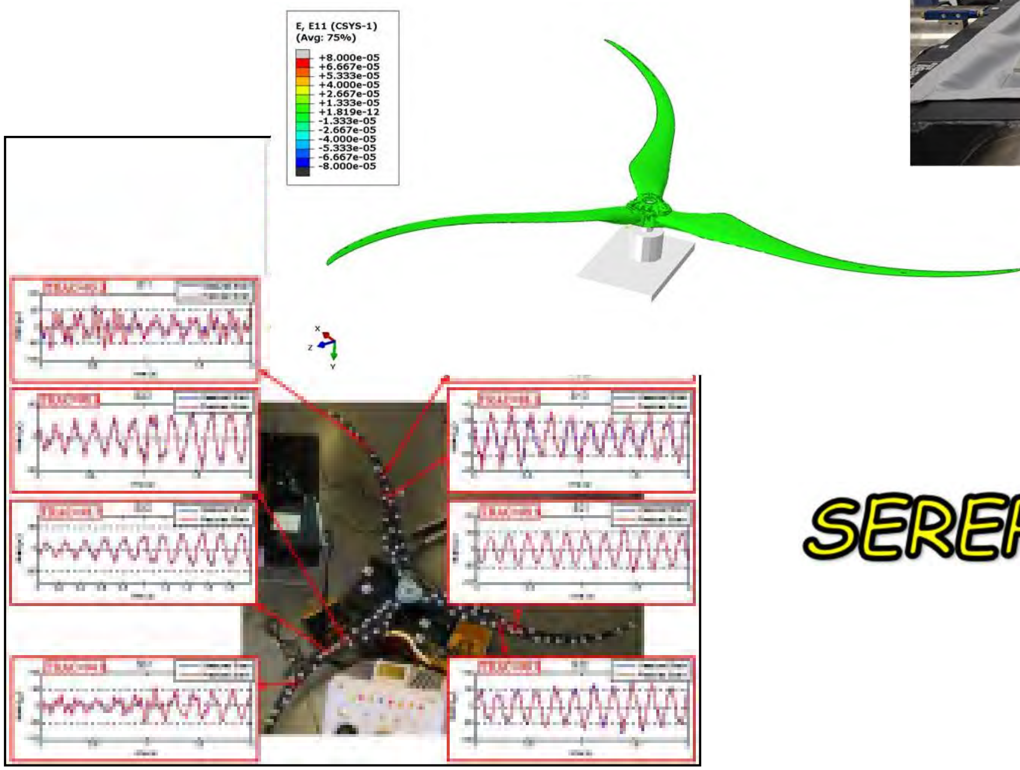
# Expansion Possibilities



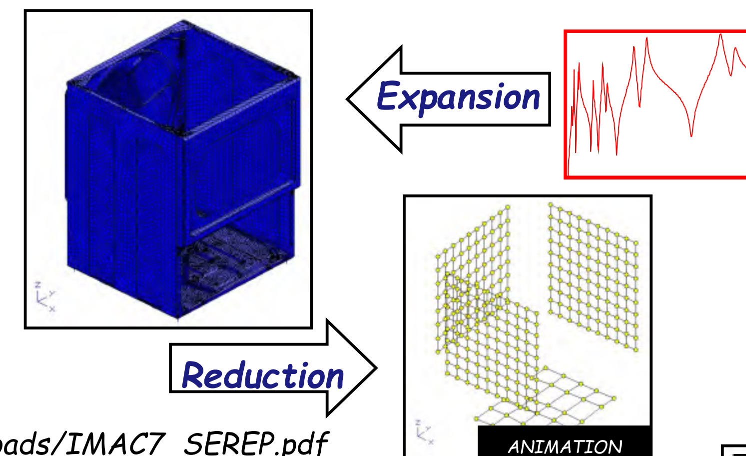

## Expansion useful in a broad range of applications.








SEREP



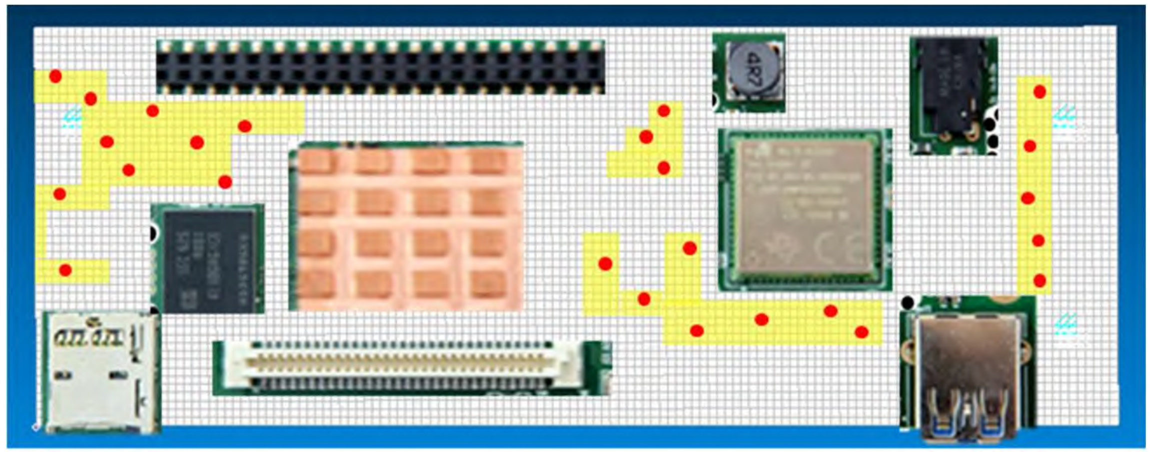

https://faculty.uml.edu//pavitabile/downloads/IMAC7_SEREP.pdf

# Direct Strain Expansion – Analytical Studies

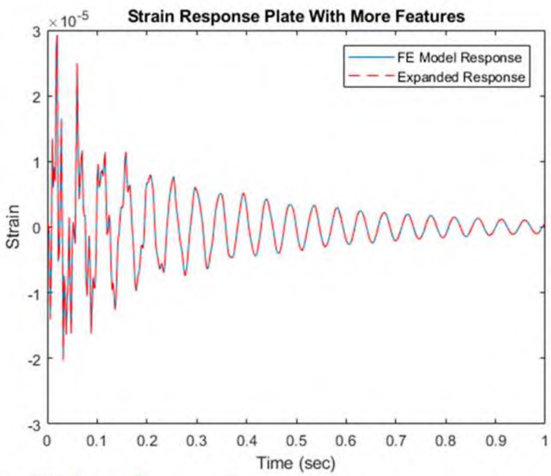
## Limited Line of Sight – Strain Expansion offers enhancements:

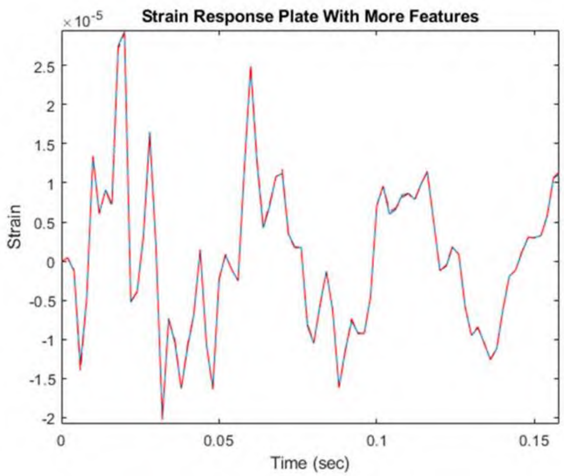
**Structure Speckled for DIC**

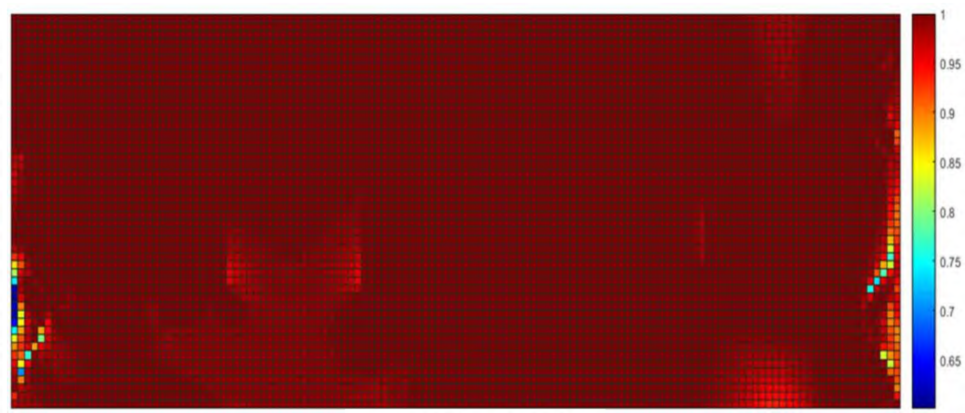

**Corresponding Area on Modified Model**




**25 Active Elements - Average TRAC = 0.9956**

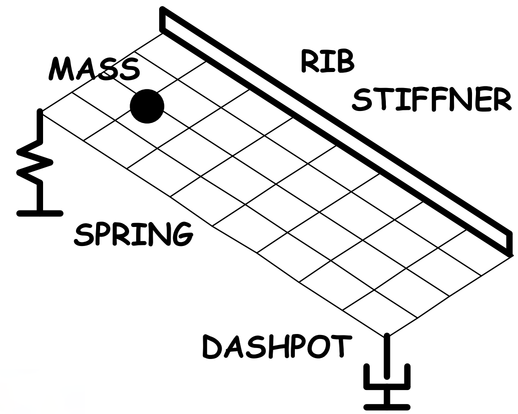


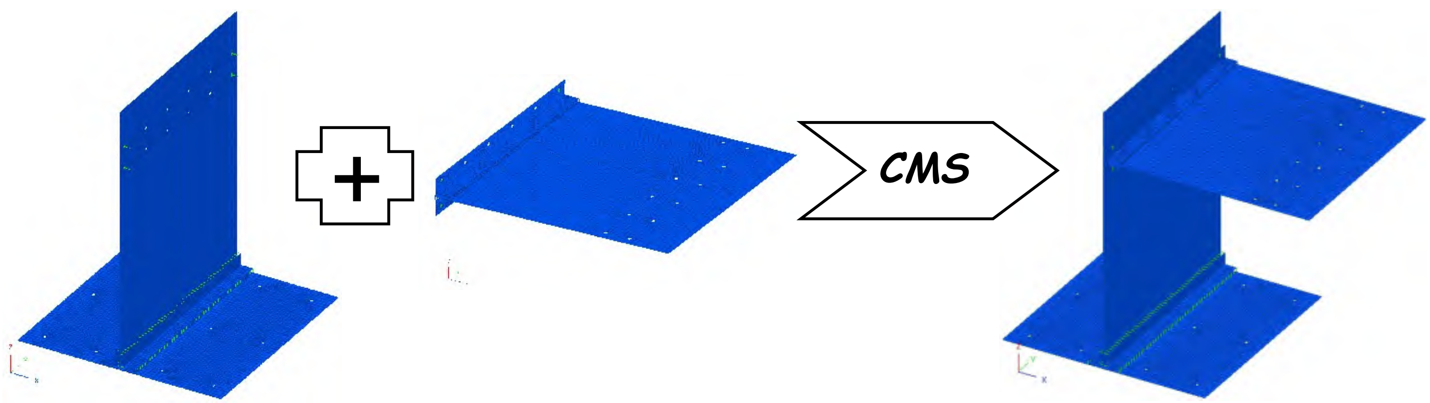




**SEREP**

# Structural Dynamic Modification and System Modeling




https://faculty.uml.edu//pavitabile/downloads/IMAC20_SDM20_MACL.pdf

1970s

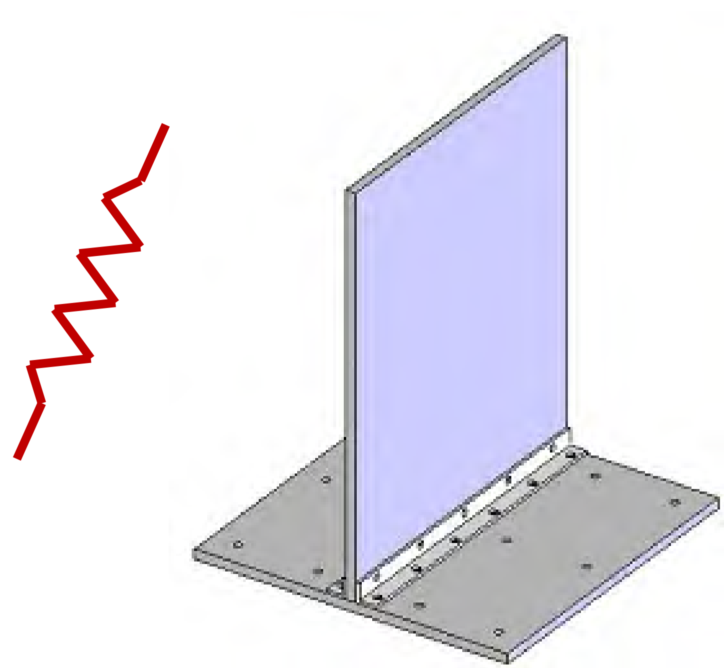
# Predict Changes in Mass and Stiffness

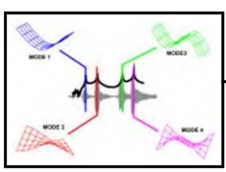
***Can the existing frequencies and mode shapes be used to predict new frequencies and mode shapes due to changes in mass and stiffness***

$$\left[\begin{bmatrix}\ddots & & \\ & \overline{M}_1 & \\ & & \ddots\end{bmatrix} + \left[\Delta\overline{M}_{12}\right]\right]\{\ddot{p}_1\} + \left[\begin{bmatrix}\ddots & & \\ & \overline{K}_1 & \\ & & \ddots\end{bmatrix} + \left[\Delta\overline{K}_{12}\right]\right]\{p_1\} = [0]$$

$$\left[\Delta\overline{M}_{12}\right] = [U_1]^T\left[\Delta M_{12}\right][U_1]$$

$$\left[\Delta\overline{K}_{12}\right] = [U_1]^T\left[\Delta K_{12}\right][U_1]$$





# Eigenvalue Modification Technique - BORN

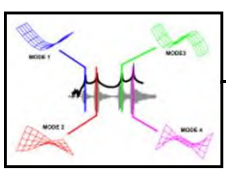
The modal projection is used to recast the equations as

$$\left[\begin{bmatrix}\ddots & & \\ & \overline{M}_1 & \\ & & \ddots\end{bmatrix} + \left[\Delta\overline{M}_{12}\right]\right]\{\ddot{p}_1\} + \left[\begin{bmatrix}\ddots & & \\ & \overline{K}_1 & \\ & & \ddots\end{bmatrix} + \left[\Delta\overline{K}_{12}\right]\right]\{p_1\} = [0]$$

where $\left[\Delta\overline{M}_{12}\right] = [U_1]^T[\Delta M_{12}][U_1]$ $\quad$ $\left[\Delta\overline{K}_{12}\right] = [U_1]^T[\Delta K_{12}][U_1]$

An eigensolution of an (m x m) system is required to uncouple the set of equations

$$\left[\left[\begin{bmatrix}\ddots & & \\ & \overline{K}_1 & \\ & & \ddots\end{bmatrix} + [U_1]^T[\Delta K_{12}][U_1]\right] - \lambda\left[\begin{bmatrix}\ddots & & \\ & \overline{M}_1 & \\ & & \ddots\end{bmatrix} + [U_1]^T[\Delta M_{12}][U_1]\right]\right]\{p_1\} = \{0\}$$



# Local Eigenvalue Modification Technique - FAST

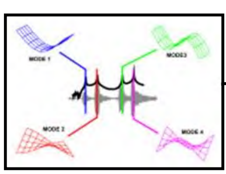
***If only one change of mass or stiffness is considered then these equations can be reduced to***

$$\left[\left[\begin{bmatrix}\ddots & & \\ & \overline{K}_1 & \\ & & \ddots\end{bmatrix} + \{v_{1k}\}\alpha_{1k}\{v_{1k}\}^T\right] - \lambda\left[\begin{bmatrix}\ddots & & \\ & \overline{M}_1 & \\ & & \ddots\end{bmatrix}\right]\right]\{p_1\} = \{0\}$$

$$\left[\left[\begin{bmatrix}\ddots & & \\ & \overline{K}_1 & \\ & & \ddots\end{bmatrix}\right] - \lambda\left[\begin{bmatrix}\ddots & & \\ & \overline{M}_1 & \\ & & \ddots\end{bmatrix} + \{v_{1m}\}\alpha_{1m}\{v_{1m}\}^T\right]\right]\{p_1\} = \{0\}$$

***The solution then reduces to a second order equation for each of the 'm' modes of the system***

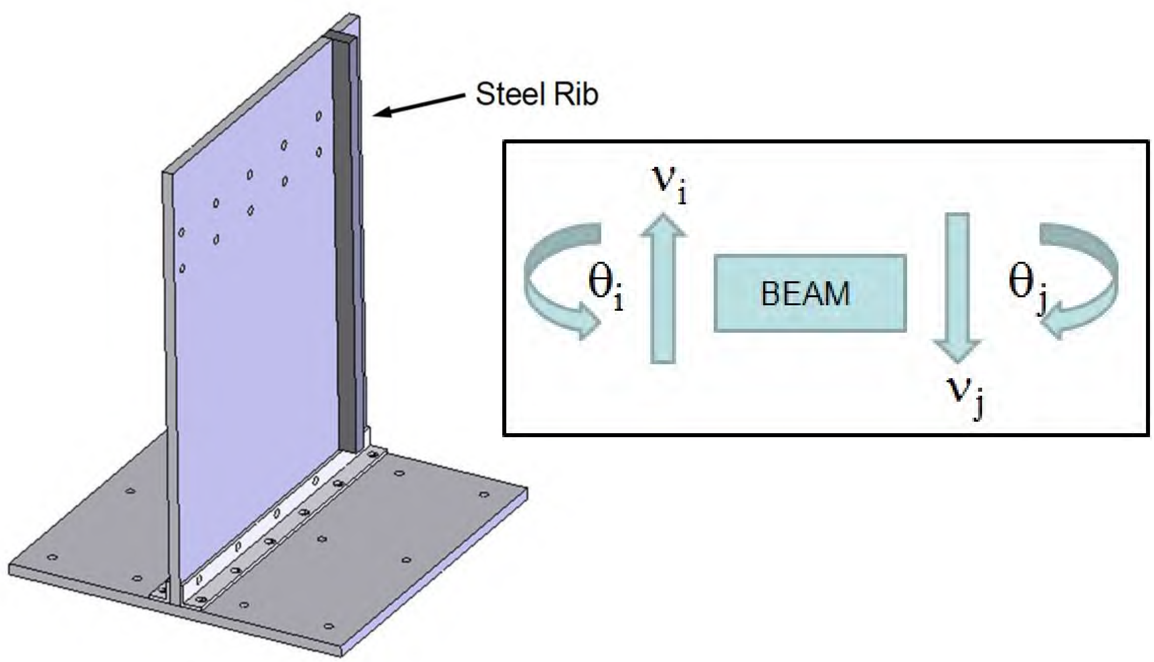
$$\frac{1}{\alpha_k} = \sum_{i=1}^{m} \frac{\left(\left\{u^{(i)}\right\}^T \{t_k\}\right)^2}{\Omega_2^2 - \omega_i^2} \qquad \frac{1}{\Omega^2 \alpha_m} = \sum_{i=1}^{m} \frac{\left(\left\{u^{(i)}\right\}^T \{t_m\}\right)^2}{\Omega_2^2 - \omega_i^2}$$



# Realistic Beam Modifications Necessary

*Realistic modifications require more than just translational degrees of freedom for modifications.*

*Rotary degrees of freedom necessary.*




**Expansion is the Tool**

# Expansion Required

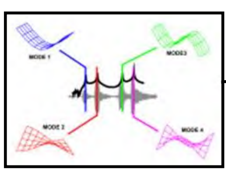
***Spline fits of data proved inadequate.***

***Equivalent Reduction (ER) developed for this.***

$$\{x_n\} = \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = [T]\{x_a\} \qquad [T_U] = [U_n][U_a]^g$$

***Note:***

***ER helped SDM for rotary modifications and allowed the path for system models to become realistic.***



# Expansion Required

*Spline fits of data proved inadequate.*

*Equivalent Reduction (ER) developed for this.*

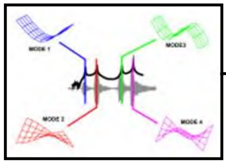
$$\{x_n\} = \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = [T]\{x_a\} \qquad [T_U] = [U_n][U_a]^g$$

*Note:*

*ER helped SDM for rotary modifications and allowed the path for system models to become realistic.*

*BUT … not too much later ER grew up and became SEREP.*

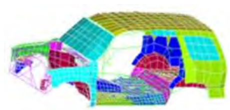
*SEREP is an extremely powerful model reduction tool.*



# System Modeling Formulation

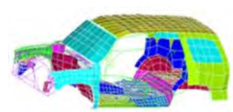
***A system model can be developed using the same approach for individual modal components***

$$[U] = \begin{bmatrix} [U^A] & \\ & [U^B] \end{bmatrix}$$

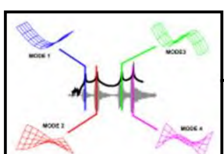


$$+ [U]^T [\Delta M][U] \begin{Bmatrix} \{\ddot{p}^A\} \\ \{\ddot{p}^B\} \end{Bmatrix}$$

$$+$$

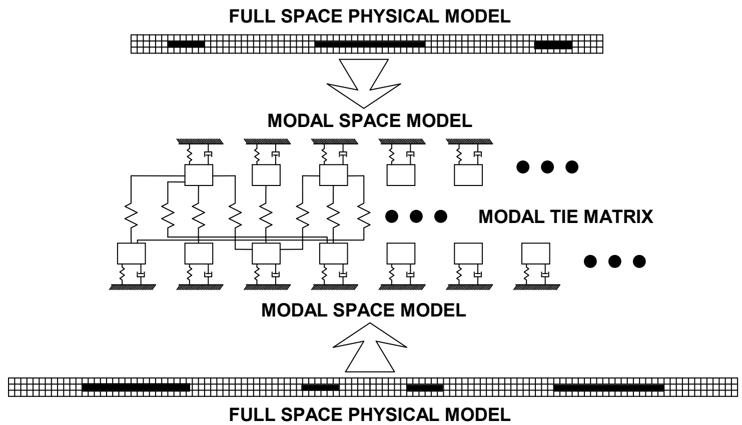


$$+ [U]^T [\Delta K][U] \begin{Bmatrix} \{p^A\} \\ \{p^B\} \end{Bmatrix} = \{0\}$$

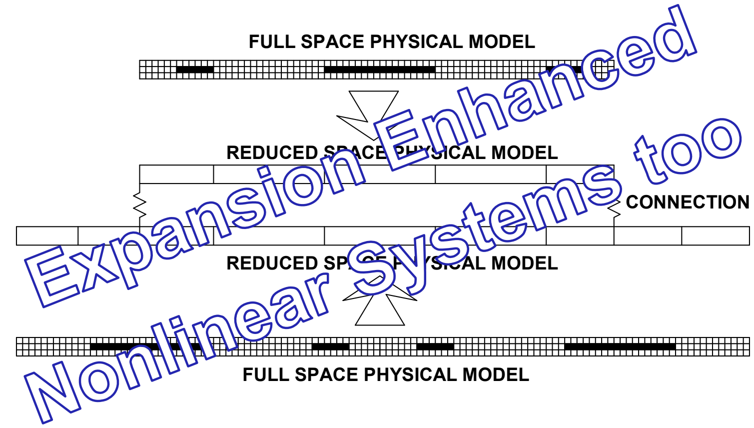


# System Modeling Possibility

## Modal Models

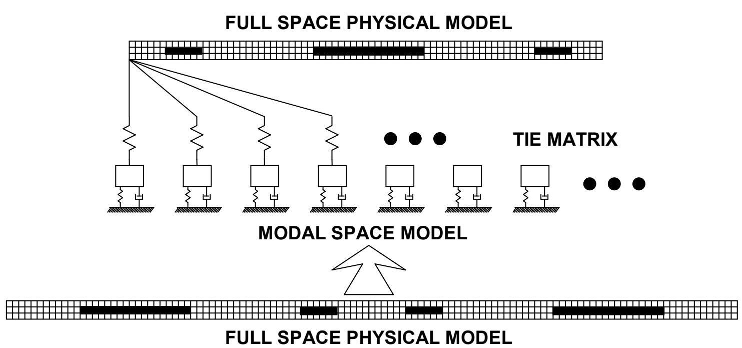



## Reduced Models

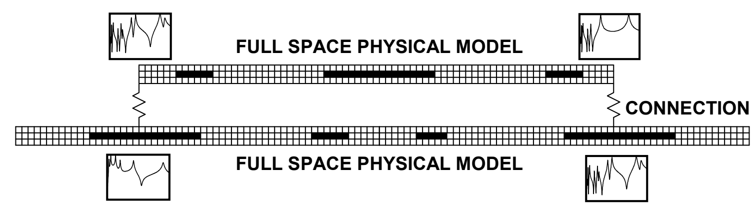


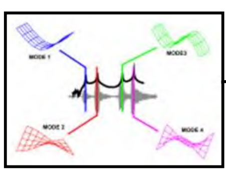

**Expansion is Needed**

## Modal/Physical Models




## Impedance Models



https://faculty.uml.edu//pavitabile/downloads/Avitabile_Svdig_Vol3_No4_Jul2001_MACL.pdf

# LEMP Quickly Outdated

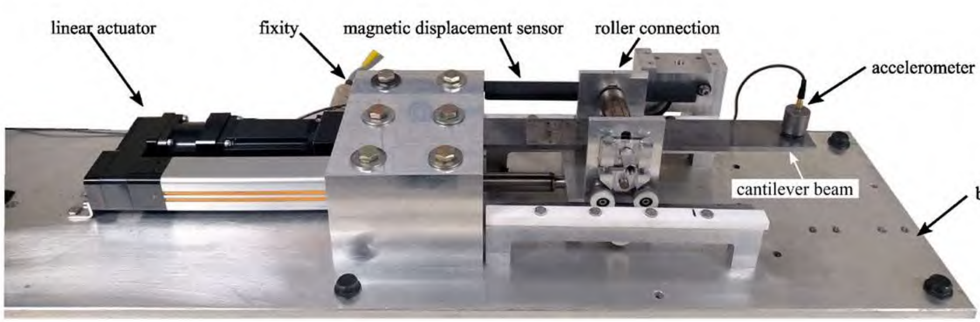
LEMP was an extremely good efficient, computational tool.

But computers got much more powerful very quickly.

LEMP was quickly retired and sent to the graveyard (early 80s).


# LEMP – brought back from the dead - 2020

## LEMP was re-introduced to help speed up eigensolutions.




*"Real-time Structural Model Updating using Local Eigenvalue Modification Procedure for Applications in High-Rate Dynamic Events", E.Ogunniyi, C.Drnek, S.Hong, Y.Wang, J.D.Bakos,* ***P.Avitabile****, J.Dodson, Mechanical Systems and Signal Processing, Volume 195, 15 July 2023, 110318,* https://doi.org/10.1016/j.ymssp.2023.110318

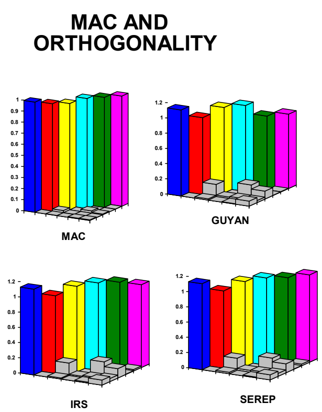
# Correlation and Model Updating

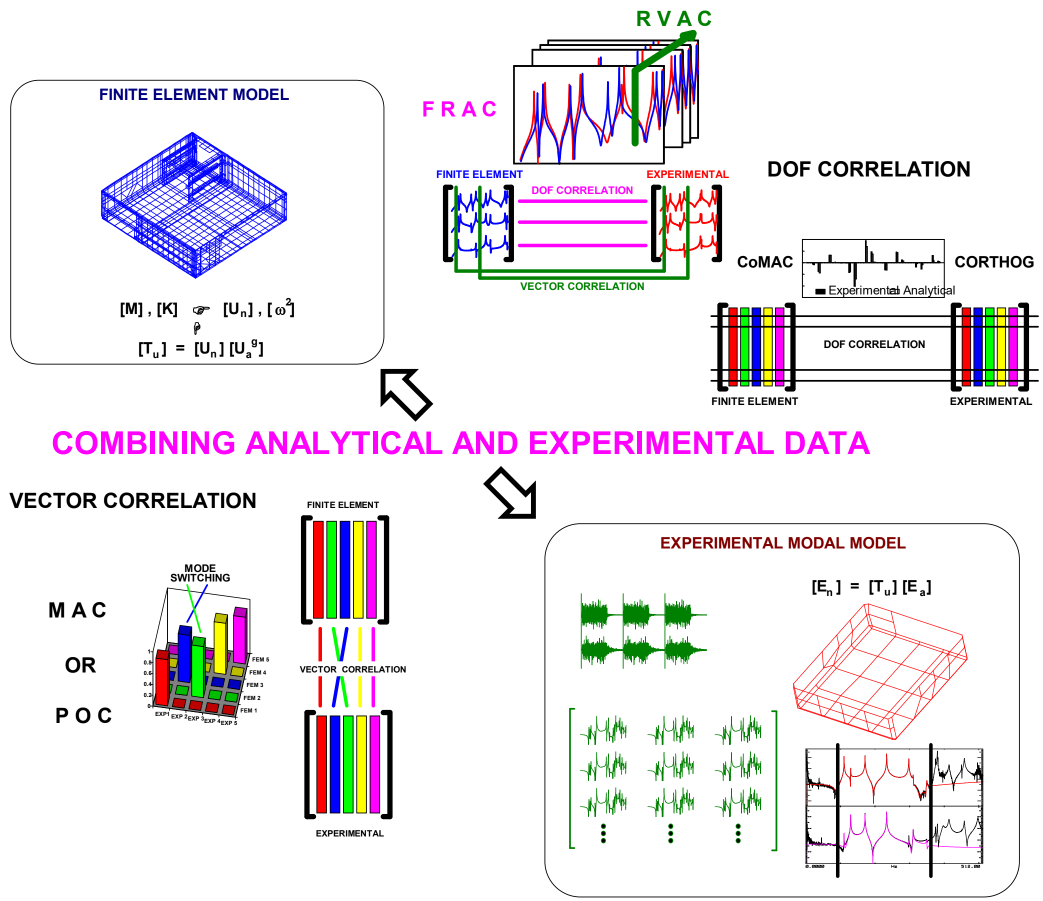
*Finite Element Model Correlation, SEM Handbook of Experimental Structural Dynamics, DOI:10.1007/978-1-4614-4547-0_34, July 2022*

# Correlation and Updating Models

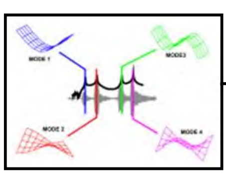
## Research, development and application of these techniques has been a continuing area of focus for many applications

**Analytical and experimental models are correlated and adjusted to provide better component and system models**

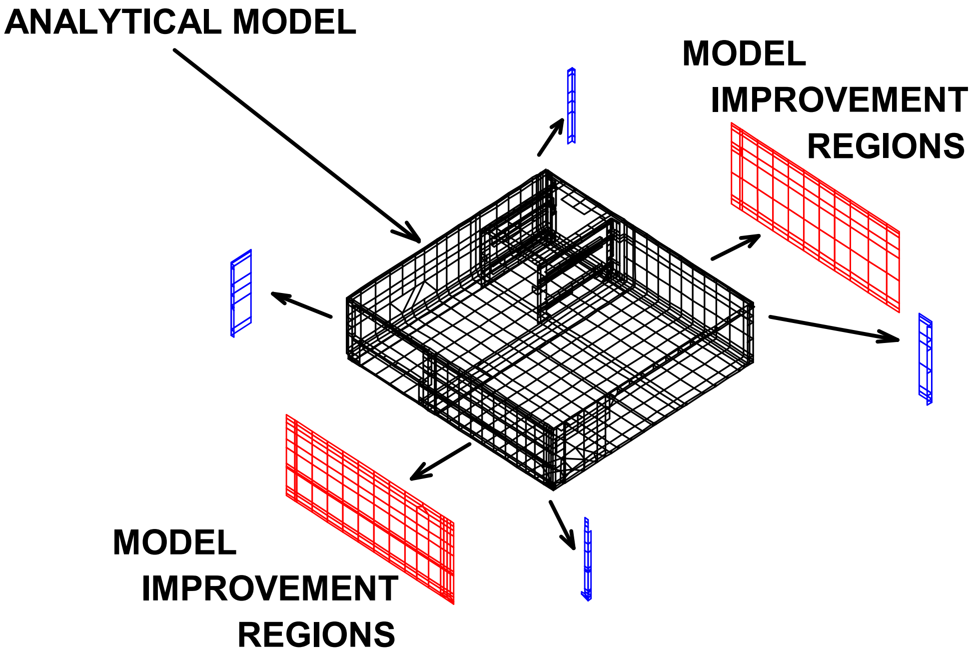


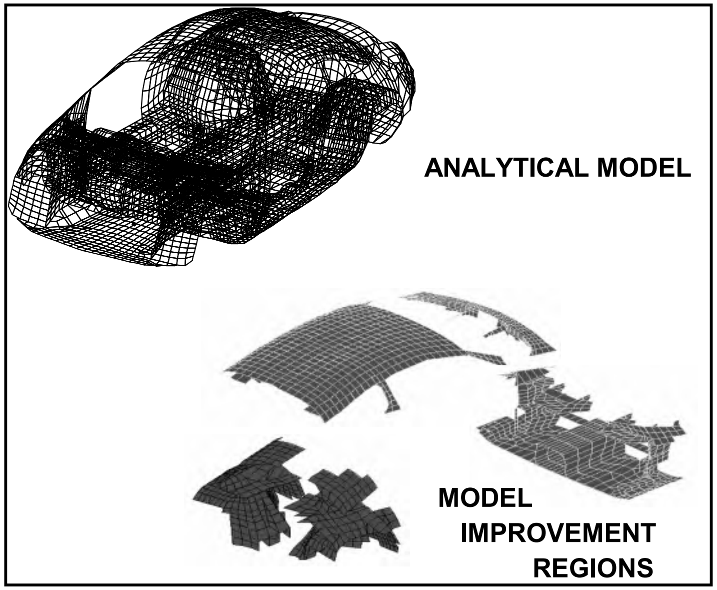




**Reduction and Expansion Needed**

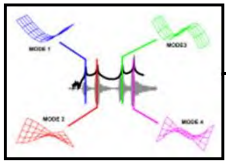


# Updating Models




- *Models can be adjusted to better reflect actual measured system characteristics*
- *Joint stiffness can be more accurately identified*
- *Simplistic modeling assumptions can be modified to reflect the actual system*



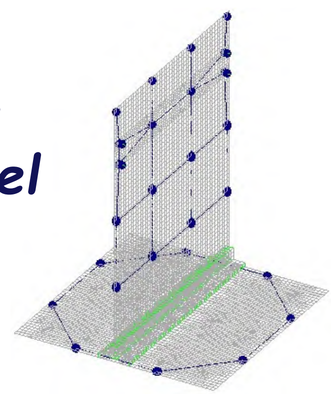
# Analytical Model Improvement (AMI)

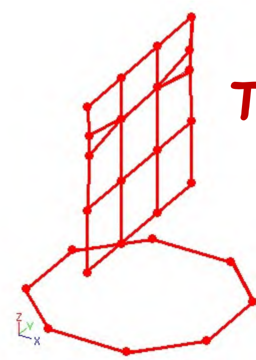
## Seed Mass and Stiffness and Target Data

$$[M_I] = [M_S] + [V]^T \left[ I - \overline{M}_S \right] [V]$$

$$[K_I] = [K_S] + [V]^T \left[ \omega^2 - \overline{K}_S \right] [V] - \left[ [K_S][E][V] \right] - \left[ [K_S][E][V] \right]^T$$

*Seed finite element model*

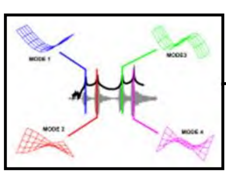




*Target test modes*

## Mass Weighted Generalized Inverse

$$[V] = \left[ [E]^T [M_S][E] \right]^{-1} [E]^T [M_S] = \left[ \overline{M}_S \right]^{-1} [E]^T [M_S]$$



# Tools For Evaluation

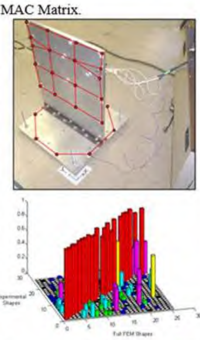
# Correlation Metrics

## Modal Assurance Criteria - MAC

$$MAC_{ij} = \frac{\left[\{u_i\}^T \{e_j\}\right]^2}{\left[\{u_i\}^T \{u_i\}\right]\left[\{e_j\}^T \{e_j\}\right]} \qquad MAC_{ij} = \frac{\left[\{e_i\}^H \{e_j\}\right]^2}{\left[\{e_i\}^H \{e_i\}\right]\left[\{e_j\}^H \{e_j\}\right]}$$

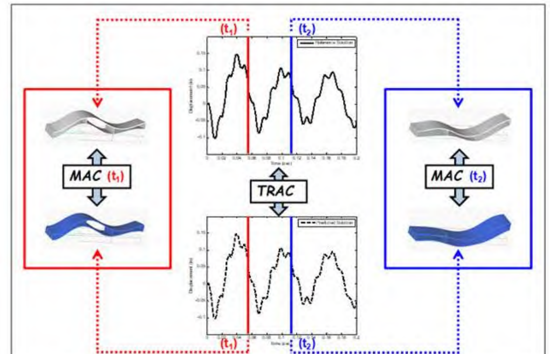
Table 3: Correlation of Upright FEA Model and Test Data along with MAC Matrix.

| FEA Mode Number | Experimental Mode Number | FEA Frequencies (Hz) | Experimental Frequencies (Hz) | Frequency % Difference | MAC Value (%) | 20 Mode POC Value |
|---|---|---|---|---|---|---|
| 1 | 1 | 26.03 | 26.17 | -0.54 | 99.8 | 1.071 |
| 2 | 2 | 70.70 | 67.34 | 4.75 | 98.5 | 0.973 |
| 3 | 3 | 77.69 | 75.89 | 2.32 | 99.3 | 1.033 |
| 4 | 4 | 108.80 | 104.48 | 3.97 | 97.1 | 1.073 |
| 5 | 5 | 158.01 | 161.55 | -2.24 | 98.9 | 1.048 |
| 6 | 6 | 269.88 | 270.00 | -0.04 | 98.9 | 1.019 |
| 7 | 7 | 304.40 | 299.93 | 1.47 | 96.5 | 1.024 |
| 8 | 8 | 350.45 | 354.60 | -1.18 | 98.5 | 0.979 |
| 9 | 9 | 419.36 | 419.22 | 0.03 | 94.3 | 0.483 |
| 10 | 10 | 457.13 | 482.15 | -5.47 | 92.7 | 0.836 |
| 11 | 12 | 541.29 | 547.84 | -1.21 | 96.6 | 0.995 |
| 12 | 11 | 563.47 | 538.20 | 4.48 | 86.3 | 0.785 |
| 13 | 13 | 777.29 | 771.76 | 0.71 | 90.3 | 1.002 |
| 14 | 14 | 779.61 | 791.18 | -1.48 | 97.7 | 0.944 |
| 15 | 15 | 862.76 | 871.90 | -1.06 | 84.3 | 0.795 |



## Test Response Assurance Criteria - TRAC

$$TRAC = \frac{\left[\{x_{n1}\}^T \{x_{n2}\}\right]^2}{\left[\{x_{n1}\}^T \{x_{n1}\}\right]\left[\{x_{n2}\}^T \{x_{n2}\}\right]}$$




## Pseudo Orthogonality Check - POC

$$POC = [U_a]^T [M_a][E_a] \qquad POC = [U_n]^T [M_n][E_n]$$


Table 3: Correlation of Upright FEA Model and Test Data along with MAC Matrix.

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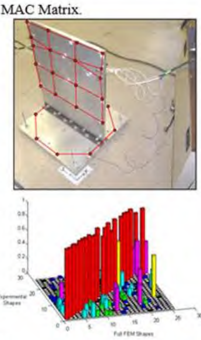
| 15 | 15 | 862.76 | 871.90 | -1.06 | 84.3 | 0.795 |

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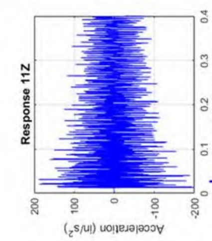
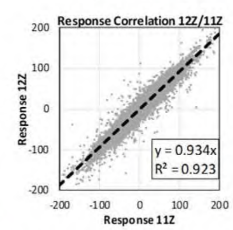
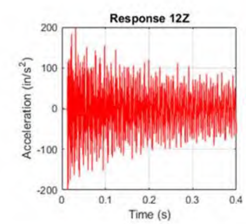
## Test Response Assurance Criteria - TRAC




**TRAC = 0.923**

$$TRAC = \frac{\left[\{x_{n1}\}^T \{x_{n2}\}\right]^2}{\left[\{x_{n1}\}^T \{x_{n1}\}\right]\left[\{x_{n2}\}^T \{x_{n2}\}\right]}$$

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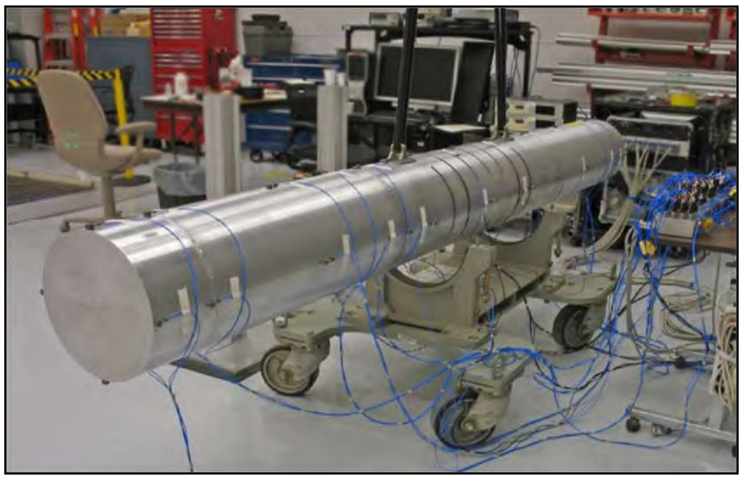
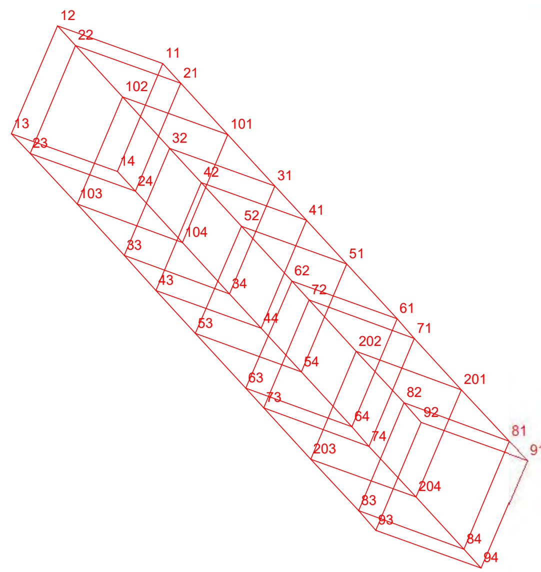
# Updated Reduced Order Model Development for Forced Response Predictions

## Sandia Monolithic Dynamic Mockup (MDM)

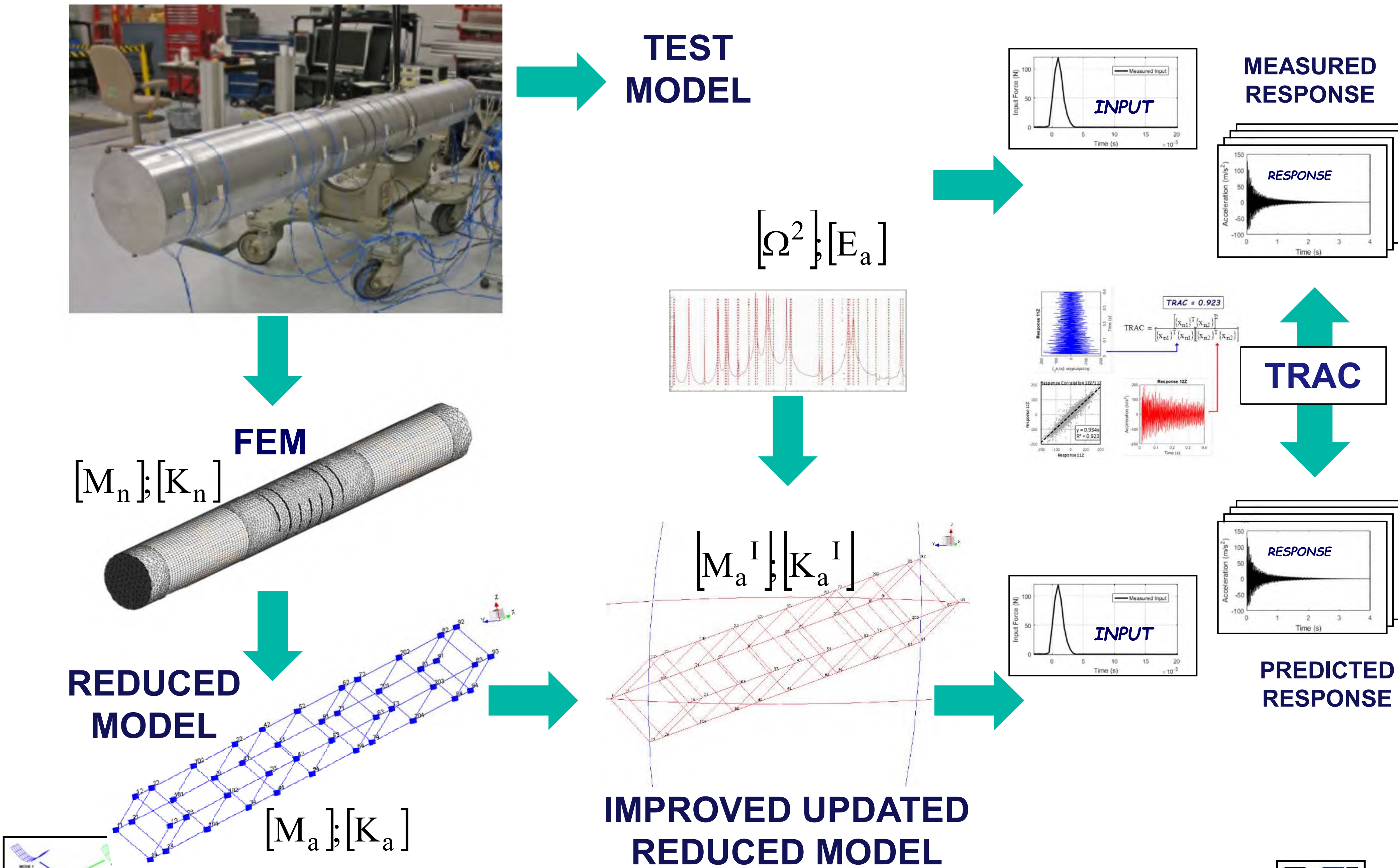




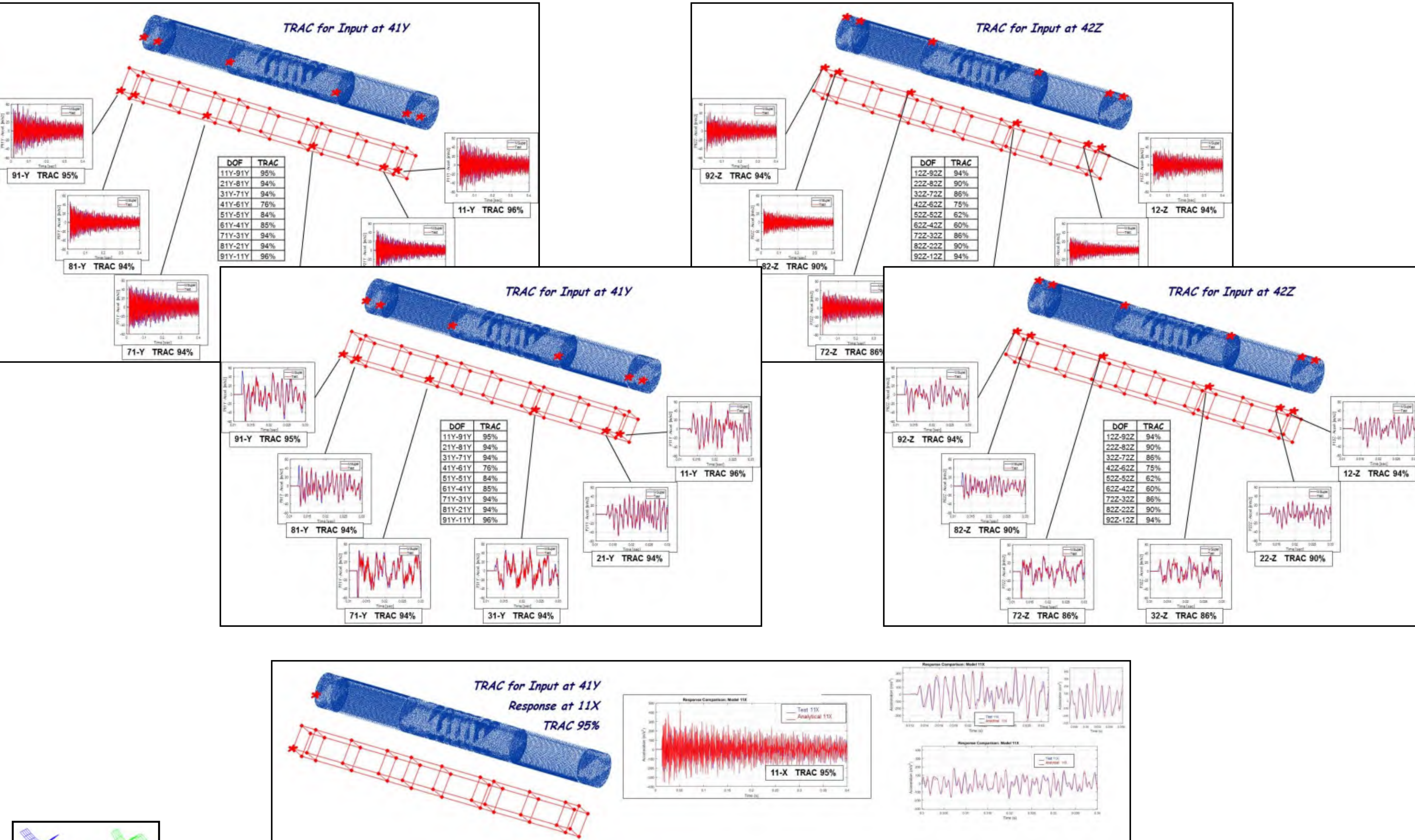

Structural Dynamics and Acoustic Systems Laboratory
University of Massachusetts Lowell

http://faculty.uml.edu/pavitabile/downloads/IMAC36_MDM_Response_101317_DRAFT.pdf

# The MDM Model – Overall Process

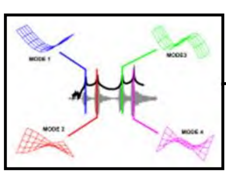


# MDM Updated Reduced Model Forced Response Predictions




| DOF | TRAC |
|---|---|
| 11Y-91Y | 95% |
| 21Y-81Y | 94% |
| 31Y-71Y | 94% |
| 41Y-61Y | 76% |
| 51Y-51Y | 84% |
| 61Y-41Y | 85% |
| 71Y-31Y | 94% |
| 81Y-21Y | 94% |
| 91Y-11Y | 96% |

| DOF | TRAC |
|---|---|
| 12Z-92Z | 94% |
| 22Z-82Z | 90% |
| 32Z-72Z | 86% |
| 42Z-62Z | 75% |
| 52Z-52Z | 62% |
| 62Z-42Z | 60% |
| 72Z-32Z | 86% |
| 82Z-22Z | 90% |
| 92Z-12Z | 94% |



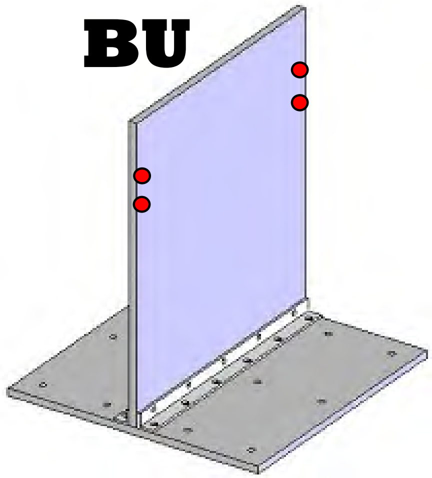
# Reduced Order Models for Efficient Response

## Redstone Apache Missile Firing Systems

***Van Zandt, Wirkkala, Butland, Nicgorski, Chipman, Avitabile***

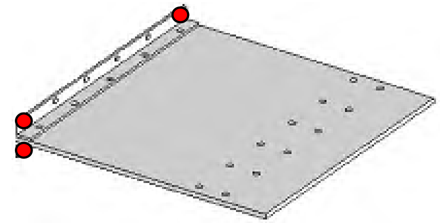
# Test Analysis Correlation – Component/System

**BU**

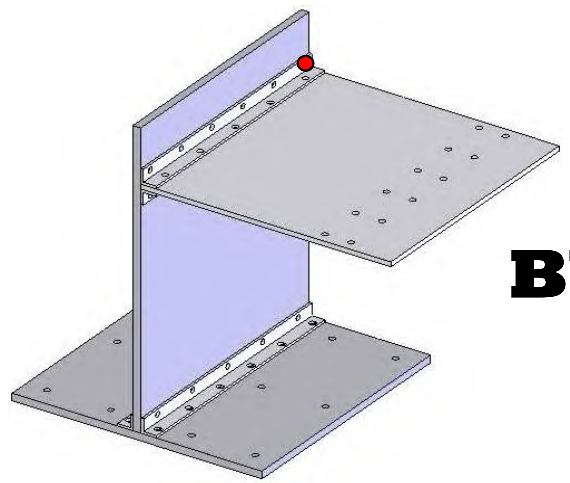


‘Base Upright’

**H**



‘Horizontal Wing’



**BUH**

“Base Upright Horizontal” assembly

# New Reduced-Order Test-Verified Approaches

**Approach 1 – Complete System Model Reduction with Reduced Model Updating:**

**(Modal Based System Response – MBSR)**

**Approach 2 – Reduced Component Model Updating from Assembled System Measured Characteristics:**

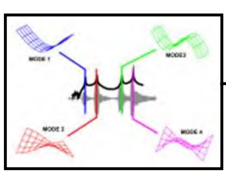
**(Modal Based Component Response – MBCR)**

**Approach 3 – Reduced Component Model Updating from Impedance Developed Model for System Characteristics:**

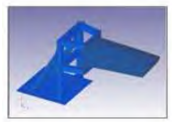
**(Frequency Based Component Response – FBCR)**

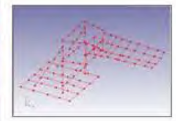
**Approach 4 – Reduced Component Model Updating for Craig Bampton Constraint Model:**

**(Craig Bampton Component Response - CBCR)**



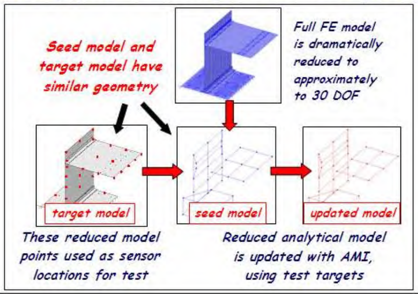
# New Reduced-Order Test-Verified Approaches

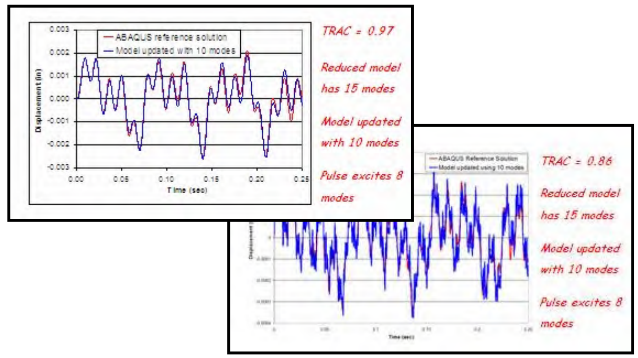




### Modal Based System Response – MBSR

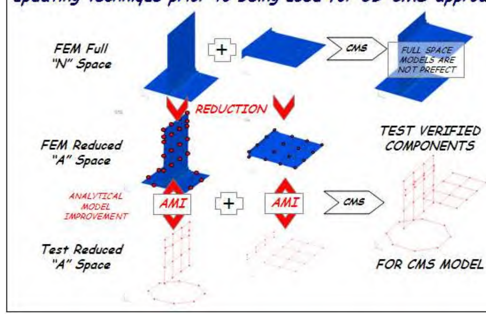
Complete System Model Reduction with Direct Updating – Modeling technique usually employed in many instances and provides very good results. Requires full model representation



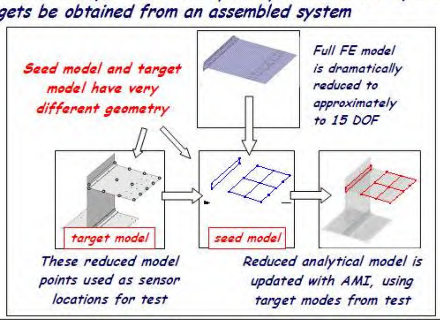

### Craig Bampton Component Response – CBCR

Test Verified, Reduced Order Component Models used to form Craig Bampton Component Mode Synthesis Models – Individual components are reduced and then updated using a direct updating technique prior to being used for CB-CMS approach



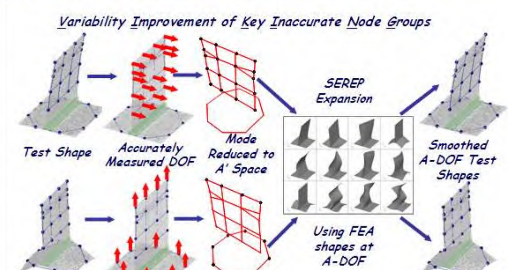

### Modal Based Component Response – MBCR

Reduced Component Model Updating from Assembled System Response – Advantage of technique is that only part of the structure needs to be modeled (such as component) but direct model updating is performed using test data from complete system assembly. This technique requires that the system targets be obtained from an assembled system



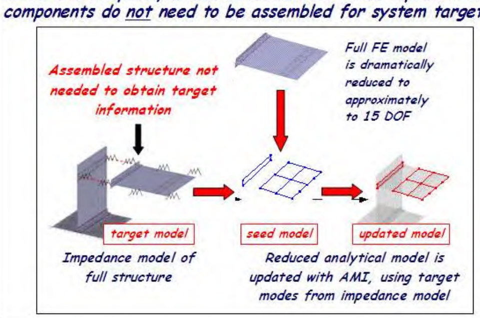

### VIKING CURES DATA IMPURITIES

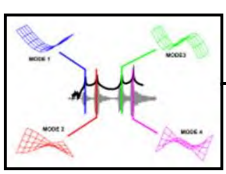
The VIKING Technique enables necessary data cleansing approaches to improve data for all processing performed. It is imperative that this be done for data collected. Early studies show that this is a major breakthrough for processing measured data and overcomes problems that have plagued these approaches for decades.



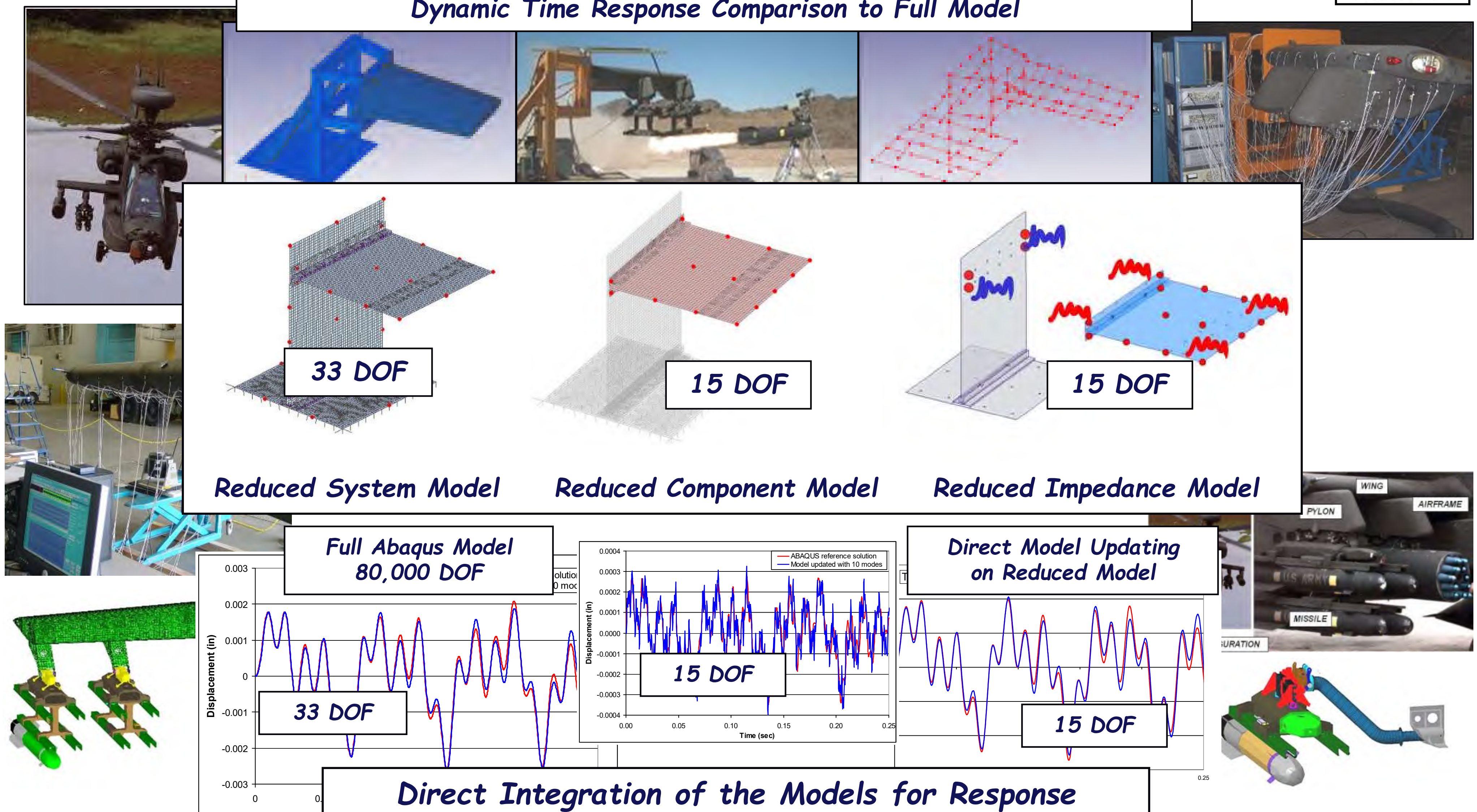

### Frequency Based Component Response – FBCR

Reduced Component Model Updating from Impedance Developed Model for System Characteristics – Similar to MBCR but the system targets are obtained from component information used to form a frequency based substructure. This implies that the components do not need to be assembled for system targets

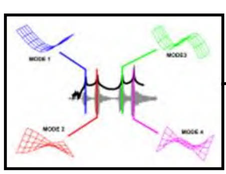


# Efficient Reduced Models for Multibody Dynamic Response Analysis




Full Abaqus (24 hr) vs 33 DOF (2 min)    Full Abaqus (24 hr) vs 15 DOF (1.5 min)



# Test Analysis Correlation – Component/System

Table 3: Correlation of Upright FEA Model and Test Data along with MAC Matrix.

| FEA Mode Number | Experimental Mode Number | FEA Frequencies (Hz) | Experimental Frequencies (Hz) | Frequency % Difference | MAC Value (%) | 20 Mode POC Value |
|---|---|---|---|---|---|---|
| 1 | 1 | 26.03 | 26.17 | -0.54 | 99.8 | 1.071 |
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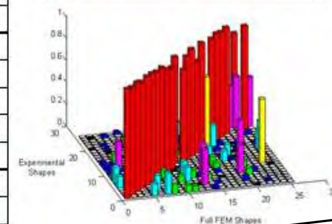
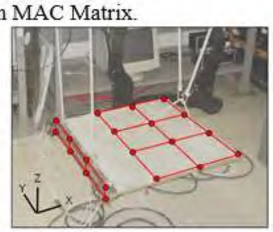
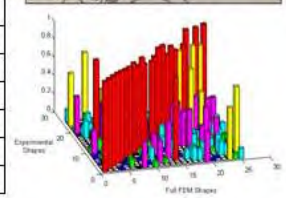


Table 2: Correlation of Wing FEA Model and Test Data along with MAC Matrix.

| FEA Flexible Mode Number | Experimental Flexible Mode Number | FEA Frequency (Hz) | Experimental Frequency (Hz) | Frequency % Difference | MAC Value (%) | 15 Mode POC Value |
|---|---|---|---|---|---|---|
| 1 | 1 | 87.87 | 90.63 | -3.14 | 99.6 | 1.013 |
| 2 | 2 | 141.94 | 148.60 | -4.69 | 99.2 | 1.026 |
| 3 | 3 | 219.27 | 228.83 | -4.36 | 99.7 | 1.078 |
| 4 | 4 | 243.55 | 253.30 | -4.00 | 99.5 | 1.048 |
| 5 | 5 | 411.43 | 427.25 | -3.85 | 98.5 | 0.992 |
| 6 | 6 | 430.80 | 446.72 | -3.70 | 97.7 | 1.056 |
| 7 | 7 | 506.55 | 528.47 | -4.33 | 96.9 | 0.989 |
| 8 | 8 | 549.54 | 568.15 | -3.39 | 99.0 | 0.958 |
| 9 | 9 | 681.00 | 673.10 | 1.16 | 93.8 | 1.022 |
| 10 | 10 | 749.30 | 783.14 | -4.52 | 98.8 | 0.993 |

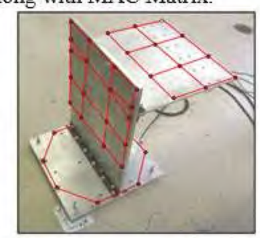


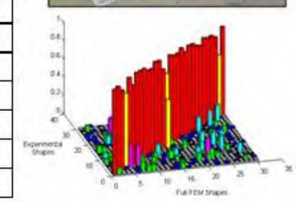


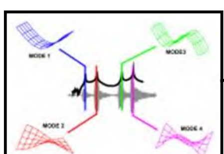
**EVEN GOOD DATA NEEDS TO BE CLEANSED ! ! !**

Table 4: Correlation of Assembly Test-Data and FEA Model along with MAC Matrix.

| Mode Number | (Fig. A 4) FEA Freq. (Hz) | Experimental Frequencies (Hz) | Frequency % Difference | MAC Value | 20 Mode POC Value |
|---|---|---|---|---|---|
| 1 | 13.06 | 13.29 | -1.76 | 98.8 | 0.993 |
| 2 | 25.2 | 26.03 | -3.29 | 99.1 | 1.055 |
| 3 | 28.55 | 28.57 | -0.07 | 99.1 | 1.075 |
| 4 | 53.41 | 52.36 | 1.97 | 98.6 | 1.019 |
| 5 | 70.25 | 72.28 | -2.89 | 99.4 | 0.983 |
| 6 | 96.12 | 93.5 | 2.73 | 99.1 | 1.033 |
| 7 | 141.63 | 145.81 | -2.95 | 99.1 | 1.052 |
| 8 | 185.54 | 191.72 | -3.33 | 98.6 | 1.010 |
| 9 | 218.57 | 226.33 | -3.55 | 97.3 | 0.998 |
| 10 | 231.27 | 237.02 | -2.49 | 93.4 | 1.015 |

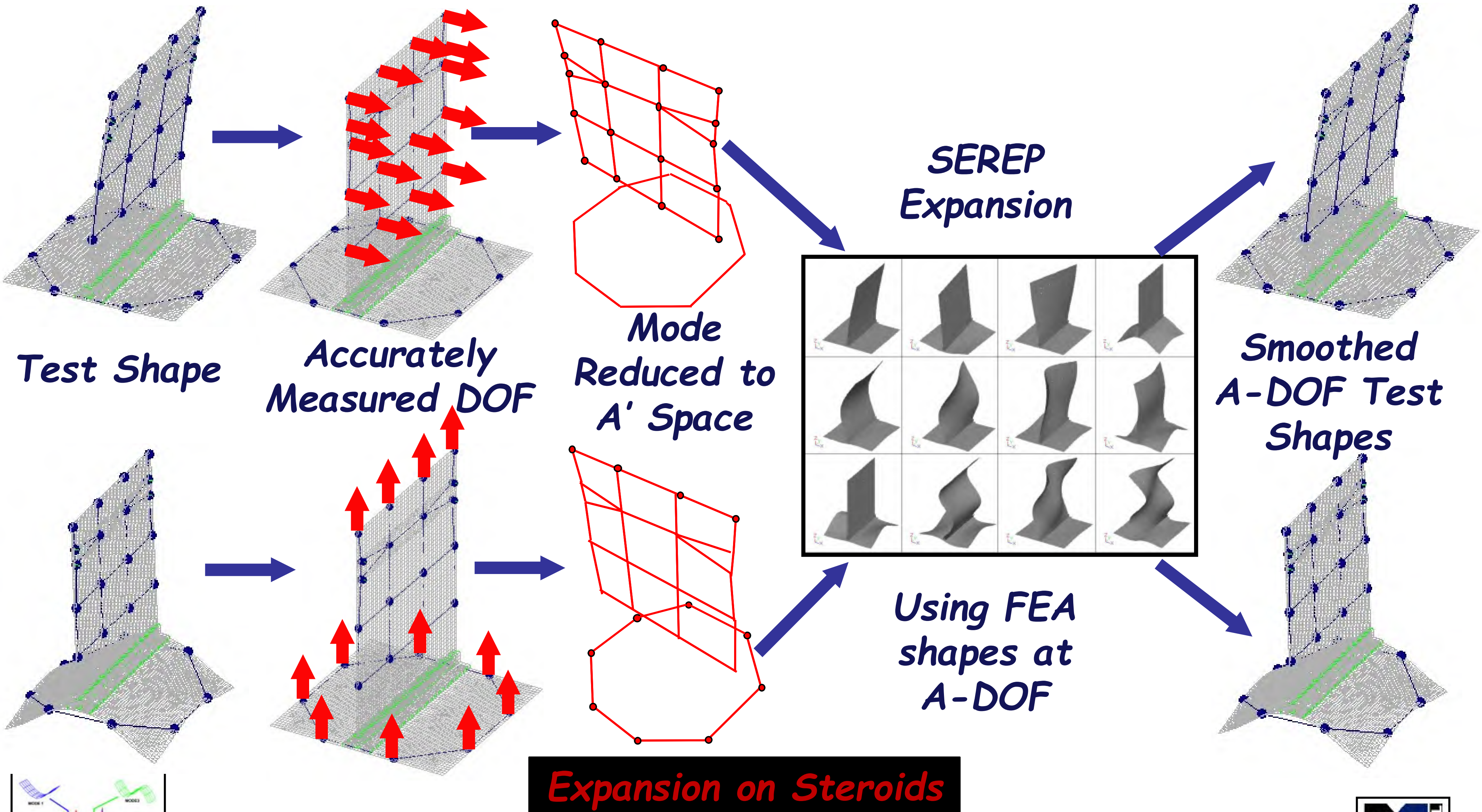






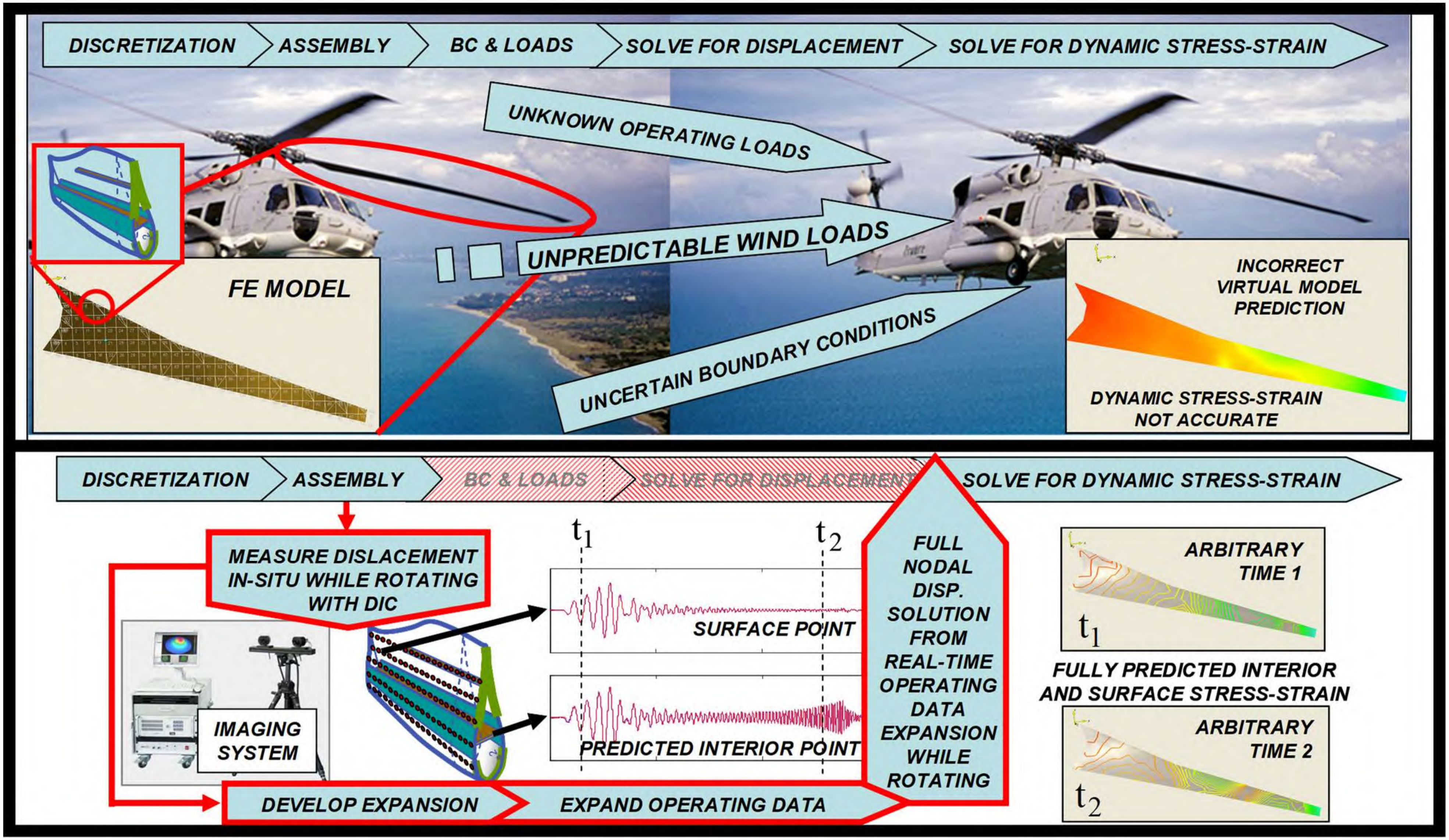
# VIKING Mode Shape Smoothing Process

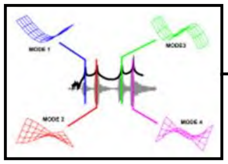
## Variability Improvement of Key Inaccurate Node Groups



# Full Field Strain Expansion from Sparsely Measured Data (and forces unknown)

# Conventional Approach vs. Alternate Approach

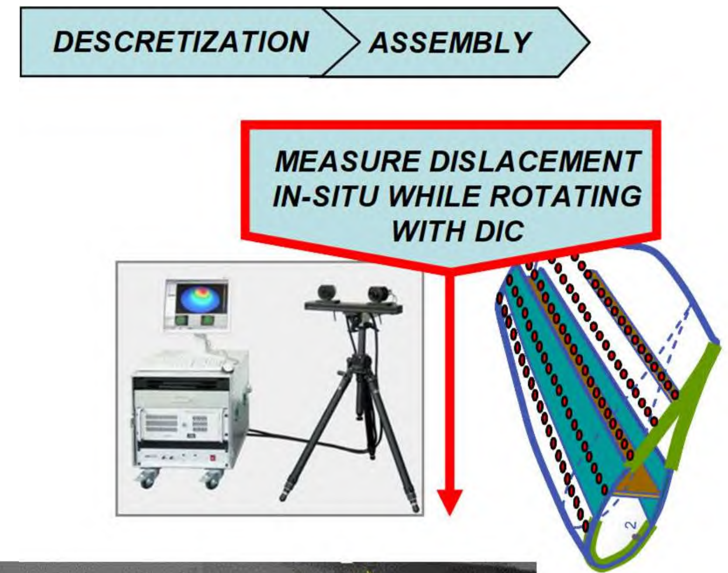


# Dynamic Response - Fundamental Change in Approach

Develop FEA model as usual

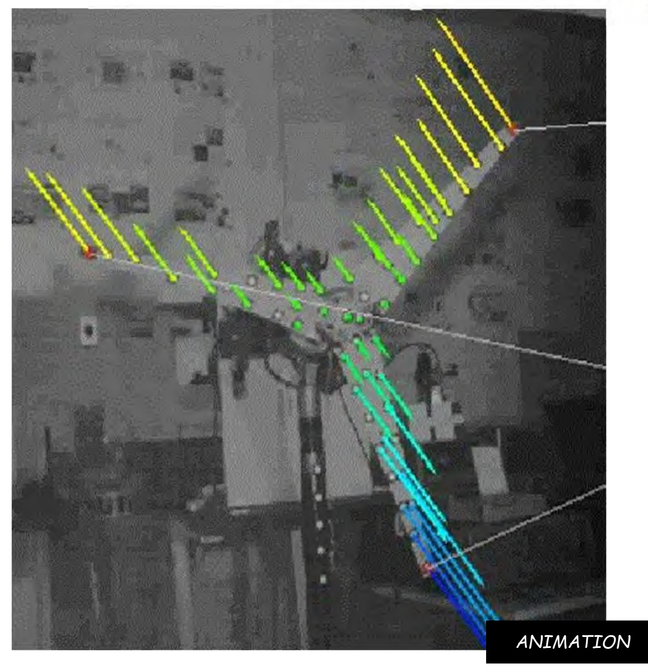
Measure response using full field approaches
- Pontos for discrete points
- Aramis for surface strain




Advantage

No assumption as to load or boundary conditions

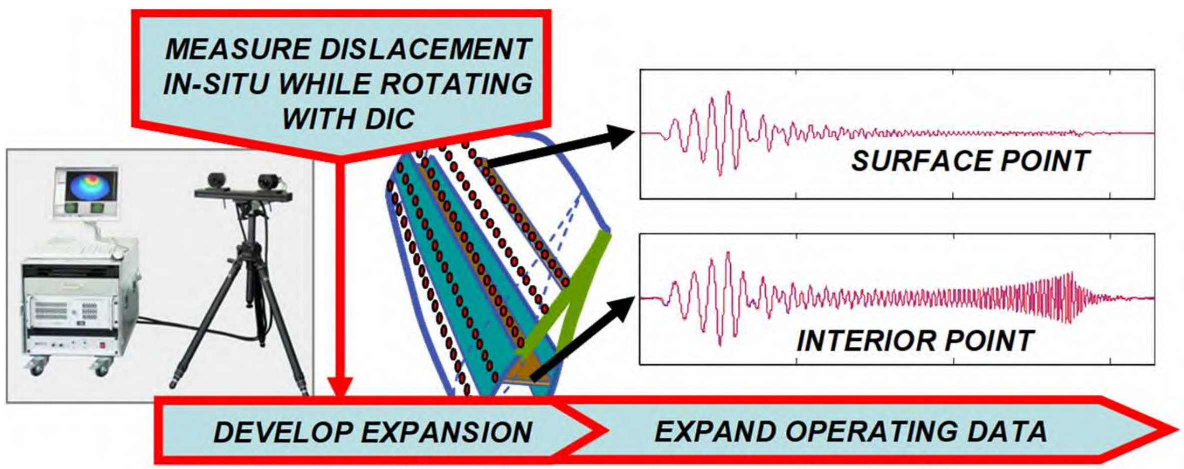
Actual displacement directly obtained



# Dynamic Response - Fundamental Change in Approach

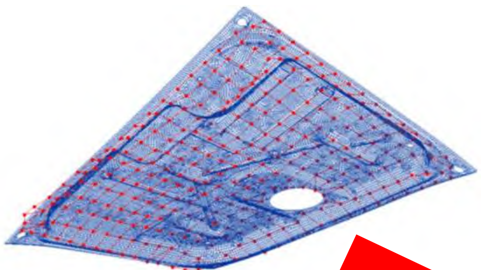
***Operating (Real-time) displacements are expanded to the full set of analytical degrees of freedom in the finite element model using orthogonal shape based expansion functions.***

***Provides full field displacement solution***

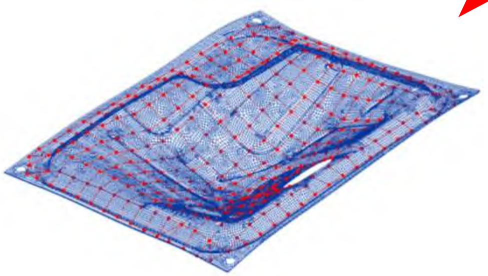



$[T_U] = [E_n][E_a]^g \qquad [RTO_n] = [T][RTO_a]$

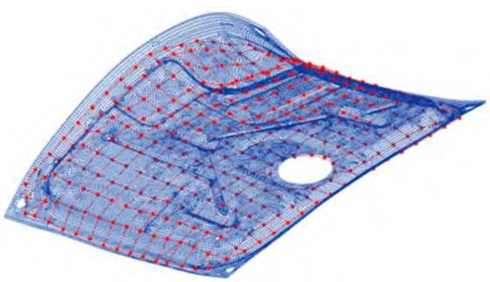
# Real-time data expansion – Redstone Arsenal



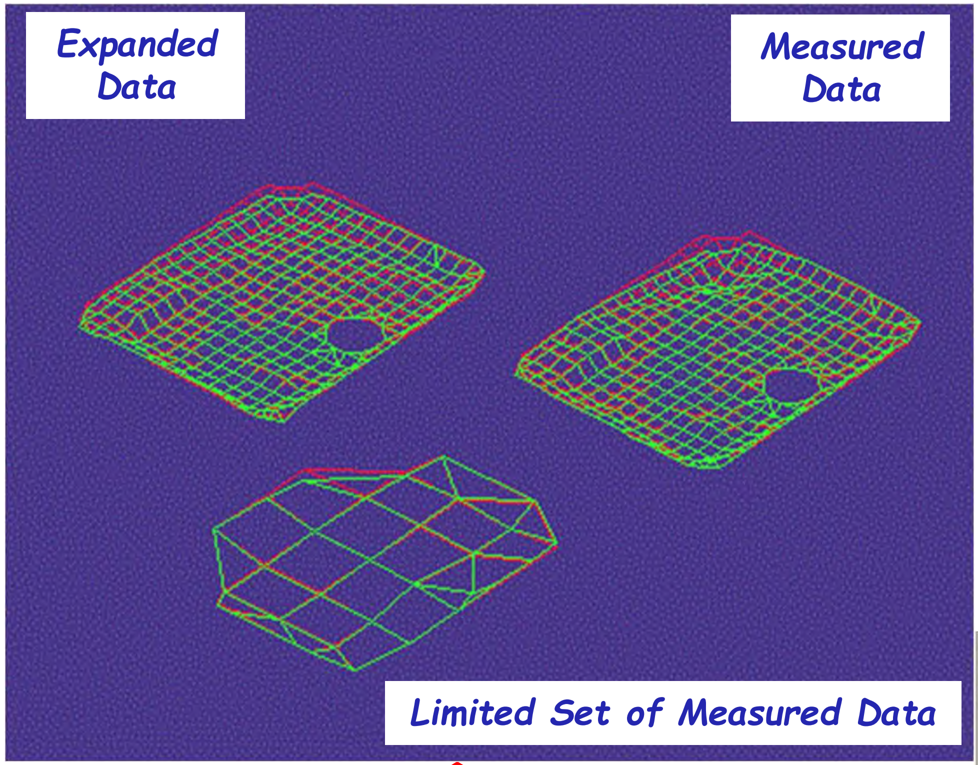
Mode 1



Mode 2



Mode 3



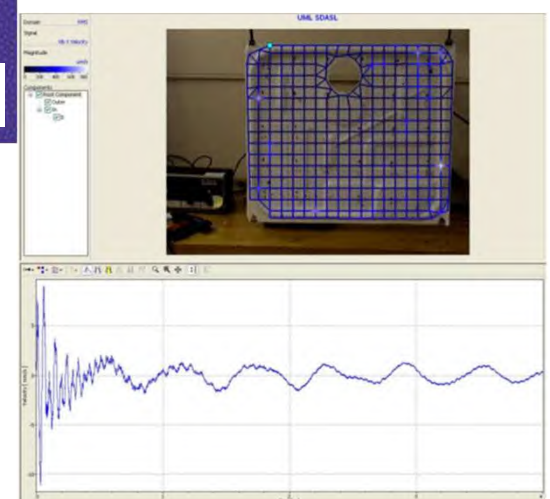

'a' set

Limited Measurement Points

$$[T_U] = [E_n][E_a]^g$$

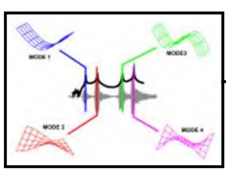
$$[RTO_n] = [T][RTO_a]$$

**Expansion is Key Here**

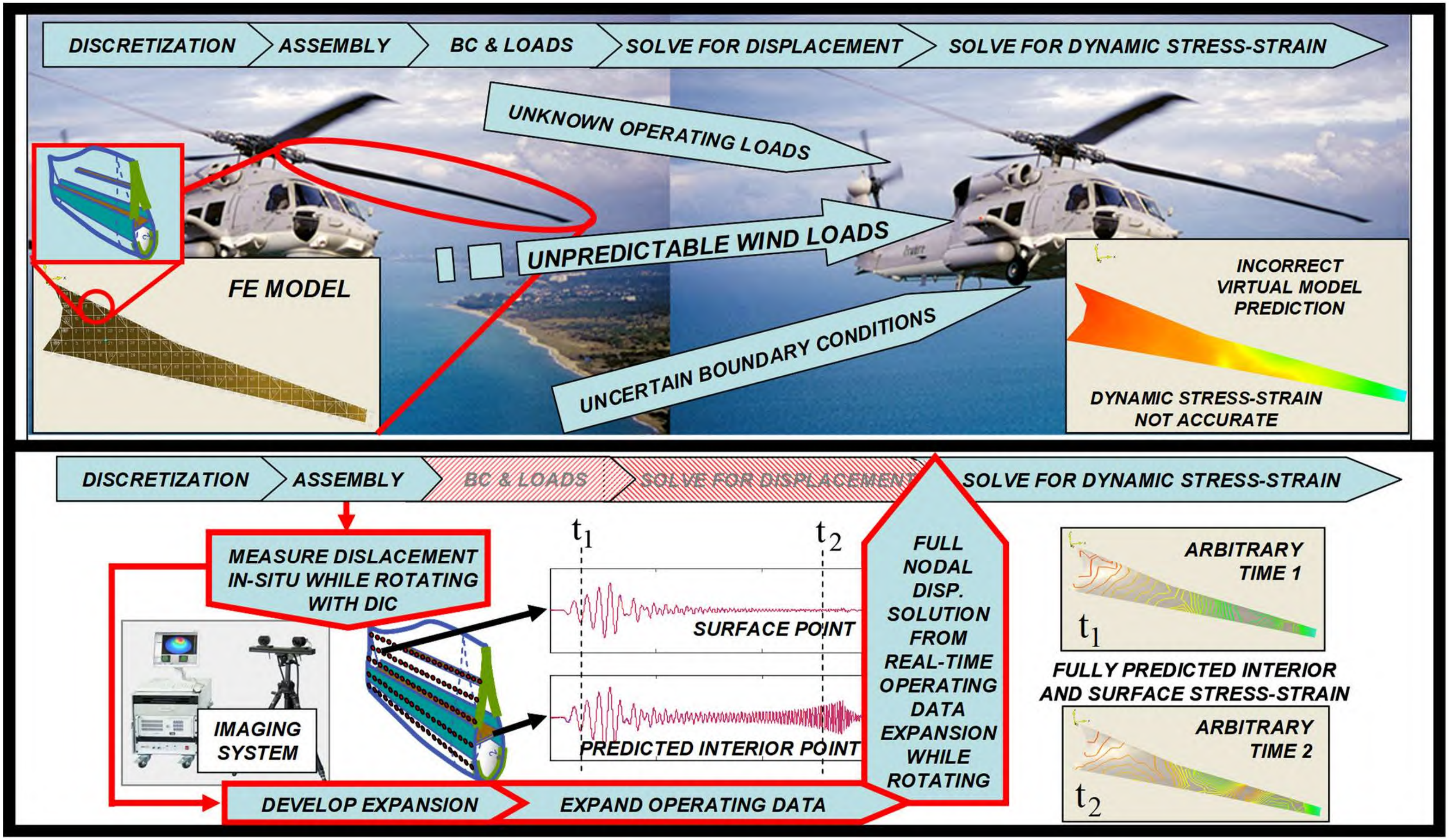


Polytec Scanning Vibrometer

ANIMATION

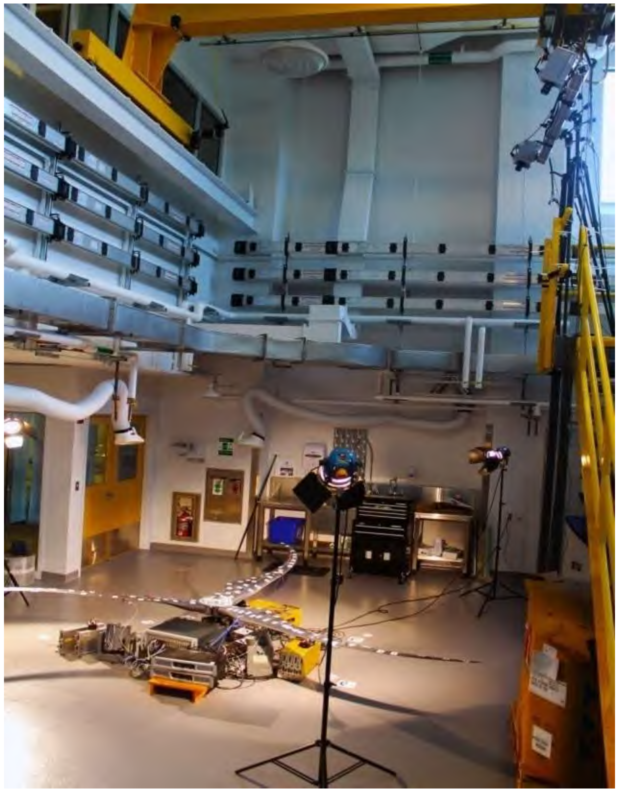


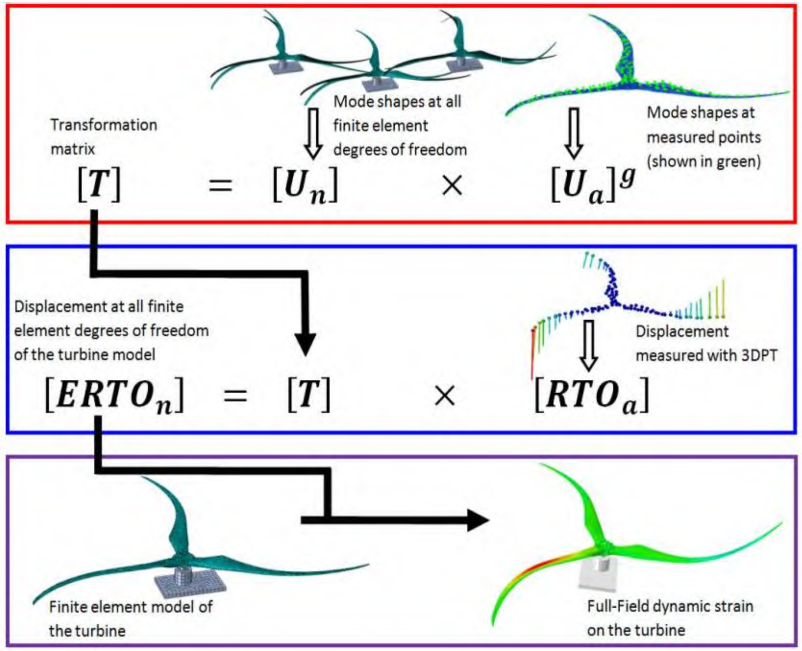
# Conventional Approach vs. Alternate Approach

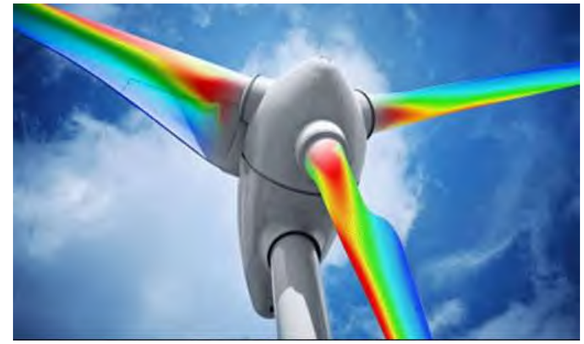


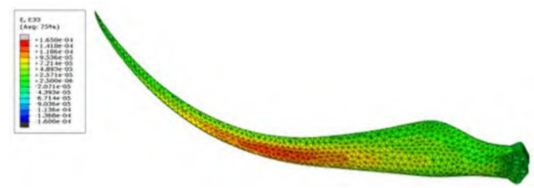
# Real Time Dynamic Strain from Limited Measurements

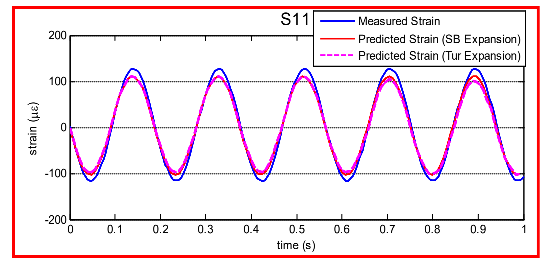
Javad Baqersad, Peter Avitabile

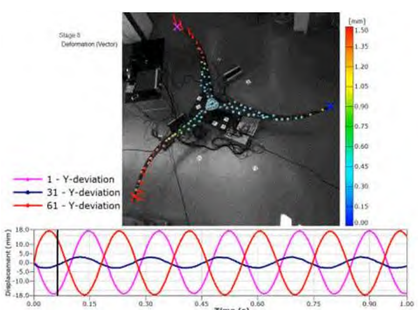


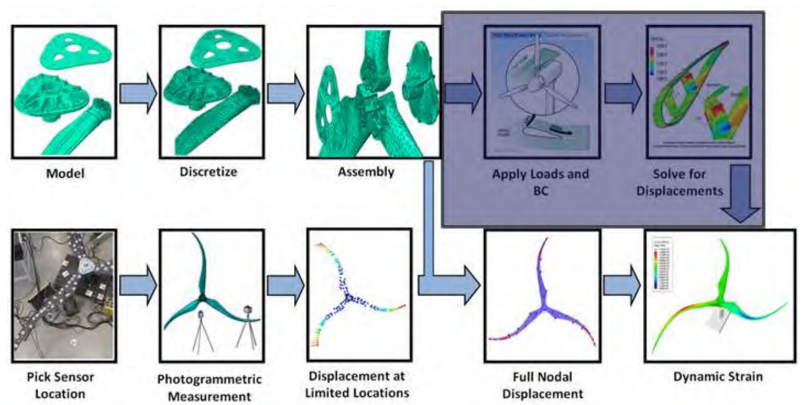


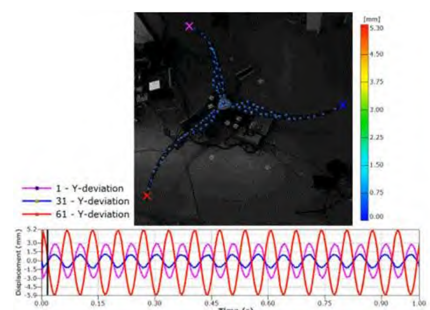



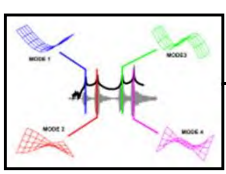




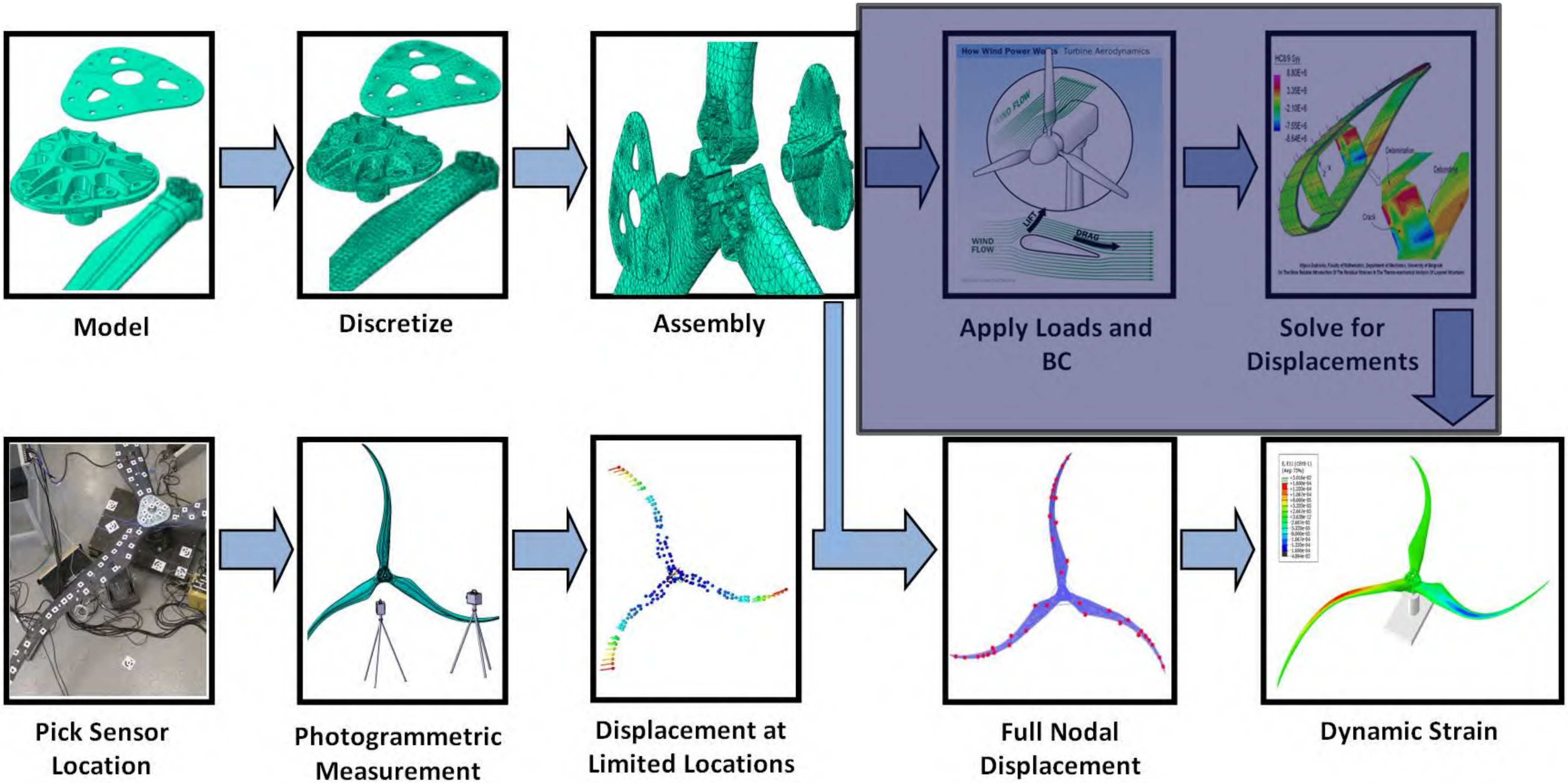



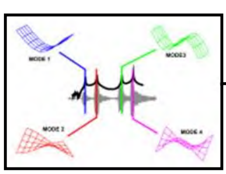



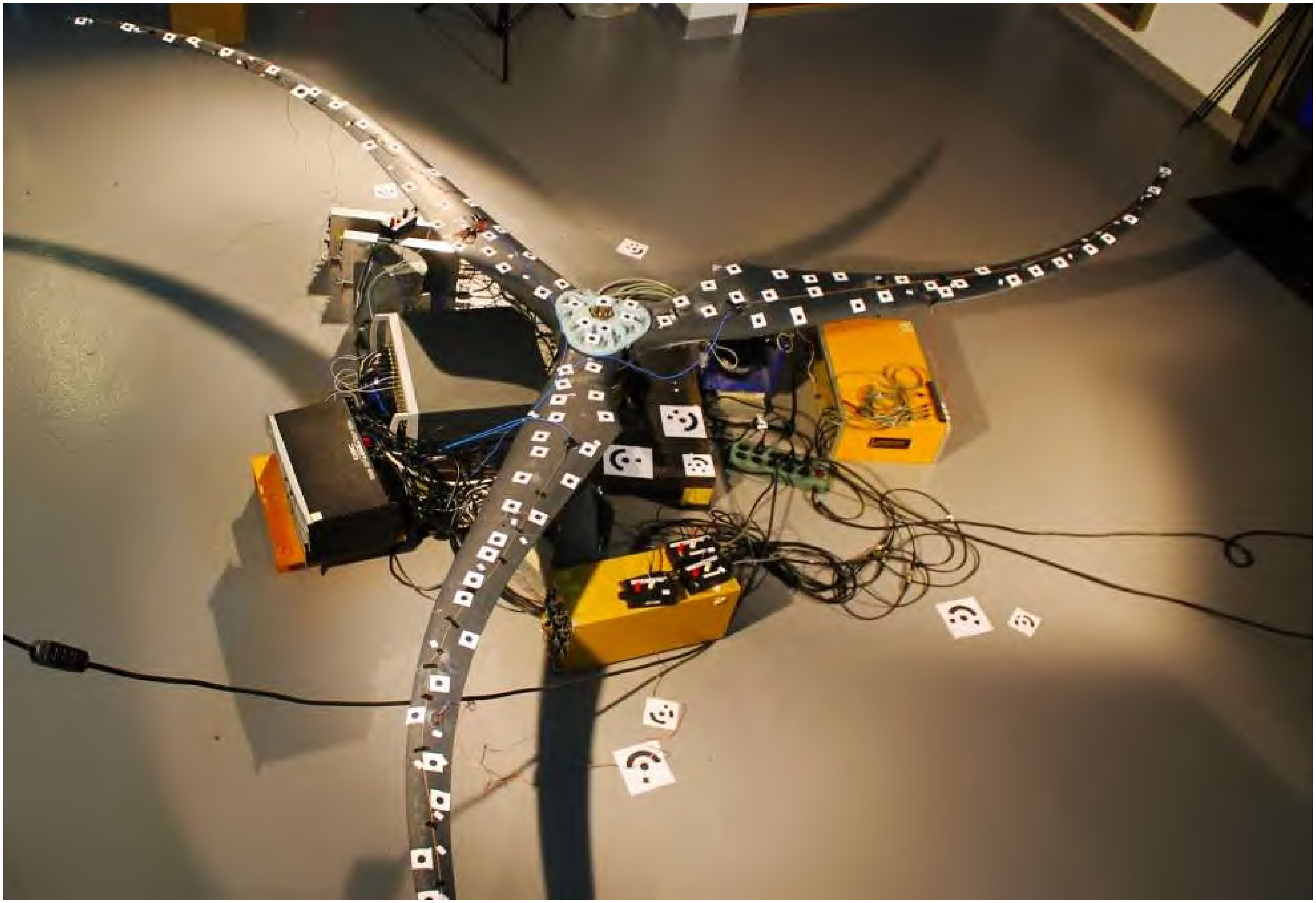
# Conventional Approach vs. Alternate Approach



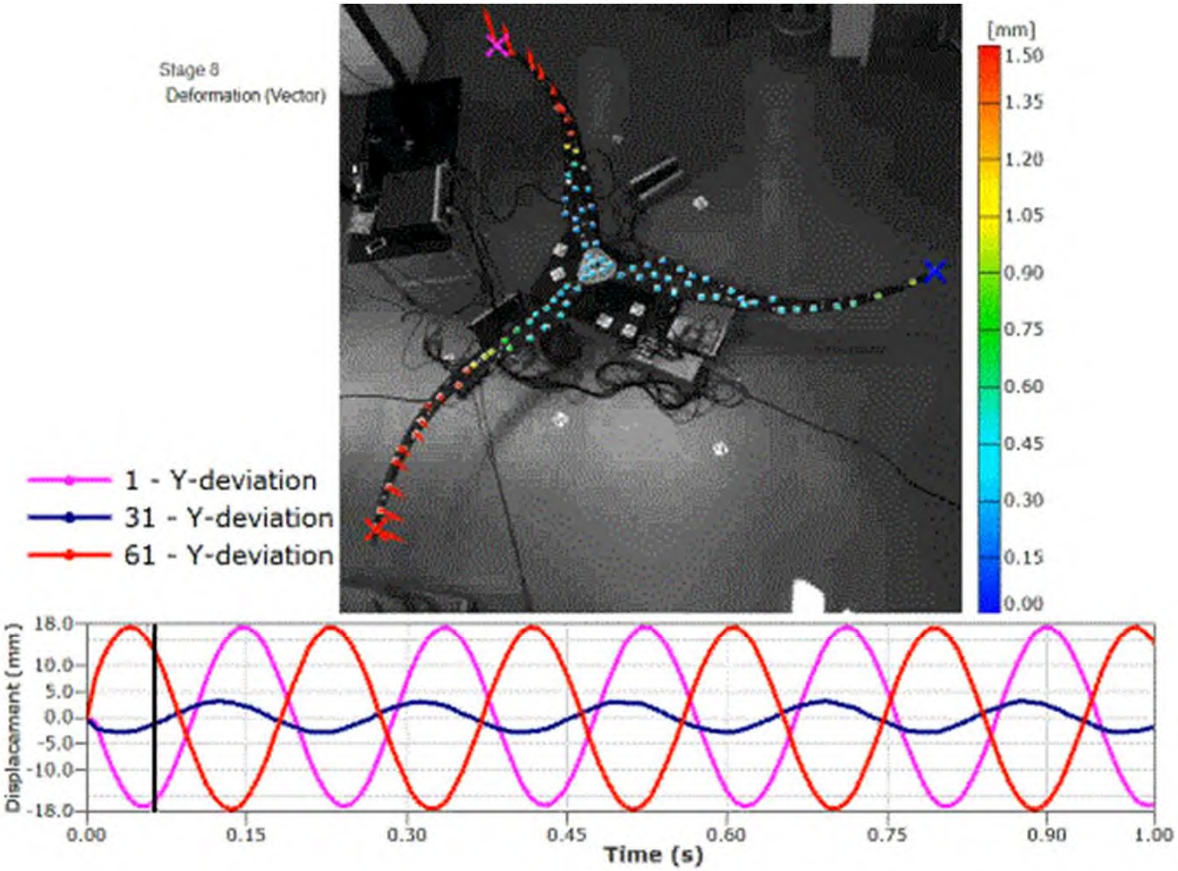

## New Approach for Full Field Strain !!!

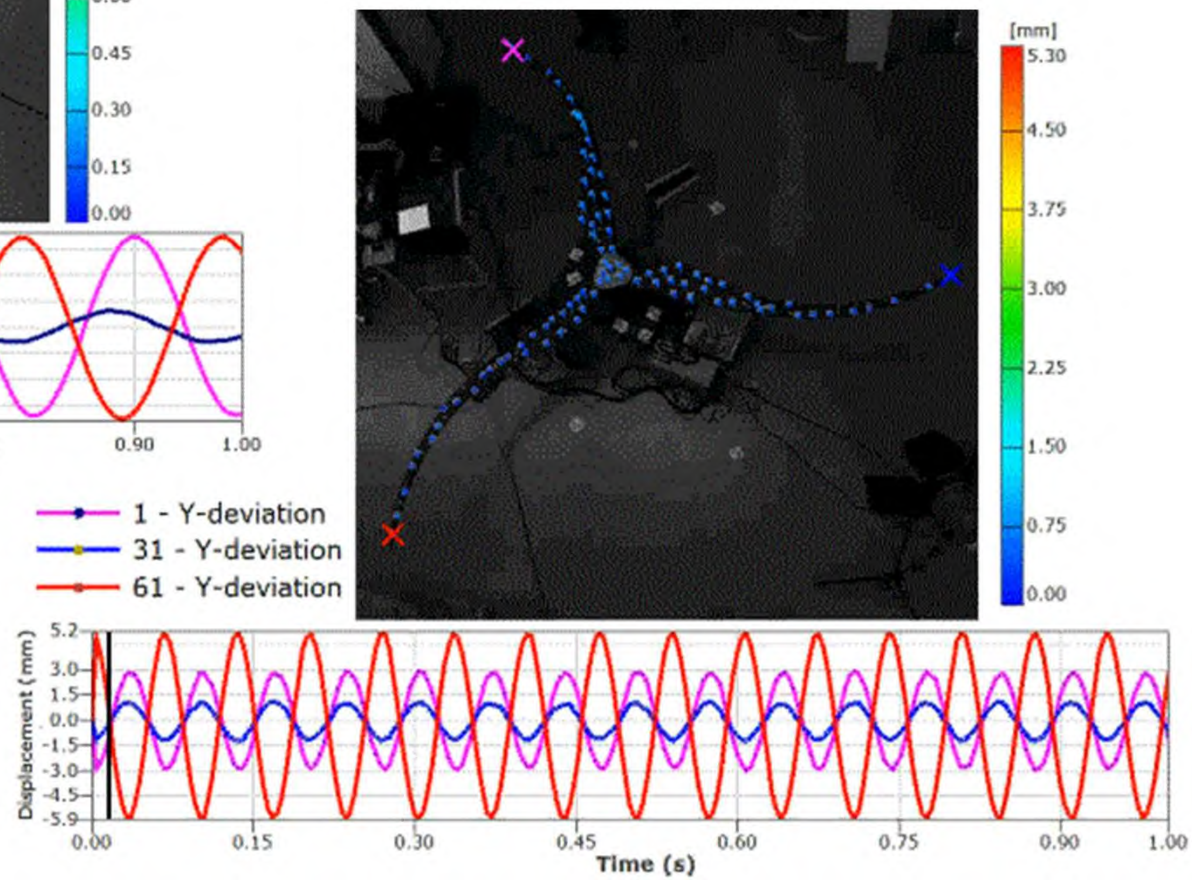


# Three Bladed System – Turbine Targets



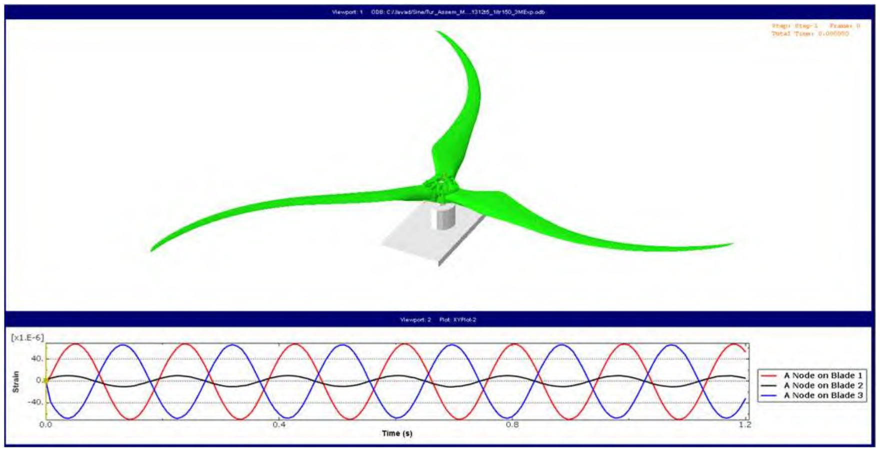
# Measured Displacement at Limited Points

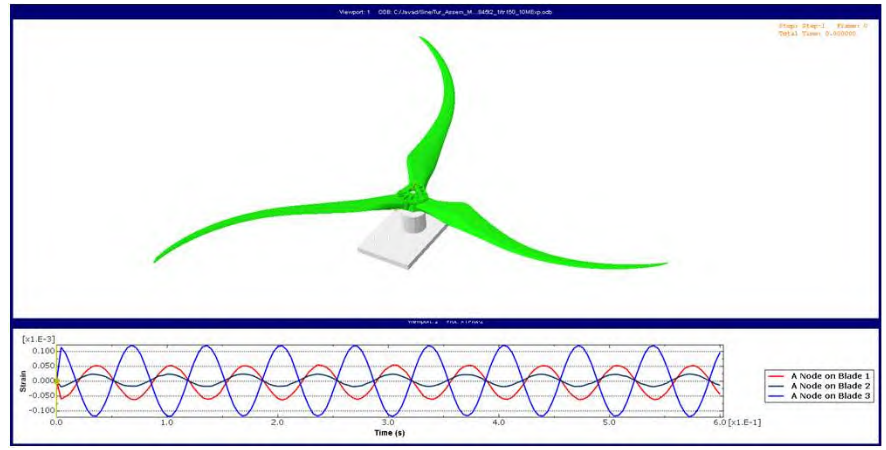


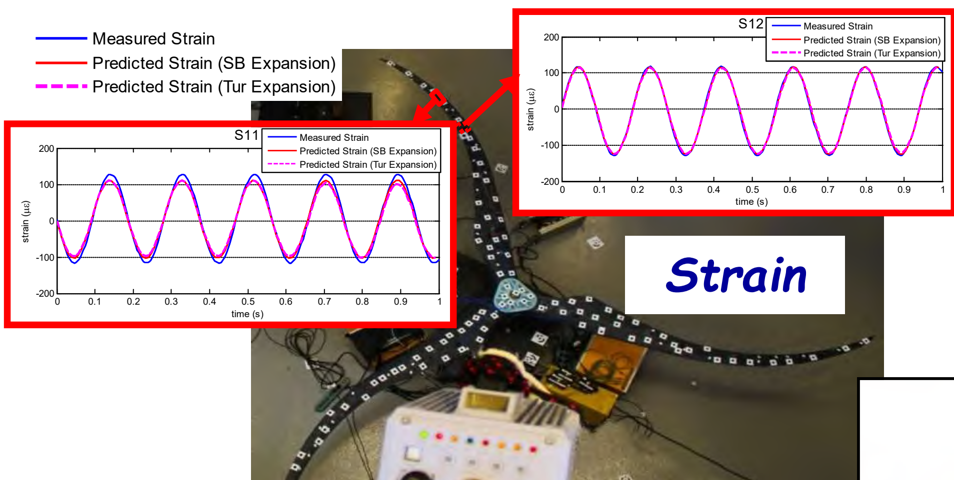



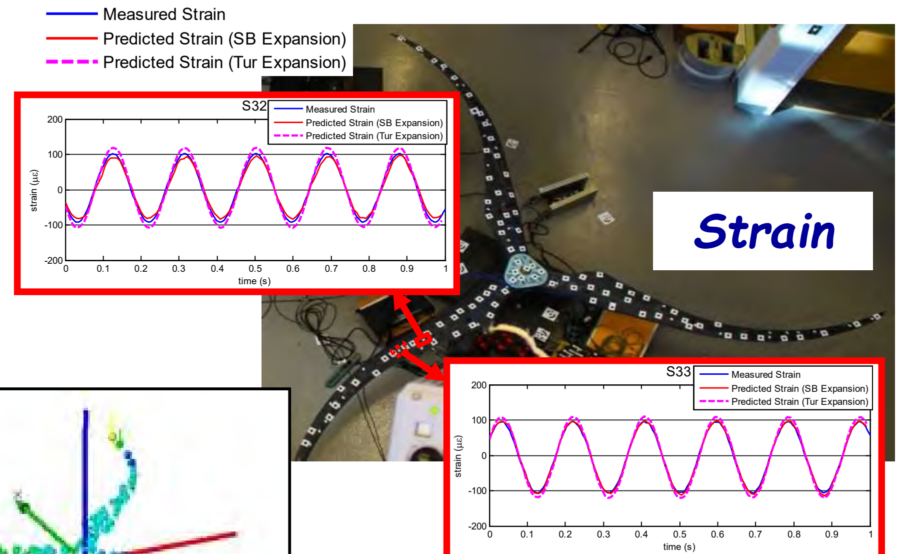

ANIMATION

# Three Bladed System – Expansion Process



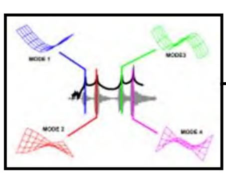



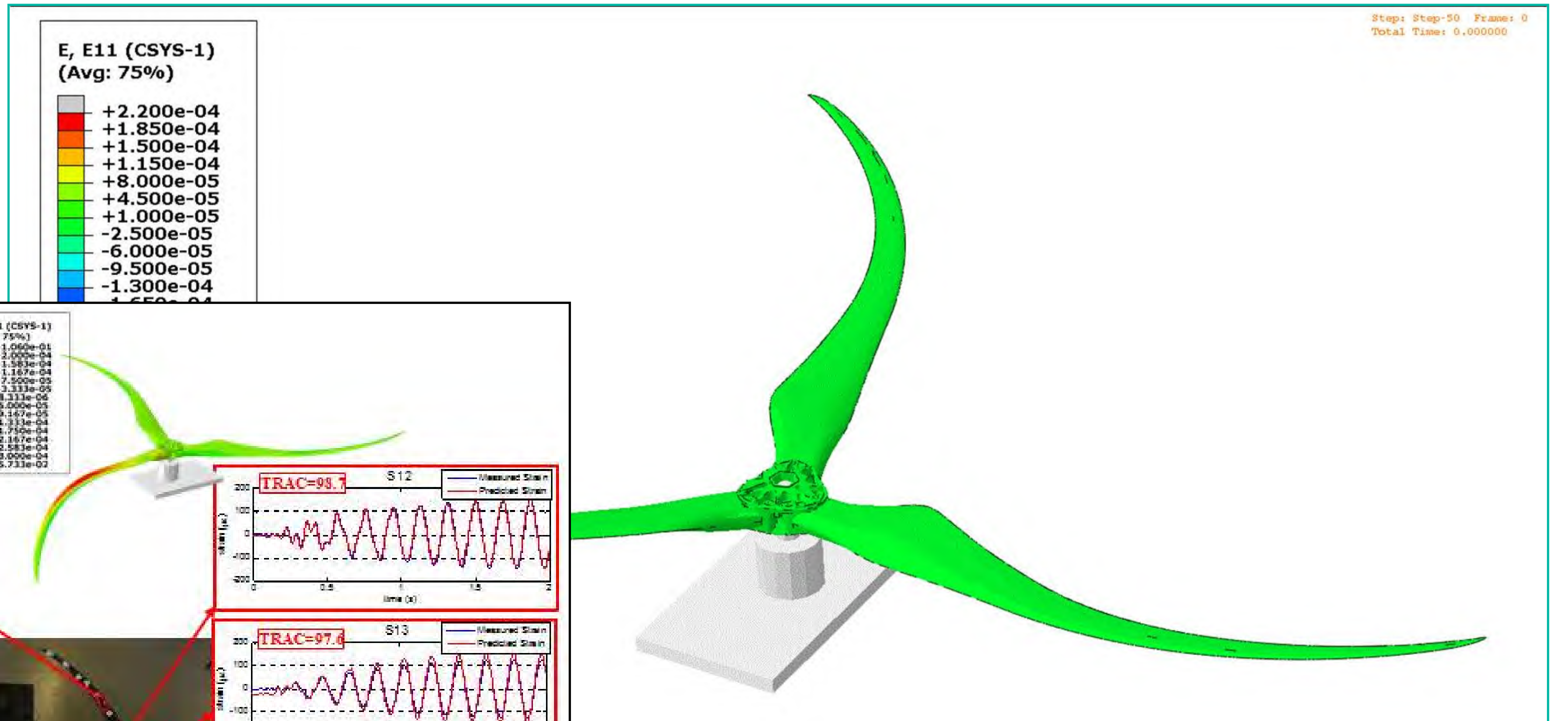





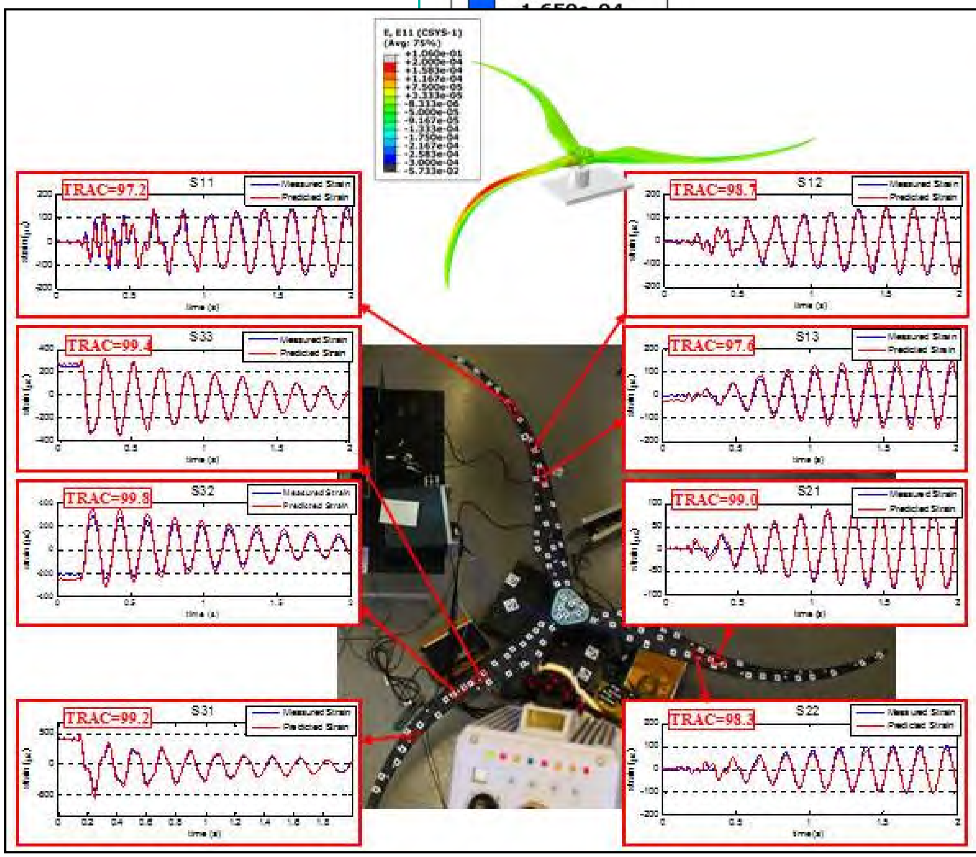
ANIMATION



# Turbine Blade – Pluck Test

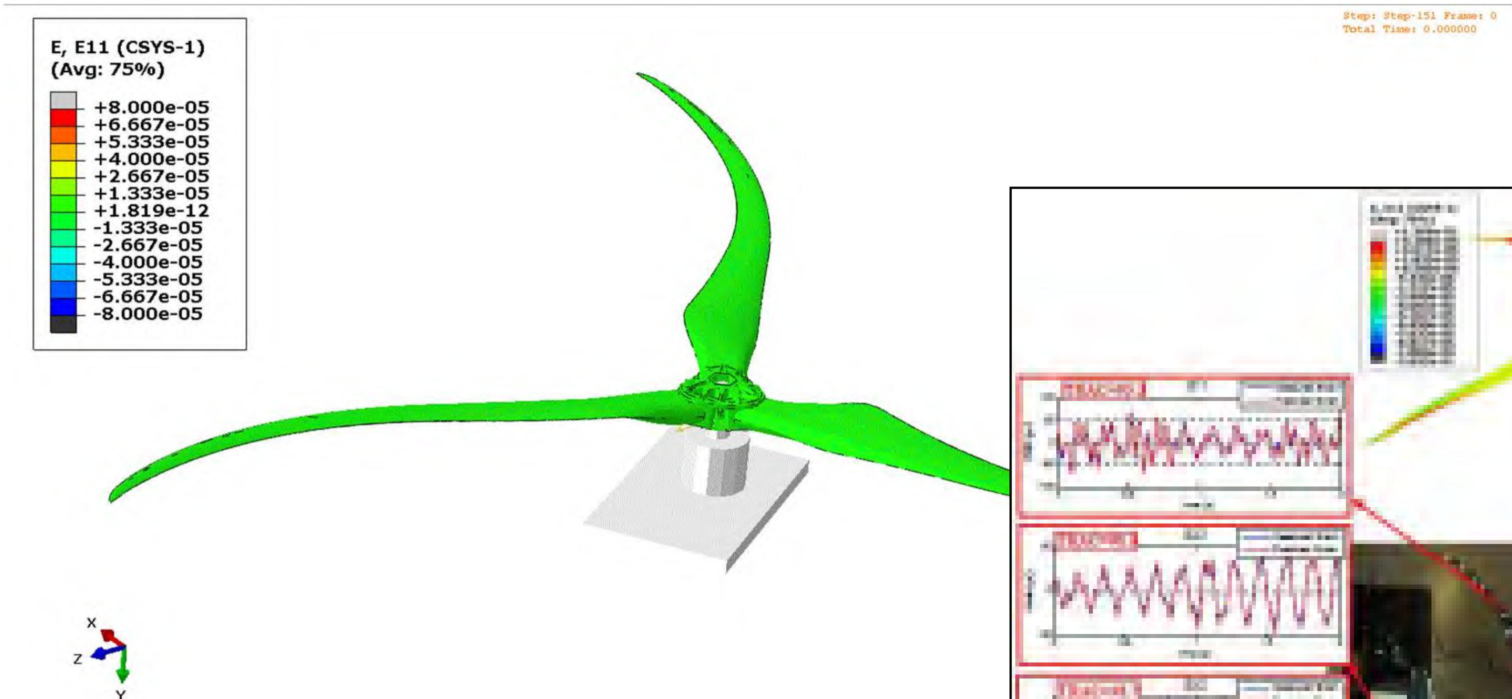



## Pluck Test

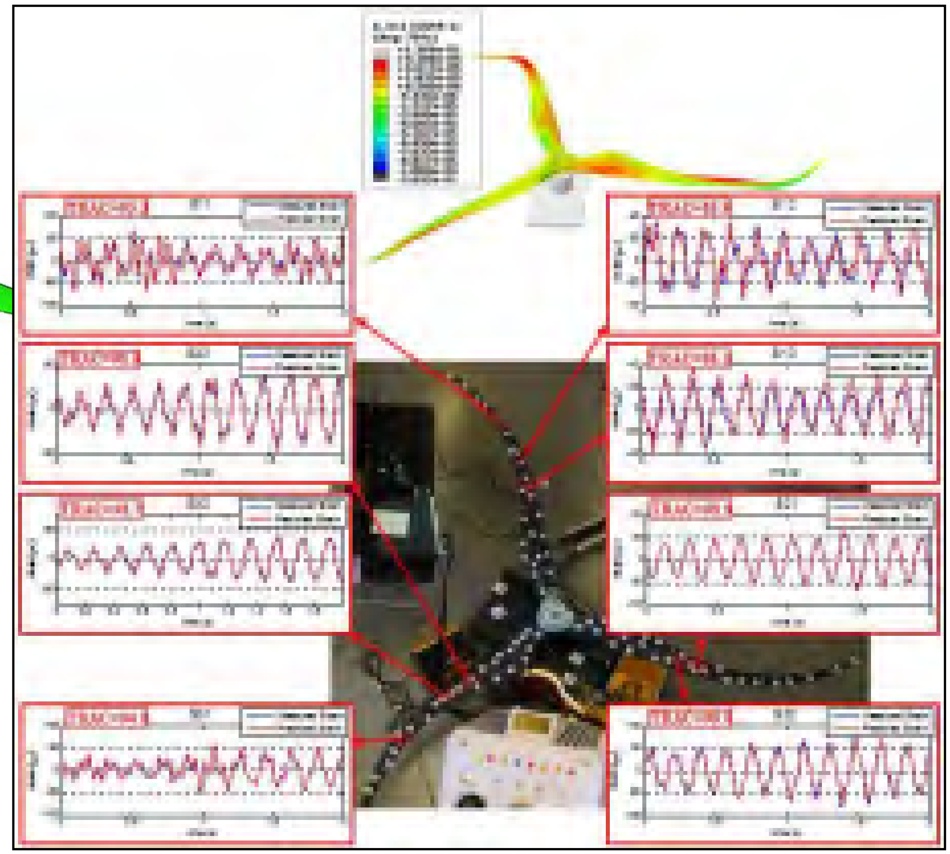



ANIMATION

# Turbine Blade – Bird Strikes

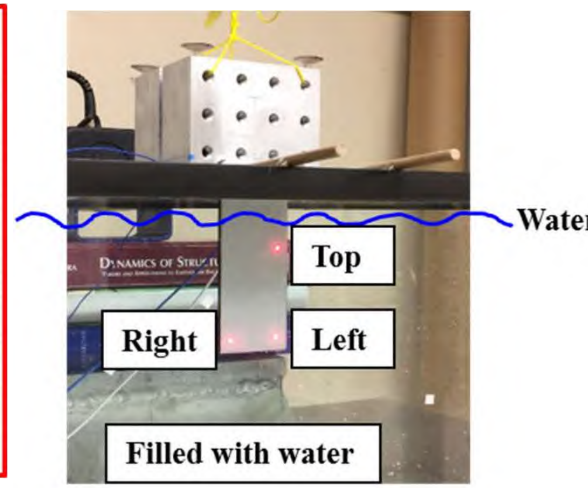


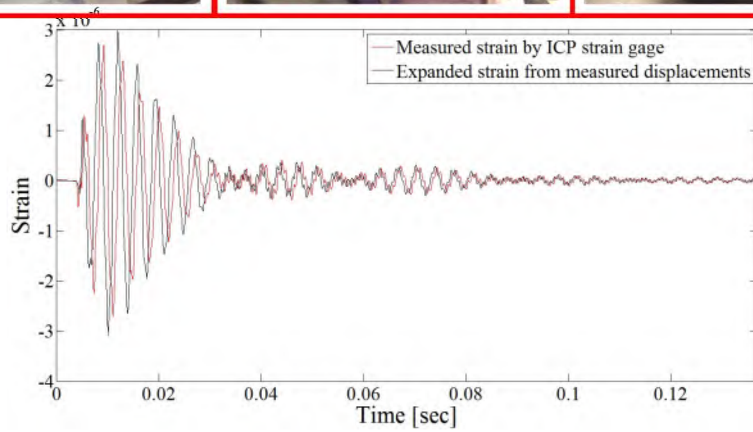

**Random Impacts**

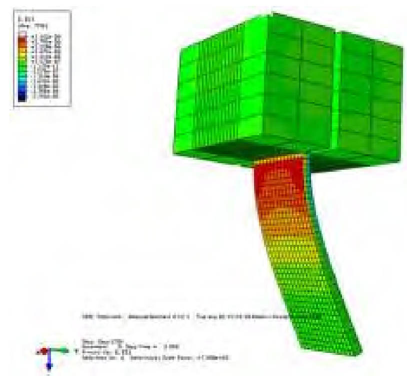


ANIMATION

# Underwater Dynamic Response at Limited Points Expanded to Full-Field Displacement Response



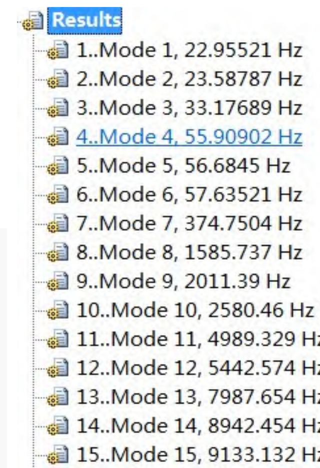
Yuanchang Chen, Peter Avitabile

http://faculty.uml.edu/pavitabile/downloads/IMAC35_Underwater_Expansion_Yuan_102016.pdf

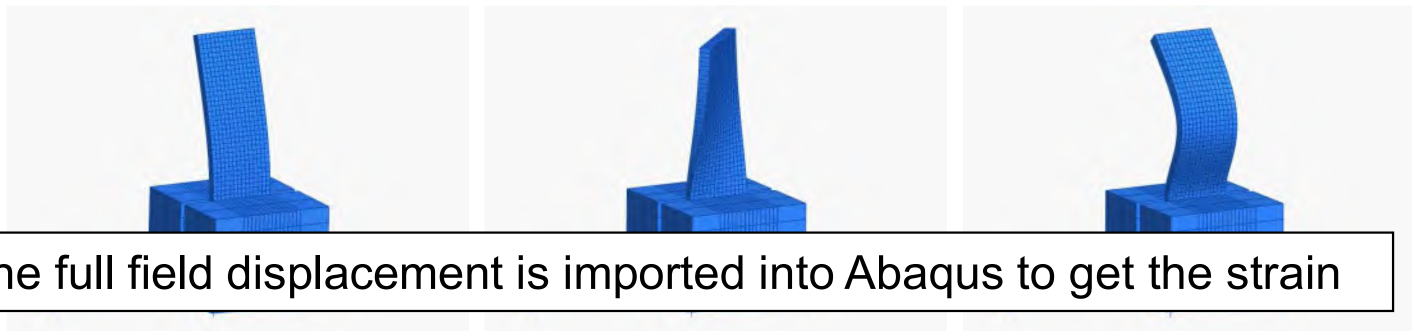
# Structure Water – expansion results

Then the displacement at three laser points is converted to the full field using the following equations

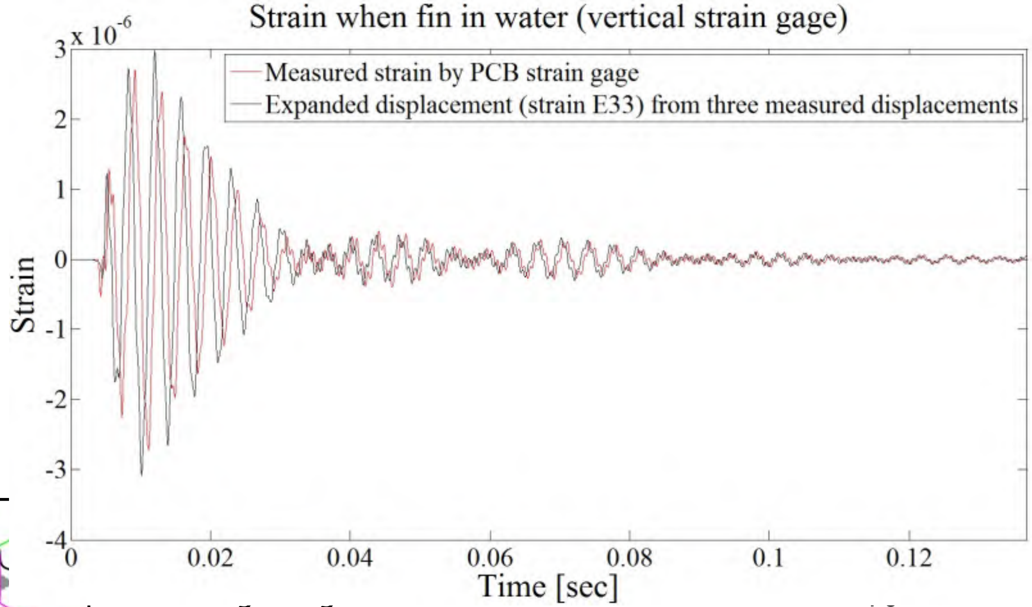
$$\{X_n\} = \begin{Bmatrix} X_a \\ X_d \end{Bmatrix} = [T]\{X_a\} \qquad [T_U] = [U_n][U_a]^g$$

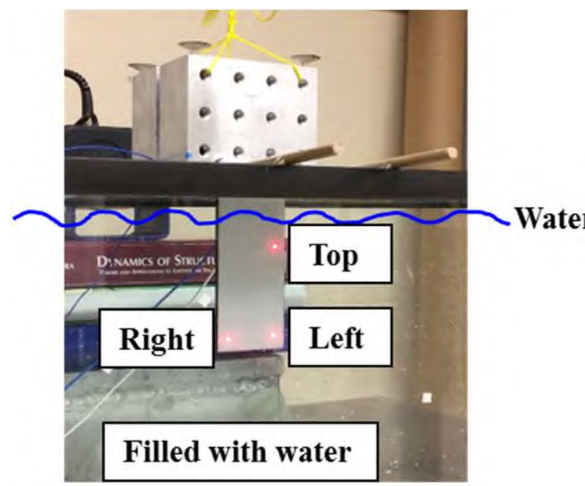



$[U_n]$ is composed of the first three flexible modes of the FE model, as follows



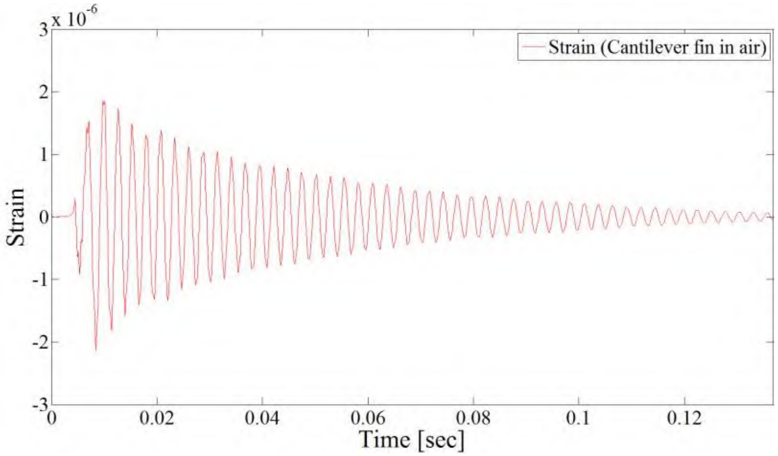
Then the full field displacement is imported into Abaqus to get the strain



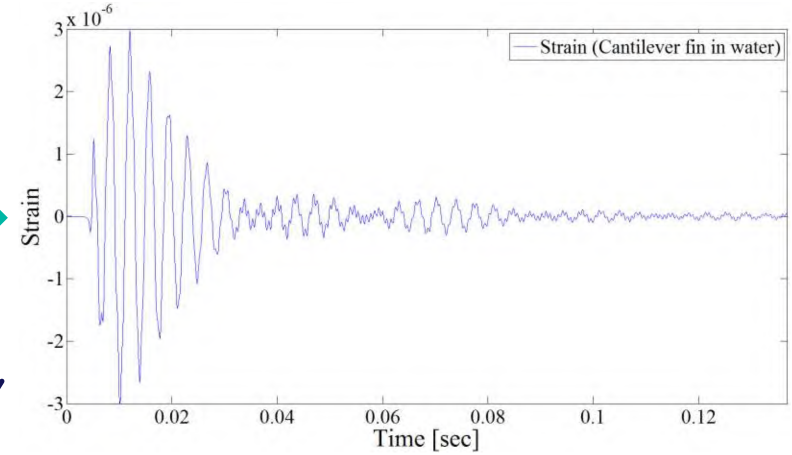



# Comparison between in air and in water

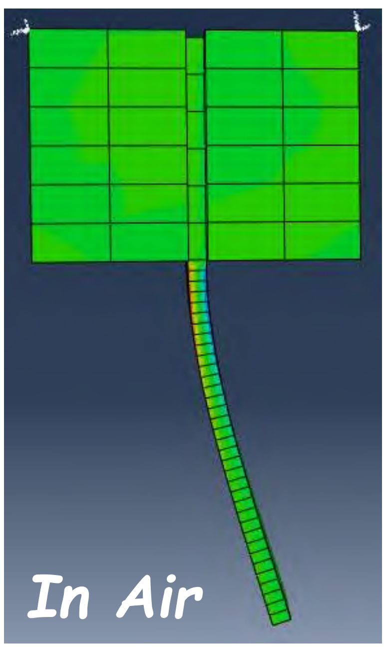
## Strain in Air

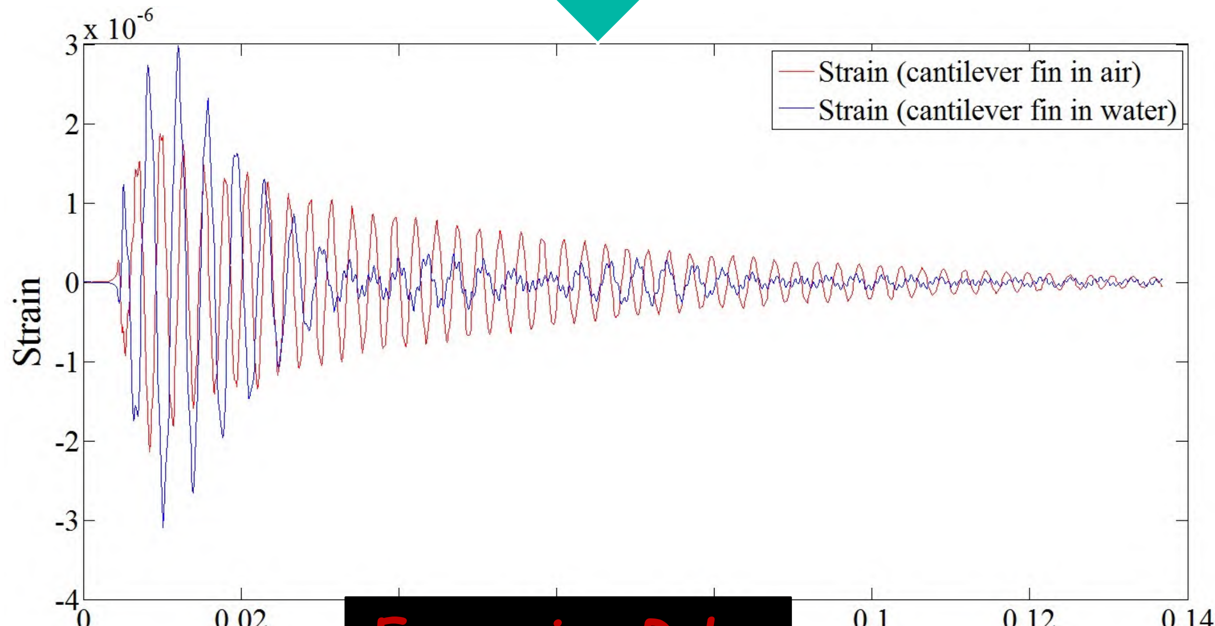


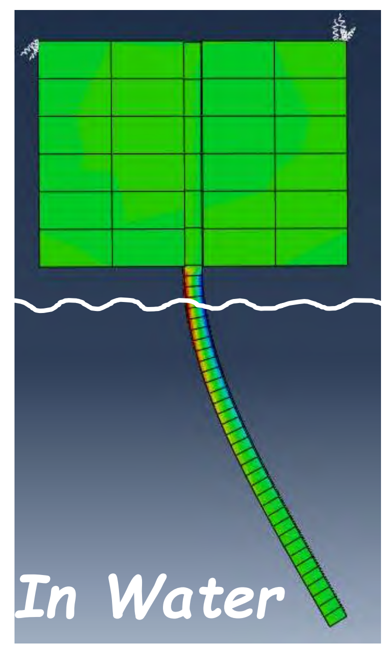

## Strain in Water




Overlay



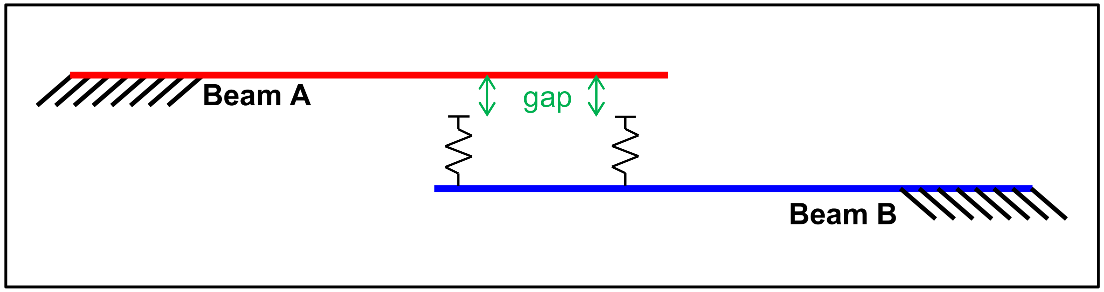
# Nonlinear Response using Linear Components with Nonlinear Connections

***Lou Thibault, Tim Marinone, Julie Harvie, Sergio Obando, Peter Avitabile***

*Linear Modal Substructuring with Nonlinear Connections, SEM Handbook of Experimental Structural Dynamics,*
*DOI:10.1007/978-1-4614-4547-0_34, July 2022*

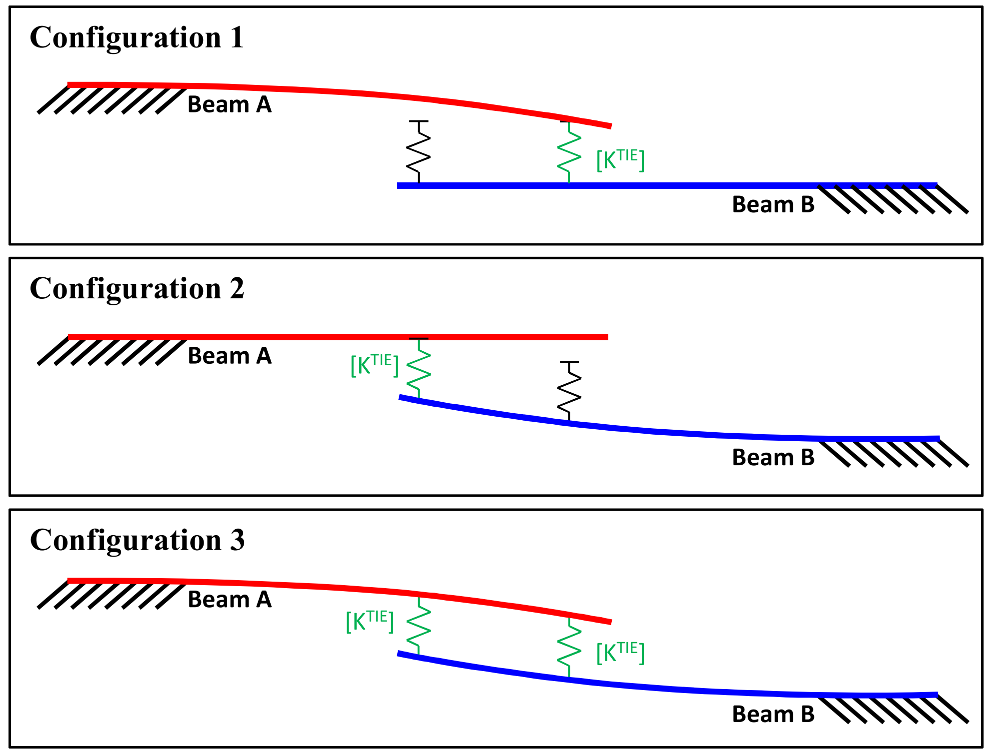
# System Modeling

## Uncoupled system

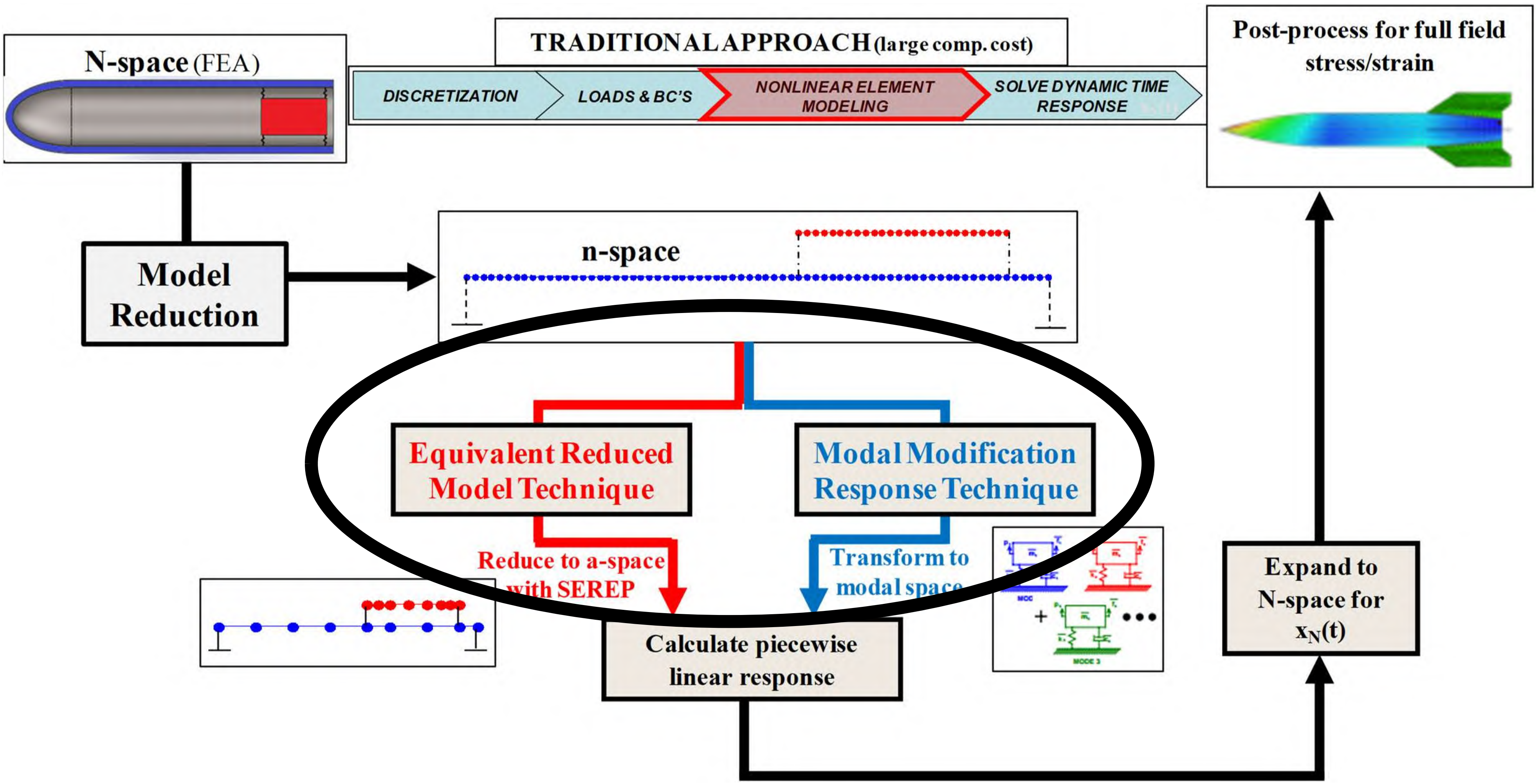



- Discrete coupling terms must be introduced to generate possible configurations

- Structural Dynamic Modifications (SDM) and Component Mode Synthesis (CMS)
- Physical System Modeling
- Mode Contribution Matrix

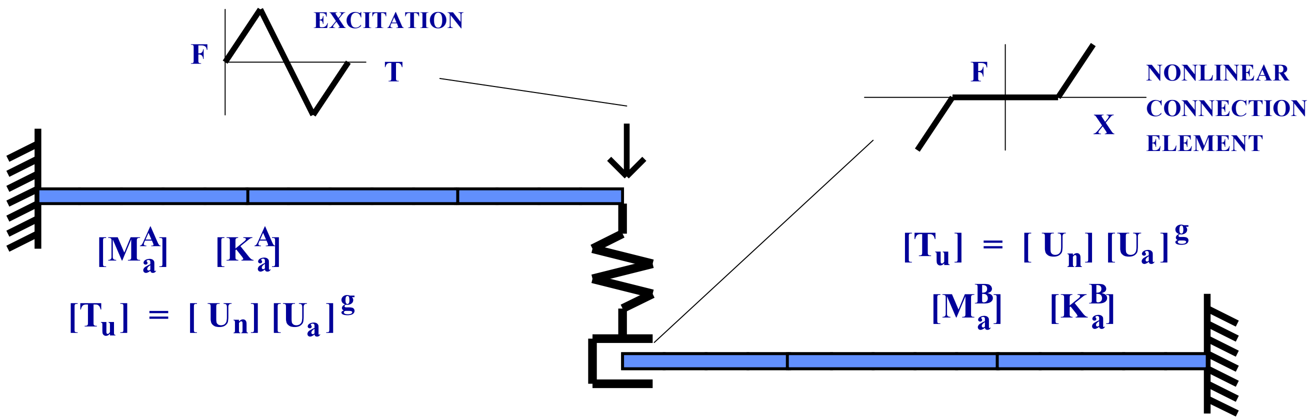


# A Piecewise Linear Approach to Nonlinear Response




Lou Thibault, Tim Marinone, Julie Harvie, Sergio Obando, Peter Avitabile

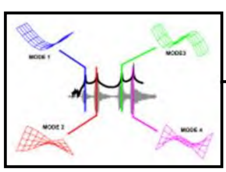
# Equivalent Reduced Order Model Technique




***Direct Integration of the Reduced Physical Equations of Motion can be performed for a reduced component model representation. Numerical integration proceeds as typically performed but with drastically reduced model size. The technique greatly improves the efficiency of the process and with optimization can integrate test.***

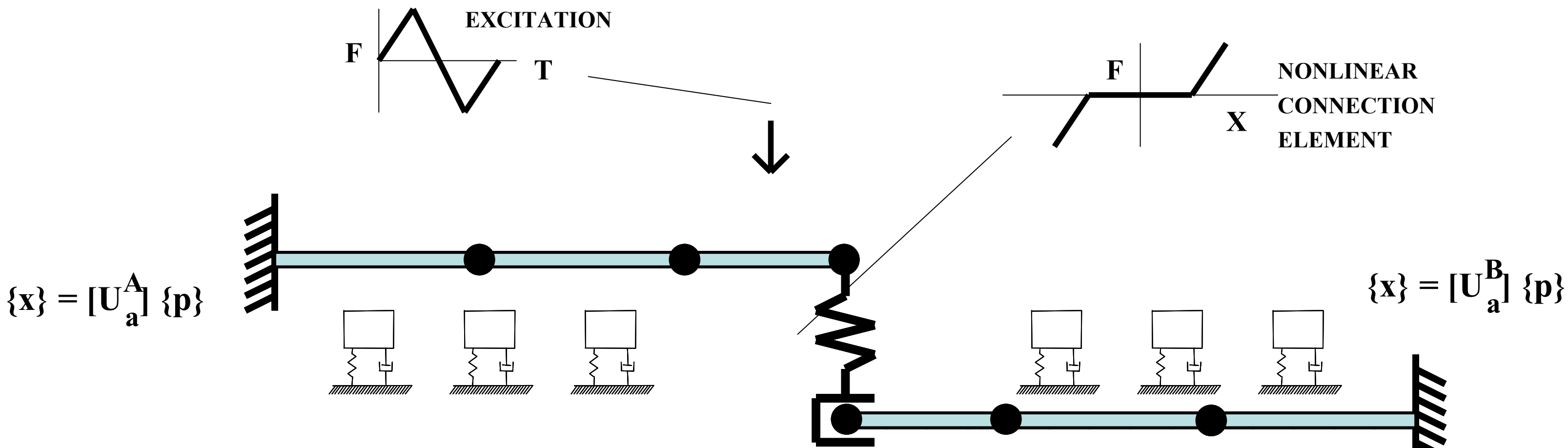
$$\begin{bmatrix} \left[M_a^A\right] & \\ & \left[M_a^B\right] \end{bmatrix} \begin{Bmatrix} \{\ddot{x}_a^A\} \\ \{\ddot{x}_a^B\} \end{Bmatrix} + \left[ \begin{bmatrix} \left[K_a^A\right] & \\ & \left[K_a^B\right] \end{bmatrix} + \left[K_{TIE}\right] \right] \begin{Bmatrix} \{x_a^A\} \\ \{x_a^B\} \end{Bmatrix} = \begin{Bmatrix} \{f_a^A\} \\ \{f_a^B\} \end{Bmatrix}$$

$$[M_a] = [U_a]^{gT}[U_a]^g \qquad [K_a] = [U_a]^{gT}\left[\Omega^2\right][U_a]^g$$



1990s

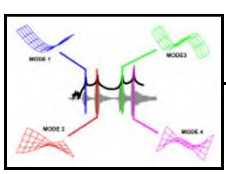
# Modal Modification Response Technique




**Mode Superposition is executed in a piecewise linear fashion depending on the "state" of the nonlinear connection element. Once the linear state changes, a structural modification is performed to update the characteristics of the system along with updated initial conditions to proceed on with the numerical integration. Current state of connection elements are monitored at each integration step**

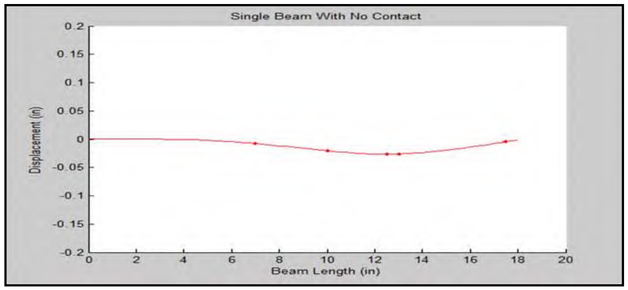
$$\begin{bmatrix} [I_m^A] & \\ & [I_m^B] \end{bmatrix} \begin{Bmatrix} \{\ddot{p}^A\} \\ \{\ddot{p}^B\} \end{Bmatrix} + \left[ \begin{bmatrix} [\Omega^{2}{}_m^A] & \\ & [\Omega^{2}{}_m^B] \end{bmatrix} + [U]^T [K_{TIE}] [U] \right] \begin{Bmatrix} \{p^A\} \\ \{p^B\} \end{Bmatrix} = \begin{Bmatrix} [U^A]^T \{f^A\} \\ [U^B]^T \{f_a^B\} \end{Bmatrix}$$

$$\{p\}_j = [U]_j^g \{x\}^{(i-1)} \quad \{\dot{p}\}_j = [U]_j^g \{\dot{x}\}^{(i-1)}$$

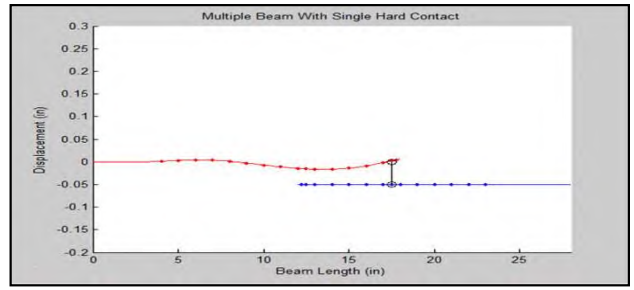


# Linear and Nonlinear Response - Highly Reduced Order Models

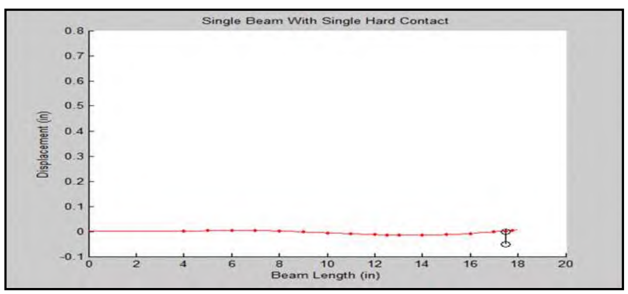
**Single Beam Linear Response**



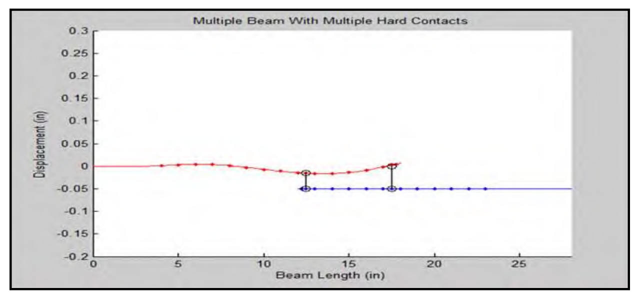

**Two Beam Single Contact**



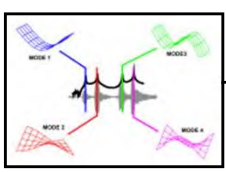

**Single Beam Single Contact**



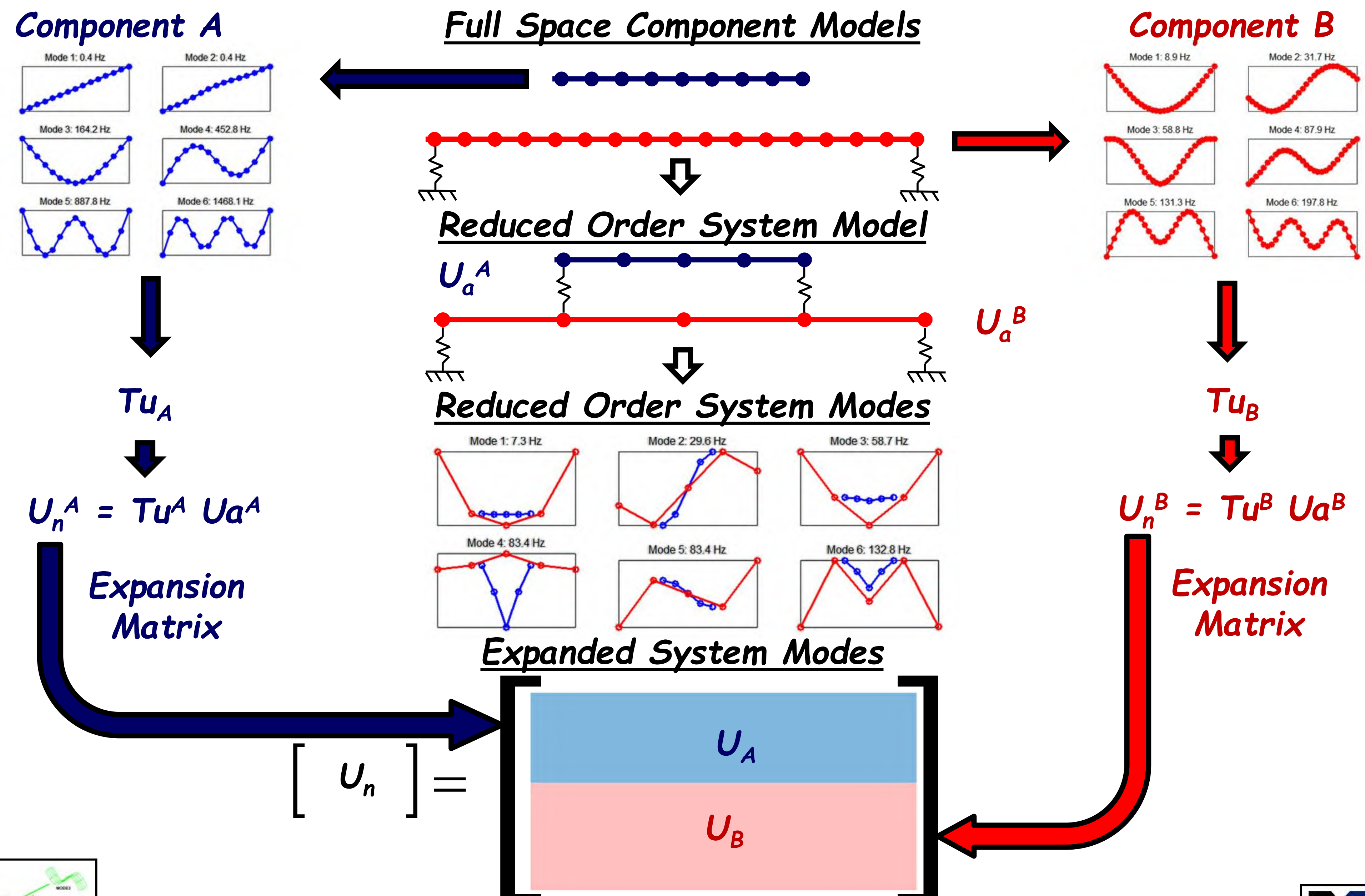

**Two Beam Multiple Contact**

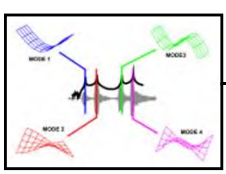



ANIMATION



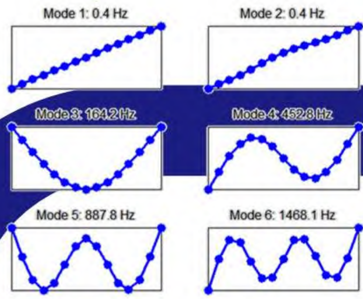
# Previous Work – Expansion of Coupled Mode Shapes



# Previous Work - Expansion Methodology

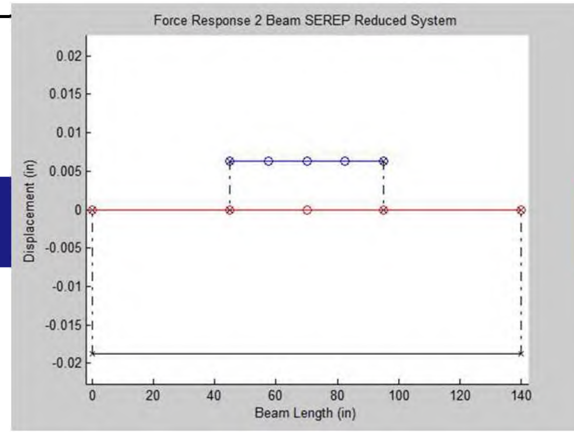
## Component A



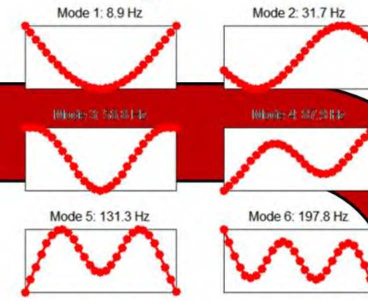

$Tu_A$

$U_n^A = Tu^A \, Ua^A$

**Expansion Matrix**




## Component B

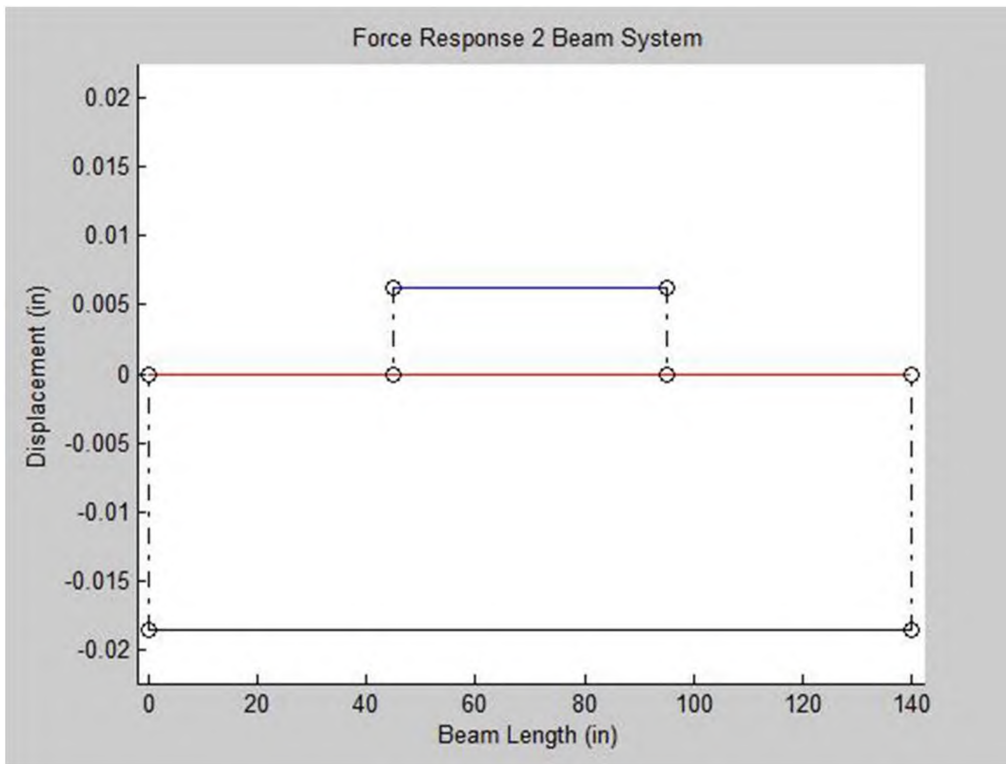



$Tu_B$

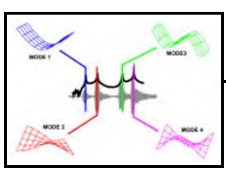
$U_n^B = Tu^B \, Ua^B$

**Expansion Matrix**

## Reduced Order System Model Response

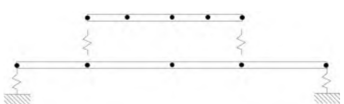



ANIMATION

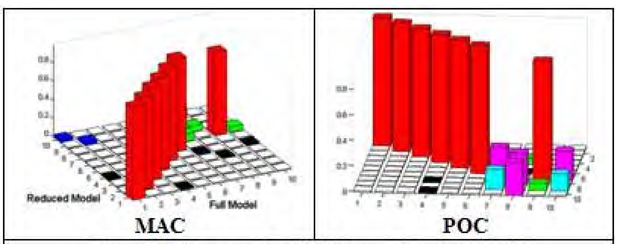


# Expansion of System Modes from Component Info

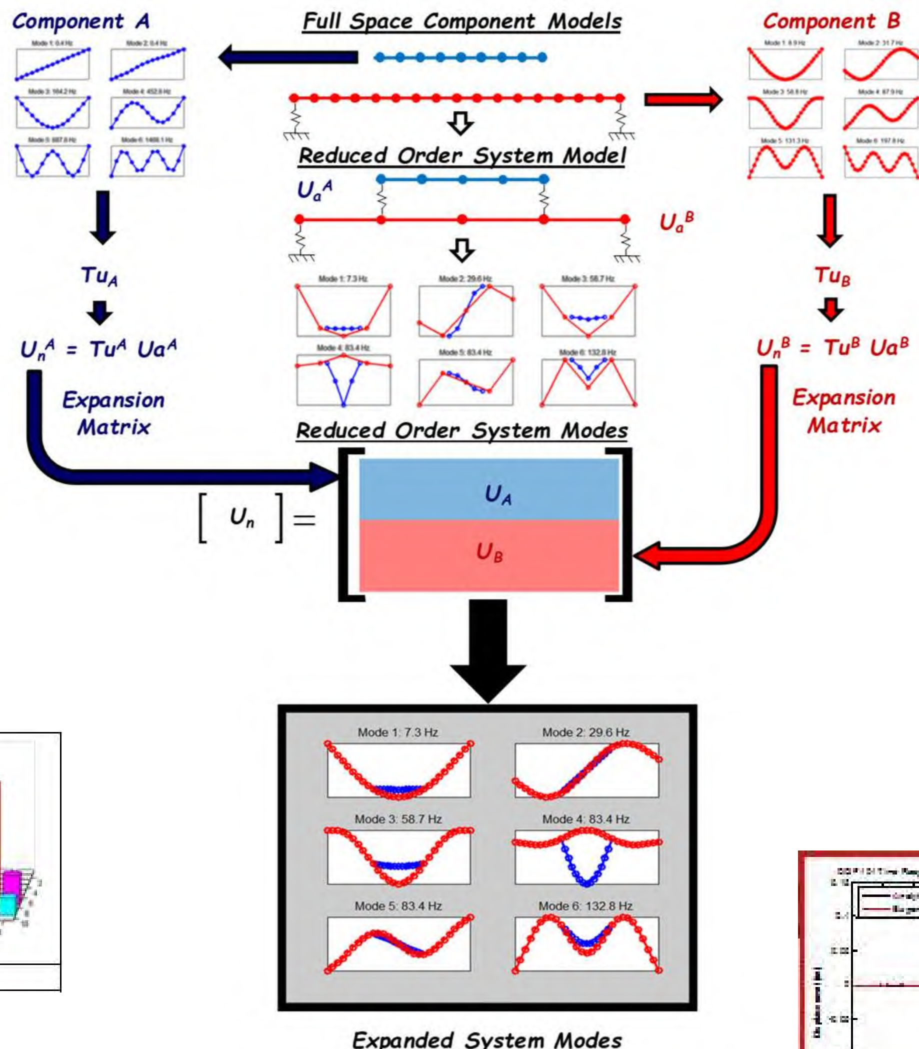
## Linear



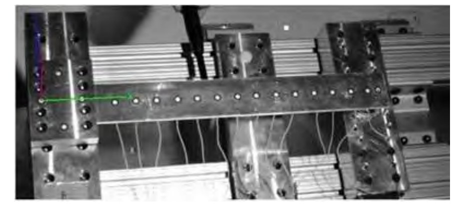
| Mode | Full Space System Model Comparison | | | | |
|---|---|---|---|---|---|
| | Frequency (Hz) | | | MAC | POC |
| | Full Physical 'n' space | Expanded model from 'a' space | % Diff | | |
| 1 | 7.3 | 7.2 | 0.01 | 1.000 | 1.000 |
| 2 | 29.5 | 29.5 | 0.02 | 1.000 | 1.000 |
| 3 | 58.7 | 58.7 | 0.01 | 1.000 | 1.000 |
| 4 | 81.7 | 83.36 | 2.00 | 0.997 | 0.998 |
| 5 | 83.3 | 83.37 | 0.10 | 1.000 | 0.999 |
| 6 | 132.8 | 132.8 | 0.00 | 1.000 | 0.999 |
| 7 | 191.8 | 316.4 | 65.0 | 0.073 | 0.253 |
| 8 | 273.8 | 680.8 | 148 | 0.064 | 0.252 |
| 9 | 306.2 | 4461.2 | 1356 | 0.002 | 0.058 |
| 10 | 398.7 | 4977.3 | 1148 | 0.000 | 0.000 |

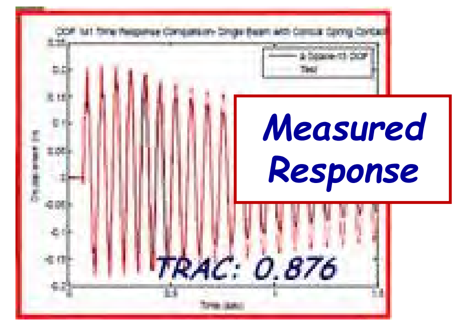


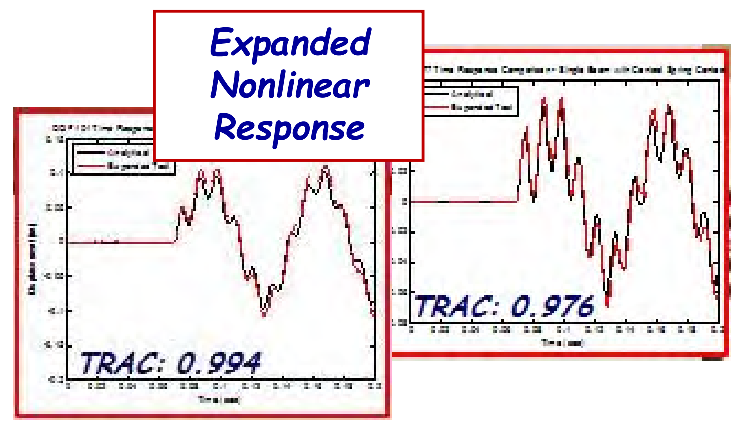

Expanded Compared to Full Space Physical Model

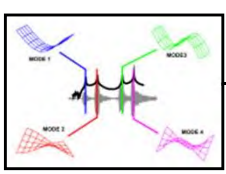



## Nonlinear



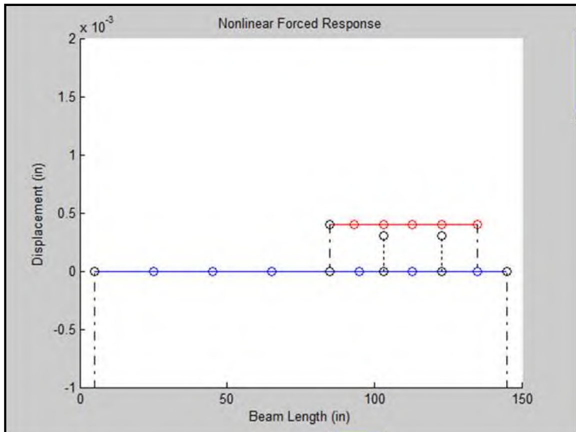


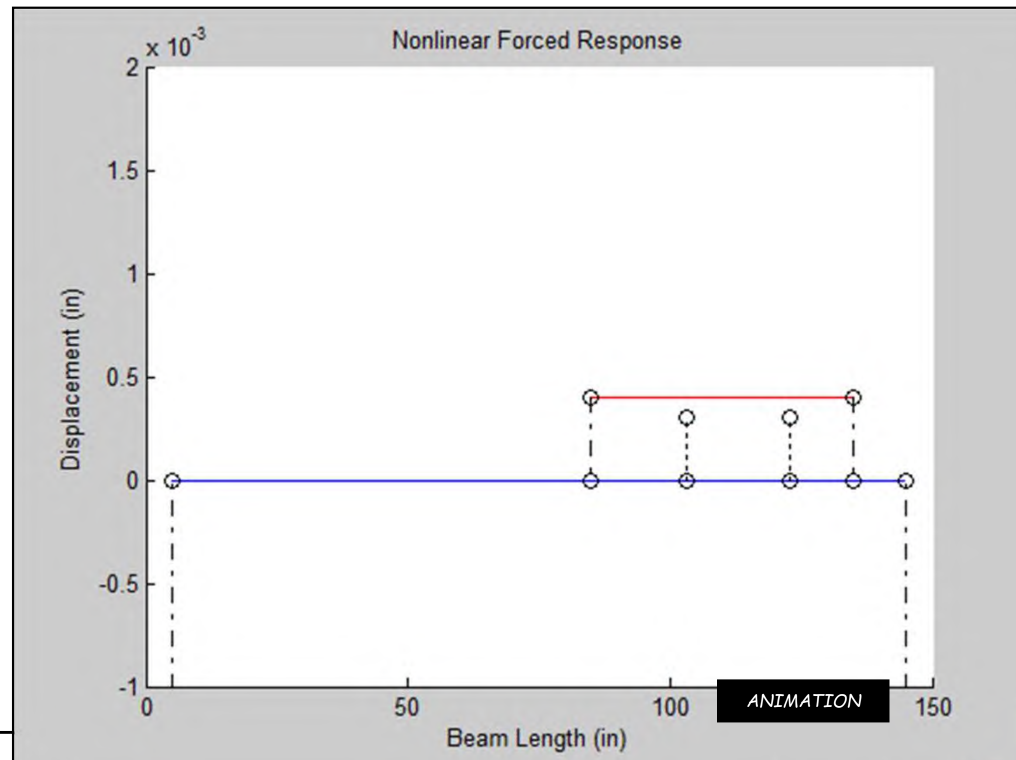



# Nonlinear Response - Highly Reduced Order Models & Expansion

Reduced Order Nonlinear Response

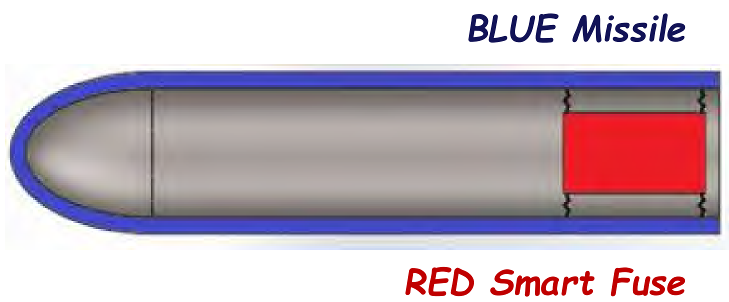


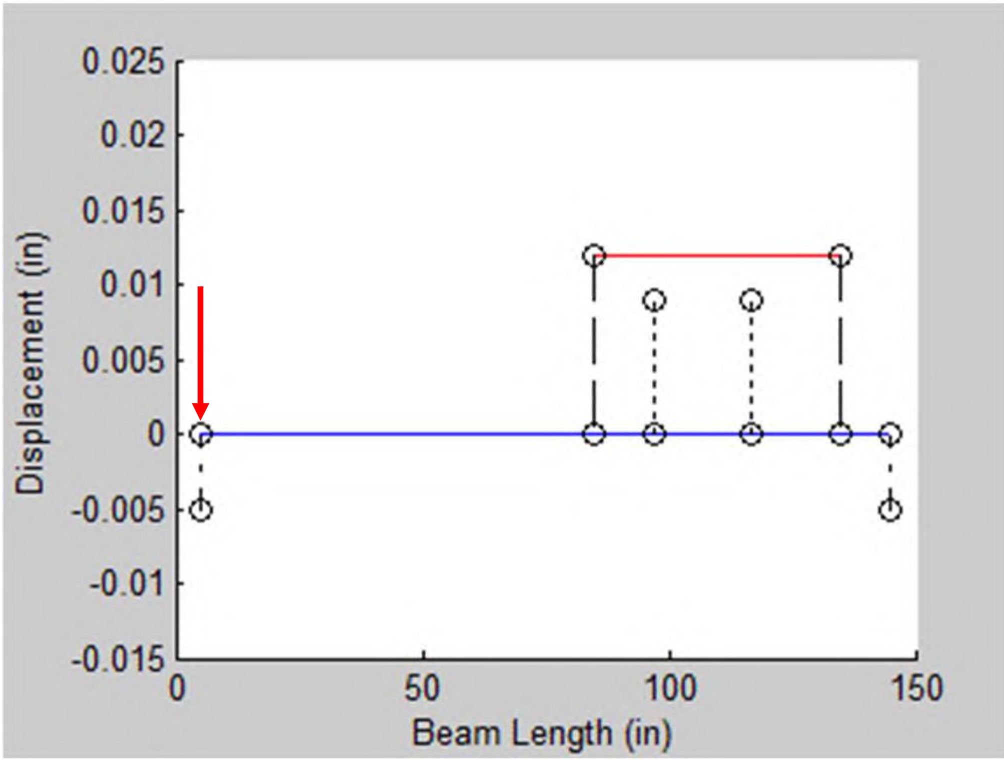

Expanded Full Field Nonlinear Response




**Expansion is Key Here**

# Typical Response Scenario

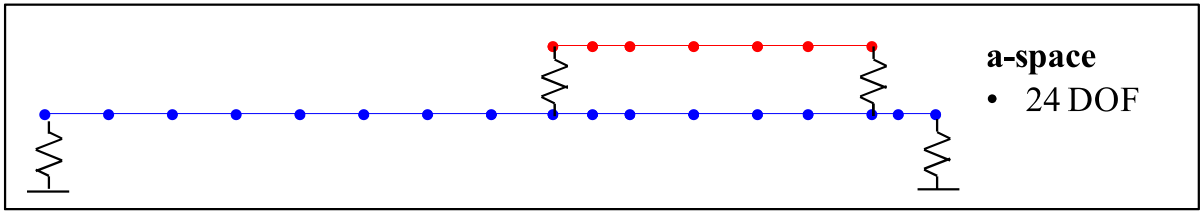



- Majority of response dominated by linear system response
- Points in time where discrete nonlinearities exist becomes an issue
- Large finite element models become computationally prohibitive to run more than a few scenarios

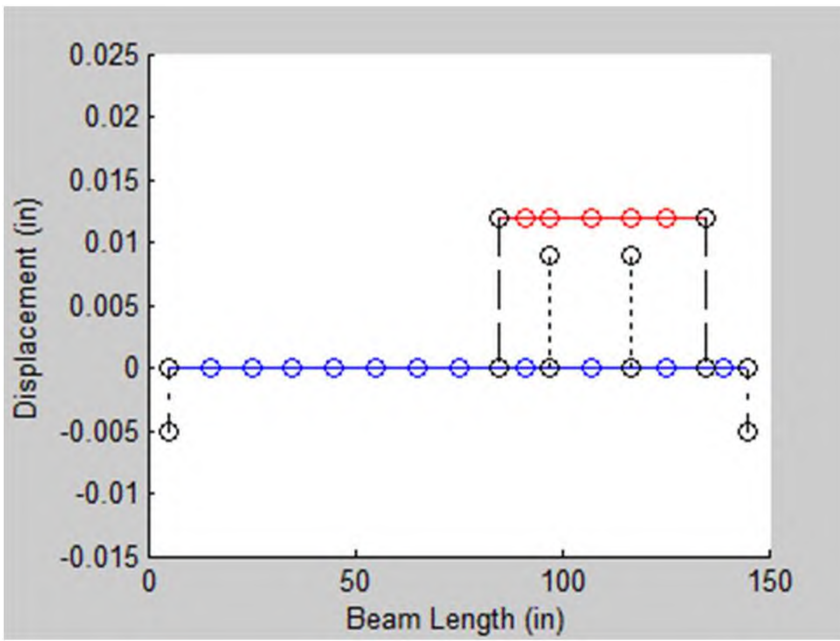
ANIMATION

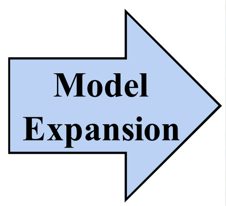
# *Case 2 – Model reduced to "a" space*



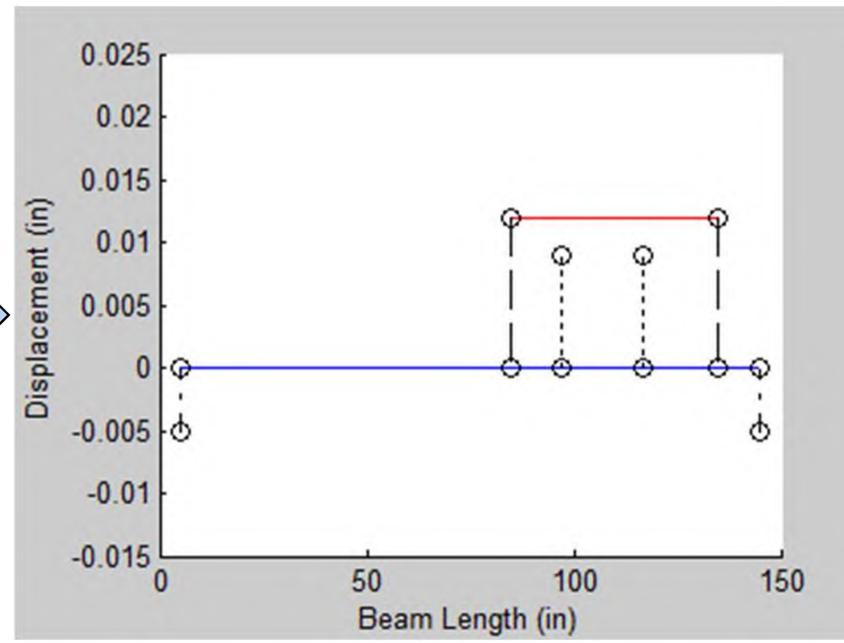

***Response calculated using ERMT with a-space model***

ERMT Response at a-space



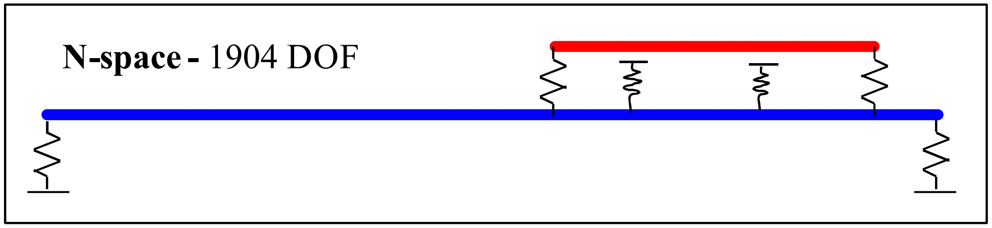

Expanded ERMT Response at N-space

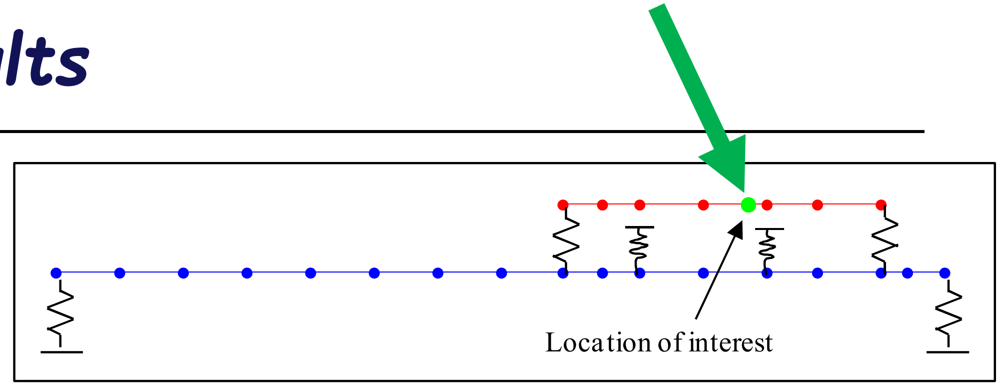


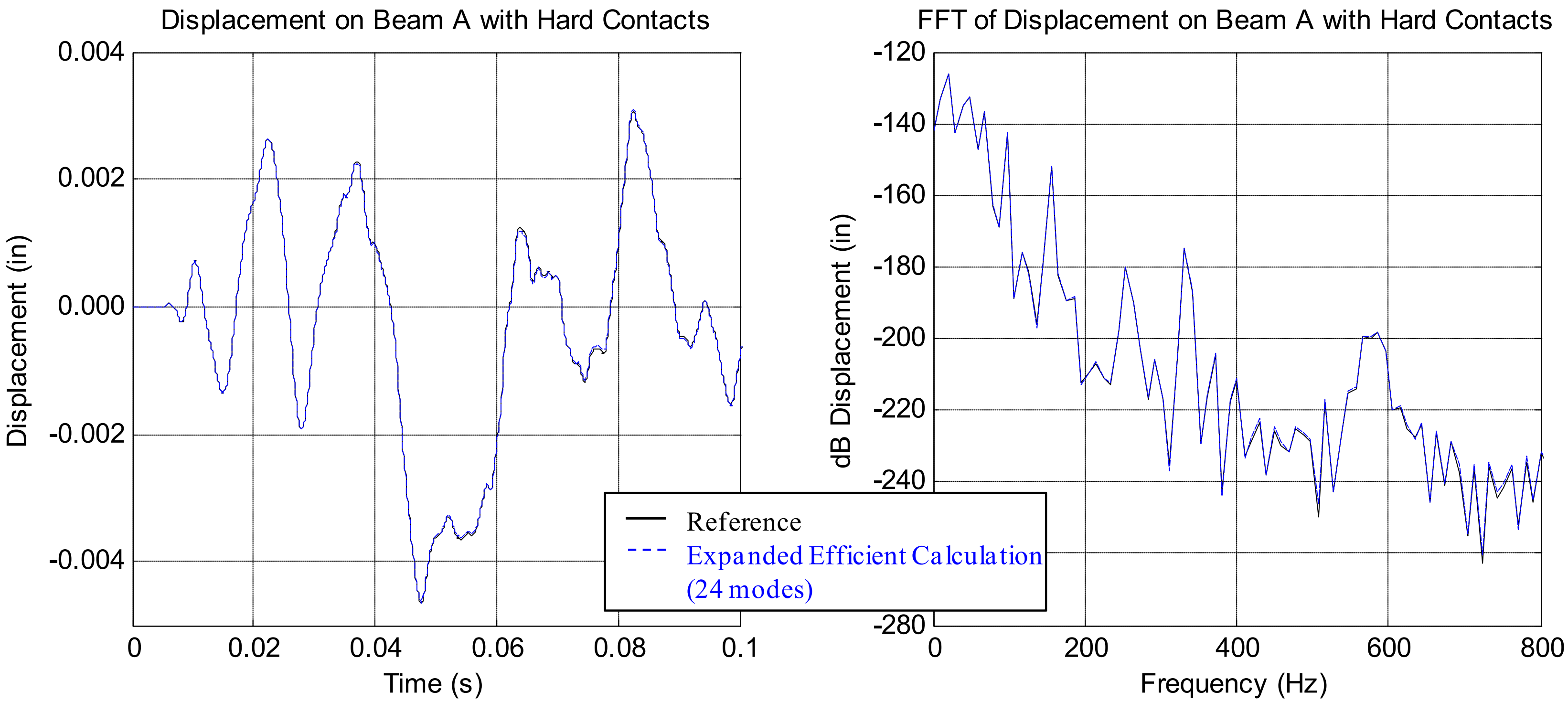

***Expansion is Key Here***

ANIMATION

# Case 2 Displacement Results

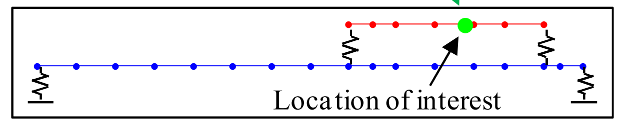


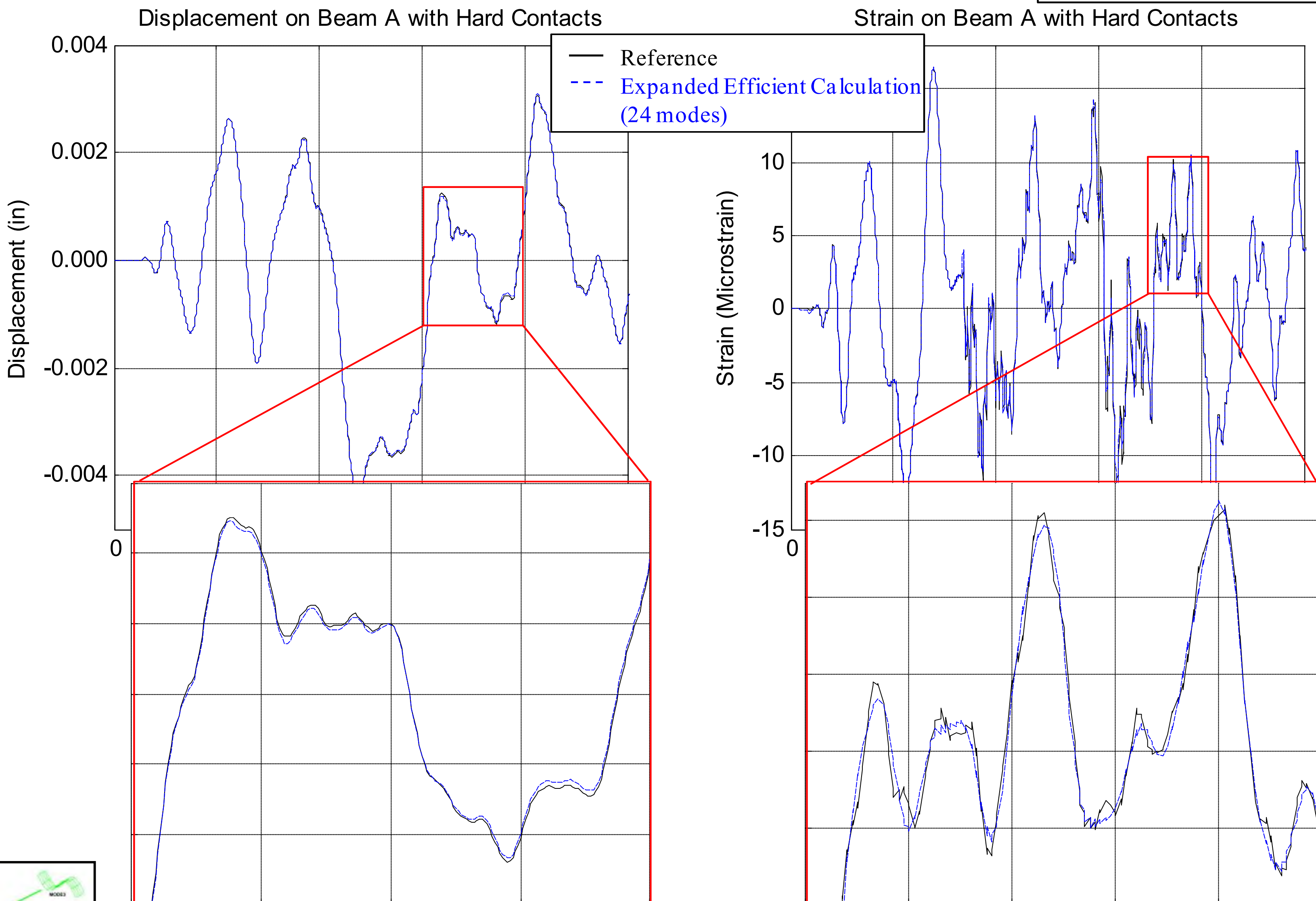



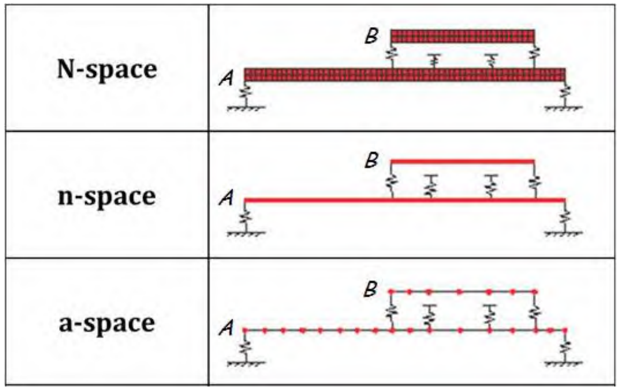

| Model | # of DOF | Solution Time (sec) | Average MAC | Average TRAC |
|---|---|---|---|---|
| Full Space | 1904 | 740.18 | 0.9998 | 0.9999 |
| Reduced | 24 | 0.28 | | |

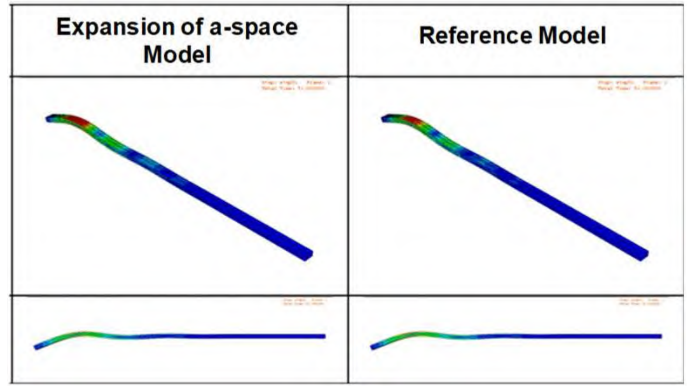
# Case 2 Strain Results

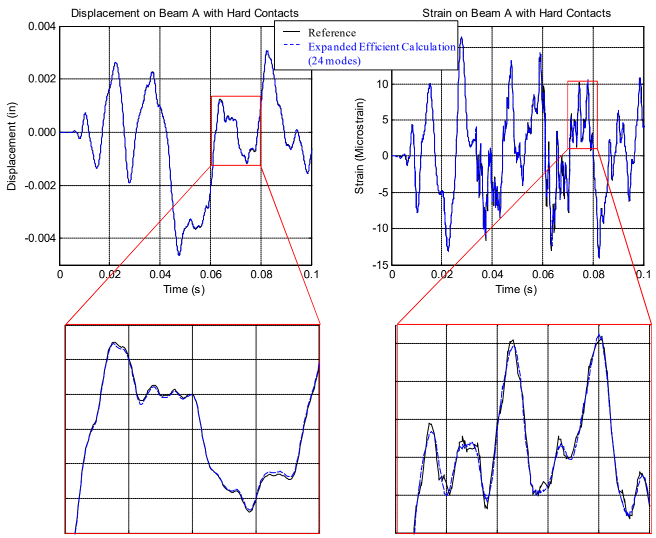


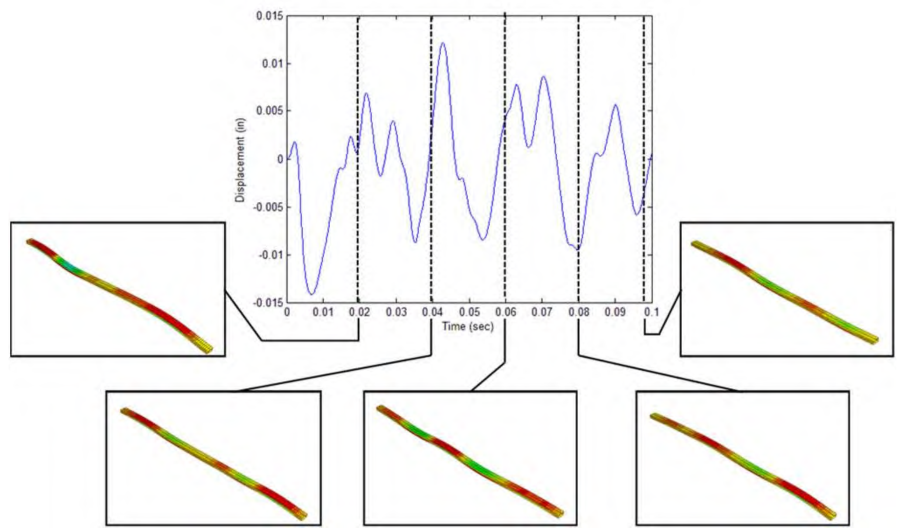



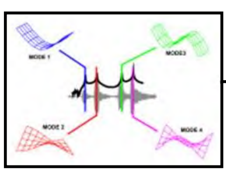
# Nonlinear Response – Expansion to Full 3D Models



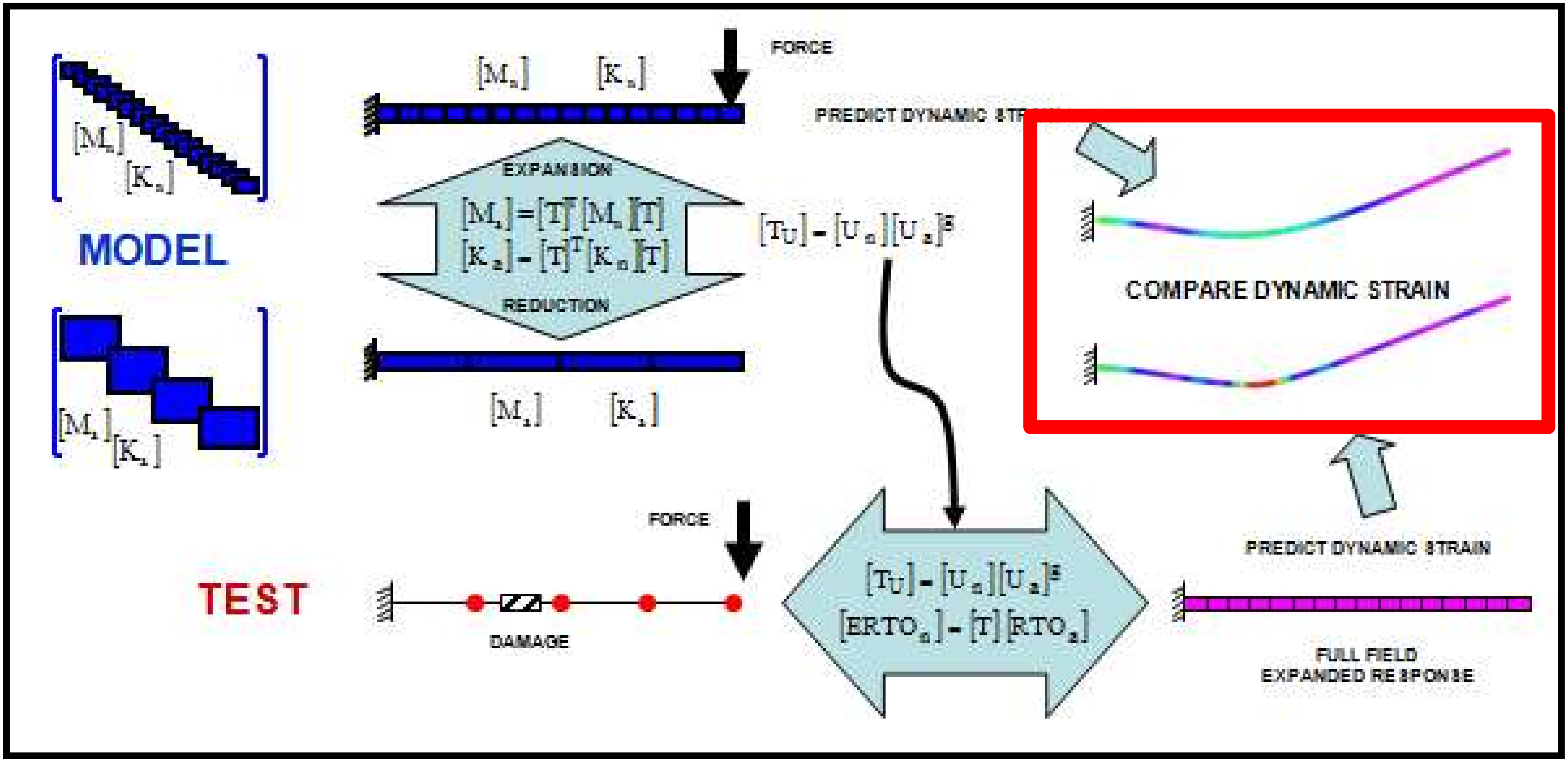



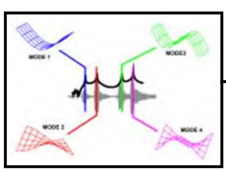
# Full Field Damage Detection

# Damage Location

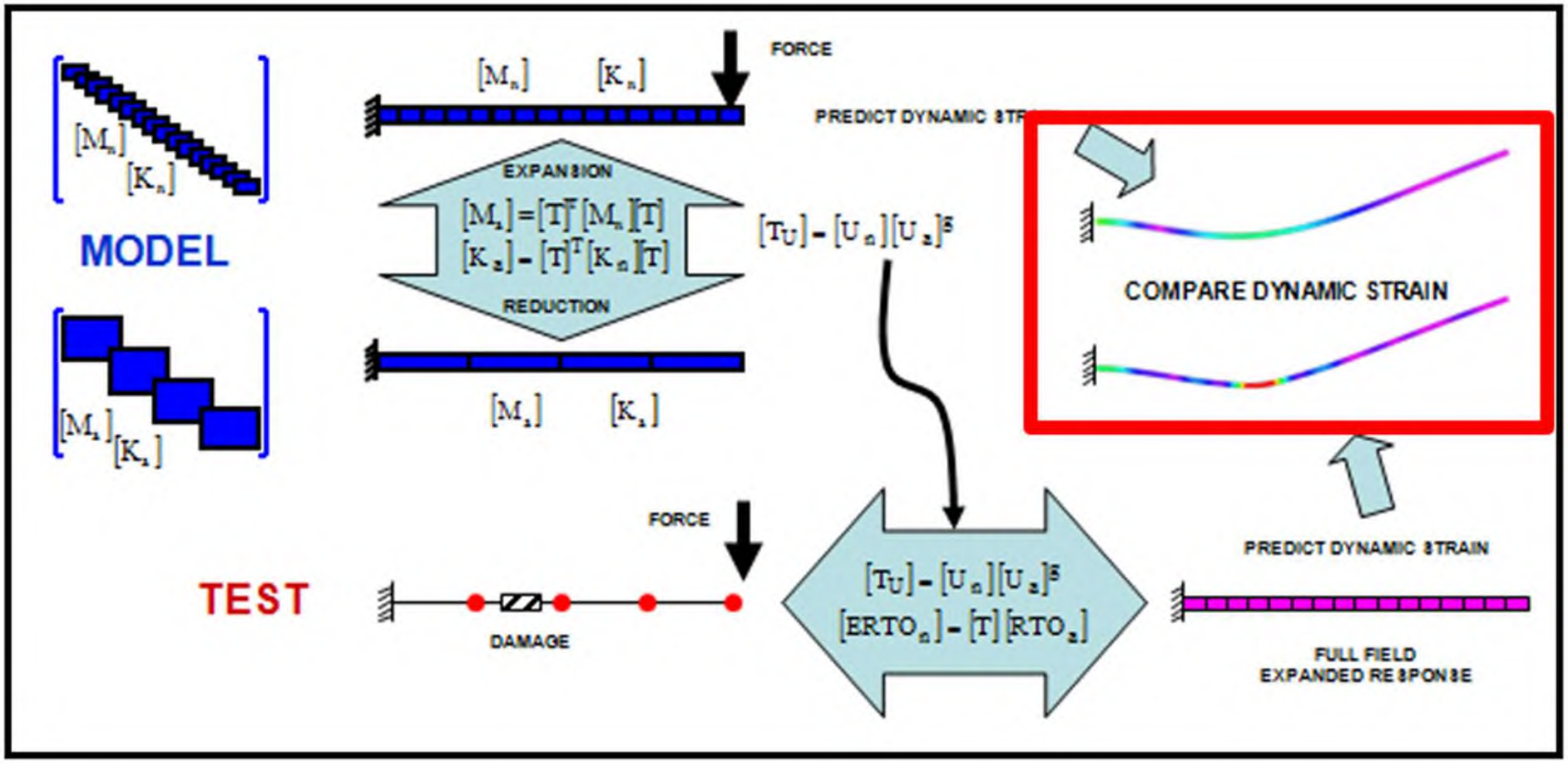
*Eric Harvey, Justin Rudduck, Brett Daniels, Peter Avitabile*

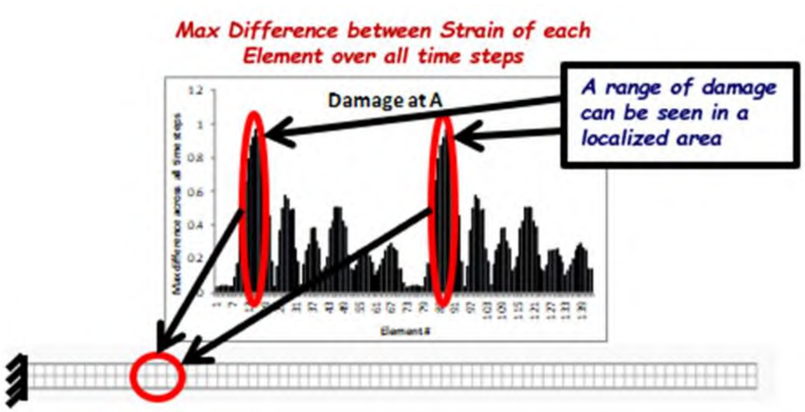


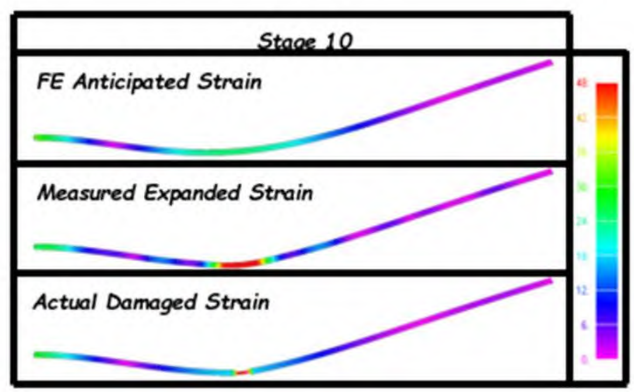

**Expansion is Key Here**

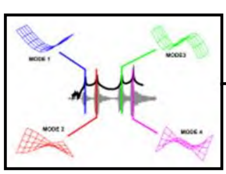


# Current Work – Damage Detection - Overview



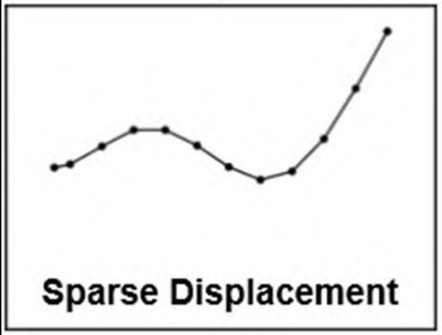



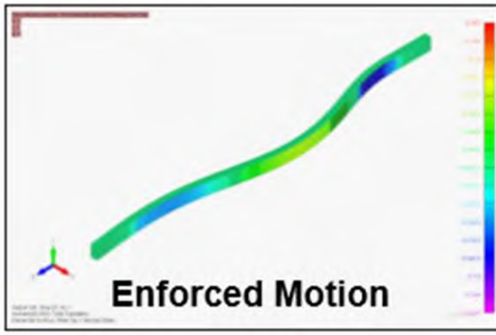
# Displacement and Strain Approaches

## Displacement Expansion Enforced Motion

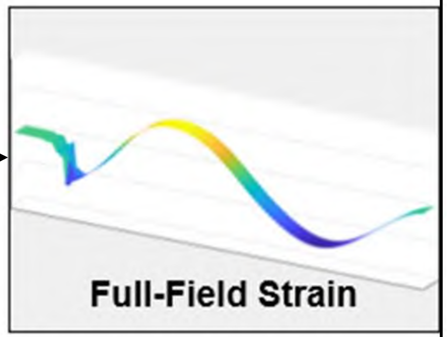
**Expand to full-field displacement**

$$[x_n]=[T_u][x_a]$$

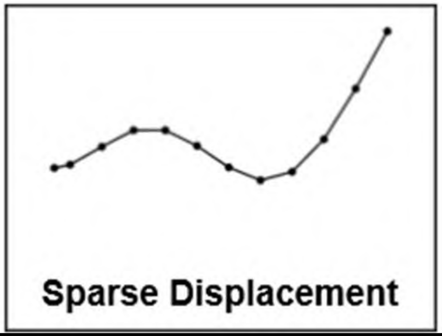



**Extract full-field strain**



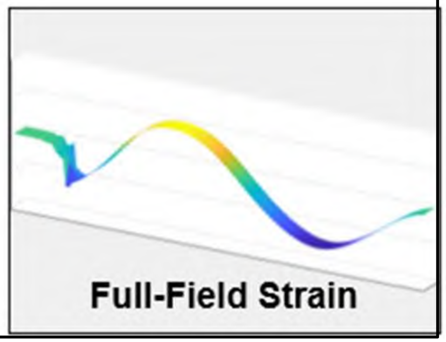

## Direct Strain Expansion

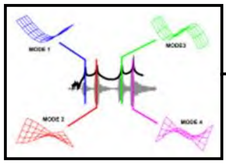



**Direct Expansion**

$$[\varepsilon_n]=[S_n][U_a]^g[x_a]$$

**from displacement to strain**

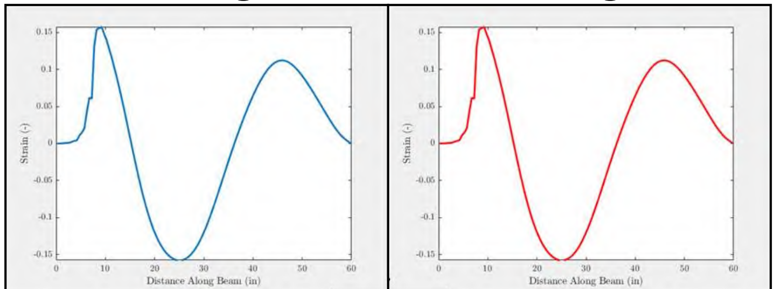


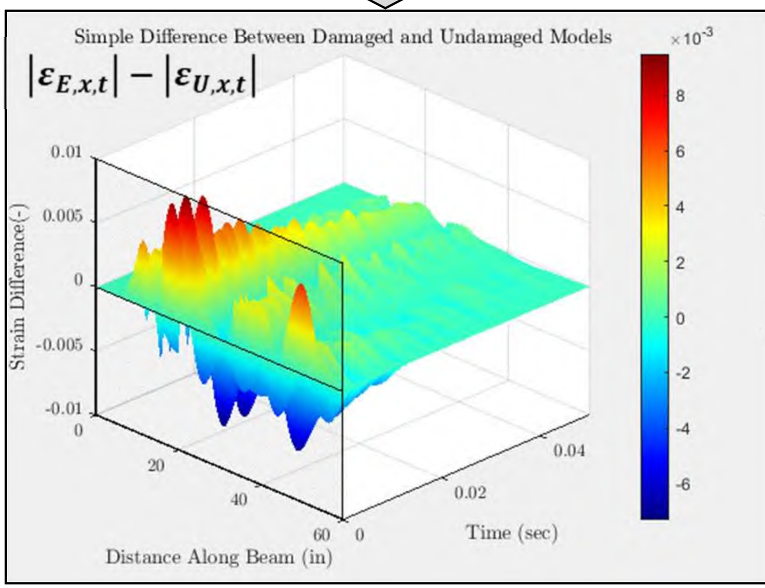



# Damage Identification Index

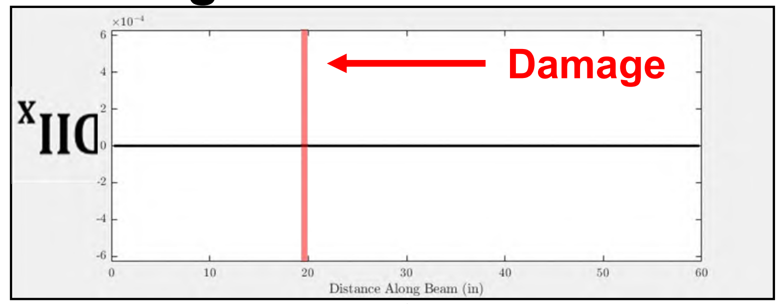
Lit Review | **Approach** | Work Completed | Planned Work

## Undamaged / Damaged

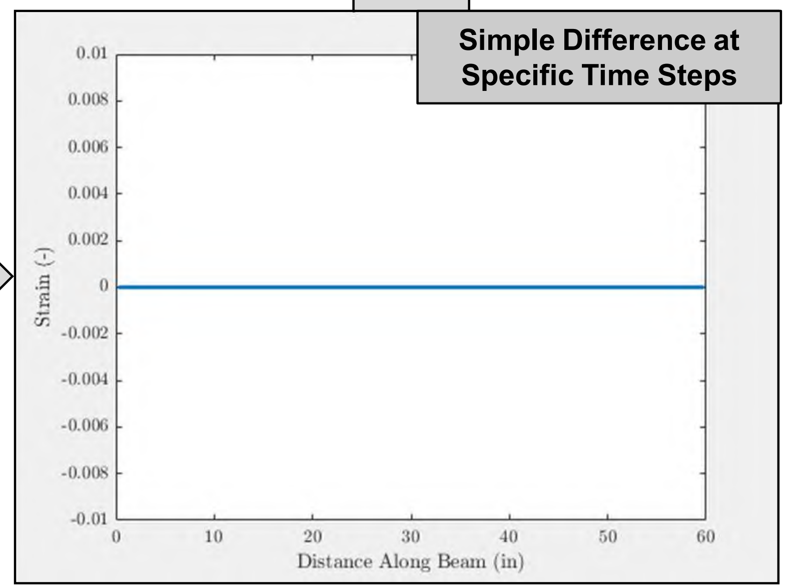


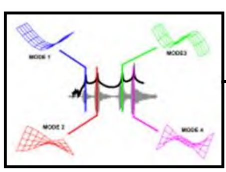




## Damage Identification Index



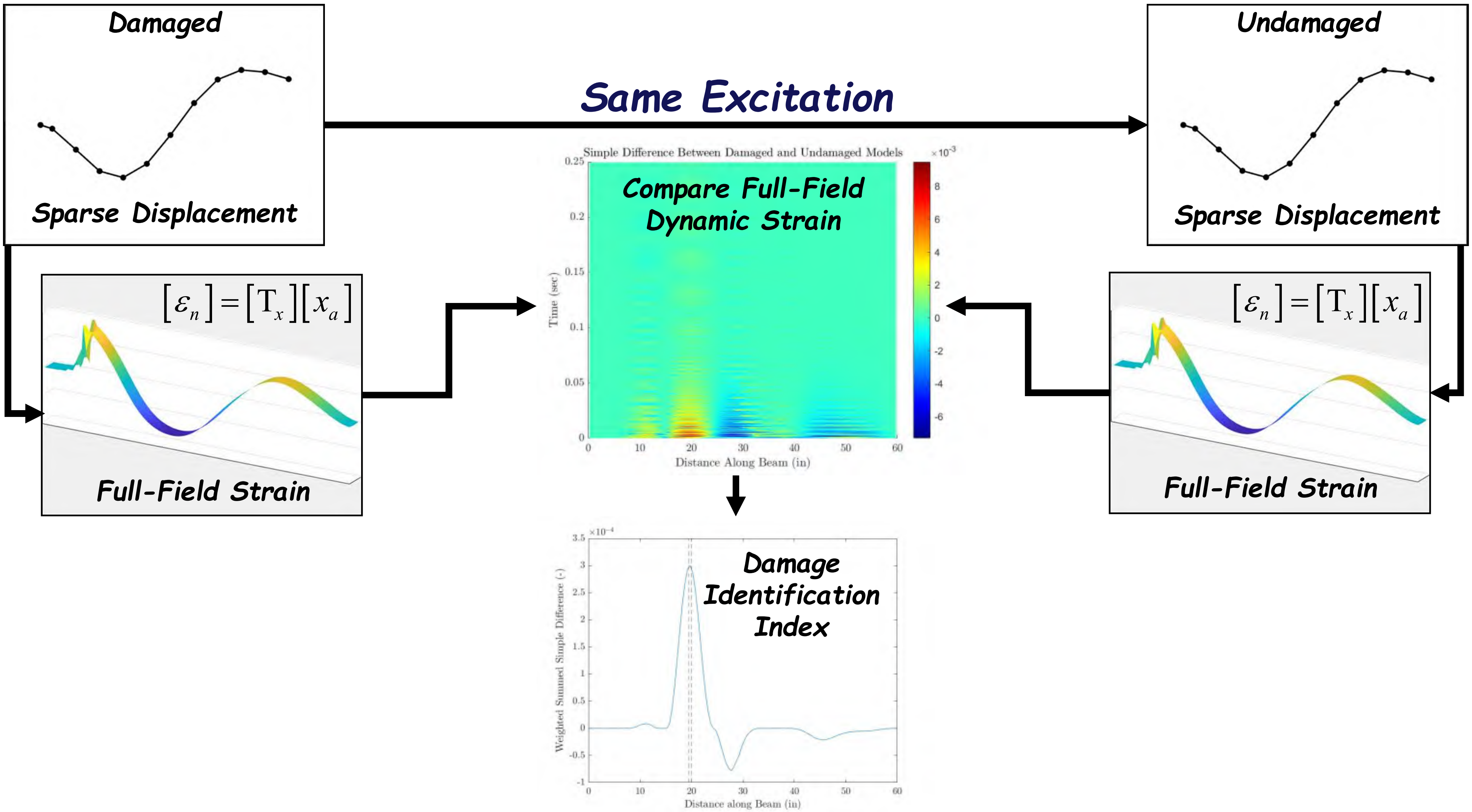

**Simple Difference at Specific Time Steps**



ANIMATION

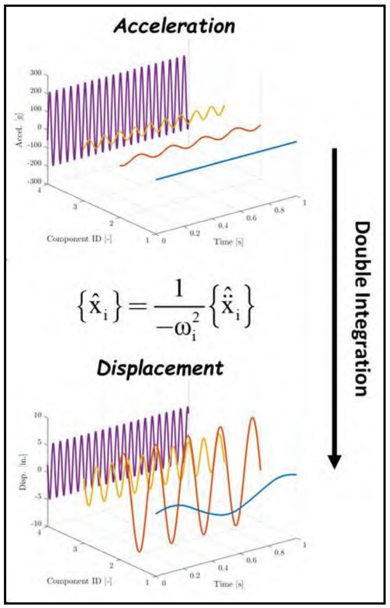
# Damage Identification Index



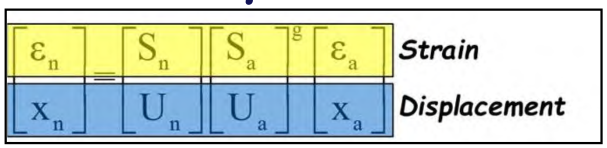
# Summary of Approaches

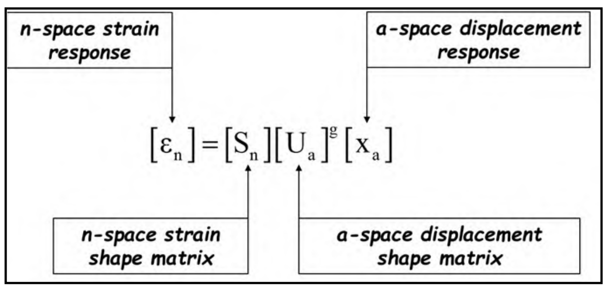
## But how does everything fit together?

### Spectral Integration



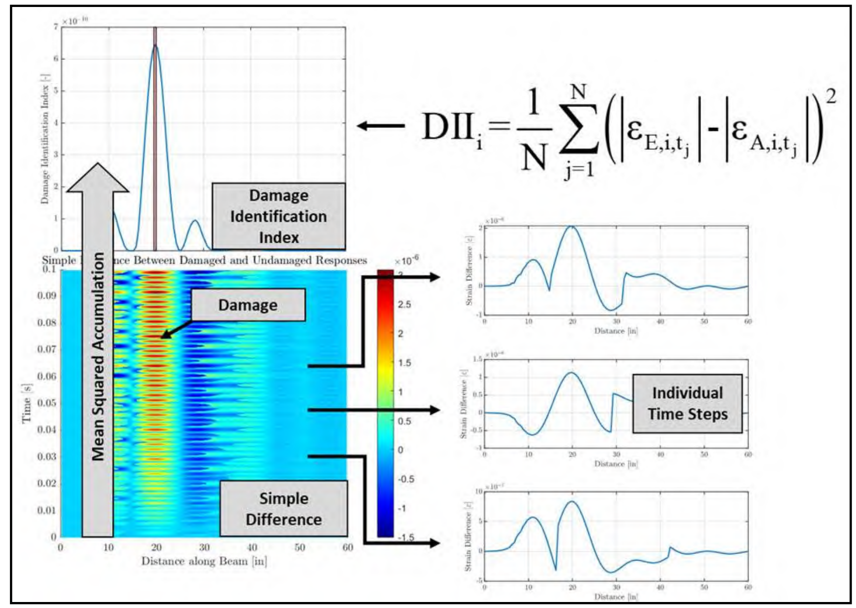

### Mixed-domain Expansion




### Displacement-Strain Expansion

### Damage Identification Index



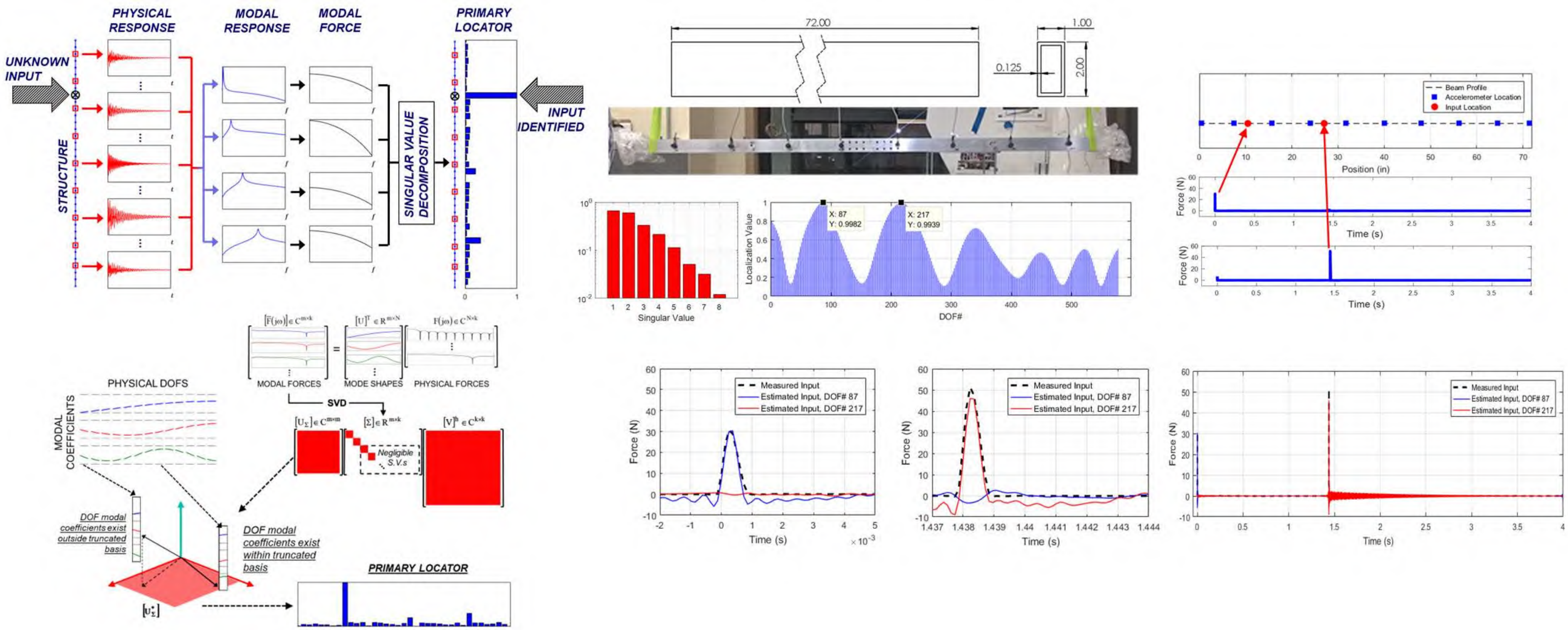
# Force Reconstruction (beyond measured points)

*"Reconstruction of Nonlinear Contact Forces Beyond Limited Measurement Locations Using an SVD Modal Filtering Approach", P.Logan, P.Avitabile, J.Dodson, Experimental Techniques (2020) accepted EXTE-D-19-00148R1, March 2020*

*"Reconstruction of External Forces Beyond Measured Points Using a Modal Filtering Decomposition Approach", P Logan, P Avitabile, J Dodson, Experimental Techniques (2019), accepted EXTE-D-18-00246R3*

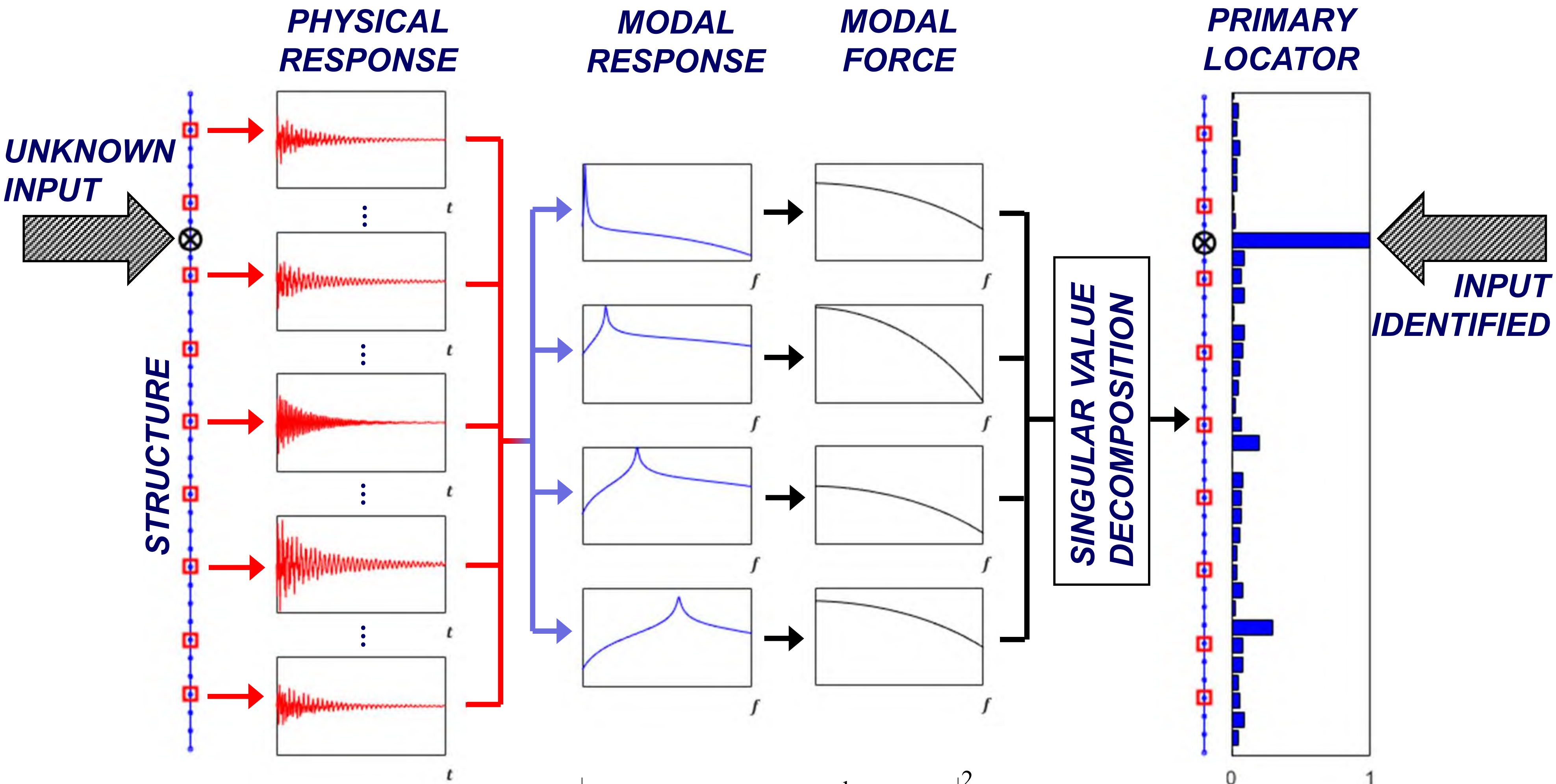
***Seriously Relies on Expansion***

# Force Reconstruction Beyond Measured Points

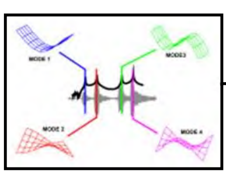



*Patrick Logan, Peter Avitabile*

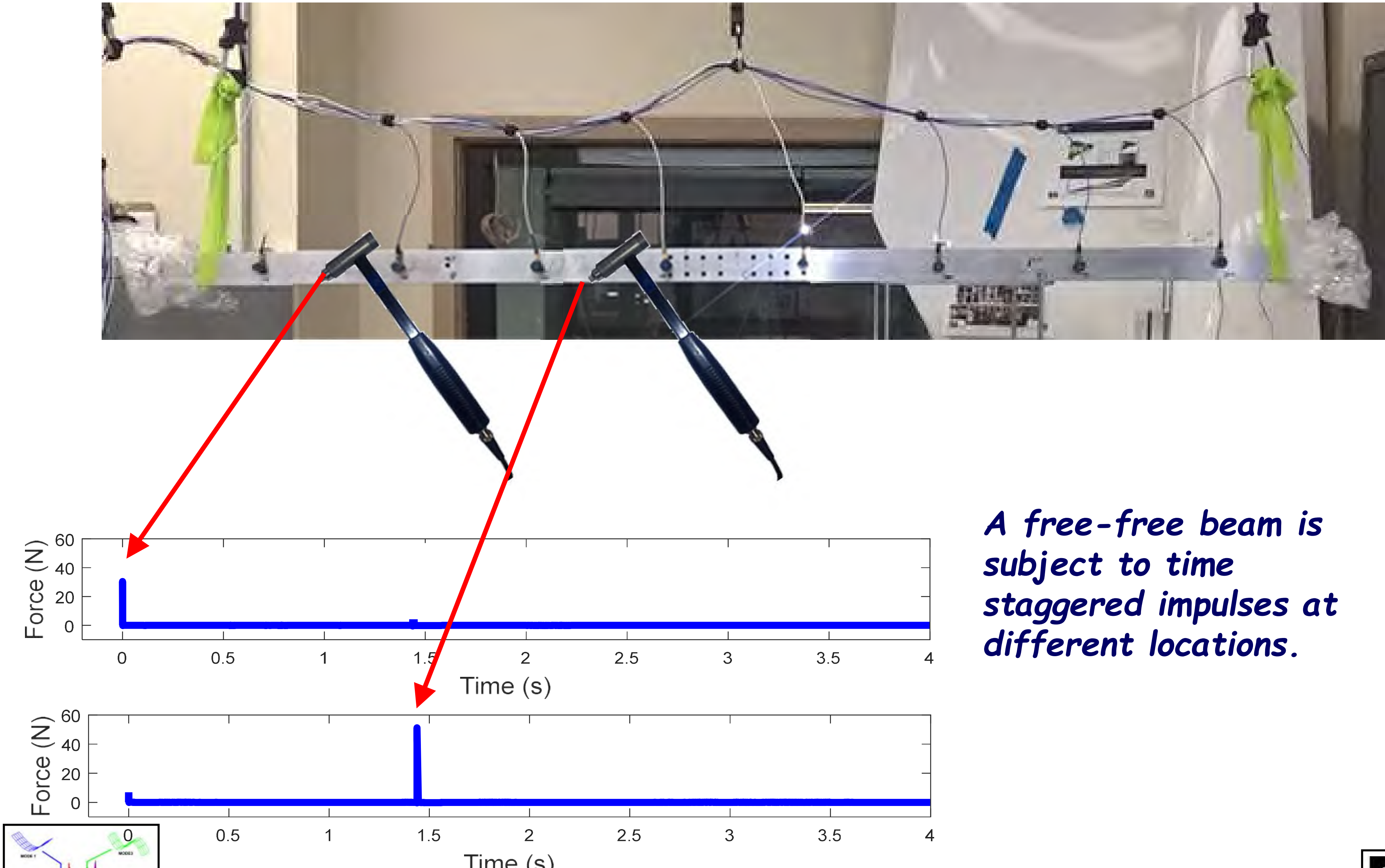
# Source Identification from Modal Forces



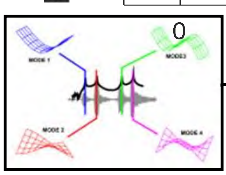

$$PL_i = \frac{\left| \left( [U_\Sigma^*][U_\Sigma^*]^\dagger [U_{i,m}]^T \right)^h [U_{i,m}]^T \right|^2}{\left( \left( [U_\Sigma^*][U_\Sigma^*]^\dagger [U_{i,m}]^T \right)^h [U_\Sigma^*][U_\Sigma^*]^\dagger [U_{i,m}]^T \right) \left( [U_{i,m}][U_{i,m}]^T \right)}$$



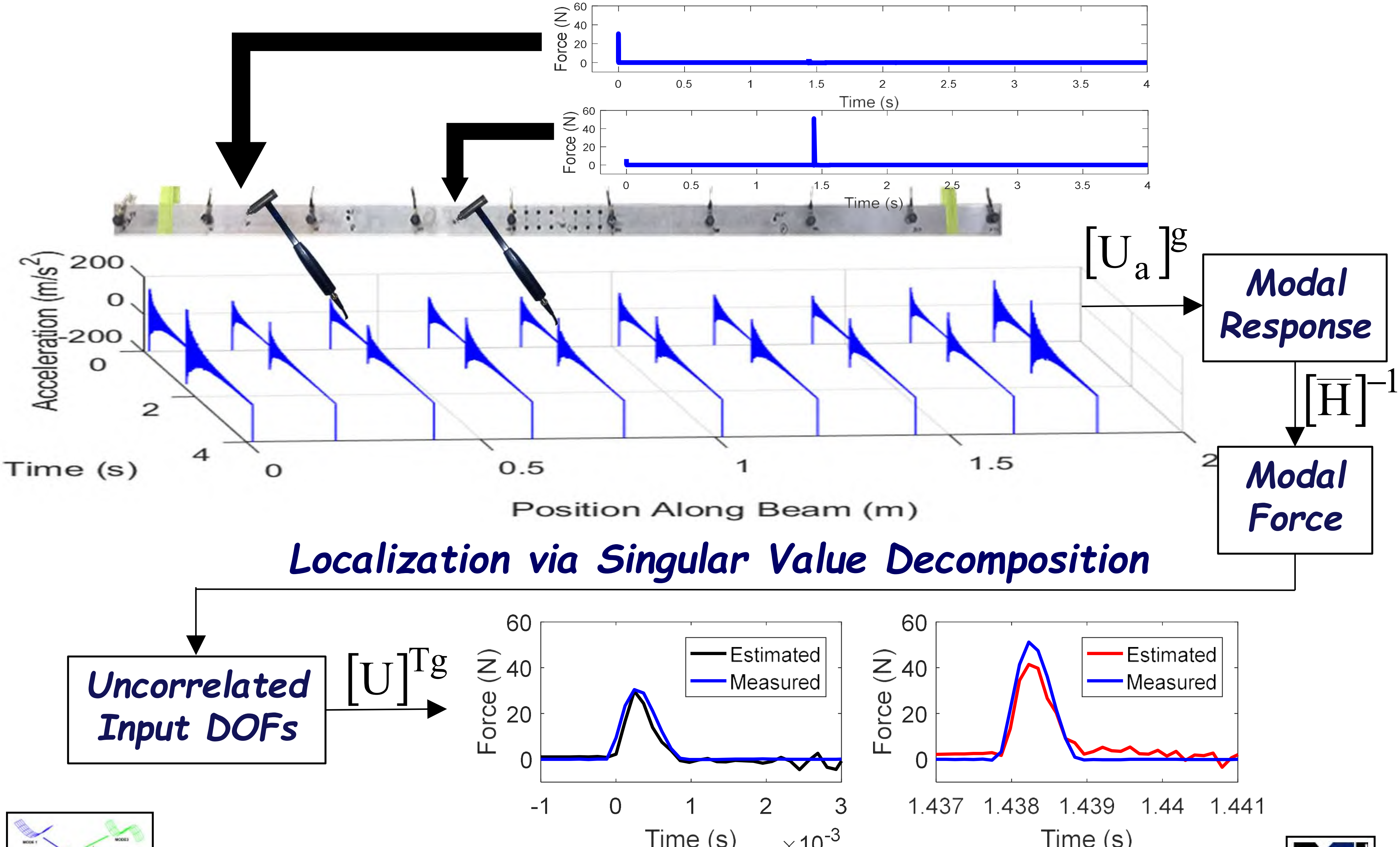
# Input Estimation with a Free-Free Beam

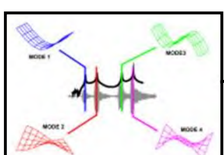



***A free-free beam is subject to time staggered impulses at different locations.***

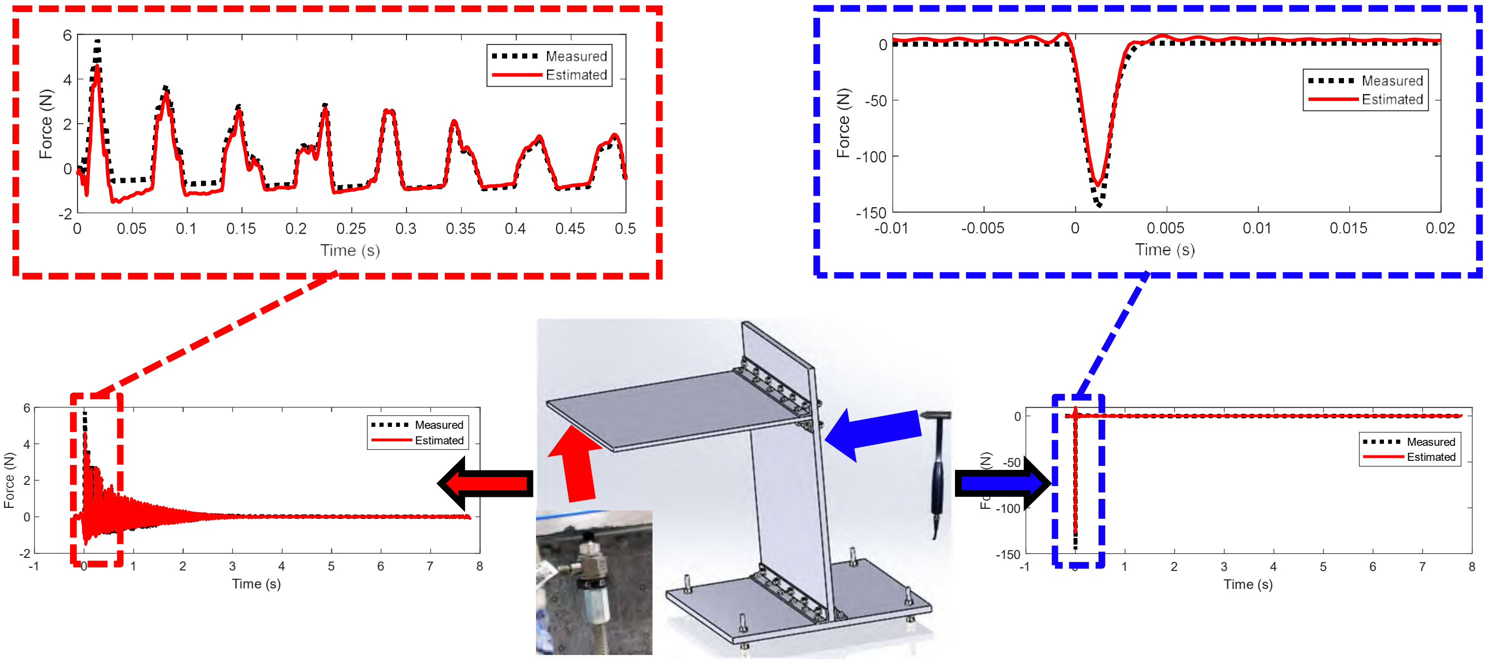


# Application of SVD Localization to a Free-Free Beam



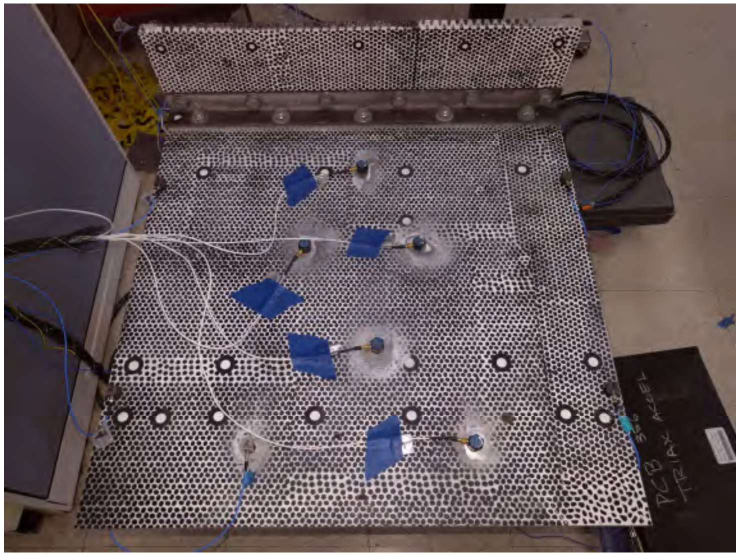



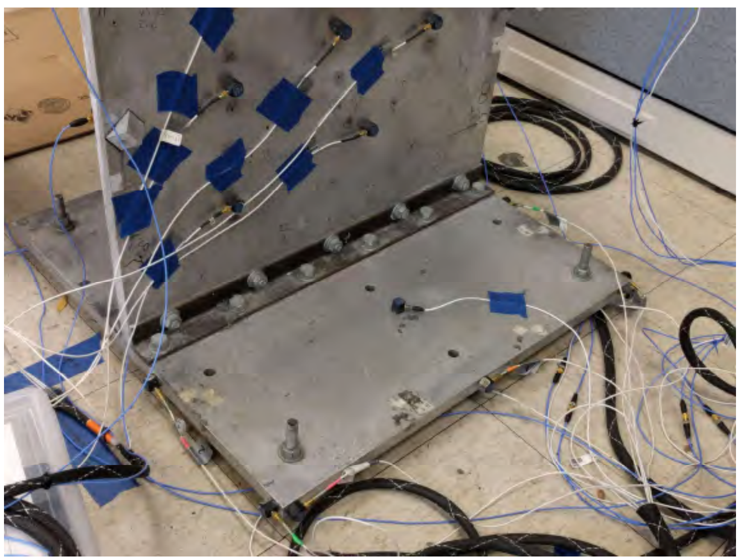
# Validation of Force Reconstruction for Linear And Nonlinear System Response

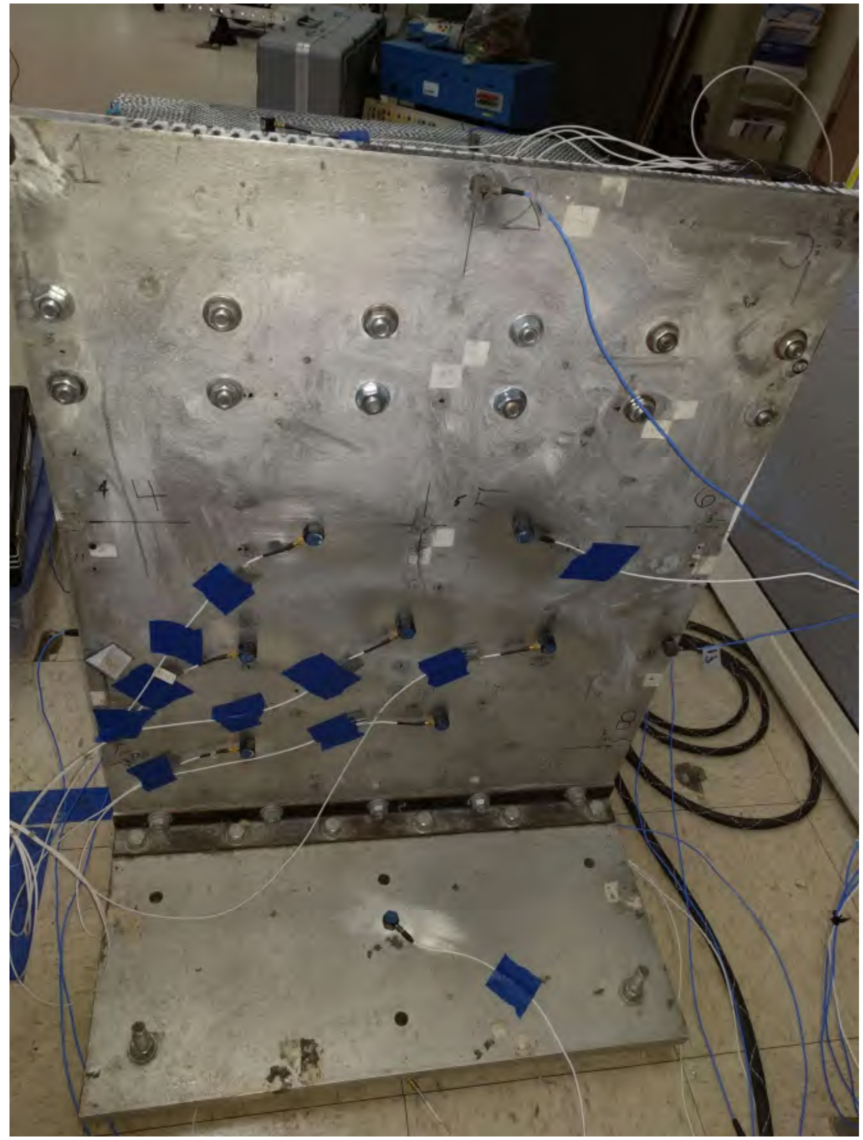



Patrick Logan, Deborah Fowler, Peter Avitabile

# Experimental Characterization of Contact Force

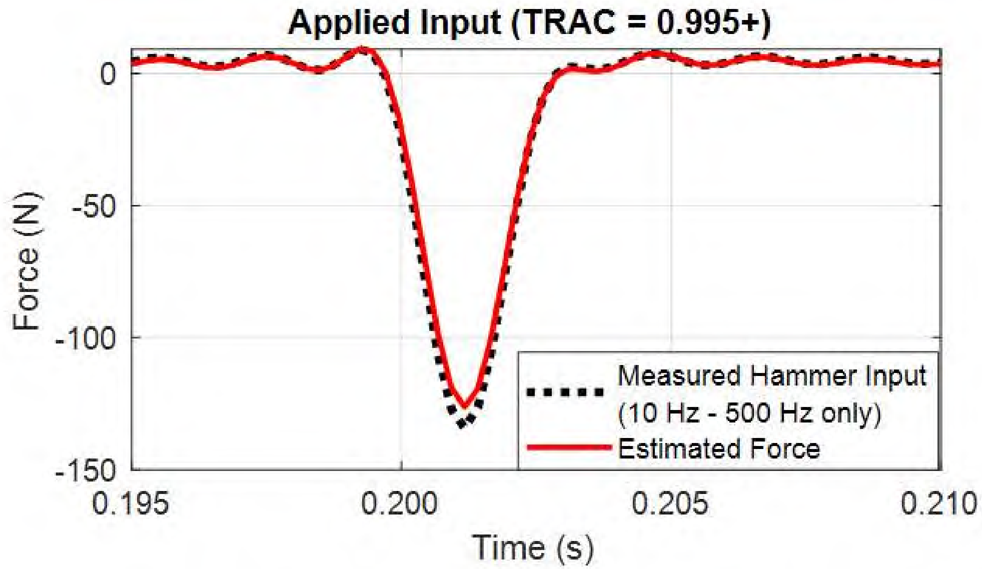


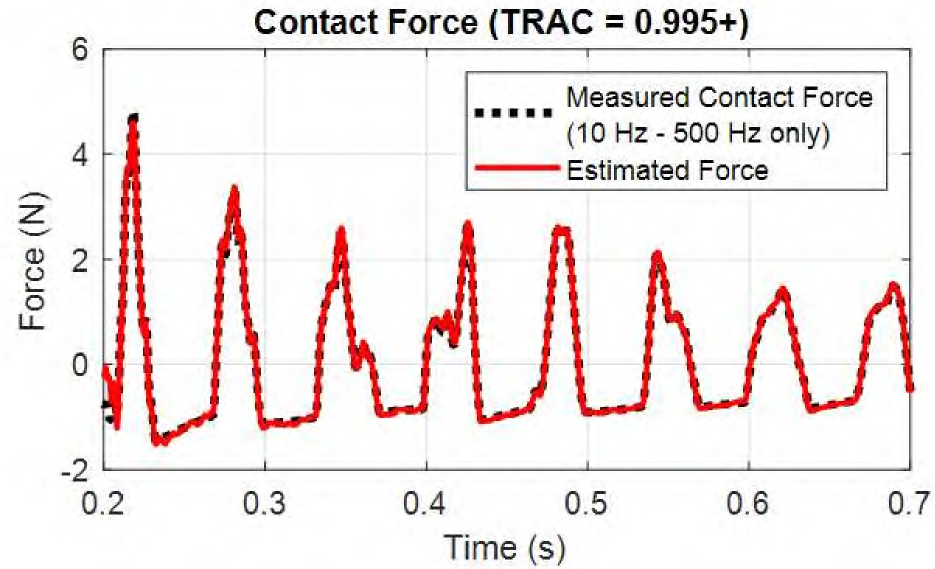


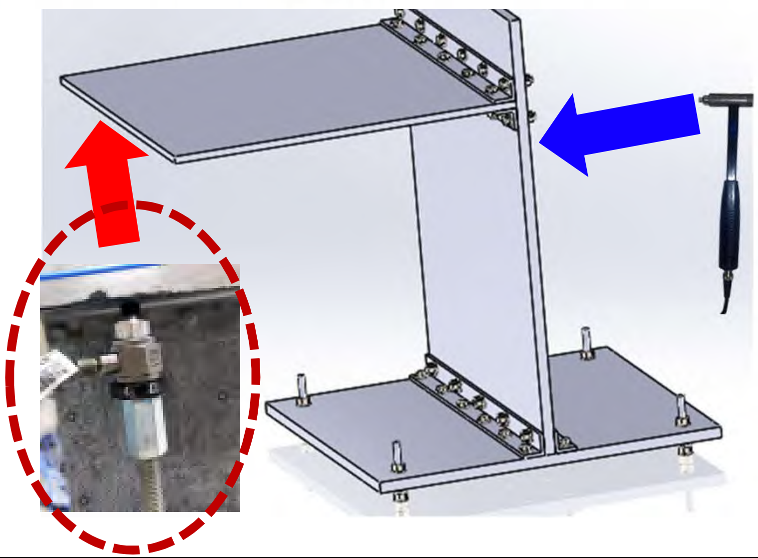


# Experimental Characterization of Contact Force

*Localization within 1% (0.35 in)*



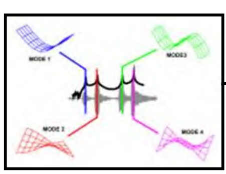
*TRAC = 93%*

*Measured Max: 5.7 N*

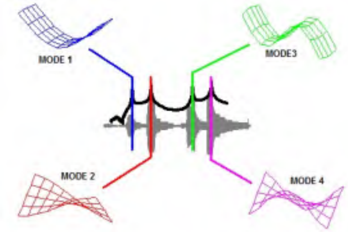
*Estimated Max: 4.6 N*


# SPECIAL THANKS TO ALL THE STUDENTS

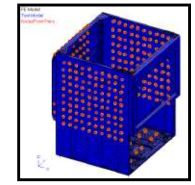
# Special Thanks to the Students of MACL/SDASL



# *Dr. Peter Avitabile – Snapshot*




*Professor, Mechanical Engineering, Co-Director, Structural Dynamics and Acoustic Systems Laboratory*
*Vice President 2014 & President 2015/2016 for Society for Experimental Mechanics*
*Associate Editor – Handbook of Experimental Structural Mechanics*

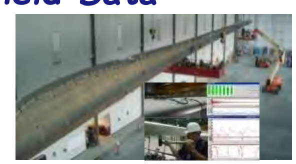


*Pete has 50 years of experience in design & analysis using FEM and experimental techniques. His main area of research is structural dynamics specializing in the areas of modeling, testing and correlation of analytical and experimental models along with advanced structural dynamic applications.*

- *Grants & Contracts: Numerous grants and contracts supporting the research in the Structural Dynamics and Acoustic Systems Laboratory including NSF Collaborative Research, Eglin AFB, DOT, DOE, ONR NUSC, Army Research Office, Pratt & Whitney, Bosch, DRS, MIT Lincoln Labs*
- *Publications: Journals since 2008 include 16 papers (3 under review and 2 in prep); conference papers published have been over 60 papers since 2008; over 40 were student presentations.*

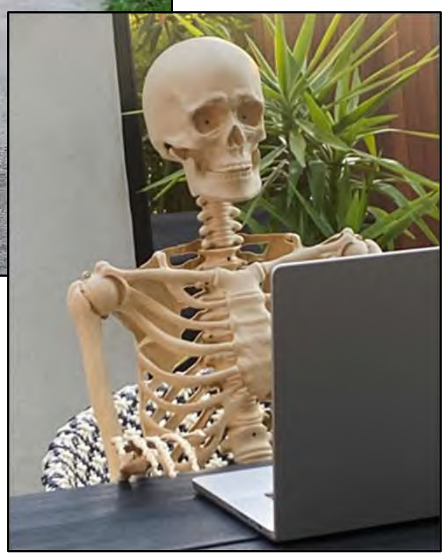
- *UML Courses taught include a wide variety of courses: Mechanical Laboratory, Structural Dynamics, Dynamic Systems, Modal Analysis, Vibrations, Experimental Modal Analysis, Numerical Methods*
- *Actively involved in the Los Alamos Dynamics Summer School at the LANL since its inception in 2000.*
- *Provide a wide assortment of industry seminars for 30 years; extremely active in Society for Experimental Mechanics and the International Modal Analysis Conference for over 30 years; provided several Keynote Speeches for Conferences Worldwide*

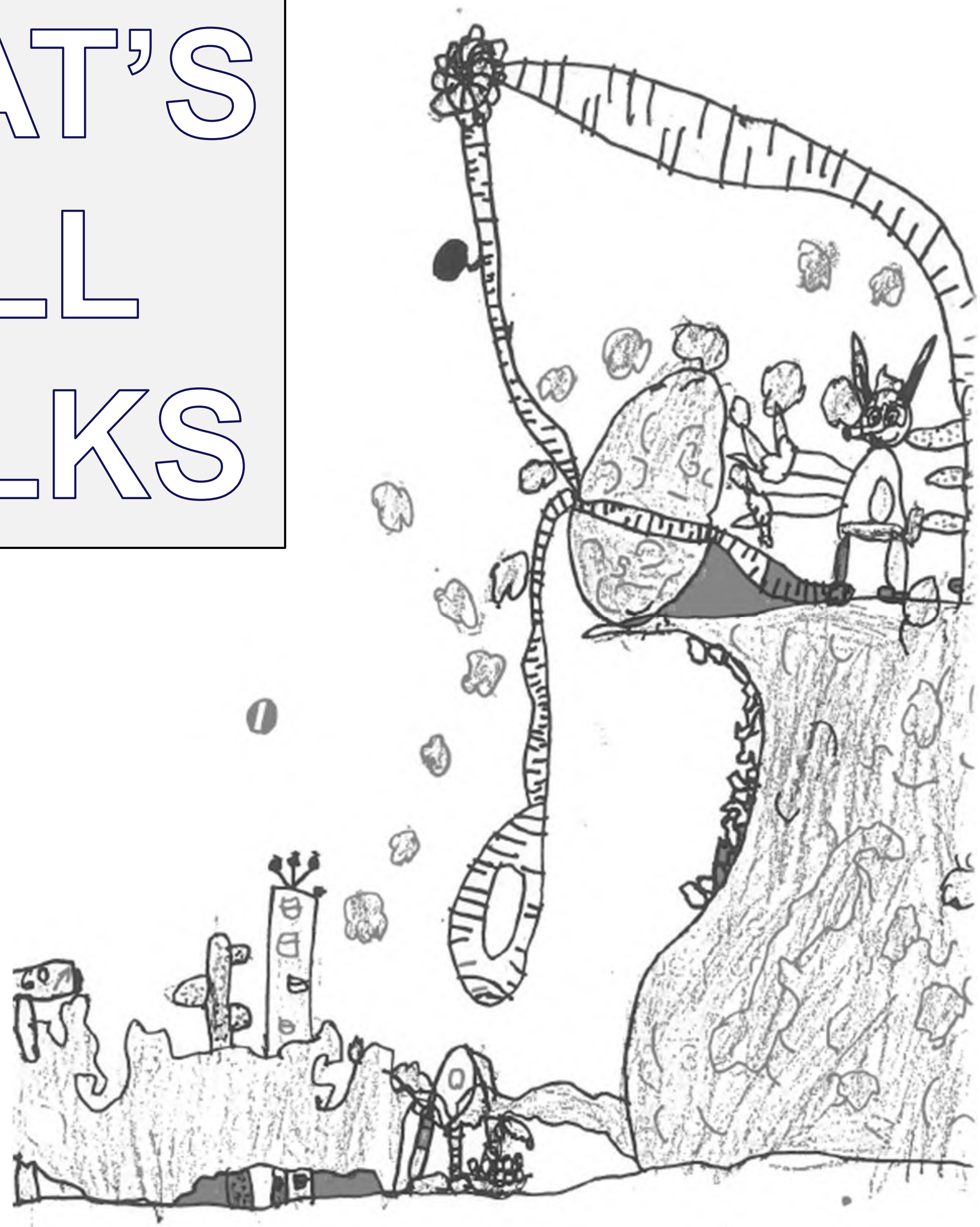
- *Development of Reduced Order, Test Verified Component and System Models for Improved/Efficient Structural Dynamic Characterization; Expansion of Real Time Operating Data for Full Field Data*
- *Integration and Full Utilization of Measured Data from Optically Based Measurement Systems for Finite Element Modeling Applications*



- *Development of Highly Reduced Order Components for Linear and Nonlinear System Model Development along with Expansion for Full Field Dynamic Displacement and Full Field Dynamic Strain*
- *Compression of Massive Optical Data Sets with Advanced Processing*

# THAT'S ALL FOLKS





## PhD Dissertations related to Model Reduction/Expansion, Nonlinear Response, Force Reconstruction, System Modeling

- **Fabio Piergentili**, *FRF Expansion for Unmeasured Translation and Rotational DOFs for Impedance Modeling Applications*
  PhD, University of Massachusetts Lowell, Lowell, MA 1999; Advisor: John O'Callahan/Peter Avitabile

- **Hiromichi Tsuji**, *Application of Optimization Procedures using Phantom Connectivity Technique*
  MS, University of Massachusetts Lowell, Lowell, MA 2002; Advisor: John O'Callahan

- **Pawan Pingle**, *Prediction of Full Field Dynamic Stress/Strain from Limited Set of Measured Data*
  PhD, University of Massachusetts Lowell, Lowell, MA 2010; Advisor: Peter Avitabile

- **Javad Baqersad**, *A Non-Contacting Approach for Full Field Dynamic Strain Monitoring of Rotating Structures using the Photogrammetry, Finite Element and Modal Expansion Techniques*
  PhD, University of Massachusetts Lowell, Lowell, MA 2015; Advisor: Chris Niezrecki/Peter Avitabile

- **Sergio Obando**, *Use of Expansion of Highly Reduced Order Models for the fir the Accurate Prediction of Full Field Dynamic Characteristics in the Forced Response of Linear and Nonlinear Systems and Components*
  PhD, University of Massachusetts Lowell, Lowell, MA 2017; Advisor: Peter Avitabile

- **Patrick Logan**, *Force Estimation – Beyond Measured Points*
  PhD, University of Massachusetts Lowell, Lowell, MA 2020; Advisor: Peter Avitabile

- **Debby Fowler**, *On the Use of Linear Dynamic Models with Limited Measured Data to Predict Nonlinear Response*
  PhD, University of Massachusetts Lowell, Lowell, MA 2021; Advisor: Peter Avitabile

- **Brett Daniels**, *Strain Deformation Control of DUT in Consideration of Field to Laboratory Inconsistencies*,
  PhD, University of Massachusetts Lowell, Lowell, MA (in progress); Advisor: Peter Avitabile & Alessandro Sabato

- **John Seymour**, *Development of a Blended Modal/Impedance System Modeling Approach for Dynamic Qualification*,
  PhD, University of Massachusetts Lowell, Lowell, MA 2024; Advisor: Peter Avitabile

## MS Theses related to Model Reduction/Expansion, Nonlinear Response, Force Reconstruction, System Modeling

- **Tracy Van Zandt**, *Development of Reduced Order Model Updating Strategies for Wing/Airframe Multibody Models*
  MS, University of Massachusetts Lowell, Lowell, MA 2006; Advisor: Peter Avitabile

- **Nels Wirkkala**, *Impedance Based System Model Development for Target Mode Identification Reduced Model Updating*
  MS, University of Massachusetts Lowell, Lowell, MA 2007; Advisor: Peter Avitabile

- **Adam Butland**, *A Hybrid Component Mode Synthesis Approach for System Modeling Applications*
  MS, University of Massachusetts Lowell, Lowell, MA 2008; Advisor: Peter Avitabile

- **Dana Nicgorski**, *Investigation on Experimental Issues for Frequency Based Substructuring*
  MS, University of Massachusetts Lowell, Lowell, MA 2008; Advisor: Peter Avitabile

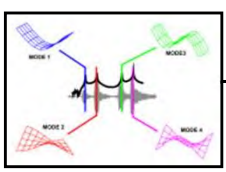
- **Chris Chipman**, *Expansion for Real Time Operating Data*
  MS, University of Massachusetts Lowell, Lowell, MA 2009; Advisor: Peter Avitabile

- **Lou Thibault**, *Development of Equivalent Reduced Order Model Technique for Linear Modal Models Interconnected with Nonlinear Connection Elements*
  MS, University of Massachusetts Lowell, Lowell, MA 2012; Advisor: Peter Avitabile

- **Tim Marinone**, *Efficient Computational Nonlinear Dynamic Response using Modal Modification Response Technique*
  MS, University of Massachusetts Lowell, Lowell, MA 2012; Advisor: Peter Avitabile

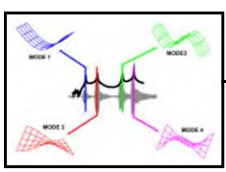
- **Julie Harvie**, *Computationally Efficient Response for Full Field Dynamic Strain Predictions*
  MS, University of Massachusetts Lowell, Lowell, MA 2013; Advisor: Peter Avitabile

- **Tyler Doveno**, *Strain Shape Expansion with Motion Magnification*
  MS, University of Massachusetts Lowell, Lowell, MA 2019; Advisor: Peter Avitabile



# PhD Thesis related to FINE, Full Field Expansion, Strain Expansion, Optical

- **Jesus ReyesBlanco**, *Adjustment of Input Excitation to Account for Fixture-Test Article Dynamic Coupling Effects*, PhD, University of Massachusetts Lowell, Lowell, MA 2017; Advisor: Peter Avitabile
- **Tyler Doveno**, *Strain Shape Expansion with Motion Magnification*, MS, University of Massachusetts Lowell, Lowell, MA 2019; Advisor: Peter Avitabile & Zhu Mao
- **Brandon Zwink**, *Dynamic Response Matching from Field to Laboratory Replication Methodology*, PhD, University of Massachusetts Lowell, Lowell, MA 2020; Advisor: Peter Avitabile
- **Ryan Schultz**, *Improving Efficiency of Multi-Shaker and Combined Shaker-Acoustic Vibration Tests*, PhD, University of Massachusetts Lowell, Lowell, MA 2020; Advisor: Peter Avitabile
- **Christopher Page**, *A Unified Substructuring Approach to Fixture Neutralization*, PhD, University of Massachusetts Lowell, Lowell, MA 2021; Advisor: Peter Avitabile
- **Daniel Rohe**, *Image Processing Techniques for Structural Dynamic Tests using Radiographic Images*, PhD, University of Massachusetts Lowell, Lowell, MA 2022; Advisor: Peter Avitabile & Zhu Mao
- **Brett Daniels**, *Strain Deformation Control of DUT in Consideration of Field to Laboratory Inconsistencies*, PhD, University of Massachusetts Lowell, Lowell, MA (in progress); Advisor: Peter Avitabile & Alessandro Sabato
- **John Seymour**, *Development of a Blended Modal/Impedance System Modeling Approach for Dynamic Qualification*, PhD, University of Massachusetts Lowell, Lowell, MA 2024; Advisor: Peter Avitabile
- **Troy Skousen**, *Impedance and Modal Based FINE Approaches Addressing Vibration Qualification Testing*, PhD, University of Massachusetts Lowell, Lowell, MA (in progress); Advisor: Peter Avitabile & Alessandro Sabato

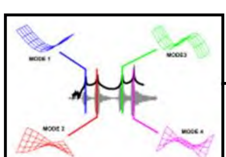


## *Papers related to FINE, Full Field Expansion, Strain Expansion, Optical*


- **"Direct Expansion from Displacement to Strain Using Equivalent Transformation of Modal Coordinates", B.Daniels, A.Sabato, R.Joshua, P.Avitabile, *Proceedings of the Forty-First International Modal Analysis Conference*, Austin, Texas, Feb 2023**
- **"Strain-based Damage Detection Using Cross-domain Hybrid Expansion Method", B.Daniels, A.Sabato, R.Joshua, P.Avitabile, *Proceedings of the Forty-First International Modal Analysis Conference*, Austin, TX, Feb 2023**
- **"System Modeling Through the Coupling of Impedance and Modal Based Models", J.Seymour, R.Joshua, P.Avitabile, *Proceedings of the Forty-First International Modal Analysis Conference*, Austin, TX, Feb 2023**
- **"A New Residual Compensation Method for Estimating Effects of Residual Modes for FRF Synthesis", J.Seymour, R.Joshua, P.Avitabile, *Proceedings of the Forty-First International Modal Analysis Conference*, Austin, Texas, Feb 2023**
- **"FRF Enhancement for Frequency Based Substructuring", J.Seymour, R.Joshua, P.Avitabile, *Proceedings of the Forty-First International Modal Analysis Conference*, Austin, Texas, Feb 2023**
- **"Vibration Control and Strain Expansion of Dynamic Strain Response Measurements", B.Daniels, P.Avitabile, Proceedings of the Fortieth International Modal Analysis Conference, Orlando, Florida, Feb 2021**
- **"FRF Model Updating for Accurate Laboratory Simulations of Actual Field Time Response", J.Seymour, P.Avitabile, Proceedings of the Fortieth International Modal Analysis Conference, Orlando, Florida, Feb 2021**
- **"FRF Target Updating for BARC Fixture Design", P.Logan, P.Avitabile, Proceedings of the Fortieth International Modal Analysis Conference, Orlando, FL, Feb 2021**
- **"A Mixed Impact Shaker MIMO Test Technique – MIS-MIMO", P.Avitabile, Proceedings of the Fortieth International Modal Analysis Conf., Orlando, FL, Feb 2021**
- **"Replicating Structure Born Excitation of a Component under Different Boundary Conditions", B.Zwink, G.Tipton, P.Avitabile, Proceedings Thirty-Ninth International Modal Analysis Conference, Orlando, FL, Feb 2020**
- **"On the Use of Optical Data in Conjunction with Phase Based Motion Magnification and Strain Expansion", B.Daniels, A.Sabato, Z.Mao, P.Avitabile, Proceedings of the Thirty-Ninth International Modal Analysis Conference, Orlando, Florida, Feb 2020**
- **"Dynamic Response Expansion using a Hybrid Transformation Approach", P.Logan, P.Avitabile, Proceedings of the Thirty-Ninth International Modal Analysis Conference, Orlando, Florida, Feb 2020**
- **"Practical Considerations for Response Tailoring using Fixtrue Neutralization Method – An Experimental Study", C.Page, J.Reyes-Blanco, P.Avitabile, Proceedings of the Thirty-Ninth International Modal Analysis Conference, Orlando, Florida, Feb 2020**
- **"Shape-Constrained Input Estimation for Multi-Shaker Vibration Testing", R.Schultz, P.Avitabile, Proceedings of the Thirty-Eighth International Modal Analysis Conference, Houston, Texas, Feb 2020**

## *Papers related to FINE, Full Field Expansion, Strain Expansion, Optical*

- **"Shape-Constrained Input Estimation for Multi-Shaker Vibration Testing", R.Schultz, P.Avitabile, Proceedings of the Thirty-Eighth International Modal Analysis Conference, Houston, Texas, Feb 2020**
- **"Application of an automatic constraint shape selection algorithm for input estimation", R.Schultz, P.Avitabile, Proceedings of the Thirty-Eighth International Modal Analysis Conference, Houston, Texas, Feb 2020**
- **"Full-Field FEM Dynamic Strain Response from a Sparse Strain Sensor Array", T.Doveno, Z.Mao, D.Soine, P.Avitabile, Proceedings of the Thirty-Eighth International Modal Analysis Conference, Houston, TX, Feb 2020**
- **"Strain Shape Expansion Applied to Optically Measured Data", T.Doveno, Z.Mao, D.Soine,, P.Avitabile, Proceedings of the Thirty-Eighth International Modal Analysis Conference, Houston, Texas, Feb 2020**
- **"Improving Optically Measured Strain Response by Leveraging Phase-Based Motion Magnification", T.Doveno, Z.Mao, D.Soine,, P.Avitabile, Proceedings of the Thirty-Eighth International Modal Analysis Conference, Houston, Texas, Feb 2020**
- **"Boundary Condition Compensation Map (from Field to Laboratory Response)", B.Zwink, B.Daniels, D.G.Tipton, P.Avitabile, Proceedings Thirty-Eighth International Modal Analysis Conference, Houston, TX, Feb 2020**
- **"Experimental Application of Boundary Condition Compensation Map (from Field to Laboratory Response)", B.Zwink, B.Daniels, P.Avitabile, Proceedings of the Thirty-Eighth International Modal Analysis Conference, Houston, Texas, Feb 2020**
- **"Combined Shaker-Acoustic Vibration Test Techniques", Schultz,R., Avitabile,P., SAVE 2019**
- **"Implementation of the Fixture Neutralization Methodology Using Data Physics Vibration Controller", J.ReyesBlanco, P.Avitabile, Proceedings of the Thirty-Seventh International Modal Analysis Conference, Orlando, Florida, Jan 2019**
- **"Fixture Neutralization Method - Adjustment of Vibration Response to Account for Fixture-Test Article Dynamic Coupling Effects Using Measured Frequency Response Functions", J.ReyesBlanco, P.Avitabile, etal, Proceedings of IMAC37, Jan 2019**
- **"Modal Projection Matching", B.Zwink, P.Avitabile, D.G.Tipton, Proceedings of the Thirty-Seventh International Modal Analysis Conference, Orlando, FL, Jan 2019**
- **"Force Customization to Neutralize Fixture-Test Article Dynamic Interaction", J.M. Reyes, P.Avitabile, Proceedings of the Thirty-Sixth International Modal Analysis Conference, Orlando, Florida, Feb 2018**
- **"Adjustment of Vibration Response to Account for Fixture-Test Article Dynamic Coupling Effects", J.M. Reyes, P.Avitabile, Proceedings Thirty-Fifth International Modal Analysis Conference, Garden Grove, CA, Feb 2017**
- **"A Study on the Dynamic Interaction of Shock Response Fixtures and Test Payload", J.Reyes, P.Avitabile, Proceedings of the Thirty-Fourth International Modal Analysis Conference, Orlando, FL, Feb 2016**



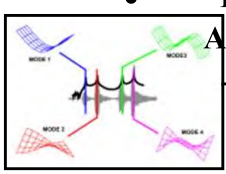
## *Journal papers related to Full Field Expansion, Reduction/Expansion, System Modeling (Nonlinear) – numerous conf papers not listed*


- **"Real-time Structural Model Updating using Local Eigenvalue Modification Procedure for Applications in High-Rate Dynamic Events", E.Ogunniyi, C.Drnek, S.Hong, Y.Wang, J.D.Bakos, P.Avitabile, J.Dodson, Mechanical Systems and Signal Processing, Volume 195, 15 July 2023, 110318, https://doi.org/10.1016/j.ymssp.2023.110318**
- **"Shape-constrained Input Estimation for Efficient Multi-shaker Vibration Testing", R.Schultz, P.Avitabile, Experimental Techniques accepted for publication https://doi.org/10.1007/s40799-020-00361-0 March 2020**

- **"Reconstruction of Nonlinear Contact Forces Beyond Limited Measurement Locations Using an SVD Modal Filtering Approach", P.Logan, P.Avitabile, J.Dodson, Experimental Techniques (2020) accepted for publication EXTE-D-19-00148R1, March 2020**
- **"Data Consistency Assessment Function (DCAF)", Y Chen, P Avitabile, J Dodson, Mechanical Systems and Signal Processing (2019), YMSSP_106688 accepted for publication January 2020**
- **"Reconstruction of External Forces Beyond Measured Points Using a Modal Filtering Decomposition Approach", P Logan, P Avitabile, J Dodson, Experimental Techniques (2019), accepted EXTE-D-18-00246R3**
- **"Underwater Dynamic Response at Limited Points Expanded to Full-Field Displacement Response", Y. Chen, P.Avitabile, accepted for publication March 2018, ASME VIB-17-1160**

- **"Proper Projection Vectors for the Expansion of Measured Experimental Modal Data", P.Avitabile, submitted 2016, Journal of Engineering Mechanics, 2018, 144(5): 04018024; DOI: 10.1061/(ASCE) EM.1943-7889.0001431**
- **"Reduced Model Approximation Approach Using Model Updating Methodologies", T.Marinone, T.Dardeno, P.Avitabile, submitted 2016, Journal of Engineering Mechanics, 2018, 144(3): 04018005; DOI: 10.1061/(ASCE) EM.1943-7889.0001422.**
- **"System Model Modes Developed from Expansion of Uncoupled Component Dynamic Data", P.Avitabile C.Nonis, S.Obando, Journal of Mechanical Engineering, Journal of Mechanical Engineering 60(2014)5, 287-297, SI 59, DOI:10.5545/sv-jme.2014.1829**

- **"Computationally Efficient Nonlinear Dynamic Analysis for Stress/Strain Applications", J.Harvie, T.Dardeno, P.Avitabile, submitted to Journal of Computational and Nonlinear Dynamics, June 2016**
- **"Developments in the Prediction of Full Field Dynamics for the Nonlinear Forced Response of Reduced Order System Models", J.Harvie, T.Dardeno, P.Avitabile, submitted to Journal of Computational and Nonlinear Dynamics, June 2016**
- **"Expansion of Transient Operating Data", P.Avitabile, C.Chipman, Mechanical Systems and Signal Processing, Volume 31, August 2012, Pages 1-12**

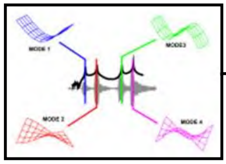



## *Journal papers related to Full Field Expansion, Reduction/Expansion, System Modeling (Nonlinear) – numerous conf papers not listed*

- **“Full-field dynamic strain prediction on a wind turbine using displacements of optical targets measured by stereophotogrammetry”, J.Baqersad, P.Poozesh, C.Niezrecki, P.Avitabile, Mechanical Systems and Signal Processing (2015), http://dx.doi.org/10.1016/j.ymssp.2015.03.021**
- **“Extracting full-field dynamic strain on a wind turbine rotor subjected to arbitrary excitations using 3D point tracking and a modal expansion technique”, J.Baqersad, C.Niezrecki, P.Avitabile, Journal of Sound and Vibration (2015), http://dx.doi.org/10.1016/j.jsv.2015.04.026**
- **"Full-Field Dynamic Strain Prediction on a Wind Turbine using Displacements of Optical Targets Measured by Stereophotogrammetry" J. Baqersad, C. Niezrecki, and P. Avitabile, Mechanical Systems and Signal Processing Journal, Accepted, In Press, doi:10.1016/j.ymssp.2015.03.021, 2015.**
- **“Full Field Dynamic Strain on Wind Turbine Blade Using Digital Image Correlation Techniques and Limited Sets of Measured Data from Photogrammetric Targets"”, J.Carr, J. Baqersad, P.Avitabile, C.Niezrecki, Experimental Techniques, November 2013, doi: 10.1111/ext.12129, 2014**
- **“Prediction of Full Field Dynamic Strain from Limited Sets of Measured Data”, P.Avitabile, P.Pingle, Shock and Vibration 19 (2012) DOI 10.3233/SAV-2012-0686, pp 765-785**

- **"Equivalent Reduced Model Technique Development for Nonlinear System Dynamic Response", Thibault L., Avitabile P., Foley J., Wolfson J., Mechanical Systems and Signal Processing, Vol. 36: pp. 422-455, 2013.**
- **“Efficient Computational Nonlinear Dynamic Analysis Using Modal Modification Response Technique”, T.Marinone, P.Avitabile, J.Foley, J.Wolfson, Mechanical Systems and Signal Processing, Vol. 31: pp. 67-93, 2012.**

- **“Reallocation of System Mass & Stiffness for Achieving Target Specifications using a Superelement/Substructure Methodology” P.Avitabile, J.O’Callahan, H.Tsuji, J.P.DeClerck, International Journal of Vehicle Noise and Vibration, Vol 1, 3/4 , 2005**
- **“Reallocation of System Mass and Stiffness for Achieving Target Specifications”, P.Avitabile, J.O’Callahan, H.Tsuji, J.P.DeClerck, International Journal of Vehicle Noise and Vibration, Vol 1, 1/2 , 2004**
- **“A Comparison of Some System Modeling Approaches”, P.Avitabile, The Shock and Vibration Digest, Vol 33, Number 4, July 2001, pp 281-291**



## *Journal papers related to Full Field Expansion, Reduction/Expansion, System Modeling (Nonlinear) – numerous conf papers not listed*


- **"Conditioning of FRF Measurements for Use with Frequency Based Substructuring", D.Nicgorski, P.Avitabile, Mechanical Systems and Signal Processing, (10.1016/j.ymssp.2009.07.013), Volume 24, Issue 2, February 2010, Pages 340-351**
- **"Experimental Issues Related to Frequency Response Functions for Frequency Based Substructuring", D.Nicgorski, P.Avitabile, Mechanical Systems and Signal Processing, (10.1016/j.ymssp.2009.09.006), Volume 24, Issue 5, July 2010, Pages 1324-1337**
- **"A Reduced Order, Test Verified Component Mode Synthesis Approach for System Modeling Applications", A.Butland, P.Avitabile, Mechanical Systems and Signal Processing, (10.1016/j.ymssp.2009.08.008), Volume 24, Issue 4, May 2010, P904-921**
- **"Development of Efficient Reduced Order Test Verified Model for Flexible Body Dynamic Simulation", P.Avitabile, T.VanZandt, N.Wirkkala, Mechanical Systems and Signal Processing, submitted for review February 2009, accepted November 2009 with revisions**

- **"Efficient Techniques for Forced Response Involving Linear Modal Components Interconnected by Discrete Nonlinear Connection Elements", P.Avitabile, J.O'Callahan, Mechanical Systems and Signal Processing, Volume 23, Issue 1, January 2009, Pages 45-67, Special Issue: Non-linear Structural Dynamics (invited publication**
- **"Frequency Response Function Expansion for Unmeasured Translation and Rotational DOFs for Impedance Modeling Applications", P.Avitabile, J.O'Callahan, Mechanical Systems and Signal Processing, V17, No 4, July 03**

- **"Twenty Years of Structural Dynamic Modification – A Review", P.Avitabile, Mechanical Systems and Signal Processing, submitted for review for publication, paper undergoing revision and expansion to accommodate reviewers comments**
- **"The Coordinate Orthogonality Check (CORTHOG)", P.Avitabile, F.Pechinsky, Mechanical Systems and Signal Processing, Vol 12, No. 3, pp 395-414, May 1998**
- **"Understanding Structural Dynamic Modification and the Effects of Truncation" P.Avitabile, J.O'Callahan, International Journal of Analytical and Experimental Modal Analysis, Vol. 6, No. 4, pp215-235, July 1991**
- **"An Efficient Method for Forced Response using a Structural Dynamic Modification Technique", P.Avitabile, J.O'Callahan, International Journal of Analytical and Experimental Modal Analysis, V6, No. 3, pp189-200, July 91**
- **"Understanding Structural Dynamic Modification and the Effects of Truncation", P.Avitabile, J.O'Callahan, Proceedings of the International Journal of Analytical and Experimental Modal Analysis, invited contribution, Oct 1991,Vol 6, No 4**
- **"An Efficient Method for Forced Nonlinear Response using a Structural Dynamic Modification Technique", P.Avitabile, J.O'Callahan, Proceedings of the International Journal of Analytical and Experimental Modal Analysis, invited contribution, Jul 1991,Vol6,No3**