CSCE 774 ROBOTICS SYSTEMS

Control Theory

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The Complex Plane (review)

Imaginary axis (j)

$$u = x + jy$$

$$\angle u \equiv \phi = \tan^{-1} \frac{y}{x}$$

Real axis

$$|u| \equiv r \equiv |\bar{u}| = \sqrt{x^2 + y^2}$$

$$\bar{u} = x - jy$$

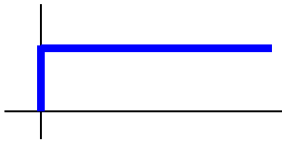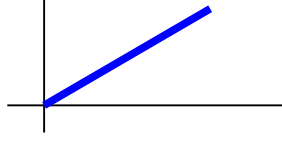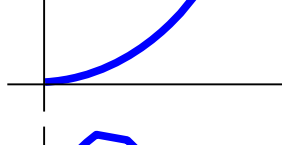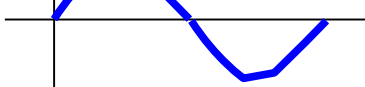(complex) conjugate
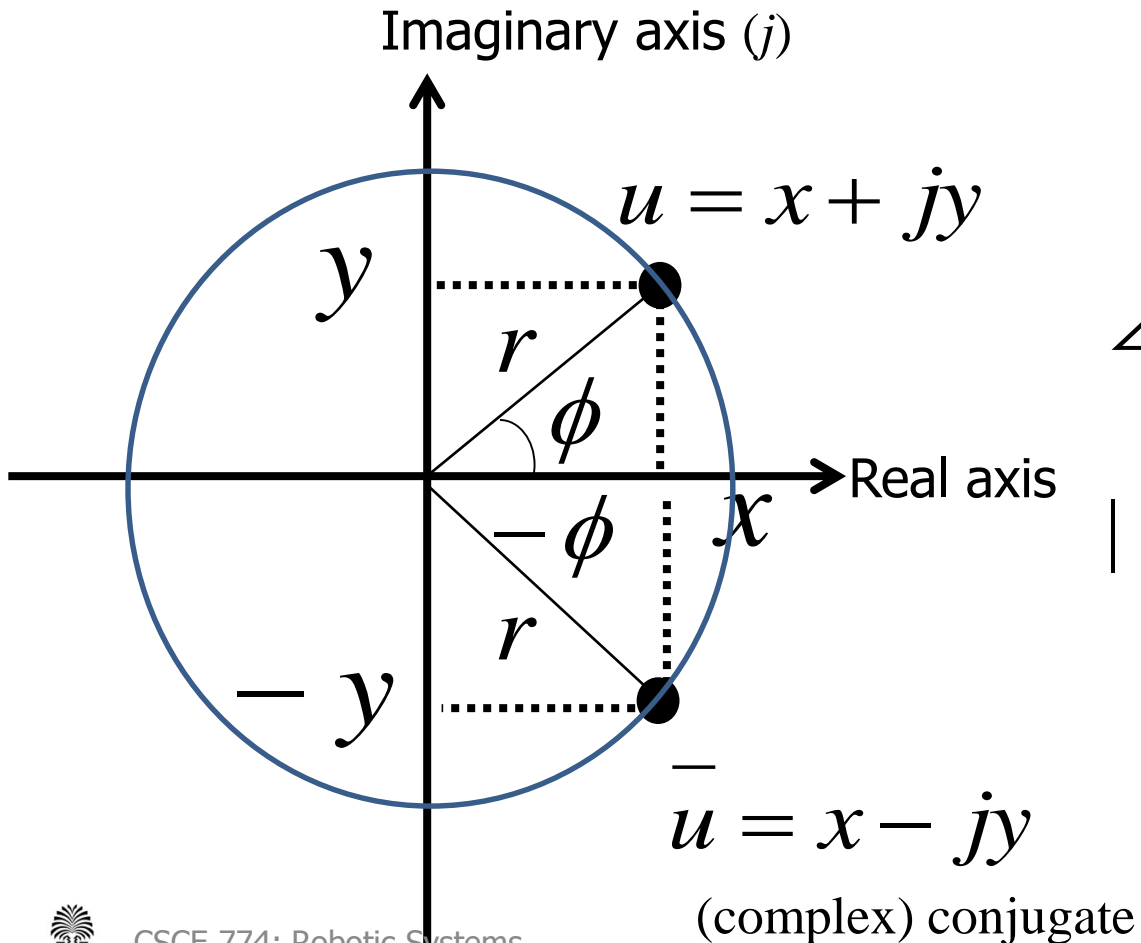
Basic Tool For Continuous Time: Laplace Transform

$$\mathbf{L}[f(t)] = F(s) = \int_0^\infty e^{-st} f(t) dt$$

- Convert time-domain functions and operations into frequency-domain
  - $f(t) \rightarrow F(s) \quad (t \in \mathbf{R}, s \in \mathbf{C})$
  - **Linear differential equations (LDE) $\rightarrow$ algebraic expression in Complex plane**
- Graphical solution for key LDE characteristics
- Discrete systems use the analogous z-transform

Laplace Transforms of Common Functions

Name	$f(t)$		$F(s)$
Impulse	$f(t) = \begin{cases} 1 & t = 0 \\ 0 & t > 0 \end{cases}$		1
Step	$f(t) = 1$		$\dfrac{1}{s}$
Ramp	$f(t) = t$		$\dfrac{1}{s^2}$
Exponential	$f(t) = e^{at}$		$\dfrac{1}{s-a}$
Sine	$f(t) = \sin(\omega t)$		$\dfrac{1}{\omega^2 + s^2}$

Laplace Transform Properties

Addition/Scaling $\quad L[af_1(t) \pm bf_2(t)] = aF_1(s) \pm bF_2(s)$

Differentiation $\quad L\left[\dfrac{d}{dt}f(t)\right] = sF(s) - f(0\pm)$

Integration $\quad L\left[\displaystyle\int f(t)dt\right] = \dfrac{F(s)}{s} + \dfrac{1}{s}\left[\displaystyle\int f(t)dt\right]_{t=0\pm}$

Convolution $\quad \displaystyle\int_0^t f_1(t-\tau)f_2(\tau)d\tau = F_1(s)F_2(s)$

Initial-value theorem $\quad f(0+) = \lim_{s\to\infty} sF(s)$

Final-value theorem $\quad \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

Differentiation

$$L\left[\frac{df(t)}{dt}\right] = sF(s)$$

$$L\left[\frac{d^2 f(t)}{dt^2}\right] = s^2 F(s)$$

$$L\left[\frac{d^3 f(t)}{dt^3}\right] = s^3 F(s)$$

Integration

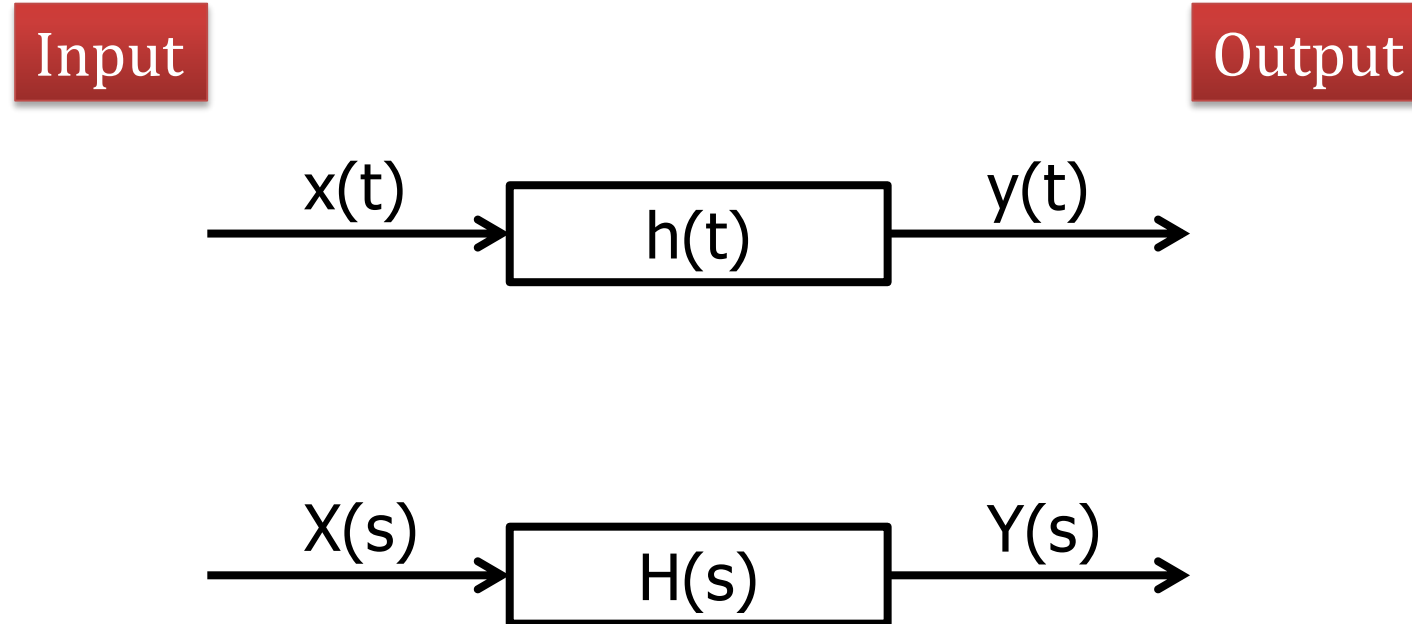$$L\left[\int_0^\infty f(t)dt\right] = \frac{F(s)}{s} + \frac{1}{s}\int_{0-}^{0+} f(t)dt$$

Insights from Laplace Transforms

- What the Laplace Transform says about f(t)
  - Value of f(0)
    - Initial value theorem
  - Does f(t) converge to a finite value?
    - Poles of F(s)
  - Does f(t) oscillate?
    - Poles of F(s)
  - Value of f(t) at steady state (if it converges)
    - Limiting value of F(s) as s->0

Block Diagram

Input

Output

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$X(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s)$$

Transfer Function

- Definition
  - H(s) = Y(s) / X(s)

$X(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s)$
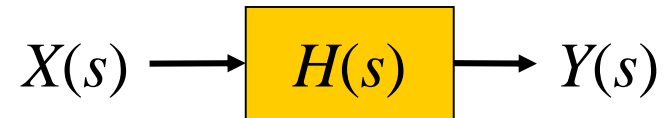
- Relates the output of a linear system (or component) to its input

- Describes how a linear system responds to an impulse

- All linear operations allowed
  - Scaling, addition, multiplication

Actuators + Sensors

- Presented actuators and sensors, e.g.:
  - Electrical motor
  - Angular position encoder
- How to get the best precision/performance?

Electric Motor

Potentiometer
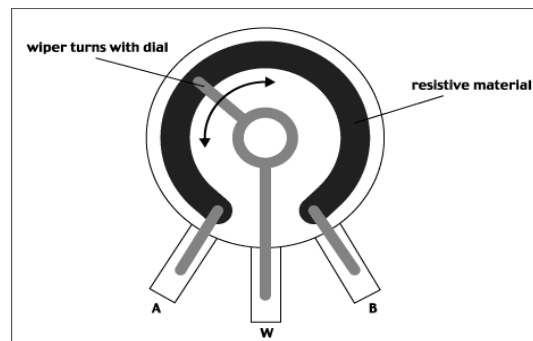
Optical Encoder
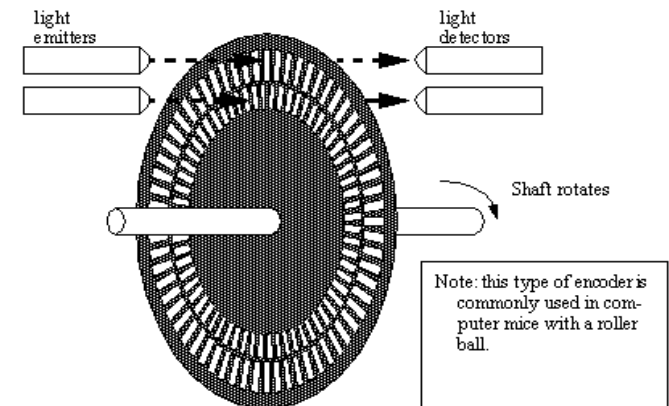
Open-loop

- Goal: move a differential drive robot
  - spin motors at a given angular velocity
- How: apply a fixed voltage to it,
  - and <u>never</u> check to see if it is rotating properly…

| Computer | Power Amplifier | Voltage | Electric Motor | Angular Velocity |
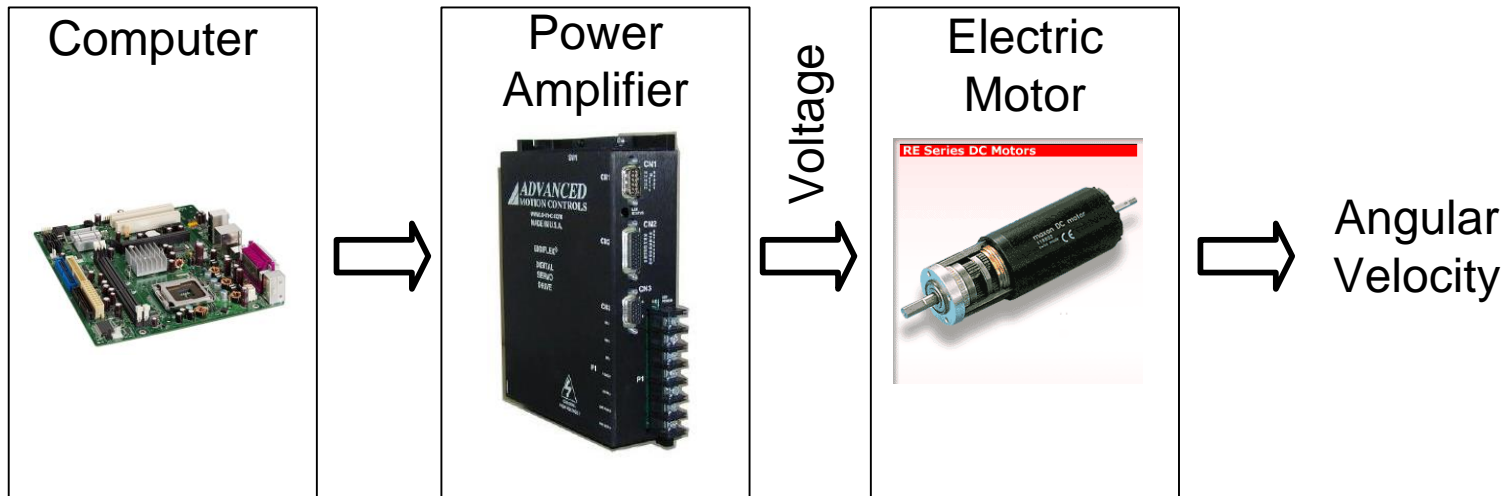
Open-loop

- Changing load on the motor?
  - Output velocity will change!

speed ω — no torque at max speed

Fixed Voltage

torque τ

stall torque
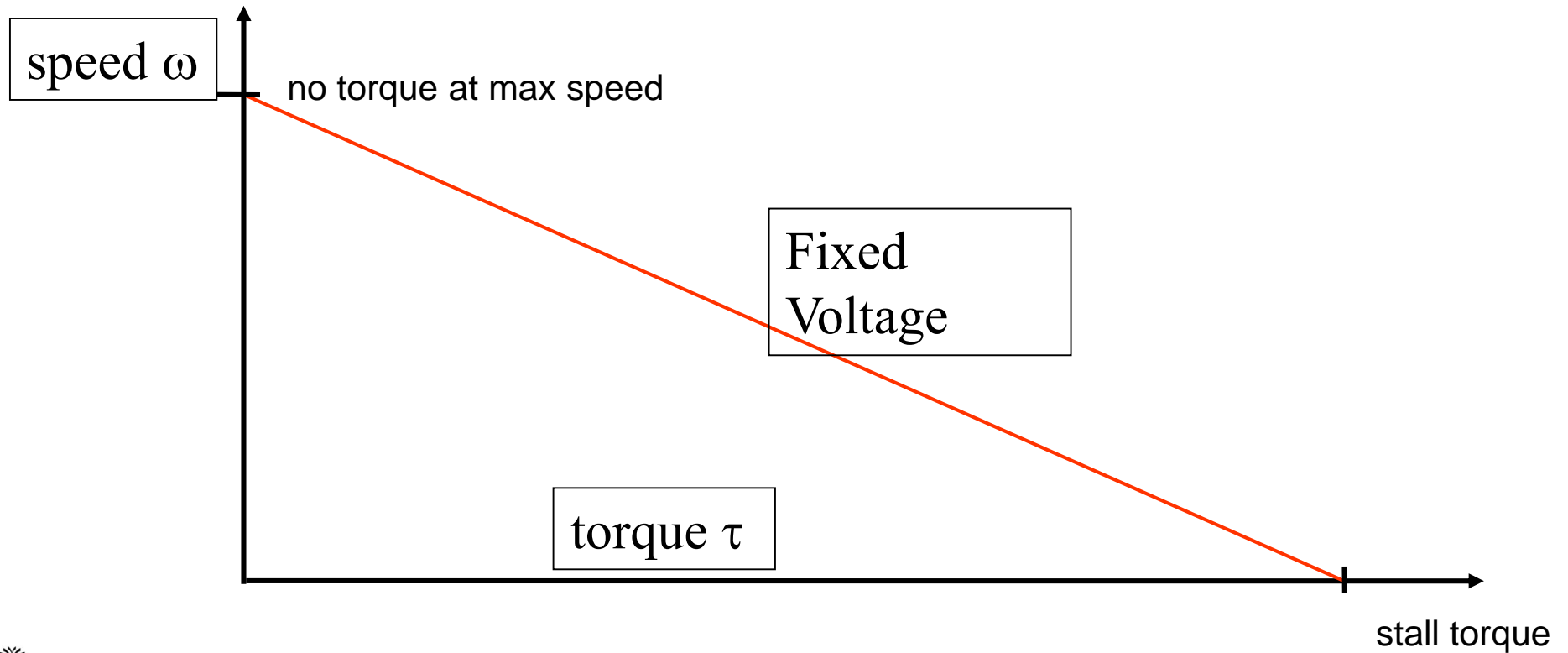
Closing the loop

- Let's measure the actual angular velocities.
- Compensate for changes in load by *feeding back* some information.
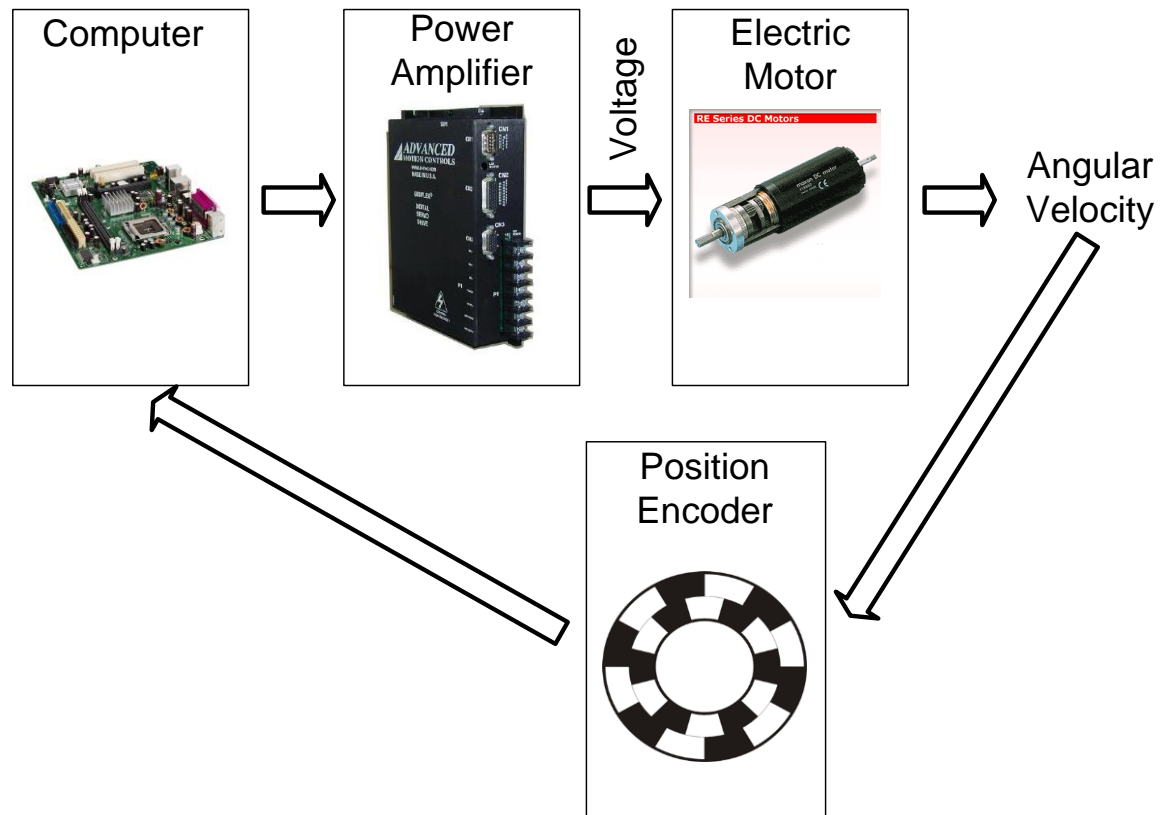
Control Theory

- Roots in another science: *Cybernetics*

> Cybernetics is the study of feedback and derived concepts such as communication and control in <u>living organisms</u>, <u>machines</u> and <u>organizations</u>.

- Expression was coined by Norbert Wiener in 1948.

(my academic great-great-grand-father)

Norbert Wiener (1913 Harvard University)
Amar Bose (1956 Massachusetts Institute of Technology)
Alan Oppenheim (1964 Massachusetts Institute of Technology)
Evangelos Milios (1986 Massachusetts Institute of Technology)
Rekleitis, Ioannis (2003 McGill University)
From http://genealogy.math.ndsu.nodak.edu/index.php
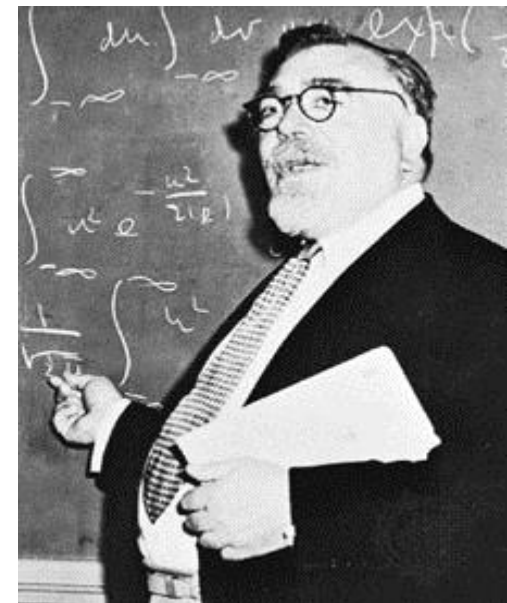
Early Example of Feedback System

- James Watt's "Centrifugal Governor" in 1788.

- Regulates the steam engine speed.
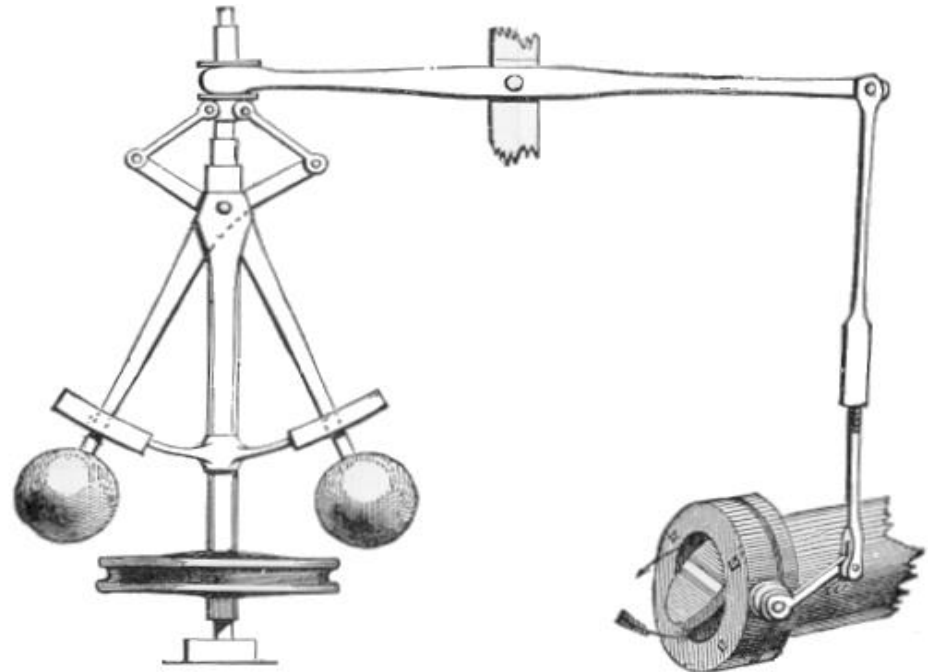
Other Examples

- Body temperature regulation
  - If cold, shiver (muscles produce heat)
  - If hot, sweat (evaporation takes away heat)
- Maintaining social peace
  - If a crime is found (sensor), the guilty party is punished (actuator).
- Cruise control in cars
- Banking industry regulation

Why Study Control Theory

- Used everywhere in robotics/mechatronics

- Systematic approach to analysis and design

- Taxonomy of controller concepts

Control Systems

- Regulation
  - Temperature (thermostat + heating)

- Tracking
  - robot joint rotation

- Process Optimization
  - maintaining best mix of chemicals

Components of Feedback Systems

- Power amplification
  - Neural signal power (μW) vs. muscle power output (tens of W)
  - Means it is an <u>active</u> system, as opposed to passive.
- Actuator
- Feedback
  - measurement (sensor)
- Error signal
- Controller

Classic Feedback Control Diagram
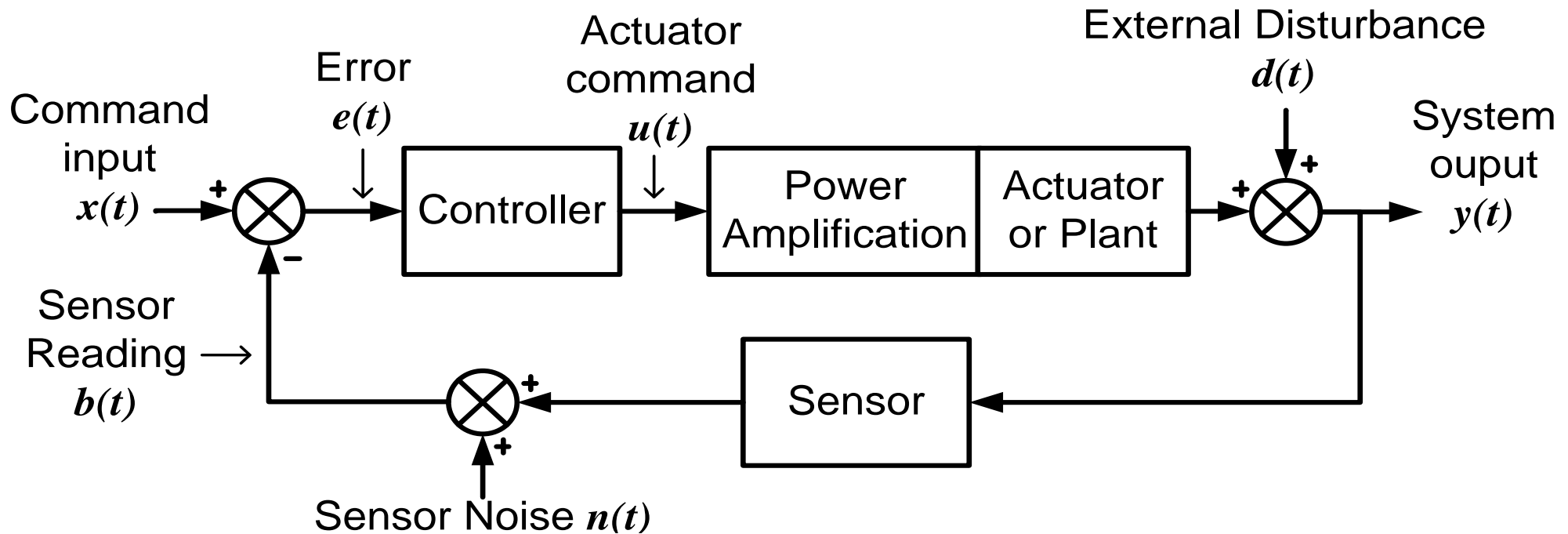
Transfer Function

- Definition: $H(s) = Y(s) / X(s)$

- Relates the output of a linear system to its input.

- Describes how a linear system responds to an impulse... called *impulse response*

Laplace Transform of Classic Feedback System

Key Transfer Functions

External Disturbance $D(s)$

Feedback

Feedforward

Error $E(s)$

Command input $X(s)$

Actuator command $U(s)$

Controller C(s)

Actuator A(s)

System ouput $Y(s)$

Sensor Reading $B(s)$
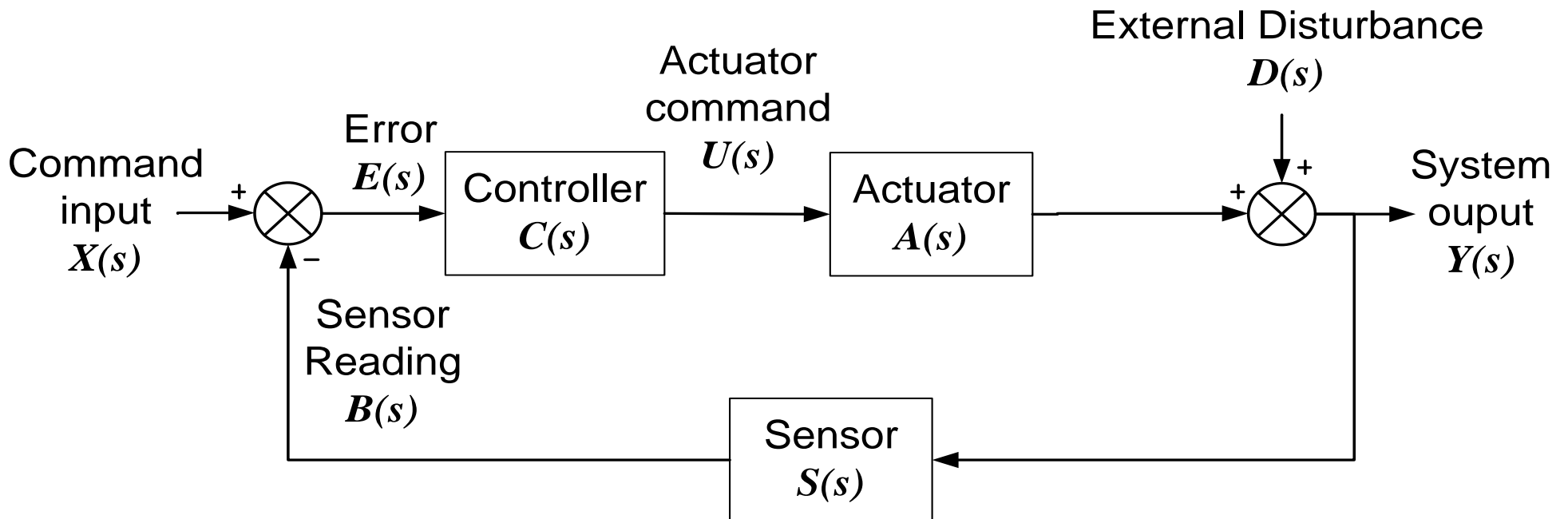
Sensor S(s)
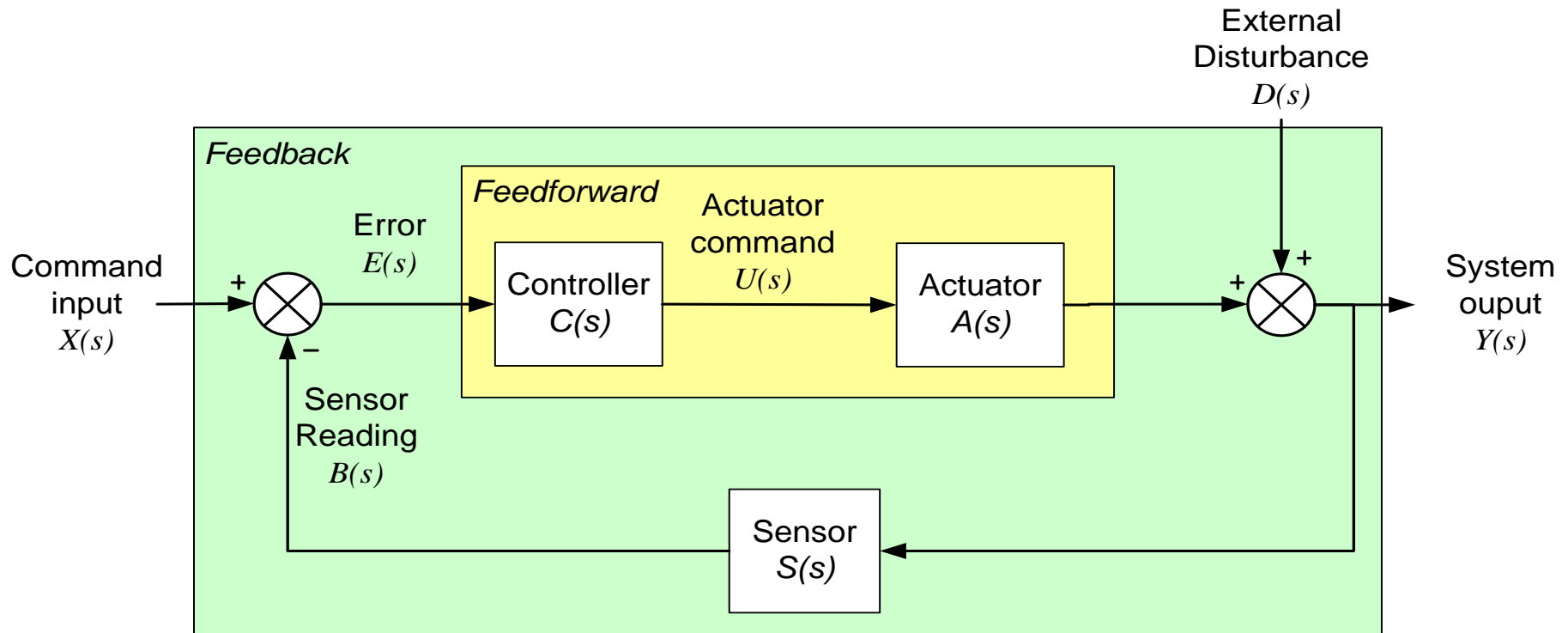
$$\text{Feedforward}: \frac{Y(s)}{E(s)} = C(s)A(s)$$

$$\text{Feedback}: \frac{Y(s)}{X(s)} = \frac{C(s)A(s)}{1 + C(s)A(s)S(s)}$$

First Order System
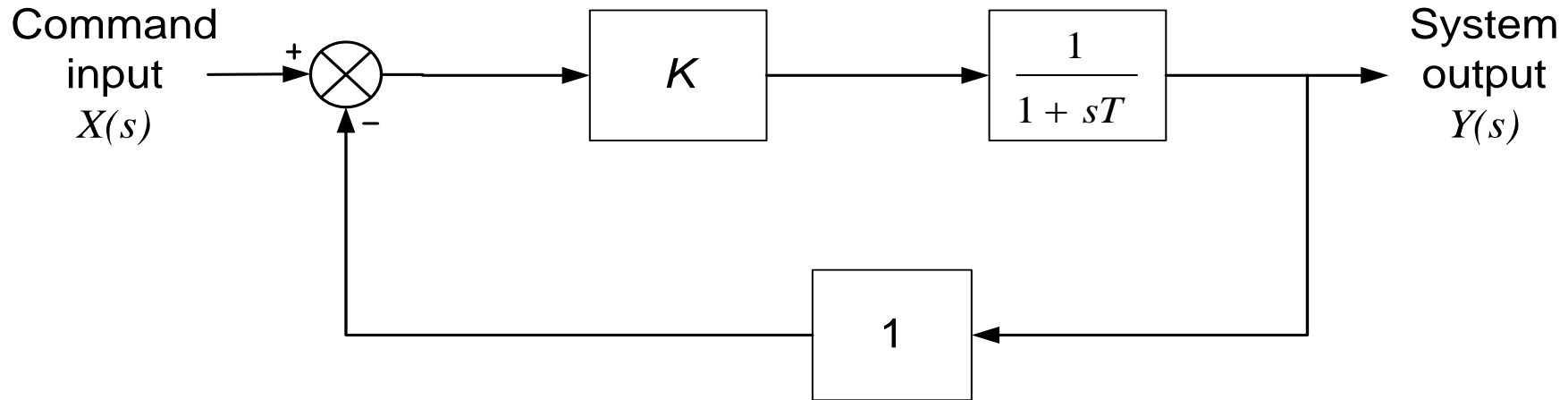
$$H(s) = \frac{Y(s)}{X(s)} = \frac{K}{1 + K + sT}$$

$u(t)$ is step function

Steady state value is given by Final Value Theorem

Steady state error is $\quad 1 - \lim_{s \to 0} sU(s)H(s)$

$$f(\infty) = \lim_{s \to 0} sF(s)$$

$$= 1 - \lim_{s \to 0} s\frac{1}{s}\{\frac{K}{1 + K + sT}\} = 1 - \frac{K}{1 + K} = \frac{1}{1 + K} \approx \frac{1}{K}$$

Response of 1ˢᵗ Order System

- Impulse $\dfrac{K}{1+K+sT}$

- Step

Time Domain

"step" function

Laplace Domain

$$\dfrac{1}{s}$$

X

$$\dfrac{K}{1+K+Ts}$$

=

$$\dfrac{K}{s(1+K+Ts)} = \dfrac{1}{s}\dfrac{K}{(1+K)} - \dfrac{K}{(1+K)\dfrac{1}{T}+s}$$
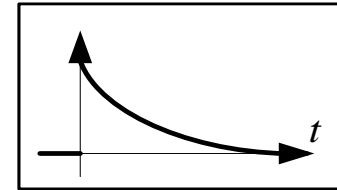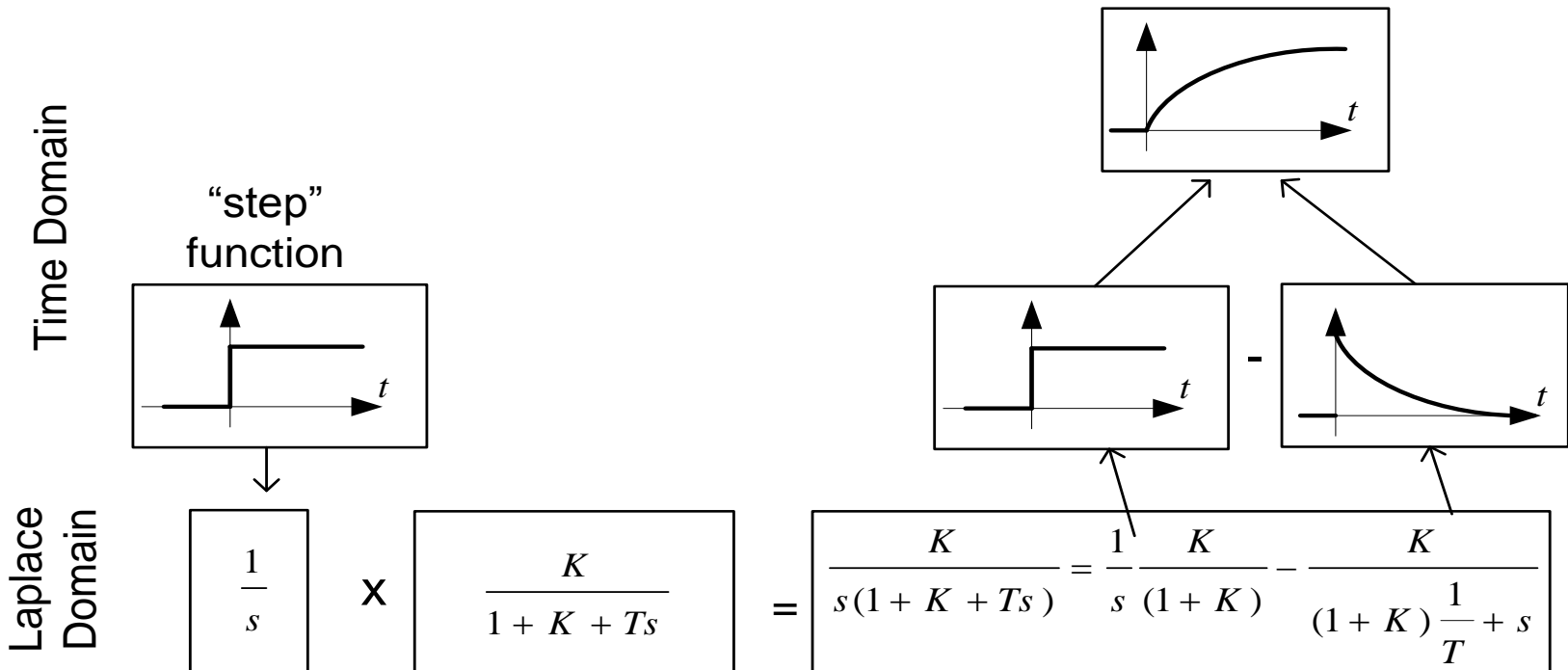
Steady-State vs. Transient

- Step Response illustrates how a system response can be decomposed into two components:

  – Steady-state part:
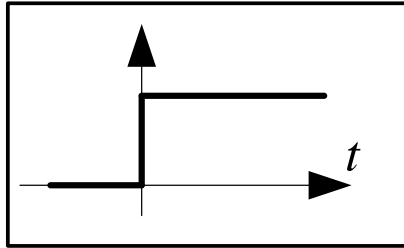
  – Transient
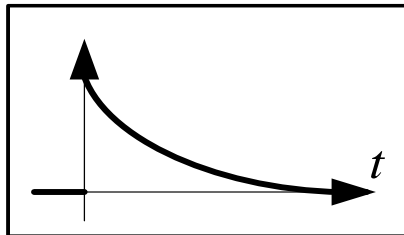
1st Order System: P Controller
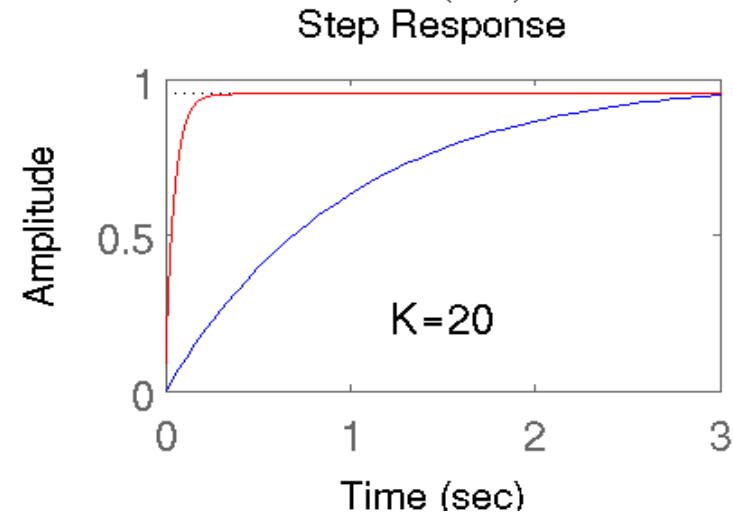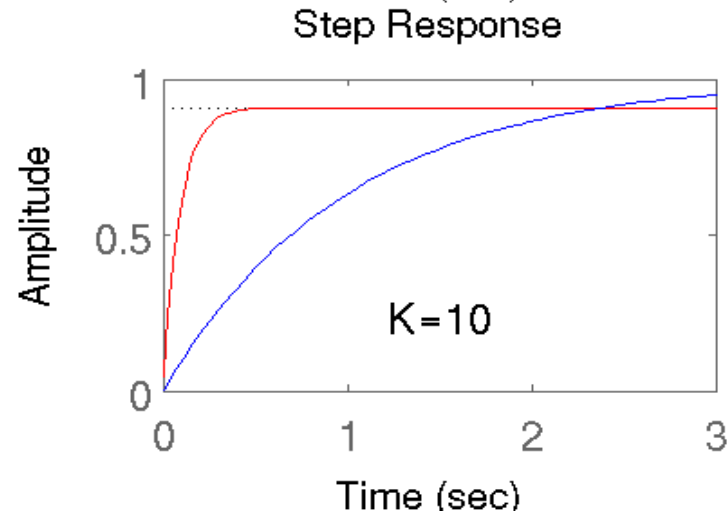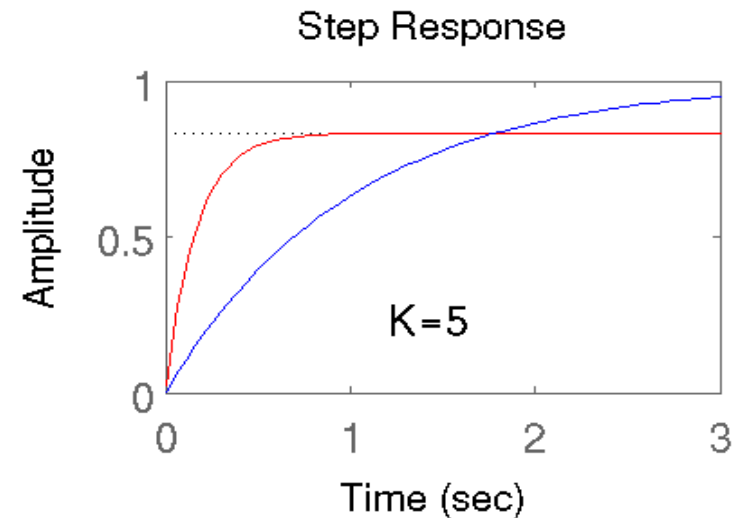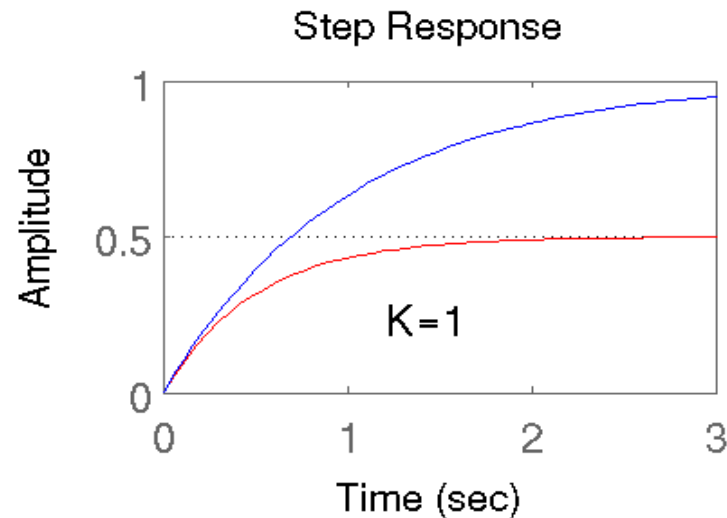
1st Order System: PD Controller
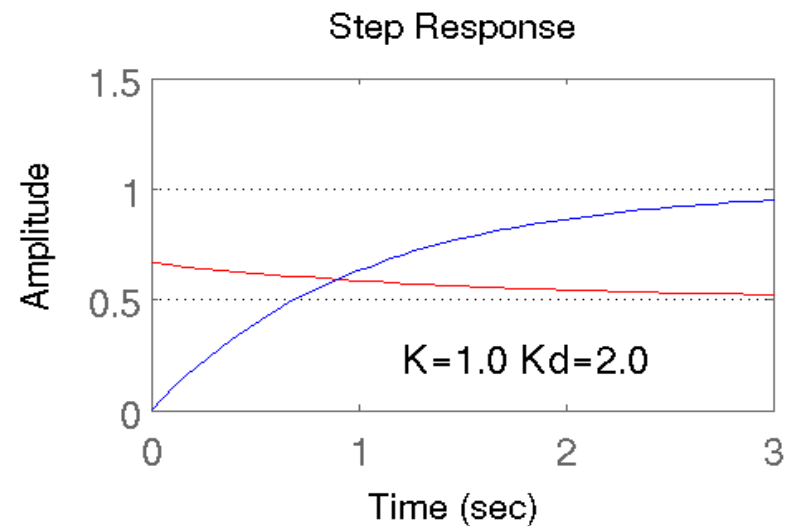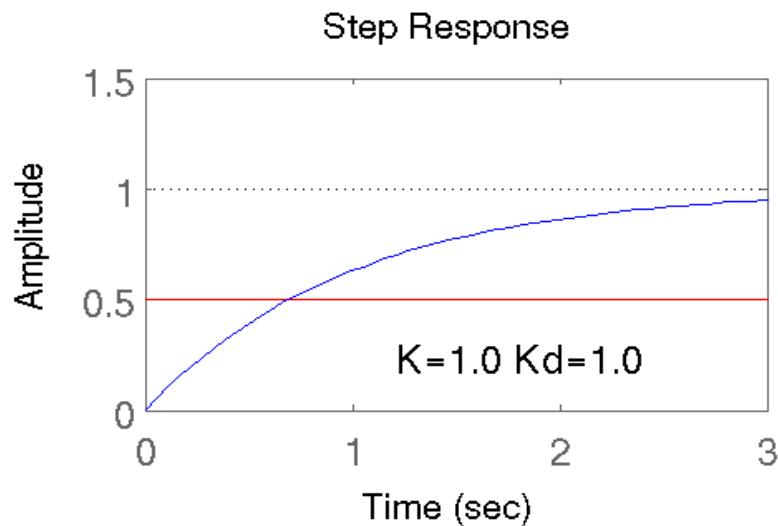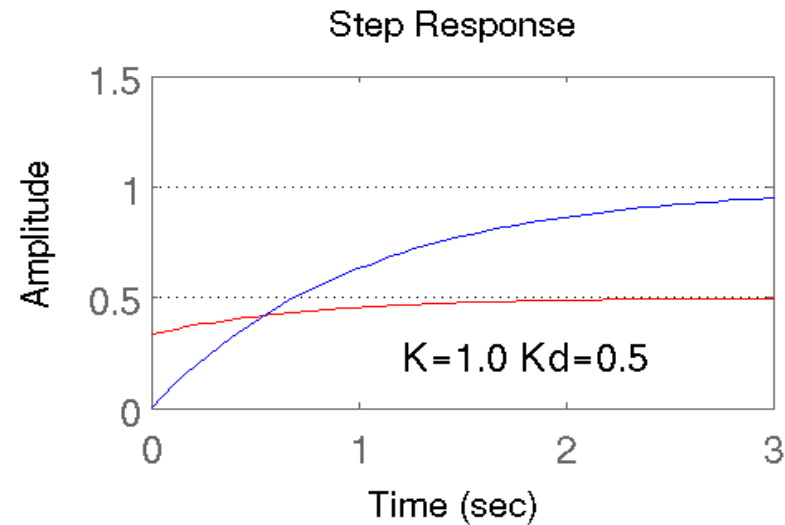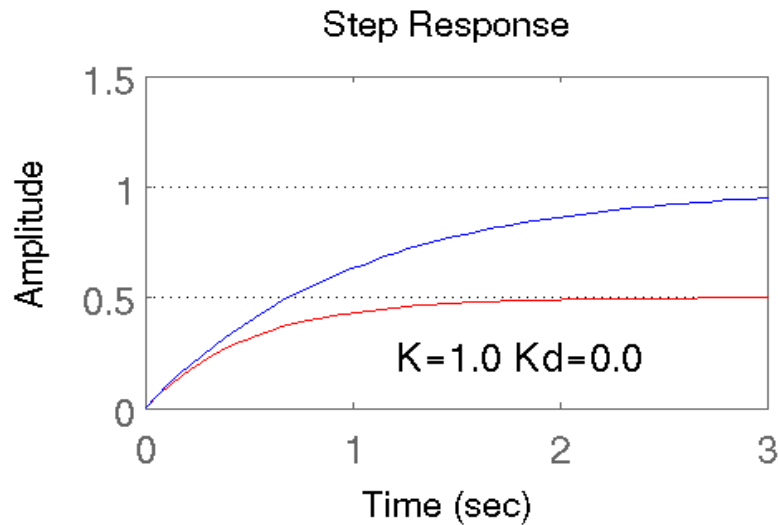
1ˢᵗ Order System: PI Controller
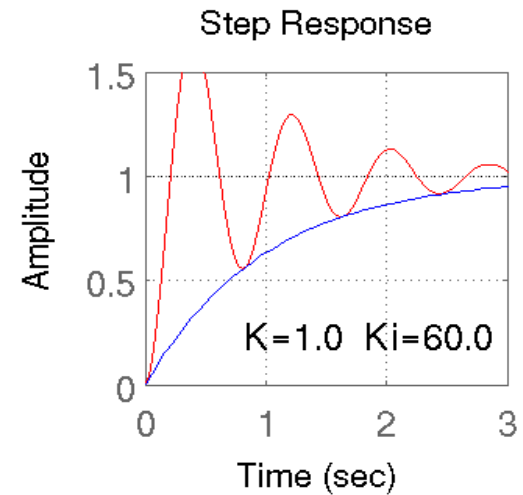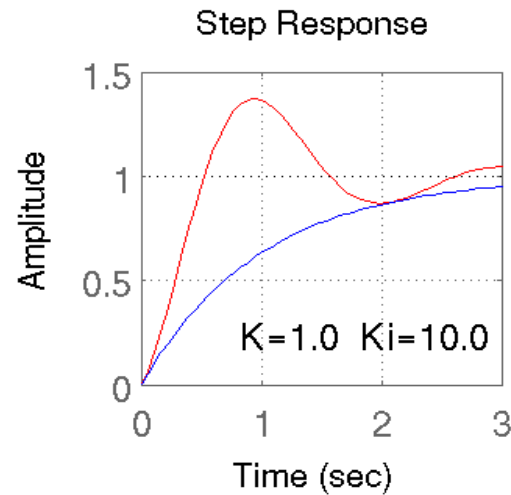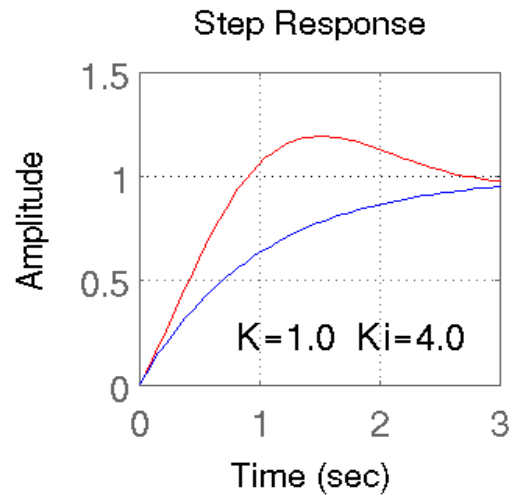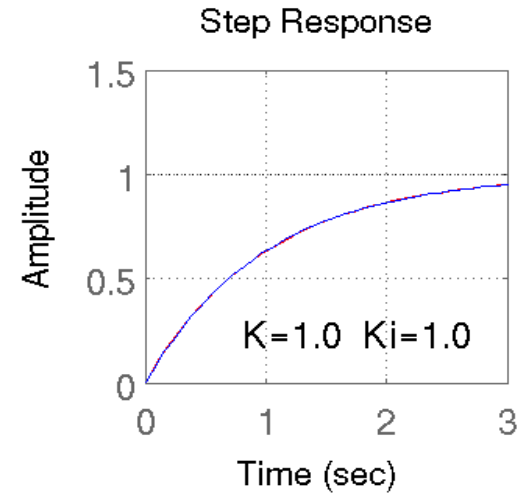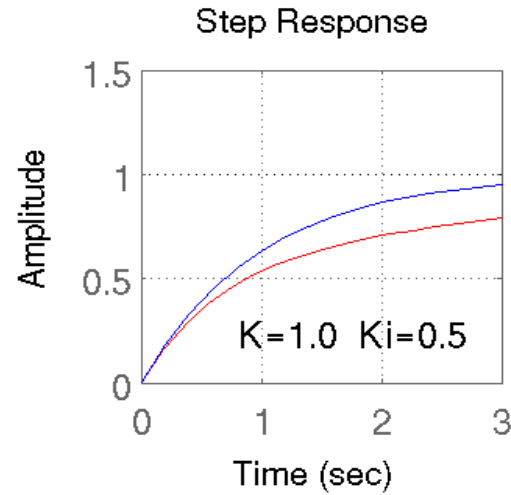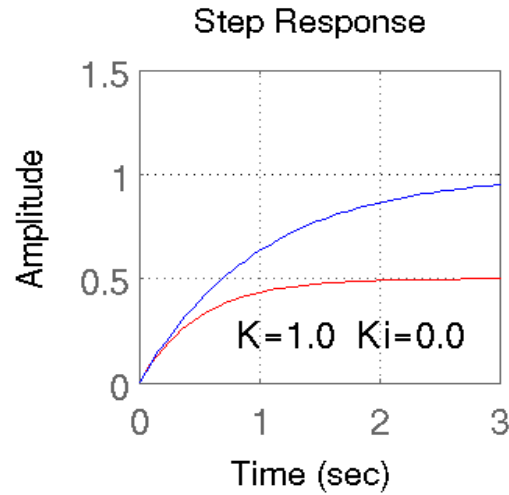
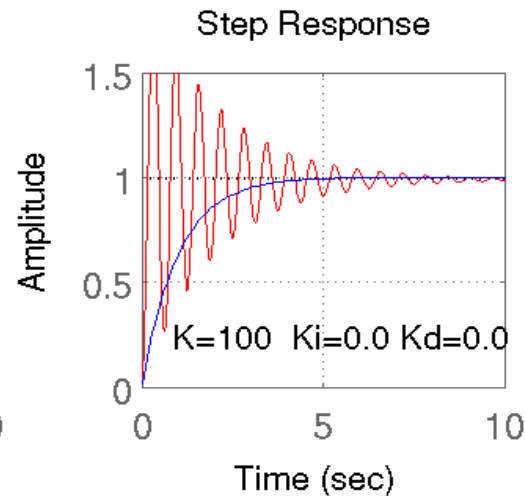# 1st Order System: PID Controller

Second Order Response

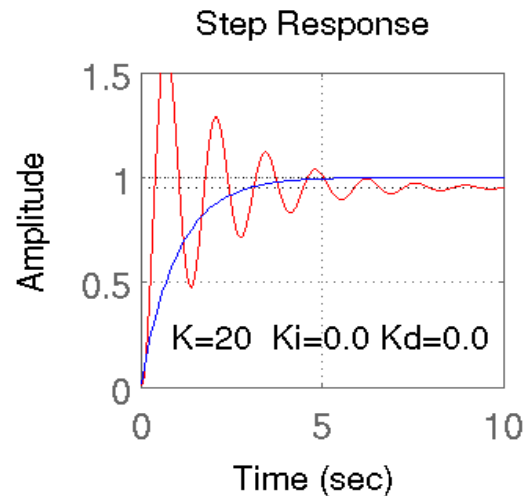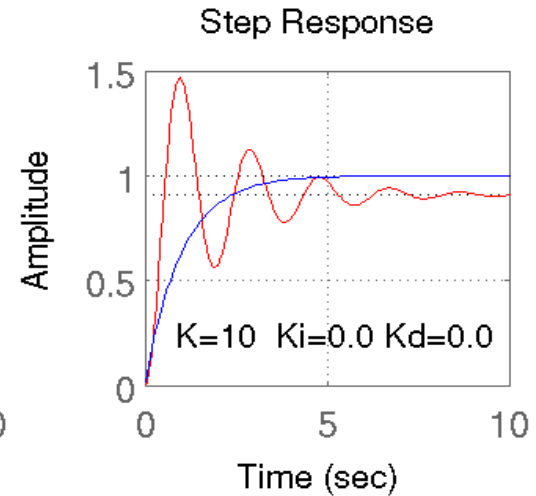

Command input $X(s)$ → + / − summing junction → K → $\dfrac{\omega_N^2}{s^2 + 2\xi\omega_N s + \omega_N^2}$ → System output $Y(s)$, with feedback gain 1
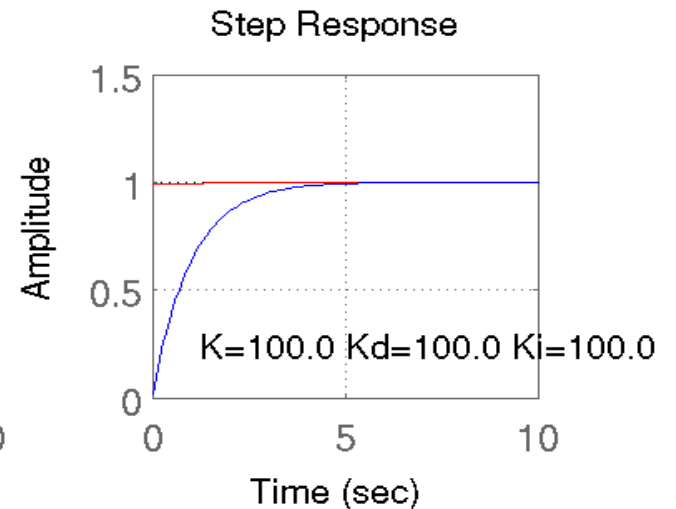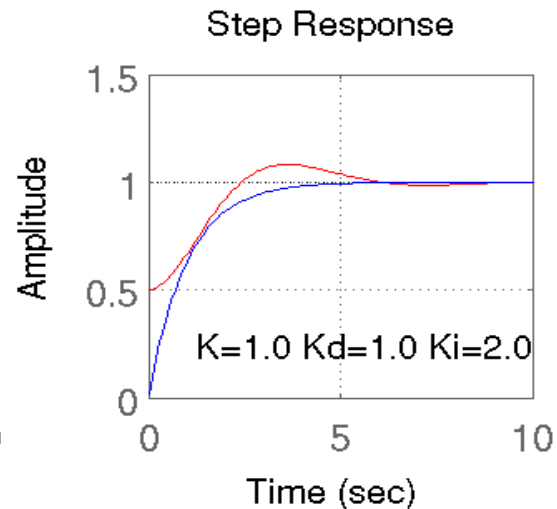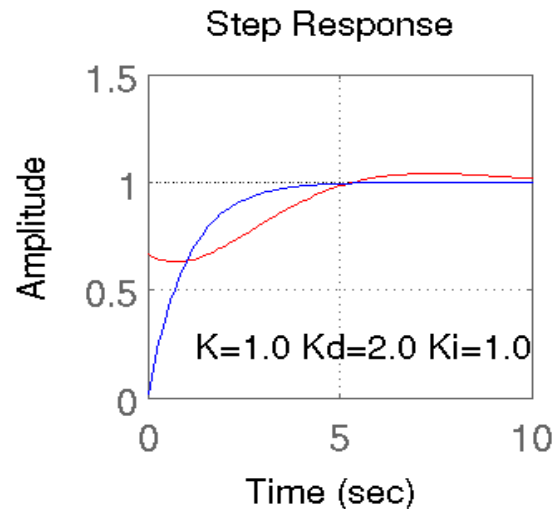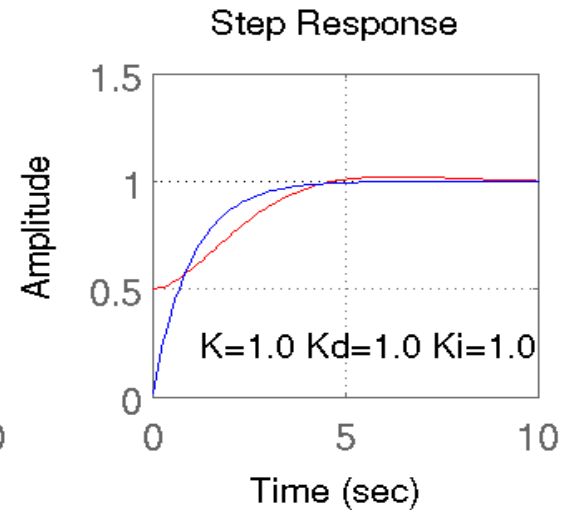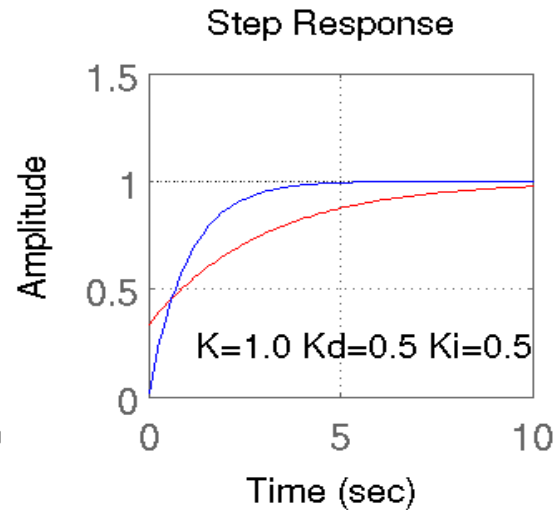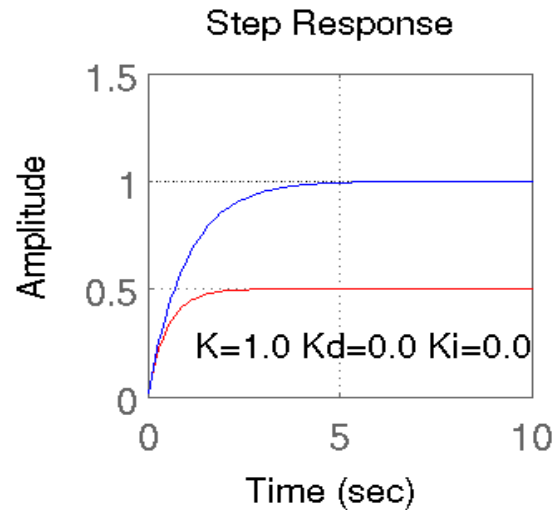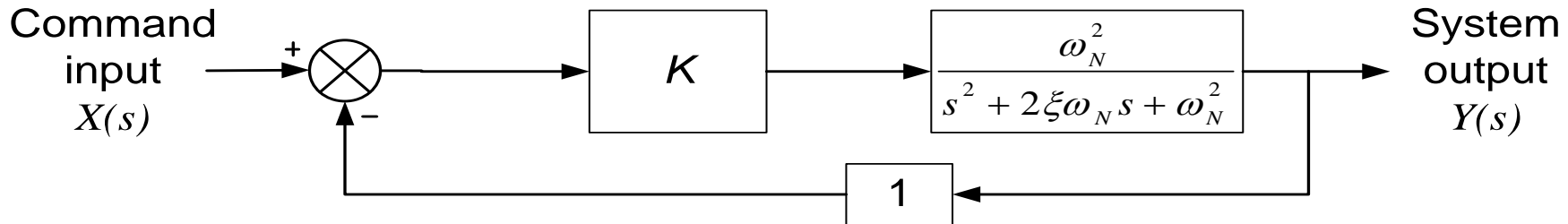
- Typical response to step input is:
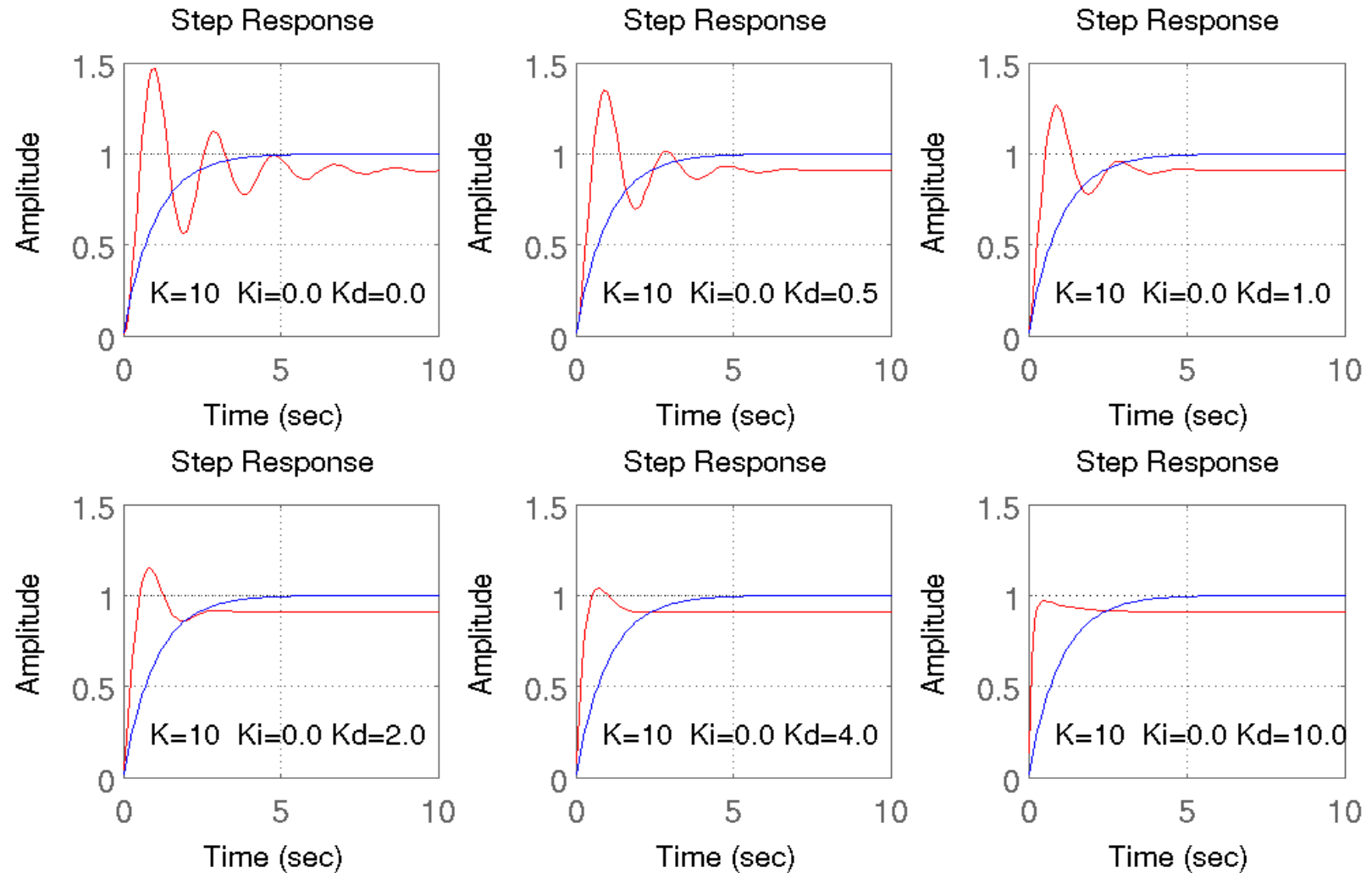
 − +

90%

overshoot

2% or 5%

settling time
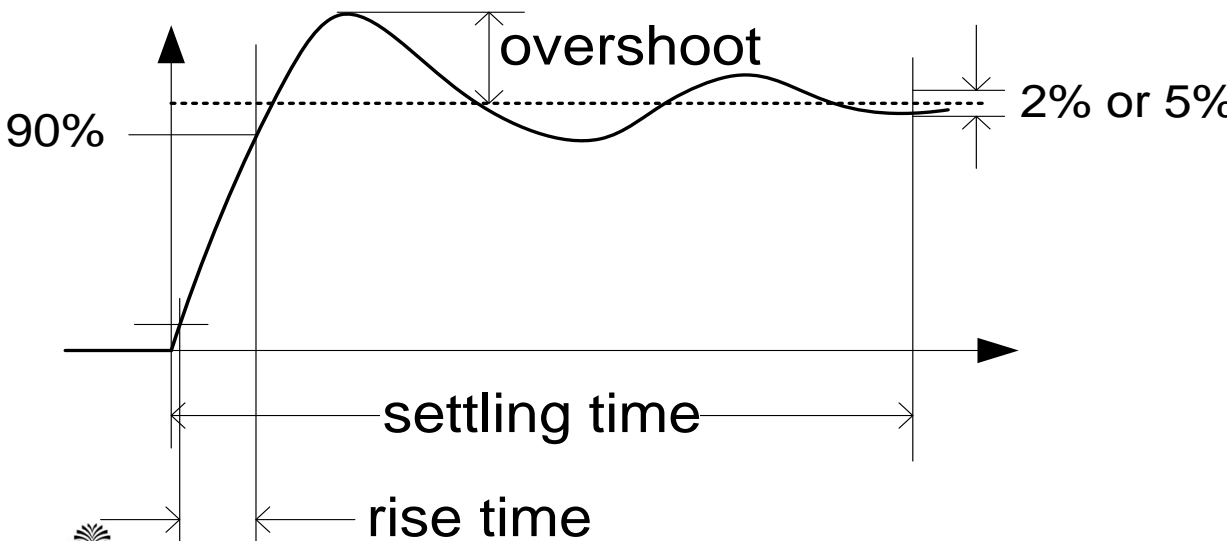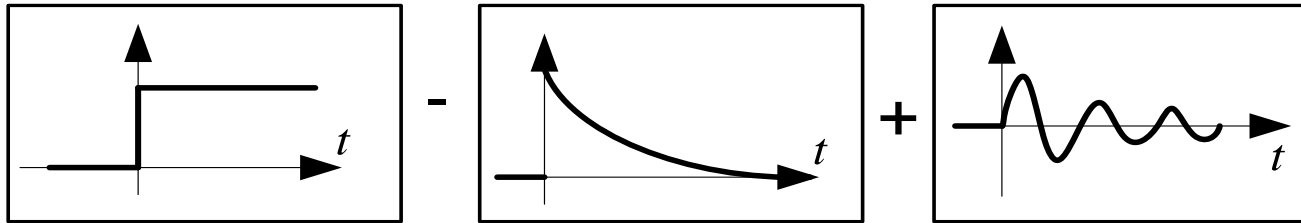
rise time

overshoot -- % of final value exceeded at first oscillation
rise time -- time to span from 10% to 90% of the final value
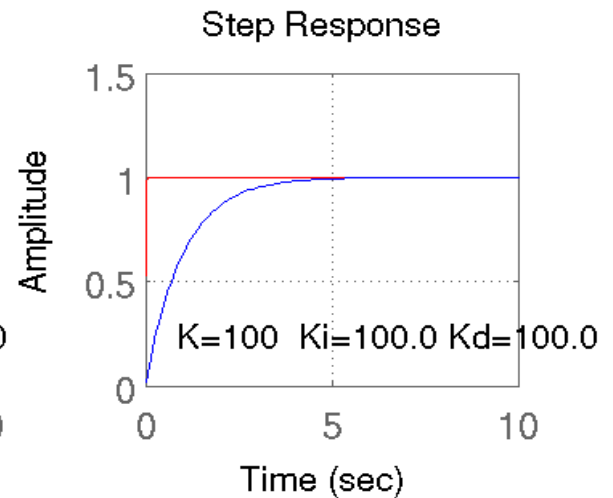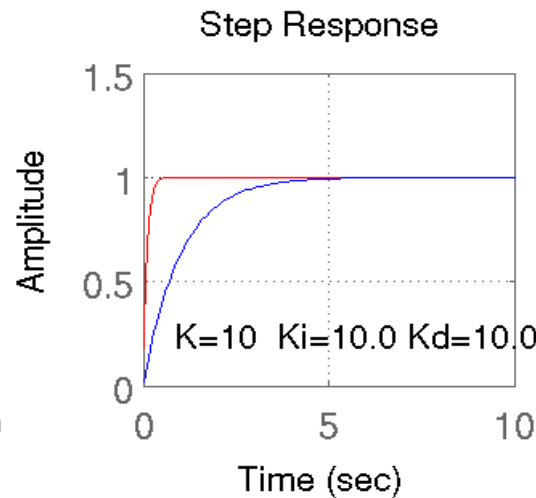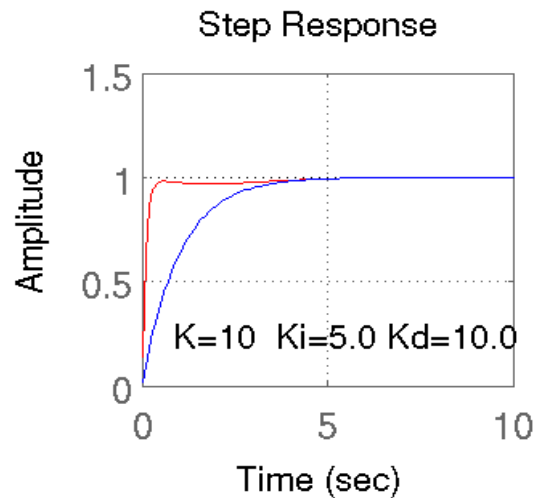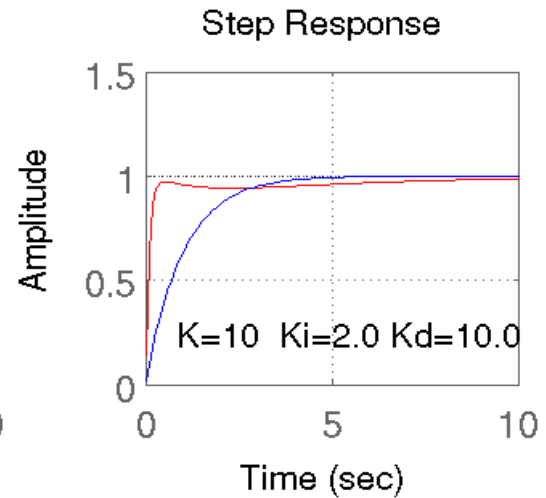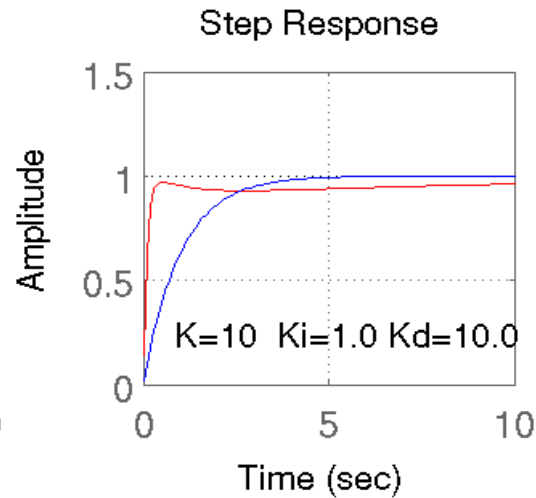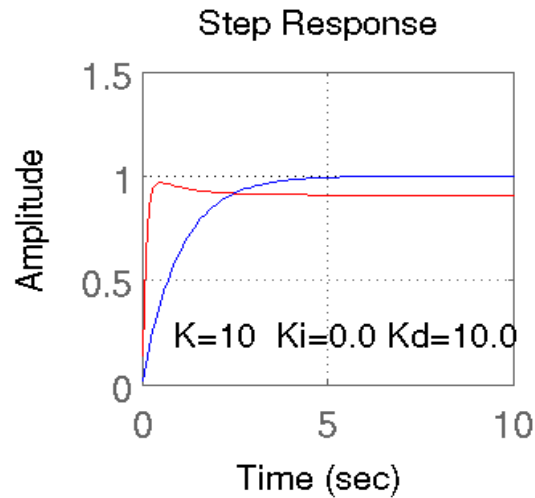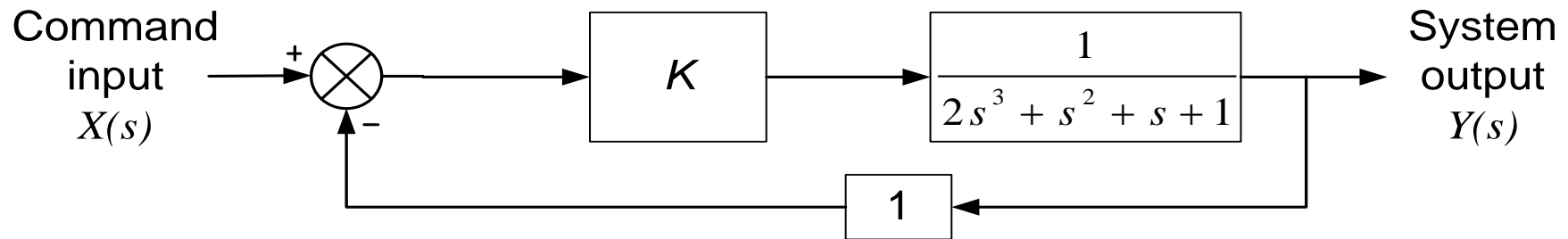**settling time** -- time to reach within 2% or 5% of the final value

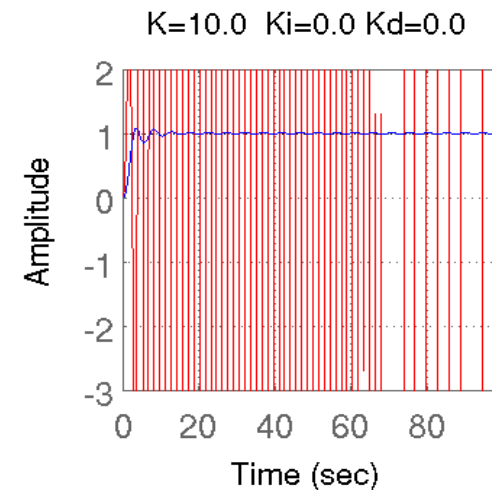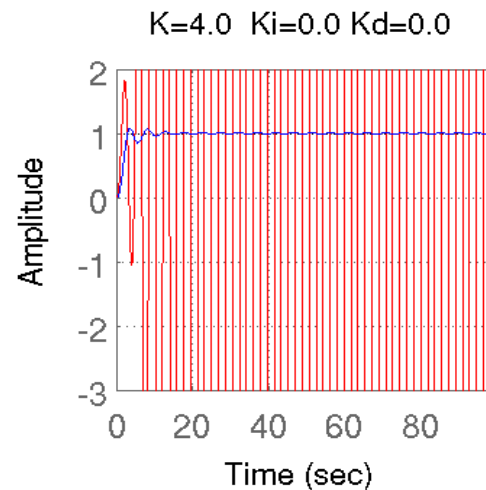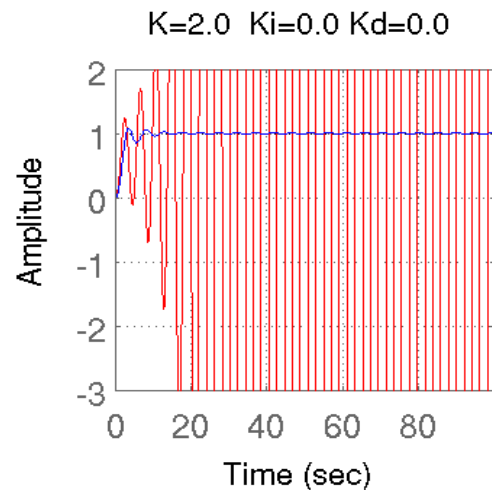# 2nd Order System: P Controller
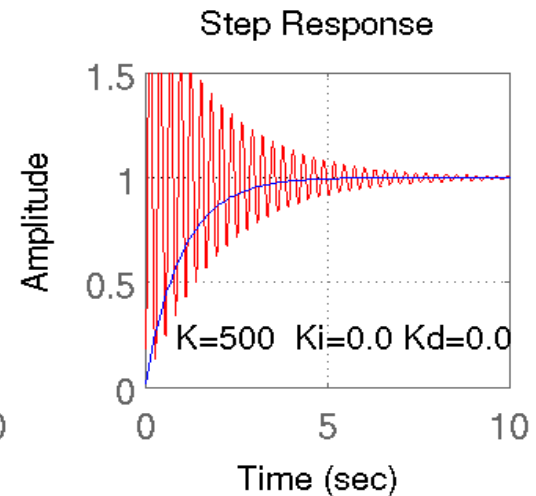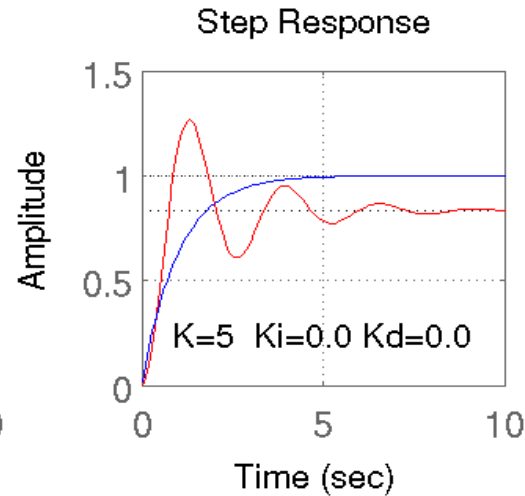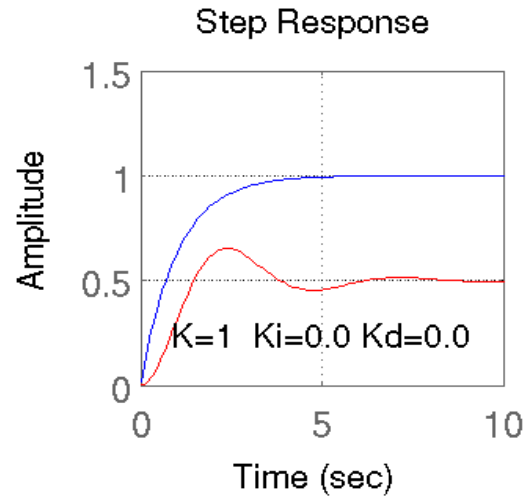
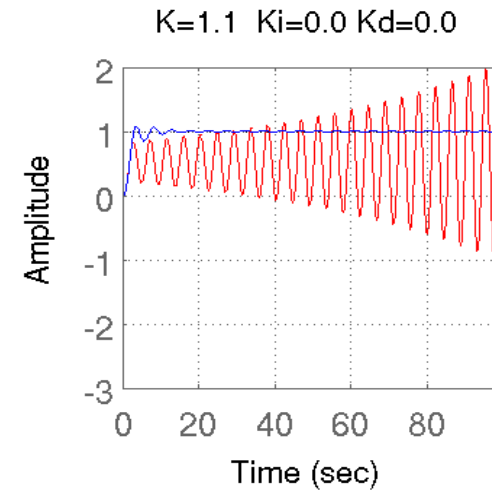# 2nd Order System: PD Controller

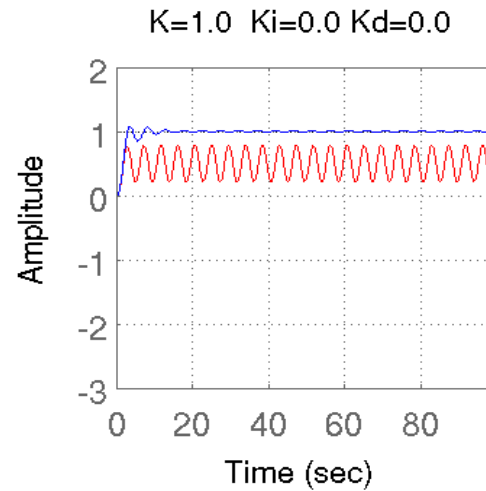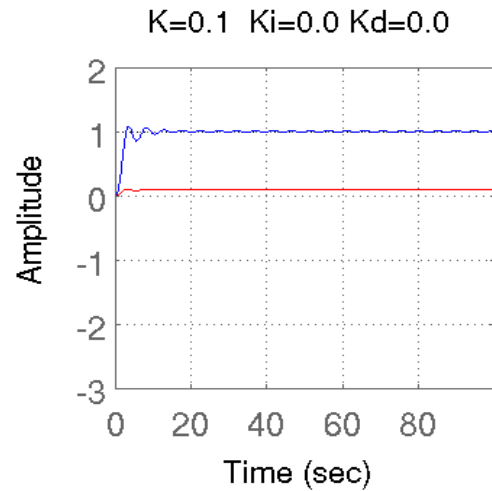# 2nd Order System: PID Controller

3rd Order System

Command
input
$X(s)$

+

−

K

$$\frac{1}{2s^3 + s^2 + s + 1}$$

1

System
output
$Y(s)$

3rd Order System: P Controller

3rd Order System: PD Controller

K=10.0 Ki=0.0 Kd=4.0

K=10.0 Ki=0.0 Kd=10.0

K=10.0 Ki=0.0 Kd=20.0

K=10.0 Ki=0.0 Kd=40.0

K=10.0 Ki=0.0 Kd=100.0

K=10.0 Ki=0.0 Kd=500.0
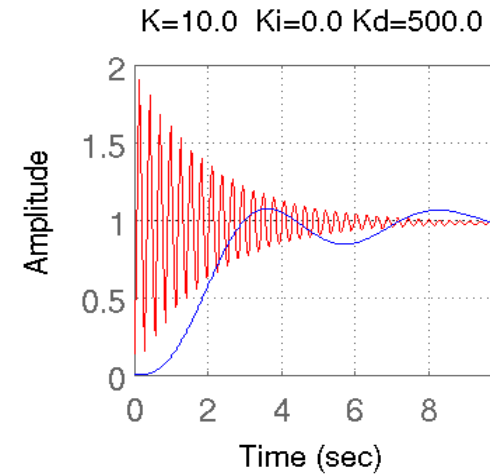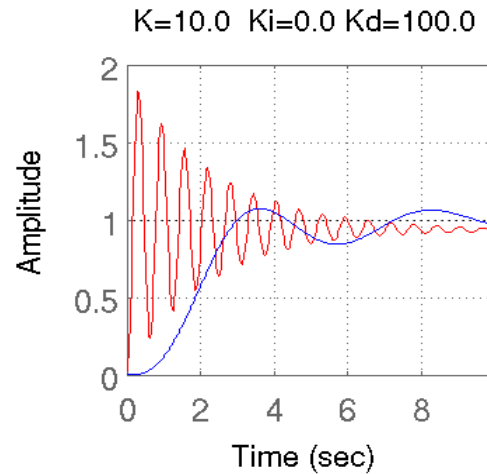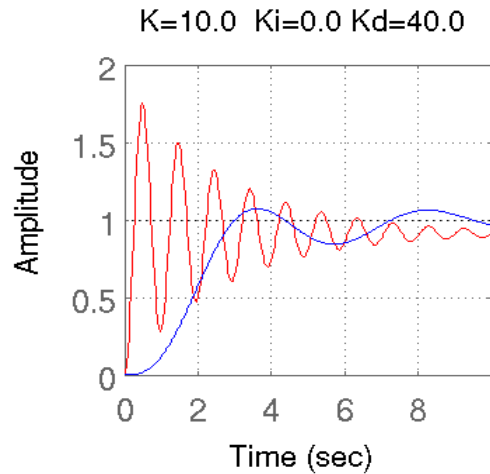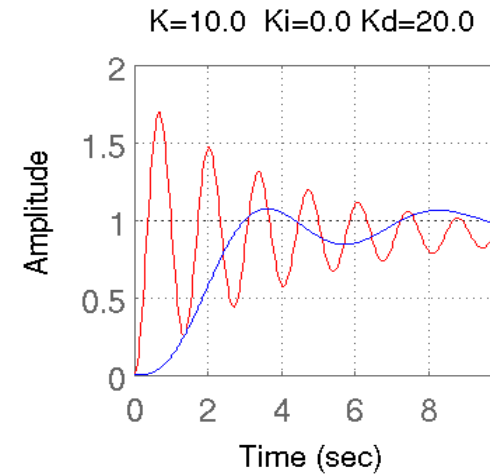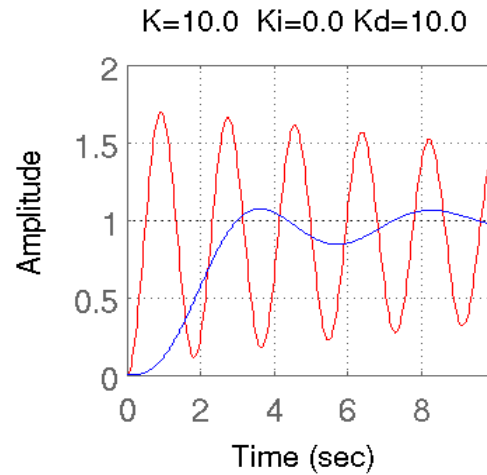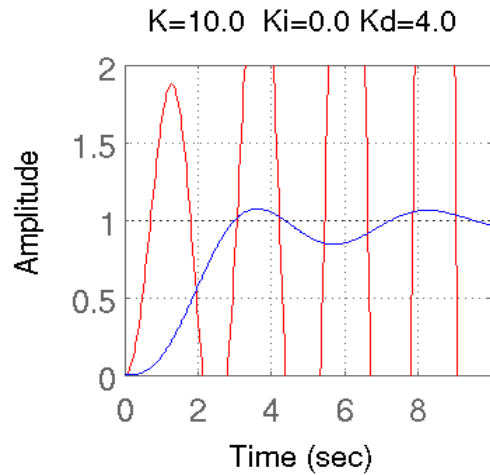
3rd Order System: PID Controller
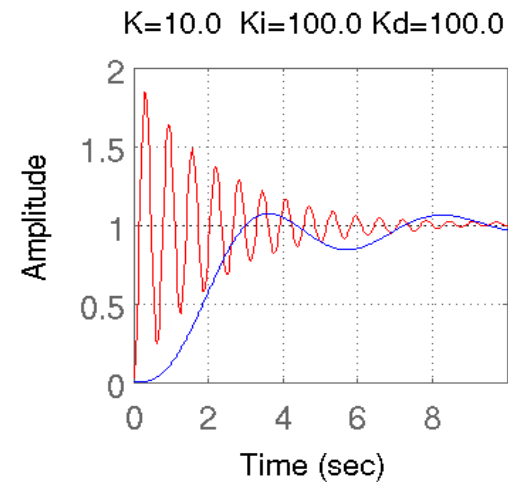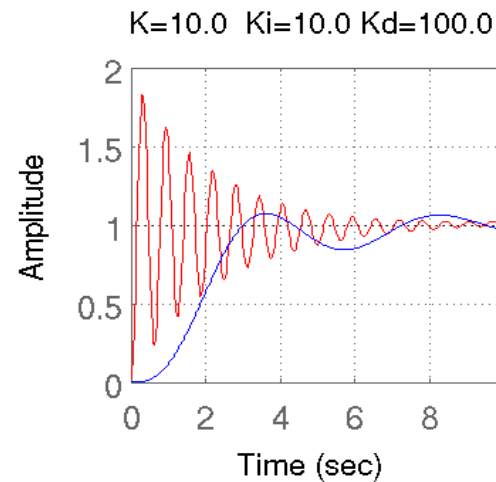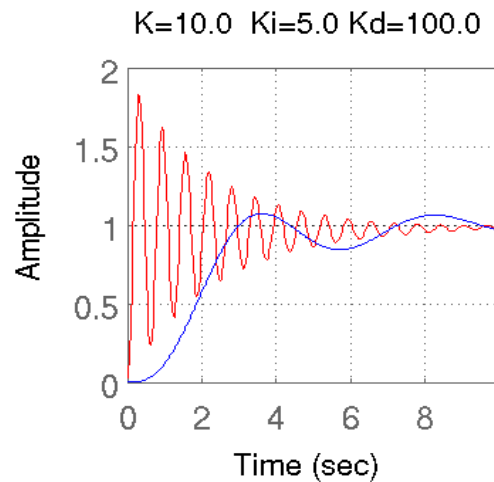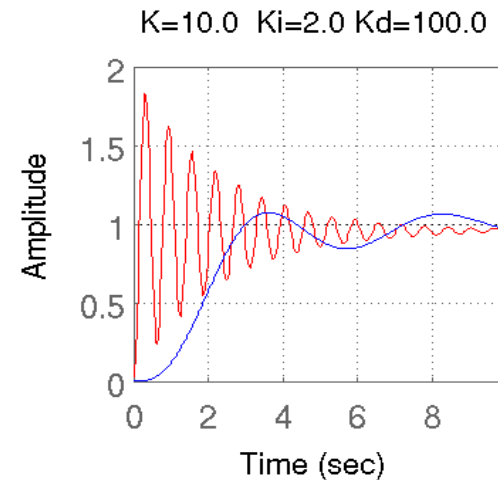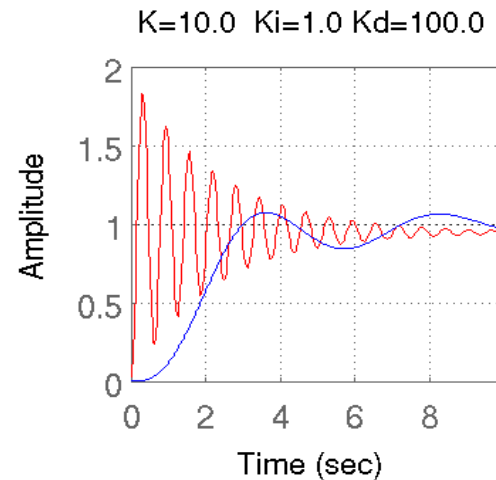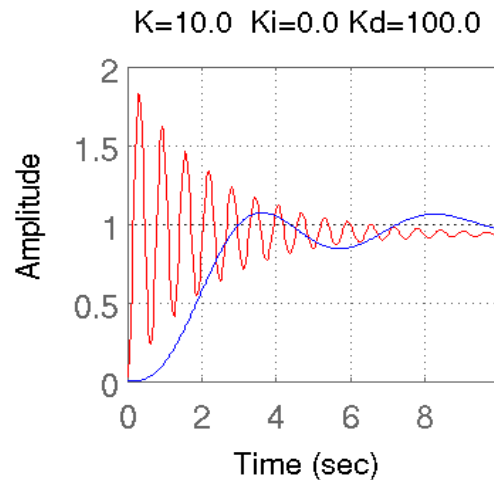
Basic Controller Functions

Proportional control: $\quad u(t) = K_p e(t) \qquad \dfrac{U(s)}{E(s)} = K_p$

Integral control: $\quad u(t) = K_i \displaystyle\int_0^t e(t)dt \qquad \dfrac{U(s)}{E(s)} = \dfrac{K_i}{s}$

Differential control: $\quad u(t) = K_d \dfrac{d}{dt} e(t) \qquad \dfrac{U(s)}{E(s)} = K_d s$

Effect of Controller Functions

- Proportional Action
  - Simplest Controller Function
- Integral Action
  - Eliminates steady-state error
  - Can cause oscillations
- Derivative Action ("rate control")
  - Effective in transient periods
  - Provides faster response (higher sensitivity)
  - Never used alone

PID Tuning

How to get the PID parameter values ?

(1) If we know the transfer function, analytical methods can be used (e.g., root-locus method) to meet the transient and steady-state specs.

(2) When the system dynamics are not precisely known, we must resort to experimental approaches.

Ziegler-Nichols Rules for Tuning PID Controller:

Using only Proportional control, turn up the gain until the system oscillates w/o dying down, i.e., is marginally stable. Assume that K and P are the resulting gain and oscillation period, respectively.

Then, use

for P control	for PI control	for PID control

$K_p = 0.5\,K$

$K_p = 0.45\,K$
$K_i = 1.2\,/\,P$

$K_p = 0.6\,K$
$K_i = 2.0\,/\,P$
$K_d = P\,/\,8.0$

Ziegler-Nichols Tuning for second or higher order systems

Advanced Control Topics

- Adaptive Control
  - Controller changes over time (adapts).
- MIMO Control
  - Multiple inputs and/or outputs.
- Predictive Control
  - You measure disturbance and react before measuring change in system output.
- Optimal Control
  - Controller minimizes a cost function of error and control energy.
- Nonlinear systems
  - Neuro-fuzzy control.
  - Challenging to derive analytic results.

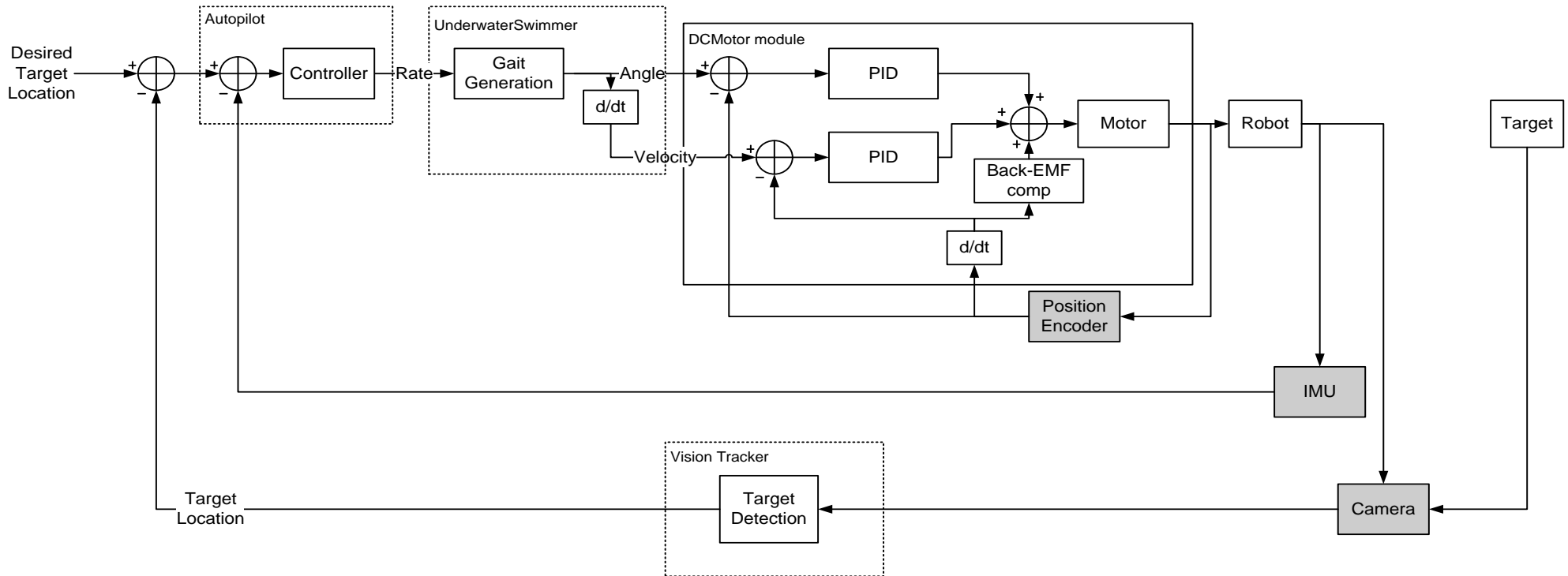# Layered Approach to Control

- Robotic systems often have a layered approach to control:

Planner

Autopilot

Attitude Controller

Flipper Position Controller
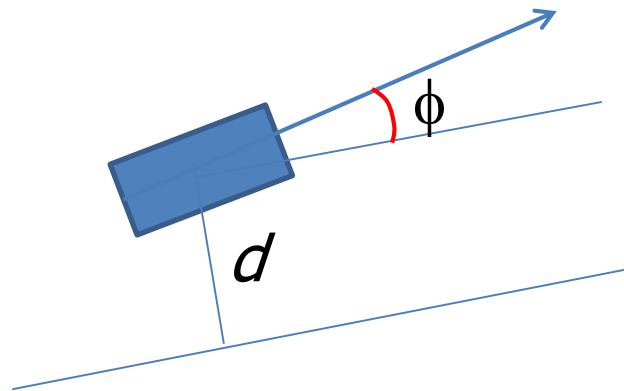
Motor
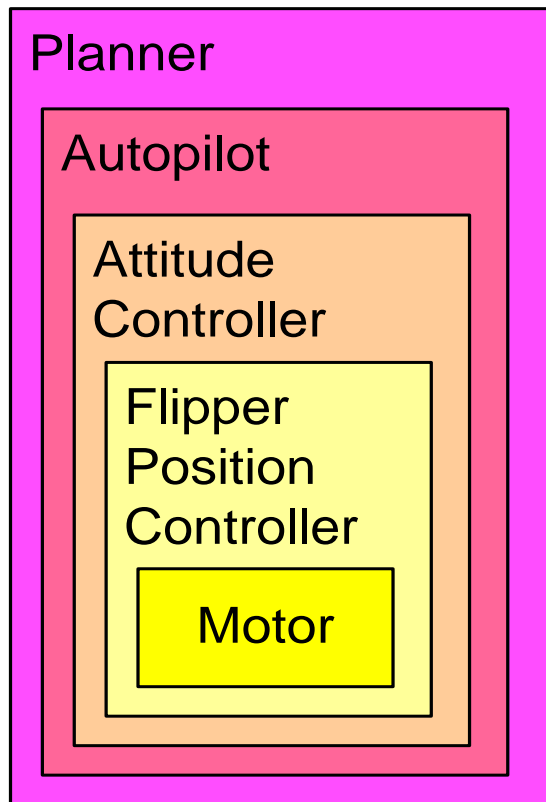
[movie](#)

Multiple Loops

Inner loops are generally "faster" that outer loop.

Line Following

- Consider a 2D robot with pose (x, y, θ) following a line. The robot is moving with constant speed v, and we control the angular velocity ω *(turnrate)*.
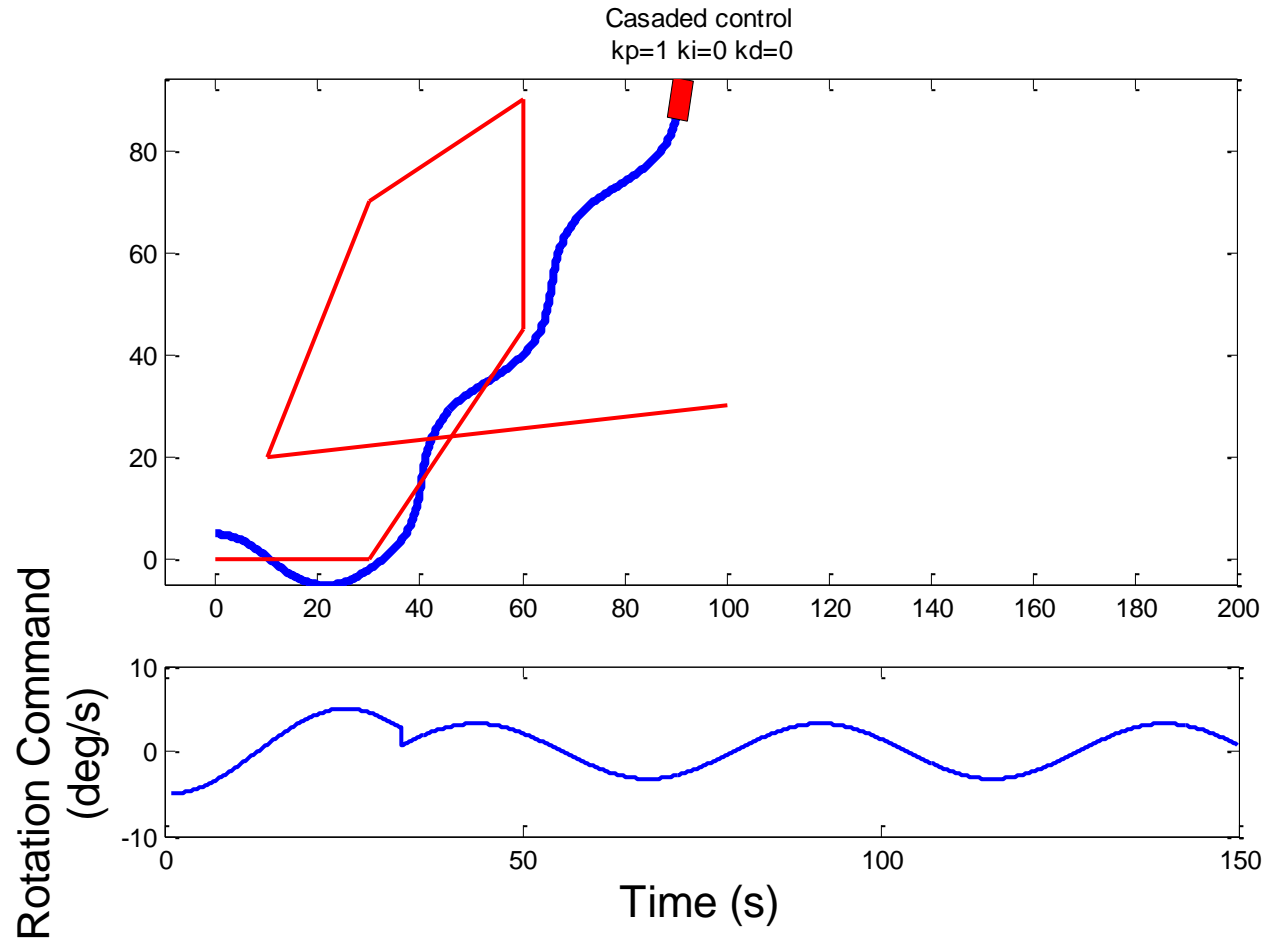
Calculating the error

- Consider two error measures,
  - d

  - ϕ

- The orientation error is used to estimate when we are heading too much in the wrong direction
  - if $|\phi| < 90°$ ω is bounded

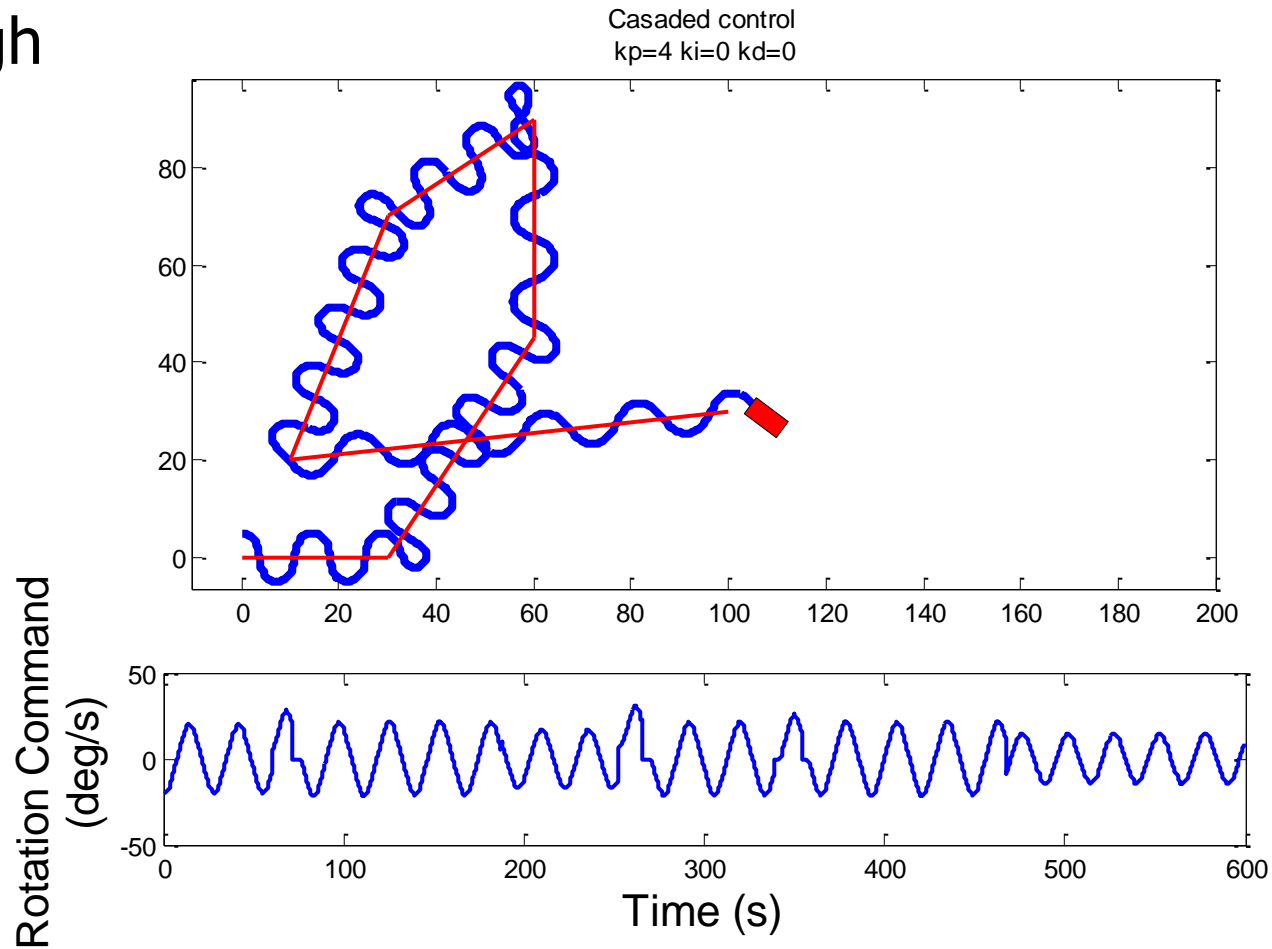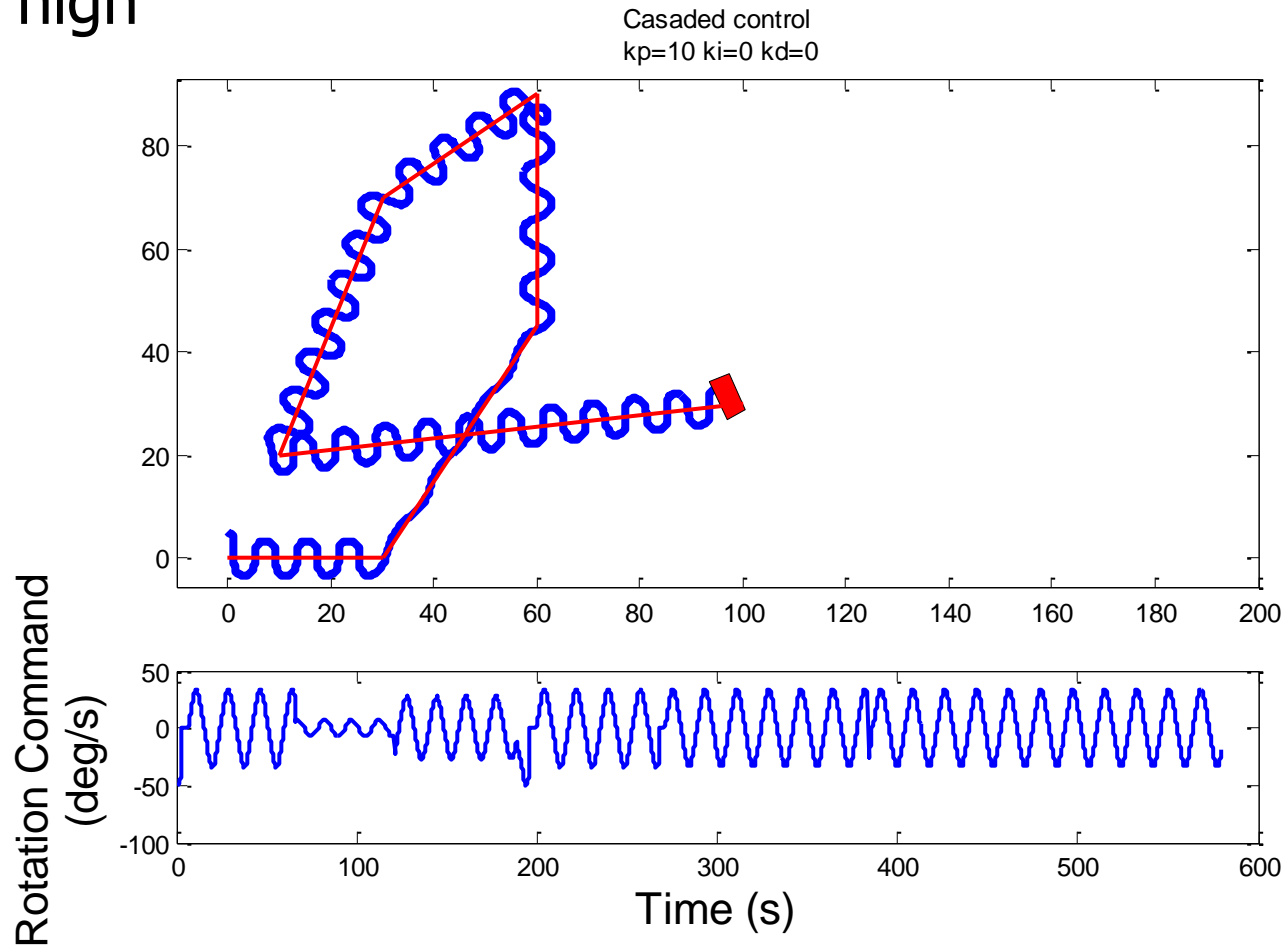- The distance error is what we feed to our controller.

P-Controller

Gain too low

Casaded control
kp=1 ki=0 kd=0

P-Controller

Gain high

Casaded control
kp=4 ki=0 kd=0

P-Controller

Gain too high

Casaded control
kp=10 ki=0 kd=0

Pd-Controller

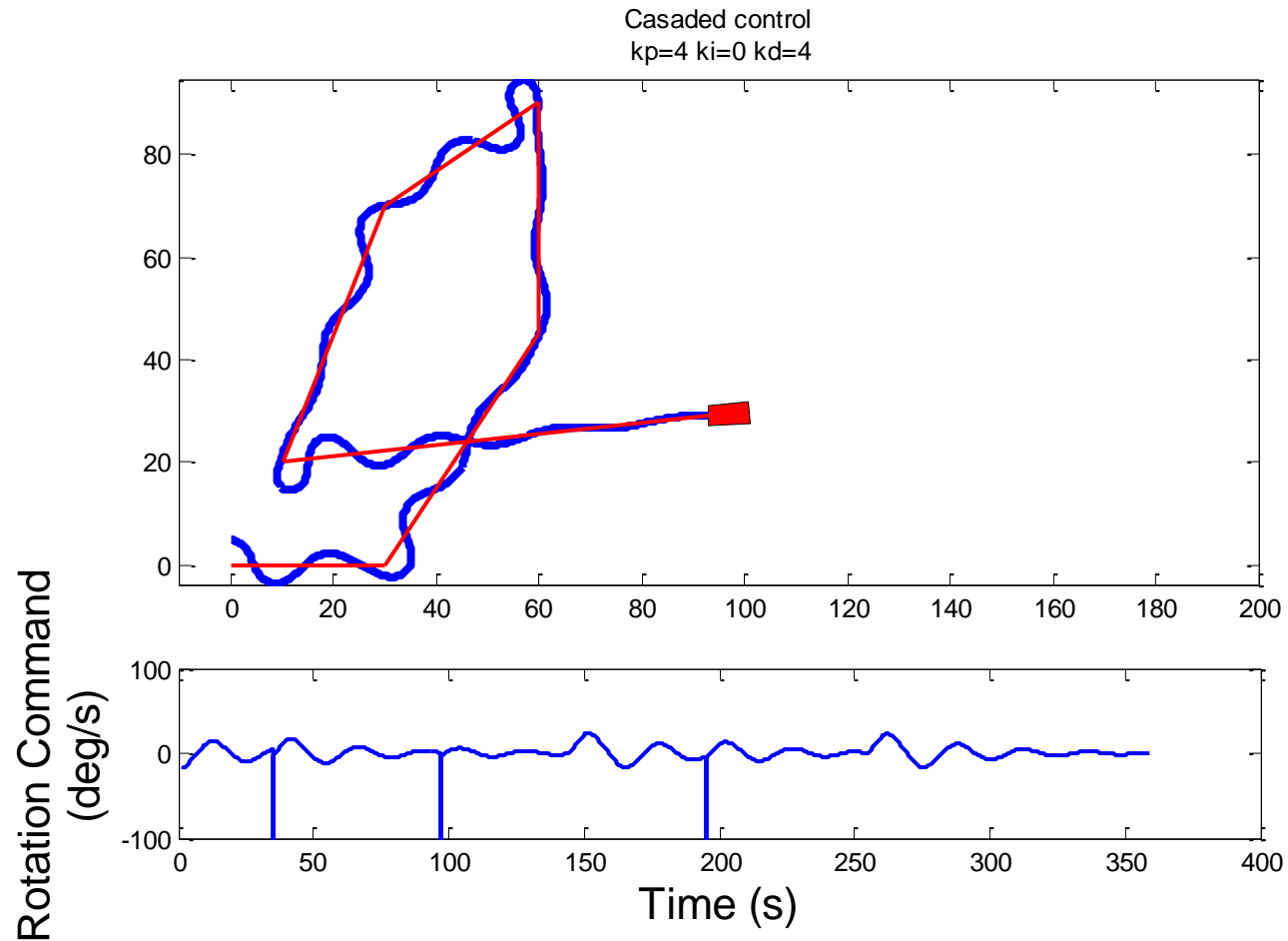- We are using also the derivative of the error:
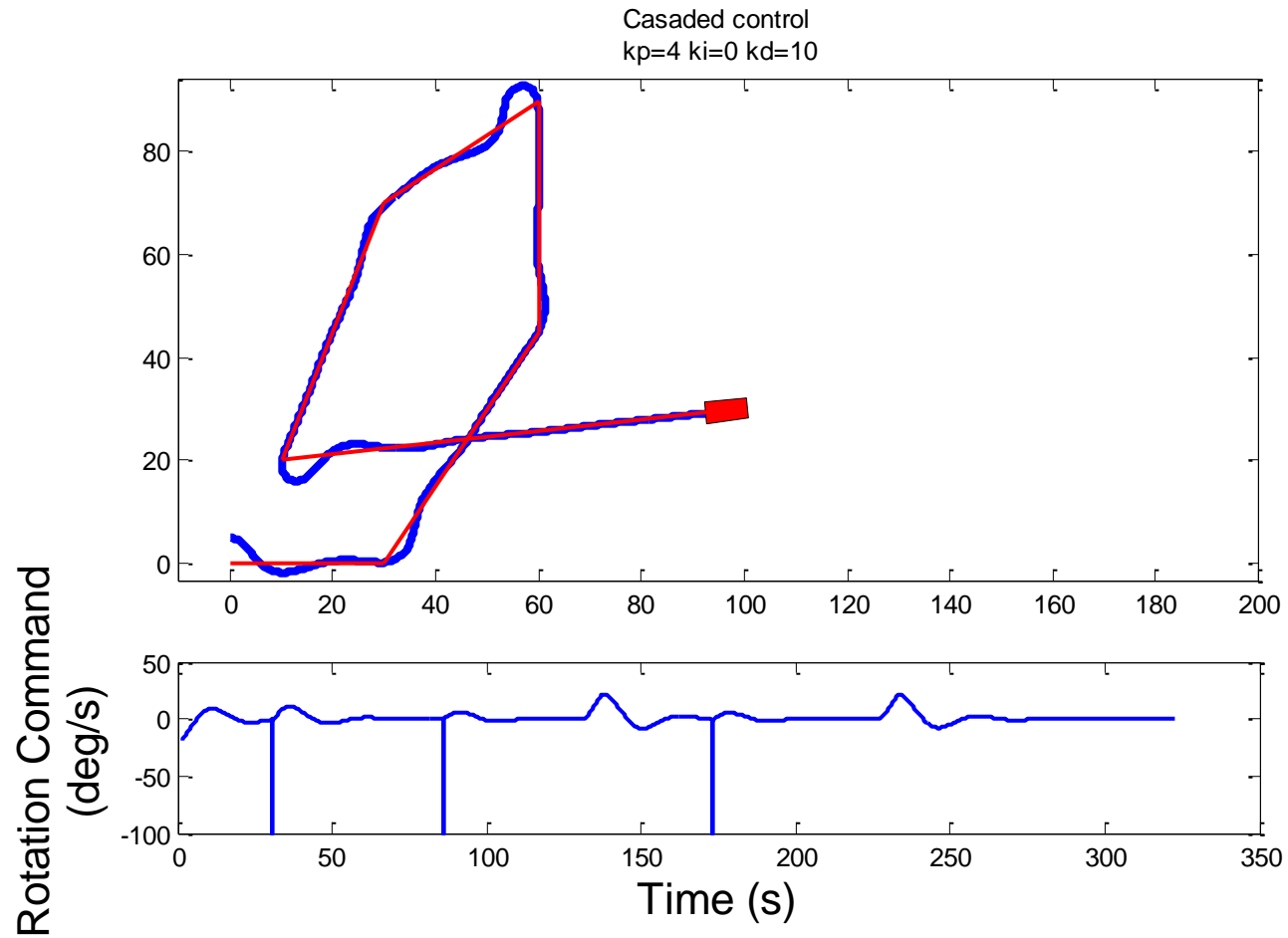 - $$\dot{e} = \frac{d_t - d_{t-1}}{dt}$$

  to calculate the turnrate ω:
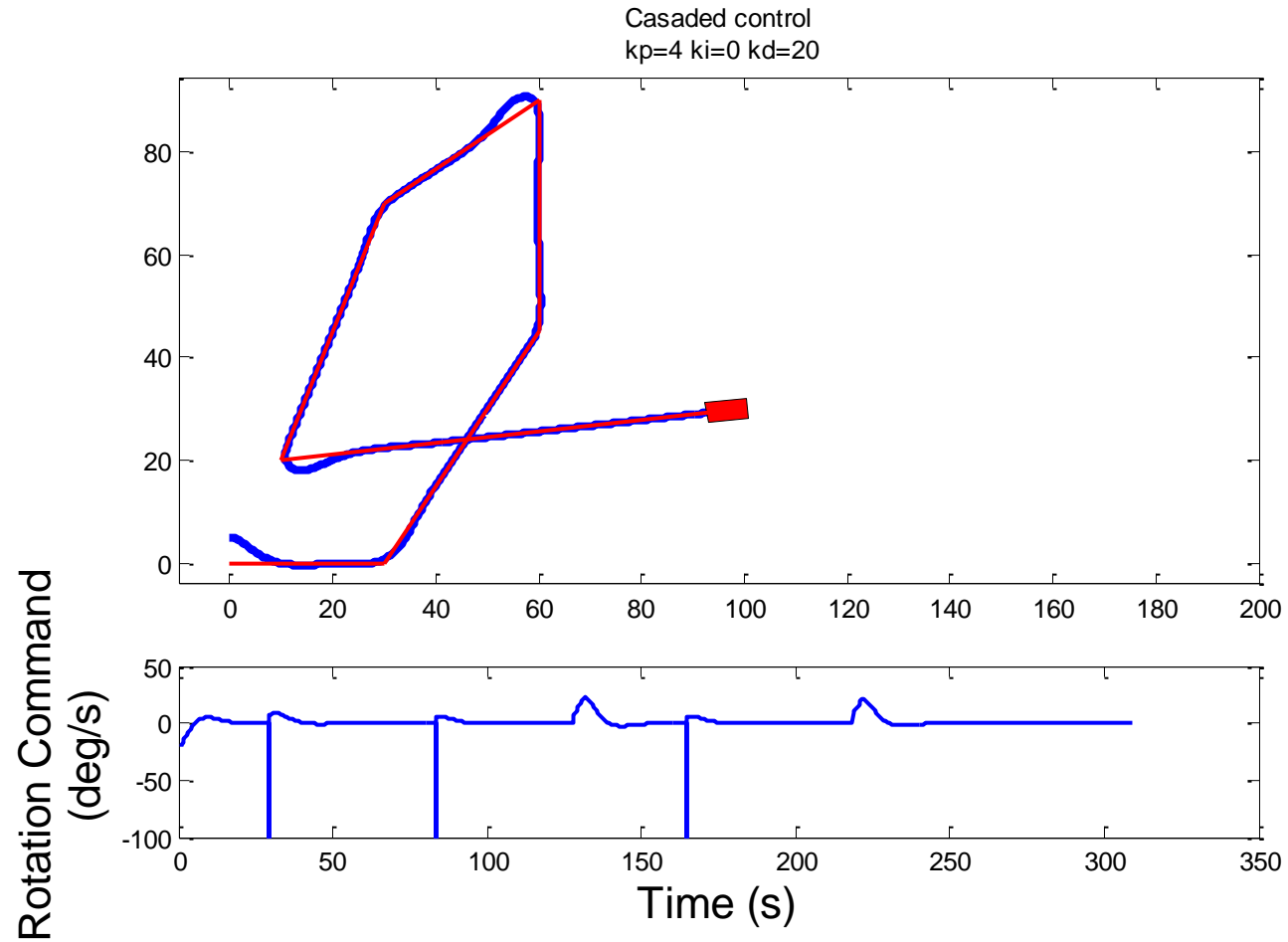
$$\omega = k_p \cdot d_t + k_d \cdot \dot{e}$$

Pd-Controller

Casaded control
kp=4 ki=0 kd=4

Pd-Controller

Casaded control
kp=4 ki=0 kd=10

Pd-Controller

Casaded control
kp=4 ki=0 kd=20

Pd-Controller

Casaded control
kp=20 ki=0 kd=50