Topological Data Analysis for Real-Time Extraction of Time Series Features

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High-Rate Structural Health Monitoring (HRSHM)

What is SHM?

- Process of implementing a damage identification strategy
- Includes:

1) observation
2) feature extraction
3) statistical analysis
4) decisions

Why SHM?

- Condition-based maintenance
  - Automatic damage detection vs. timely inspections
- Useful?
  - Early warnings for proactive actions
- Any tradeoffs?
  - Requires sophisticated hardware and software

High-Rate Structural Health Monitoring (HRSHM)

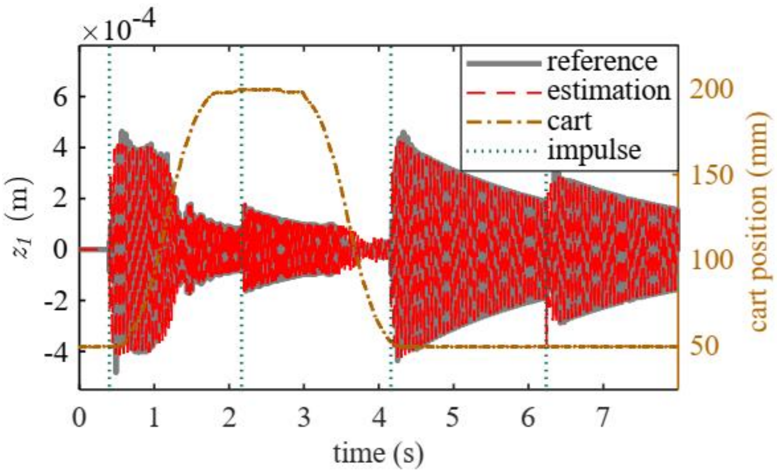
Definition

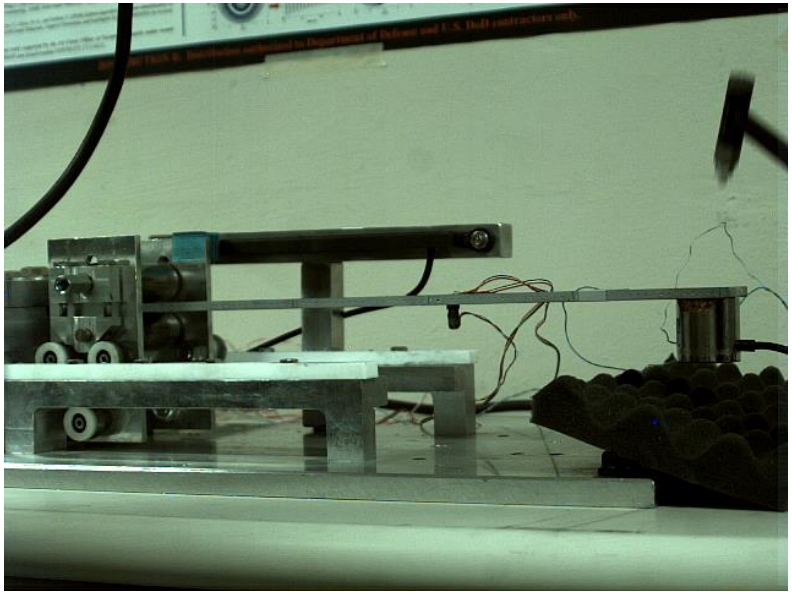
- 10 kg TNT takes 0.3 to 100 ms to travel over 1 to 40 m distance
- 100 μs at Mach 5 corresponds to 150 mm. 1.5 MHz sampling rate gives a 1 mm resolution

High-Rate Structural Health Monitoring (HRSHM)

Definition

- **Systems experiencing high-rate dynamics**
  - Accelerations higher than 100 g_n (g_n = 9.81 m/s^2) in less than 1 ms
- **Characterized by**
  - Large uncertainties in external loading
  - High levels of nonstationarity and heavy disturbance
  - Generations of unmodeled dynamics from changes in mechanical configuration

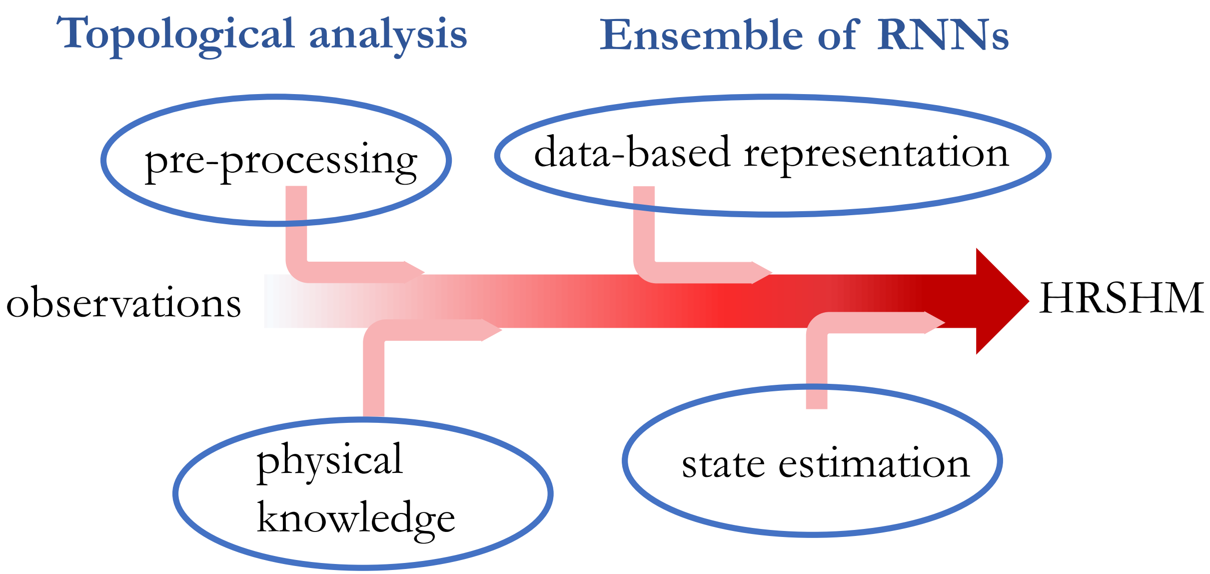




Hong, Jonathan, et al. "Variable input observer for nonstationary high-rate dynamic systems." *Neural computing and applications* 32 (2020): 5015-5026.

Research Approach

Strategy:



RNN: recurrent neural network
HRSHM: high-rate structural health monitoring

Motivation & Objectives

Goals:

- Universal framework for high-rate
- Real-time learning – sequential learning
- Easy-to-train network

Challenges:

- Very limited training data
- Nonstationary environment
- Sub-millisecond computations

HRSHM – Early Work

Control of induction motors

Technique	Speed for one update	Reference
extended Kalman filter	86 μs	Xu *et al*. 2007
adaptive sliding observer	86 μs	Xu *et al*. 2007
Luenberger observer	5 μs	Zhang *et al*. 2009
sliding mode observer	5 μs	Zhang *et al*. 2009
extended Kalman filter	100 μs	Zhang *et al*. 2009

Adaptive observers

Technique	Speed for one update	Reference
exponential parameter estimation	3 s	Khayati *et al*. 2013
modification of modulating functions	8 s	Byrski *et al*. 2014

Zhang, Yongchang, et al. "A comparative study of Luenberger observer, sliding mode observer and extended Kalman filter for sensorless vector control of induction motor drives." *2009 IEEE Energy Conversion Congress and Exposition*. IEEE, 2009.

Xu, Z., F. Rahman, and Dianguo Xu. "Comparative study of an adaptive sliding observer and an ekf for speed sensor-less dtc ipm synchronous motor drives." *2007 IEEE Power Electronics Specialists Conference*. IEEE, 2007.

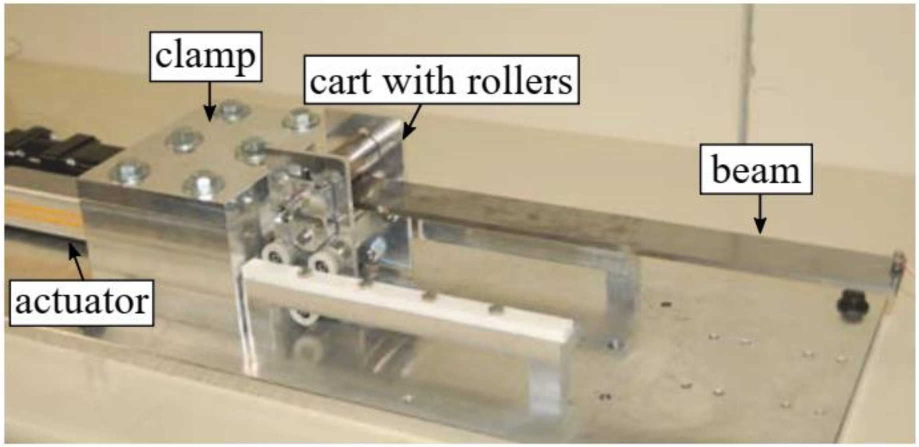
Khayati, Karim, and Jiang Zhu. "Adaptive observer for a large class of nonlinear systems with exponential convergence of parameter estimation." *2013 International Conference on Control, Decision and Information Technologies (CoDIT)*. IEEE, 2013.

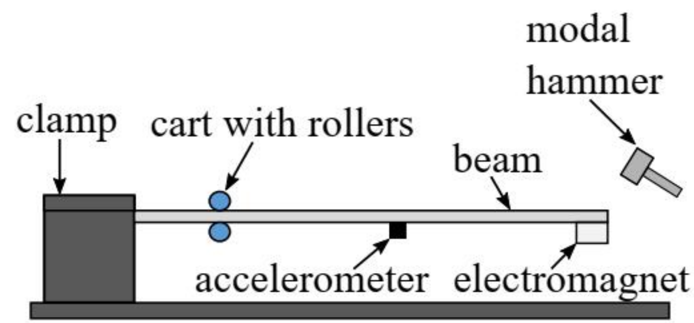
Byrski, Witold, and Jędrzej Byrski. "On-line fast identification method and exact state observer for adaptive control of continuous system." *Proceeding of the 11th World Congress on Intelligent Control and Automation*. IEEE, 2014.

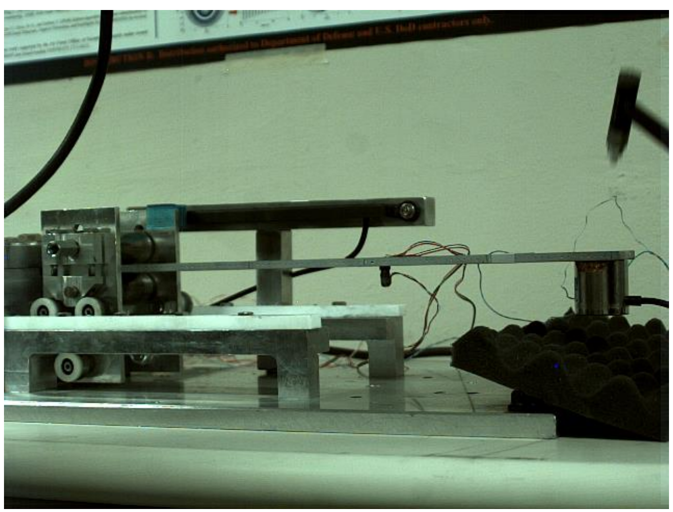
Motivation & Objectives

Fast Model Reference Adaptive System

- Dynamic Reproduction of Projectile Ballistic Environments for Advanced Research (DROPBEAR)



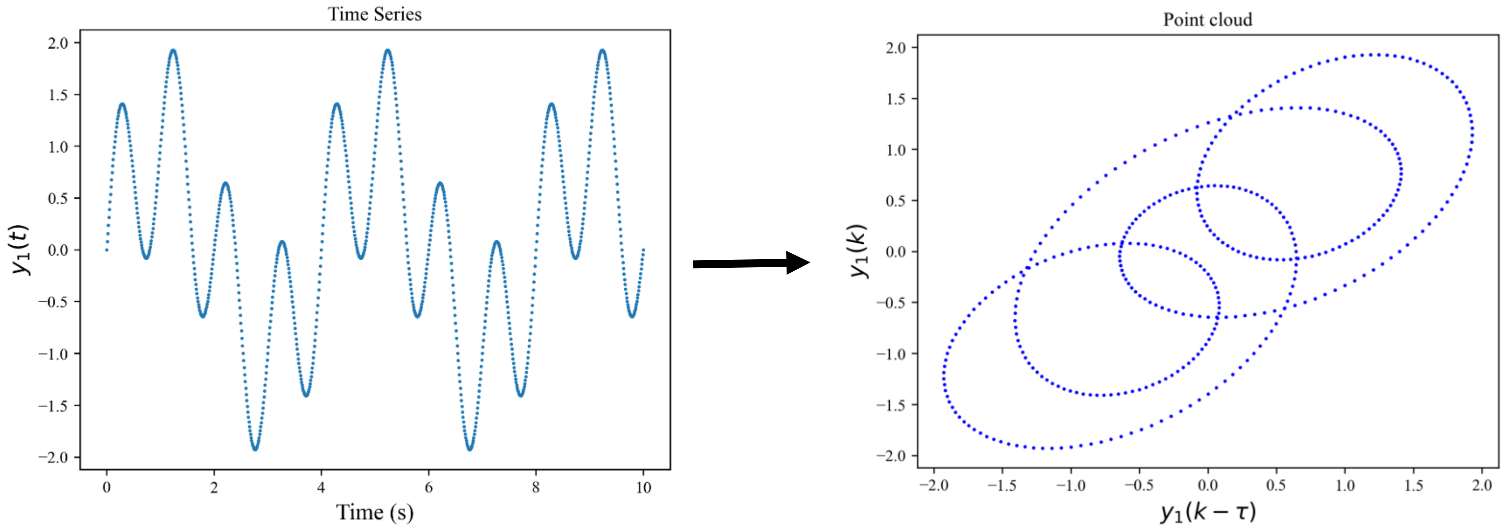




Tests	Pin position	Frequency (Hz)	Estimated frequency (Hz)	Convergence time (ms)
1-5	50 mm	17.7	17.67	780
6-10	100 mm	21.0	21.00	400
11-15	150 mm	25.0	24.99	160
16-20	200 mm	31.0	31.01	100

Data Pre-Processing

Embedding Theorem



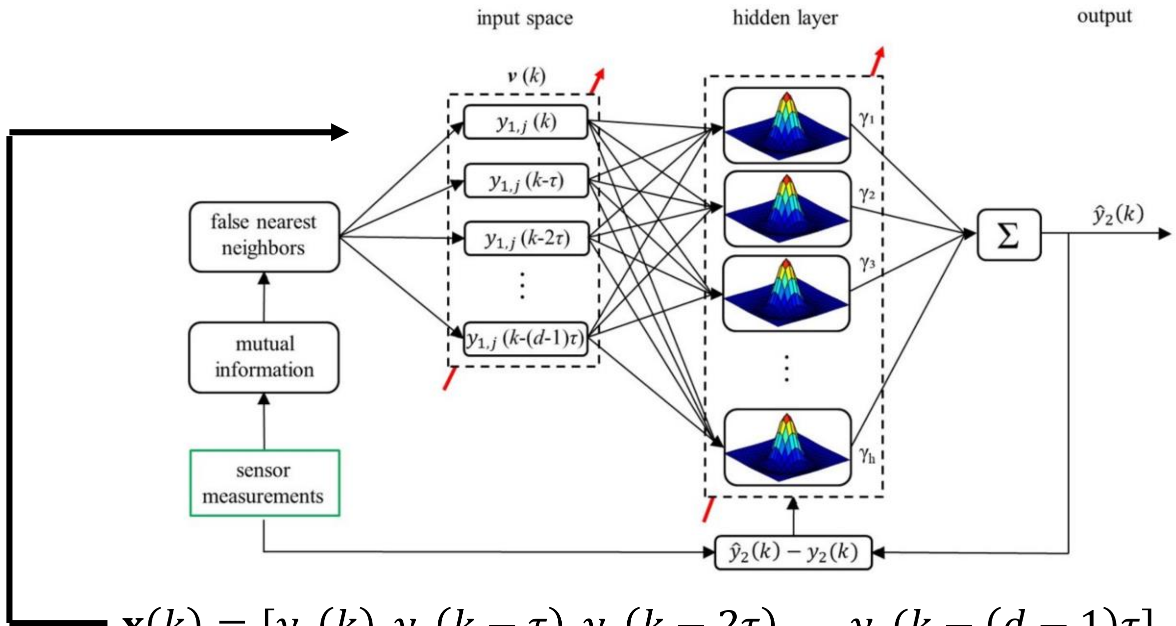
$$\mathbf{s}(k) \equiv \mathbf{x}(k) = [y_1(k), y_1(k-\tau), y_1(k-2\tau), \dots, y_1(k-(d-1)\tau]$$

Key Remarks:

- 1-to-1 mapping exists between the state vector **s** and delay vector **x**
- The delay vector **x** preserves the essential dynamics
- Minimal representation can be obtained using the essential dynamics as inputs

Previous Research

Variable Input Observer (VIO)



$$\mathbf{x}(k) = [y_1(k), y_1(k-\tau), y_1(k-2\tau), \ldots, y_1(k-(d-1)\tau]$$

Overall:

- Predicting with no pre-training possible

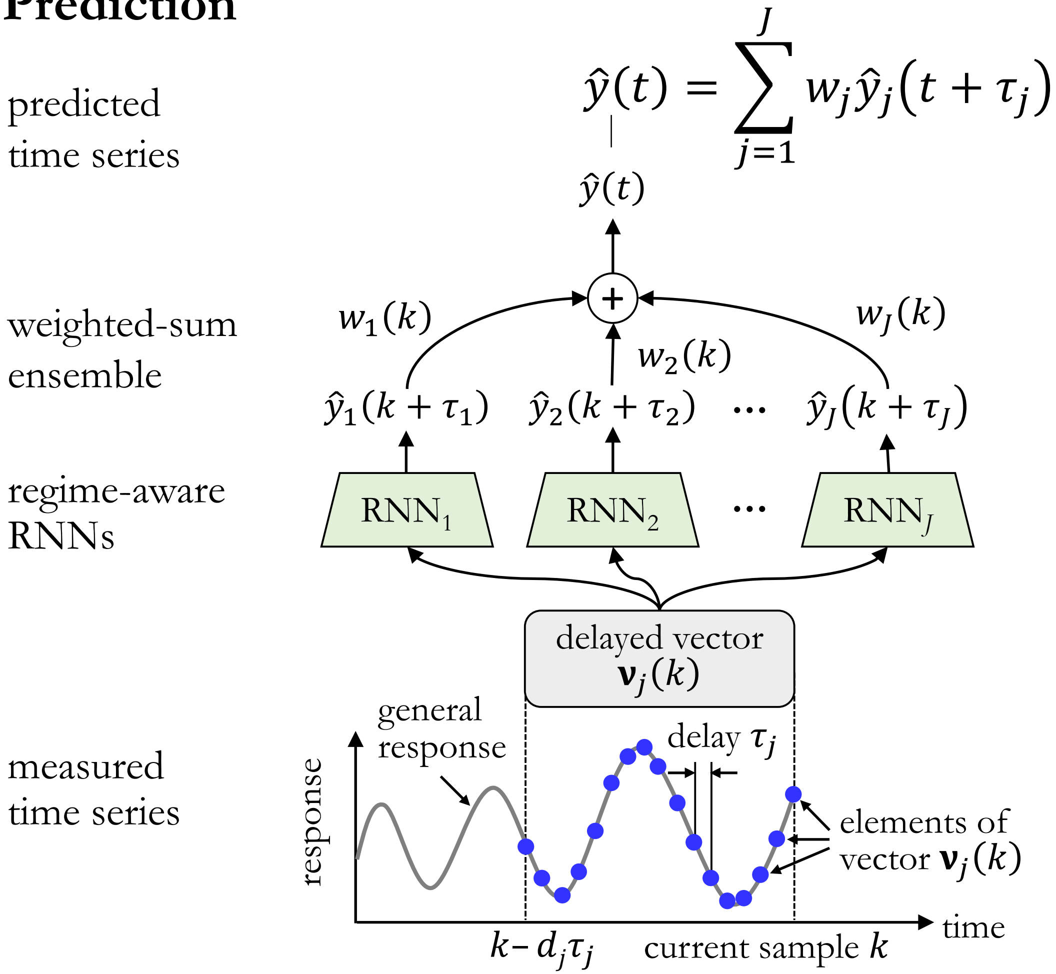
Challenges:

- Relatively long computation time
- No learning occurred

Hong, Jonathan, *et al*. "Variable input observer for nonstationary high-rate dynamic systems." *Neural computing and applications* 32 (2020): 5015-5026.

Previous Research

Ensemble of RNNs for Time Series Prediction



Key Features

- Each RNN built with different input vector (multi-rate sampling)
- Short-sequence LSTM architectures in parallel
- RNNs are trained on a single event (different sensor)
- Transfer learning to extrapolate
- Weighted-sum or “attention” layer assembles individual estimations

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Topological Data Analysis

Introduction:

- Characteristics of data that do not depend on certain details of the representation
- Infer relevant topological features from these spaces
- Using these features for further processing (data classification)
- TDA has been never used for time series prediction

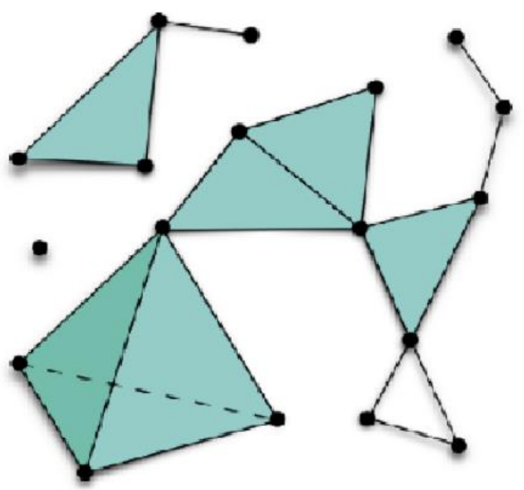
Challenges:

- No direct access to topological information
- Need for topological construction (simplicial complexes)
- Distinguish topological signal from noise
- Find a way to incorporate TDA features within neural network
- Find a fast way (shortcut) to implement TDA features.

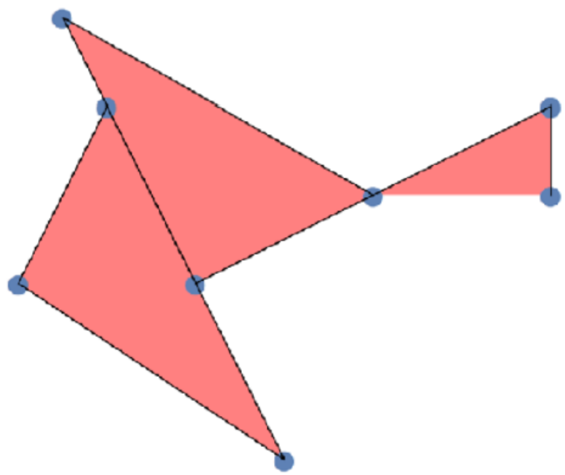
Topological Data Analysis

Simplicial Complexes

- A generalization of a graph
- A 0-simplicial complex is a set of points, a 1-simplicial complex is a graph
- An n-simplicial complex contains up to n-dimensional simplices



A simplicial complex

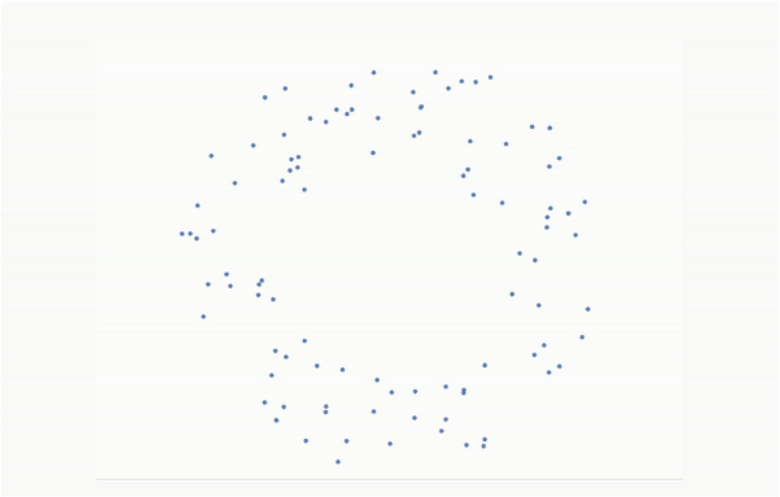


Not a simplicial complex

Topological Data Analysis

Simplicial Complexes

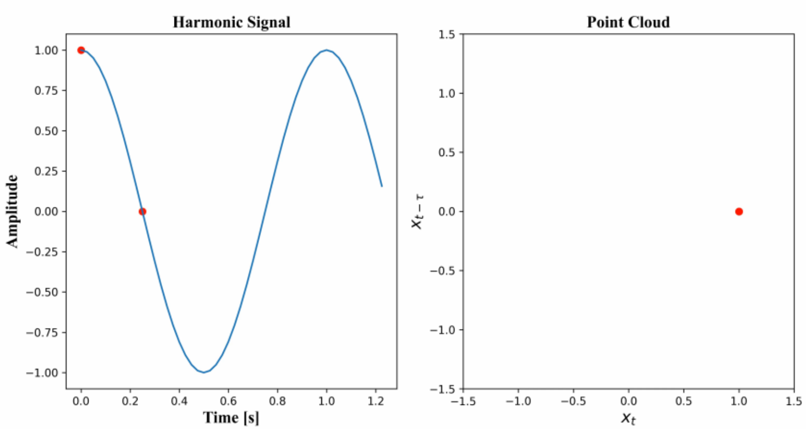
- It is not obvious what the correct radius is for the construction of our simplicial complex
- Persistent homology solved this problem by measuring topological features which persist while growing radii
- Persistence diagram keeps track of the increase/decrease in each Betti number, representing the birth and death of features as radii increase.
- Informally, the k_{th} Betti number refers to the number of k-dimensional holes on a topological surface

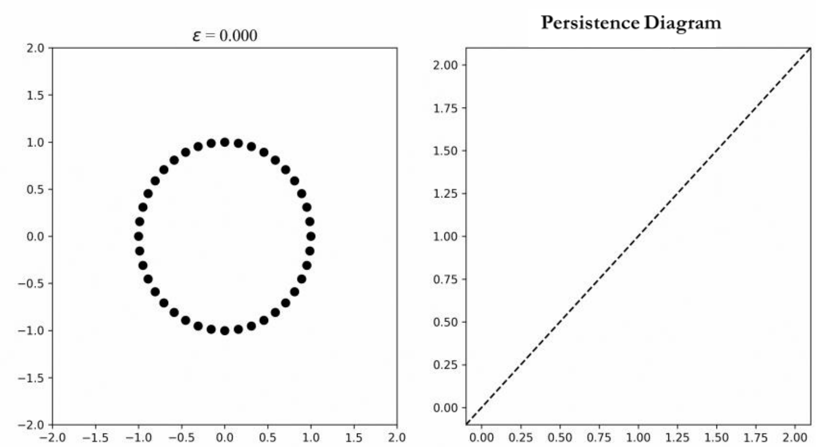


Topological Data Analysis

Persistence Diagram

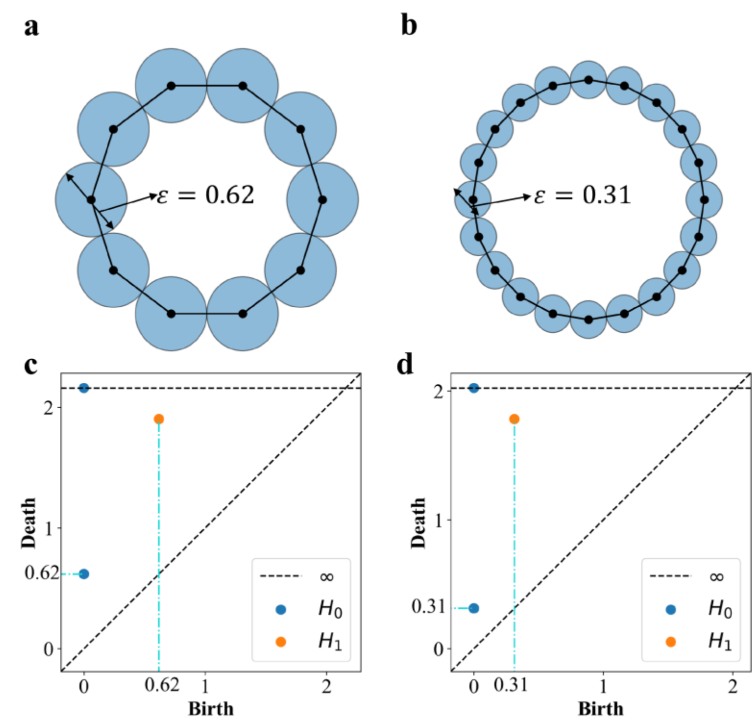
- Record the changes when increasing the threshold into a plot known as the persistence diagram
- Each point represents a hole in the point cloud





Topological Data Analysis

- TDA features on a physical context
- Application: cantilever beam with a fast-moving boundary condition
- The meaning of TDA features for a single-harmonic time series
- Suggested optimal embedding dimension is 2
- Containing information about zero-dimensional hole (H_0) and one-dimensional hole (H_1)
- H_1 relates to the frequency of harmonic signals
- H_0 does not have any physical meaning



Topological Data Analysis

Challenges:

- The embedding theorem is applicable only to stationary systems
- Our dynamics are highly non-stationary

Strategy: Windowing

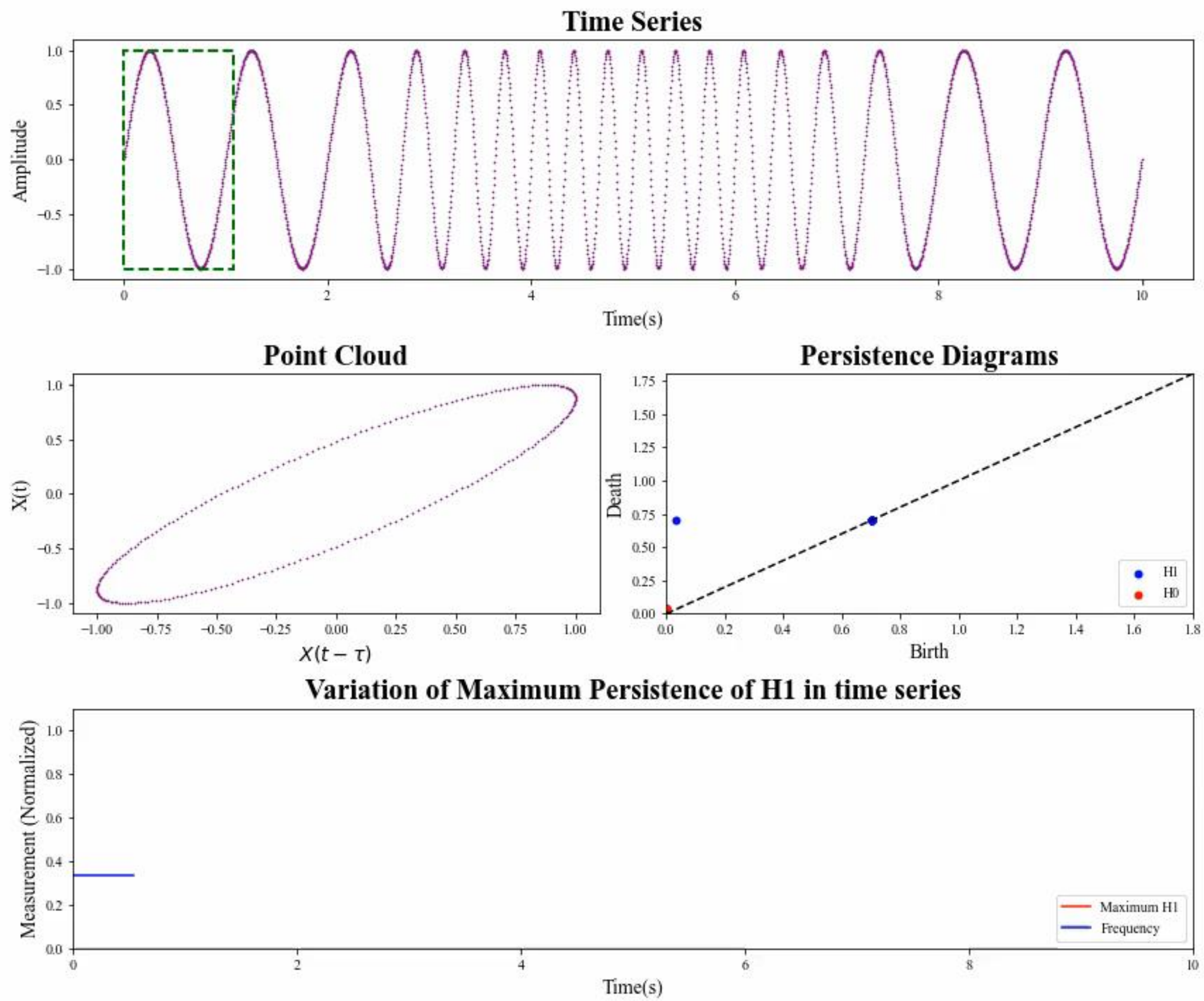
- Applying a sliding window over the dataset to extract local values for $\mathbf{H_1}$
- Maximum allowable time delay $(\tau) = \dfrac{0.25}{f_{max}}$
- Size of windowing $= \tau + \dfrac{1}{f_{min}}$ (Ensure point cloud will form a complete loop)

Case Study #1: Synthetic Cosine Data

- $x(t) = Cos(2\pi f(t)t)$
- Moving window size:

$$\tau + \frac{1}{f_{min}} = 1 + 0.03 = 1.03$$

- Time delay = 0.03 s

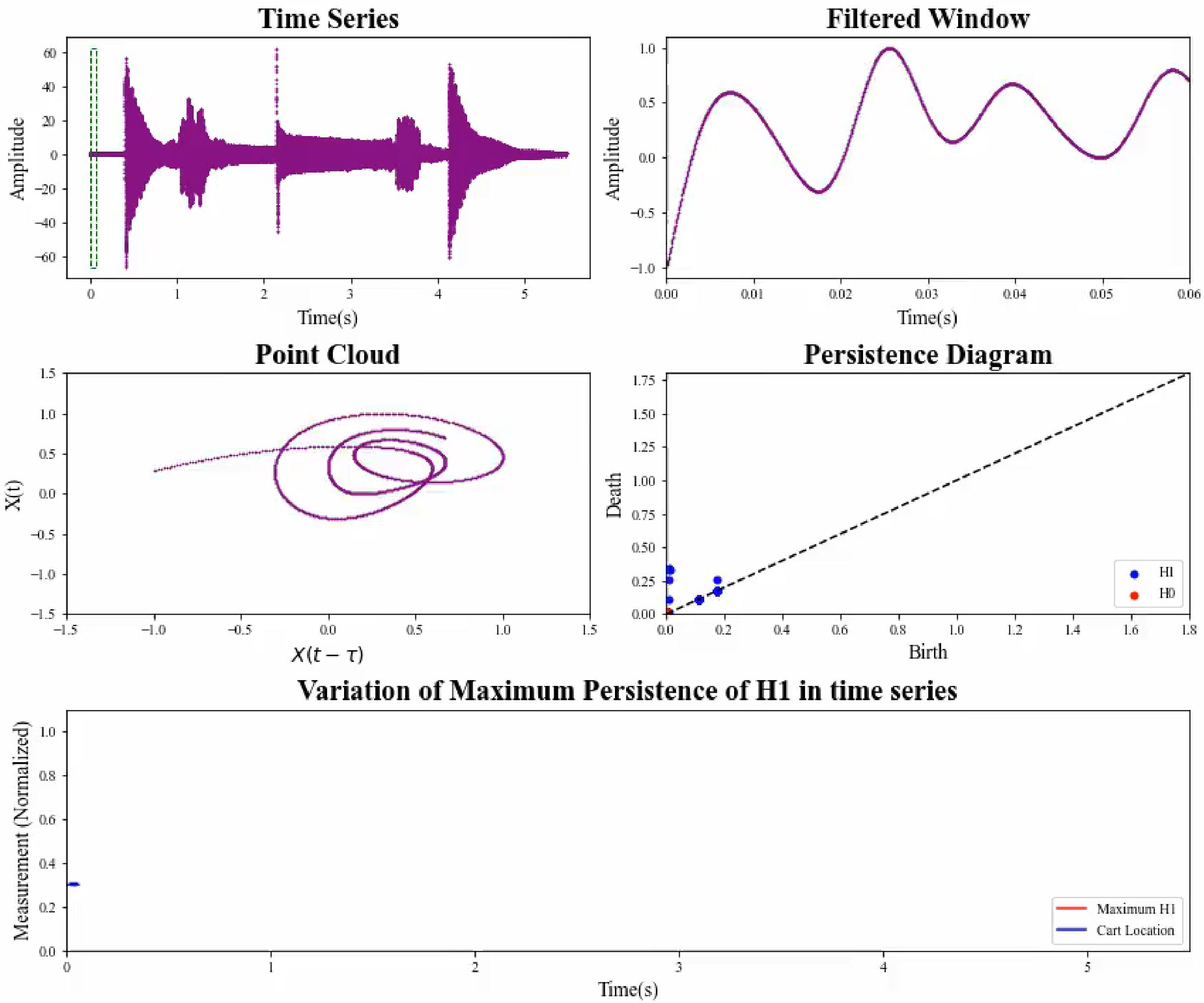


Case Study #2: Experimental Data from DROPBEAR Testbed

- Moving window size:

$$\tau + \frac{1}{f_{min}} = 0.06$$

- Time delay = 0.008 s



Topological Data Analysis

Challenges:

- Computation time (far beyond the sub-millisecond requirement)
- Rely on physical knowledge to construct window size and time delay

Goals:

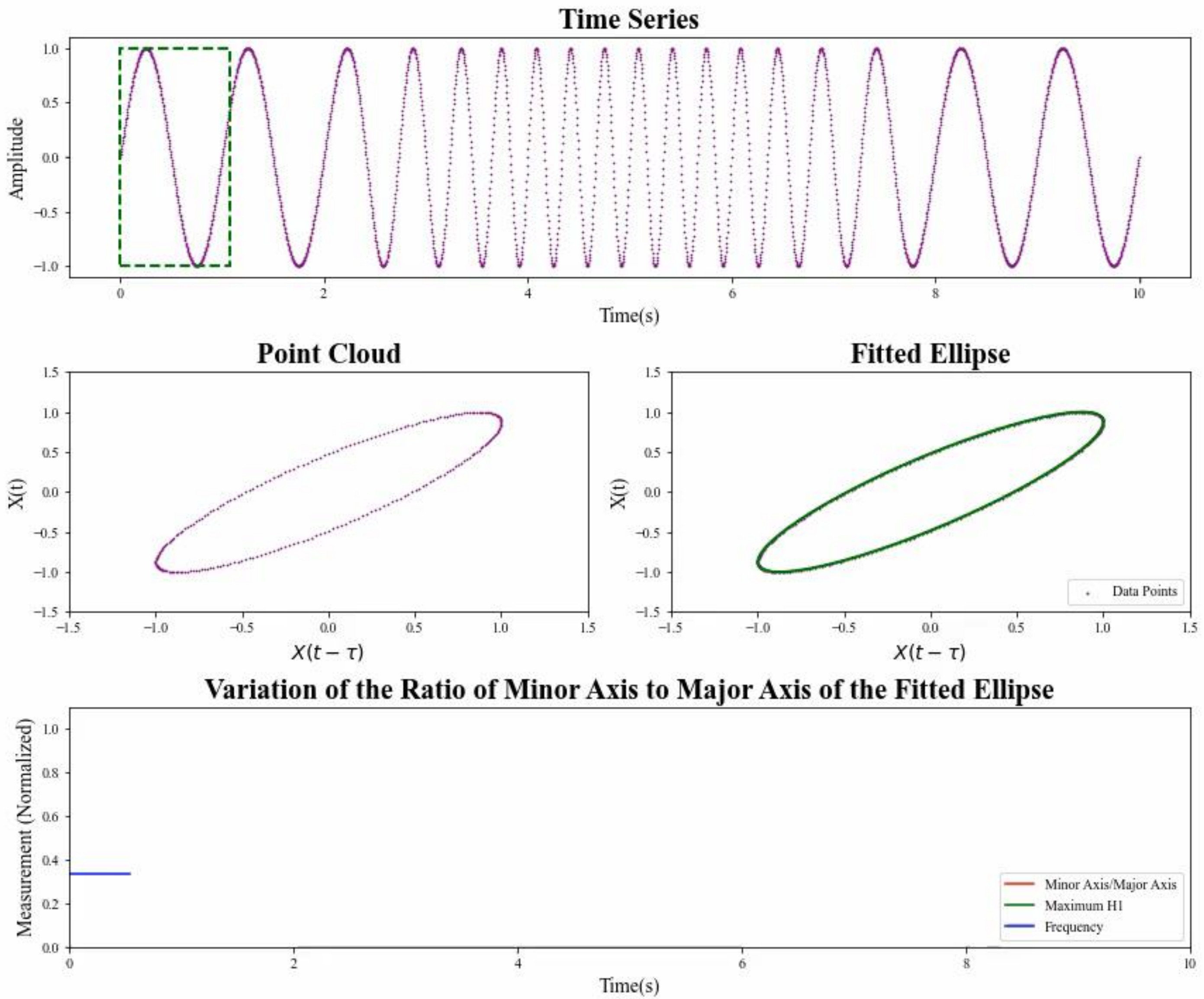
- Extracting TDA features faster
- For H_1, the ratio of the minor axis to the major axis of an ellipse determines the death time of point cloud

Fast TDA

- In each window, an ellipse is fitted through least square optimization
- Plot the ratio of the minor axis to the major axis as an indication of the persistence of the ellipse

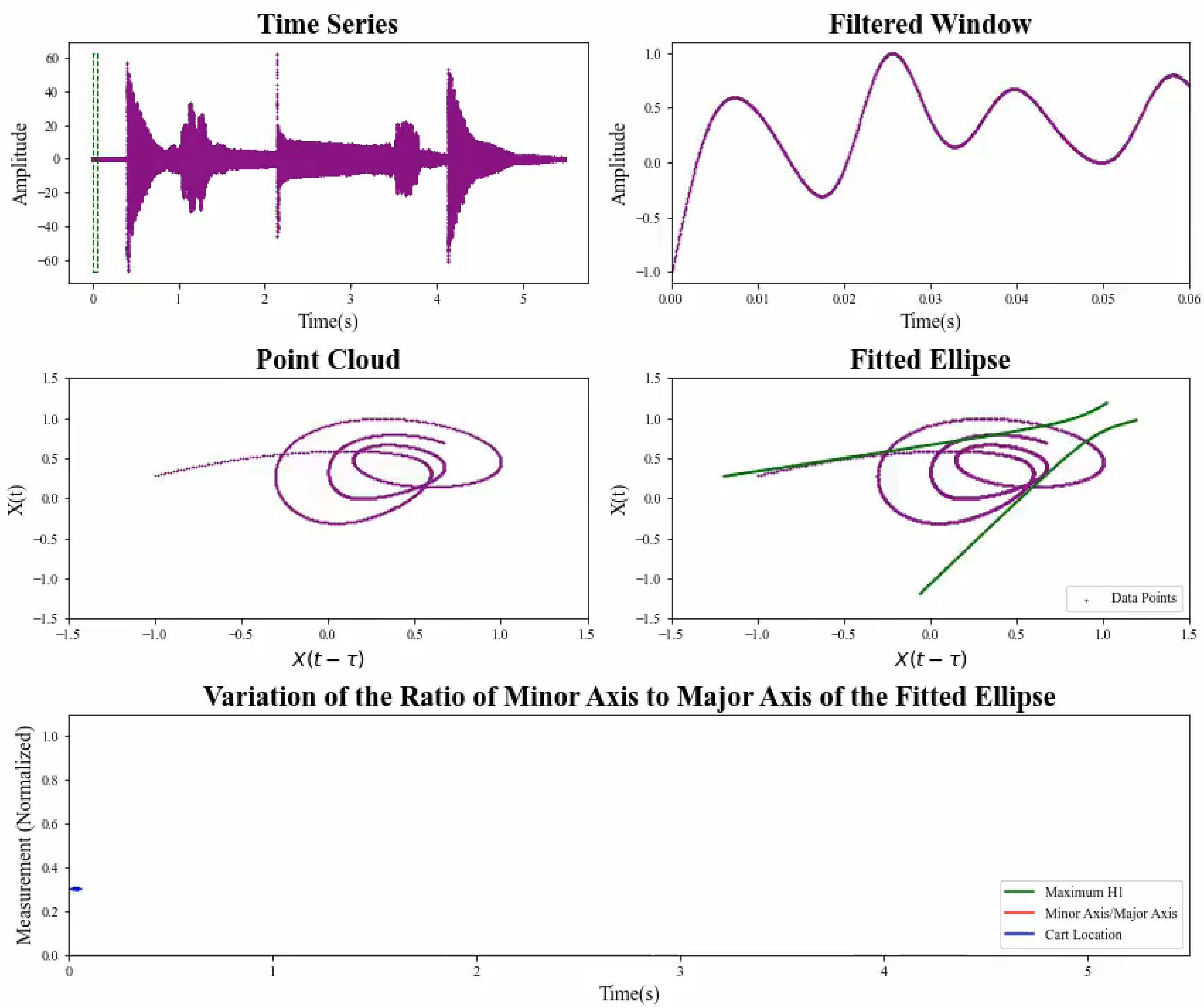
Case Study #1: Synthetic Data

- **Fast TDA**



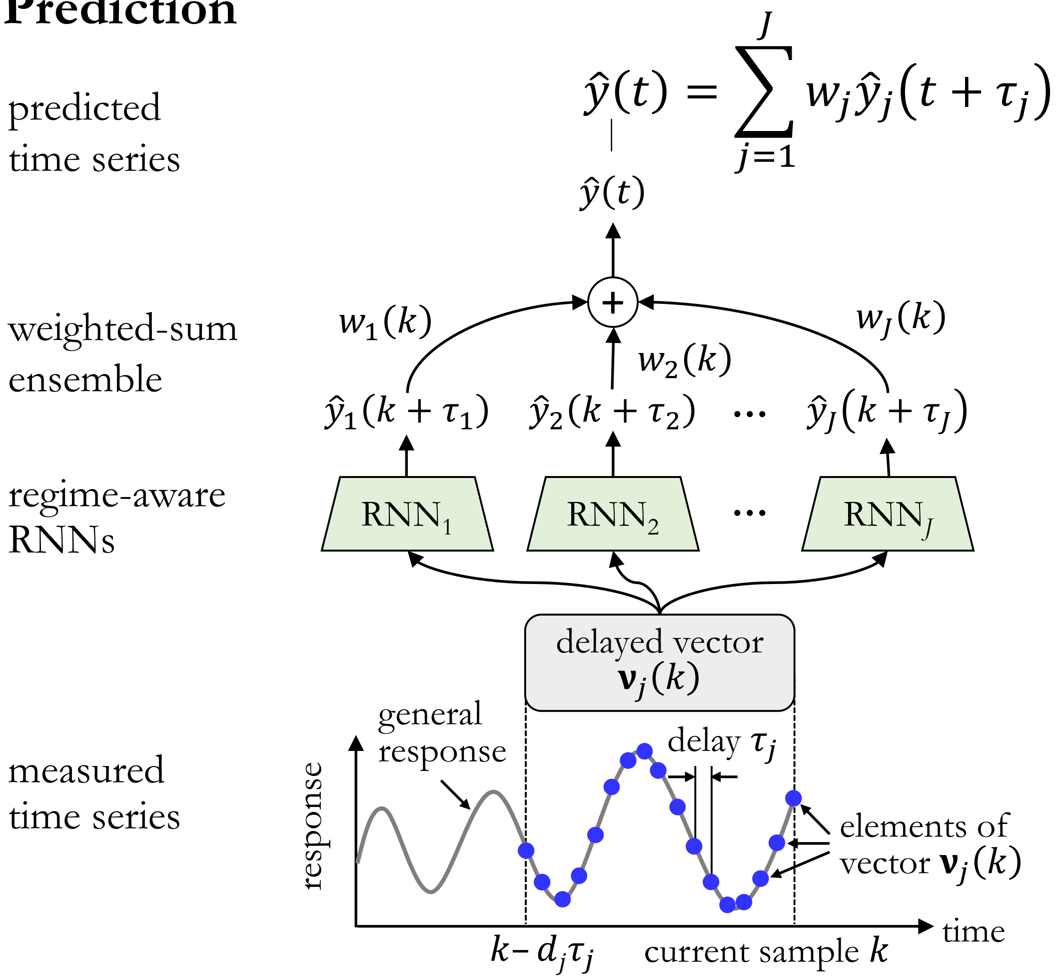
Case Study #2: Experimental Data from DROPBEAR Testbed

- **Fast TDA**



Recap on Previous Research

Ensemble of RNNs for Time Series Prediction



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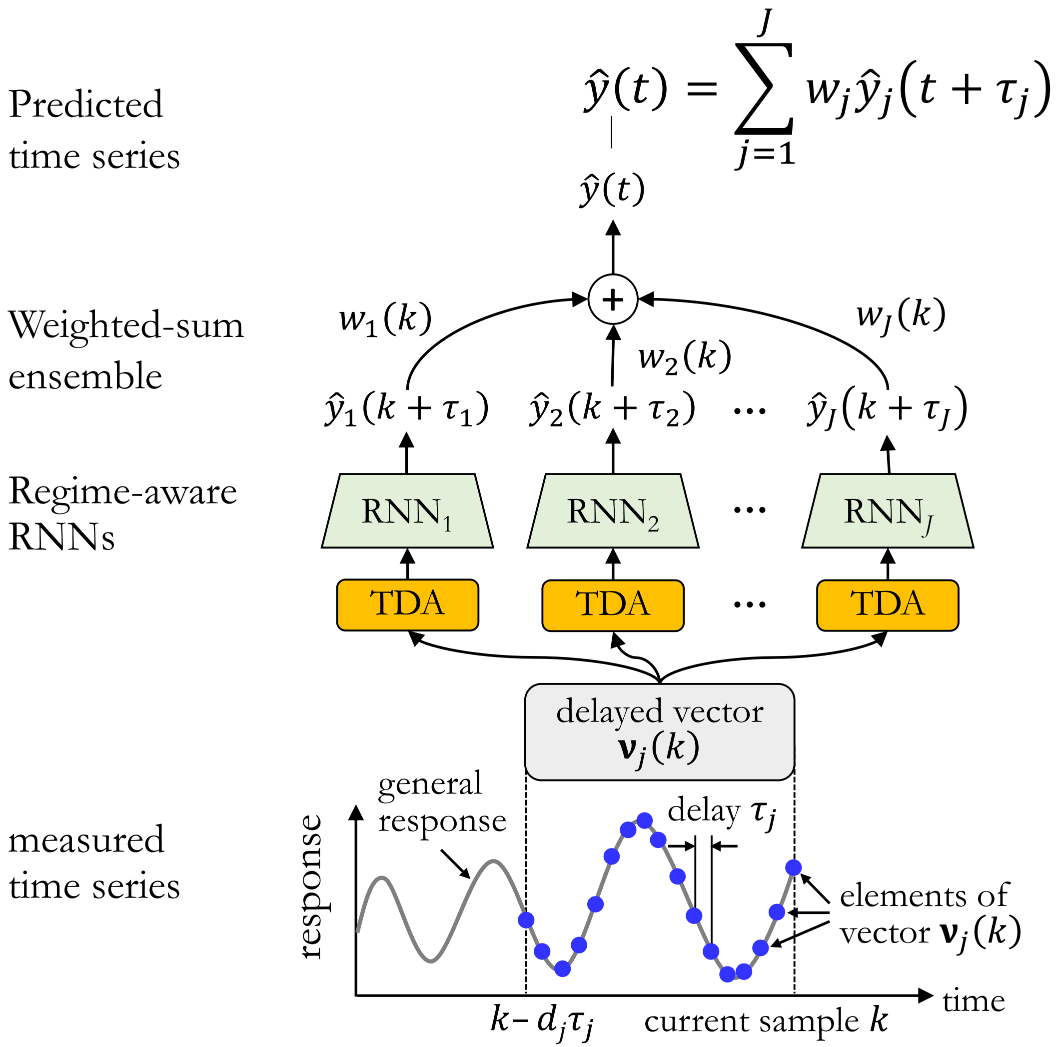
Future Research: Topology-Aware RNN Ensemble

Approach 1: Topological Input Features

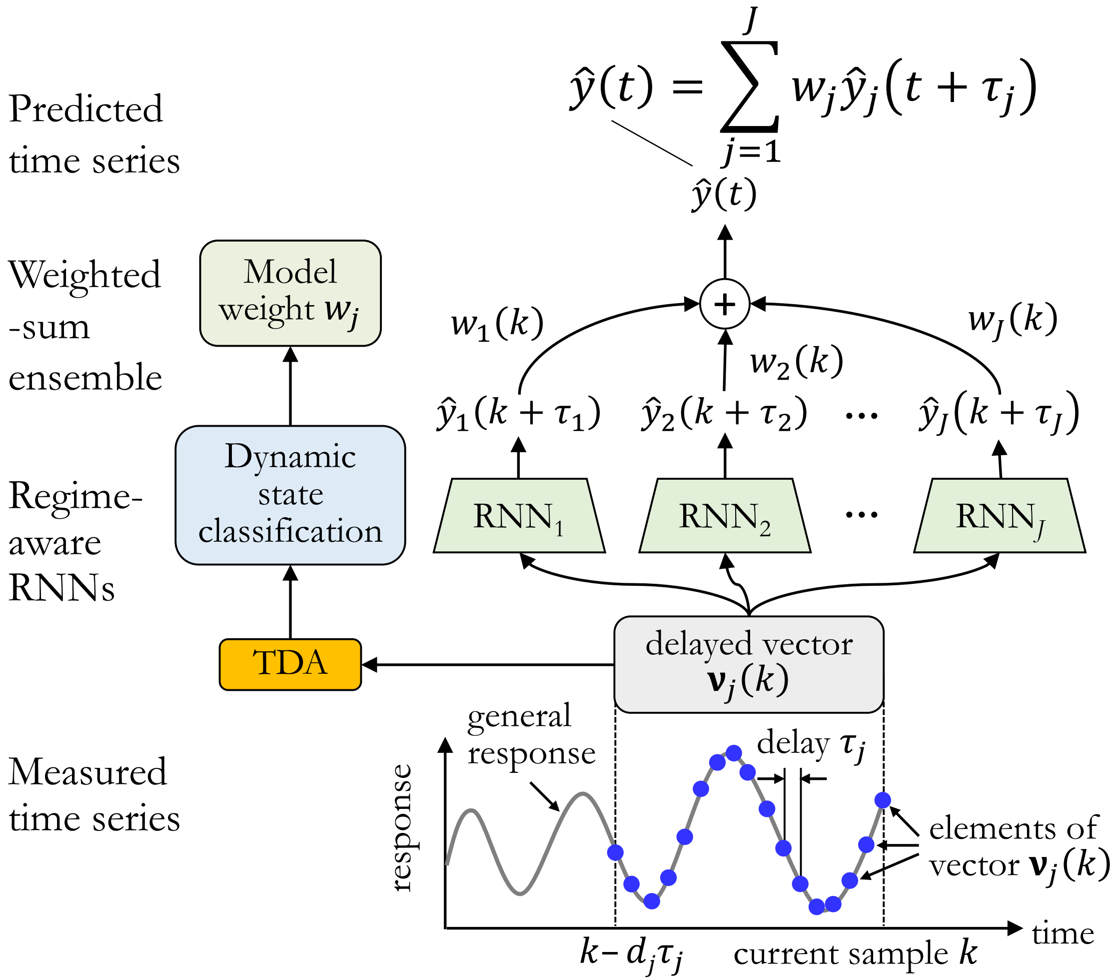


Key Features:

- TDA for signal processing to extract topological features
- Mapping from topological features to time series
- Potential extension to state (e.g., damage) estimation

Future Research: Topology-Aware RNN Ensemble

Approach 2: Topology-Aware Model Weighting



Key Features:

- TDA for dynamic state classification to weigh individual estimates in an ensemble
- Maintain the mapping from time series to time series
- Weighted-sum or "attention" layer assembles individual estimations

Summary

- **Key Outcomes:**
  - Linked TDA features to physics for a non-stationary single-harmonic
  - Sequential sub-sampling used for non-stationary applications
  - Demonstrated the new concept: Fast-TDA
- **Upcoming Work/Challenges:**
  - How to extend TDA-to-physics to more complex problems, including multiple dominating frequencies.
  - How to select on-the-spot time delays to form point clouds
  - Need to understand the relationship between sub-sampled persistence and degrees of nonlinearities
  - Adapt TDA to the sub-millisecond realm (further develop Fast-TDA)

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Questions?