



UNIVERSITY OF
SOUTH CAROLINA

CSCE 774 ROBOTICS SYSTEMS

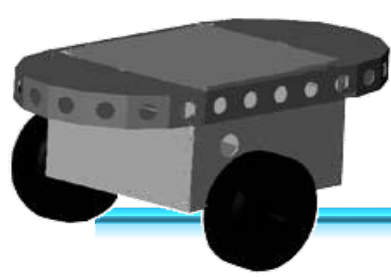
Coverage



Coverage

- A task performed quite often in everyday life:
 - Cleaning
 - Painting
 - Plowing/Sowing
 - Tile setting
 - etc.





Motivation



Humanitarian Demining

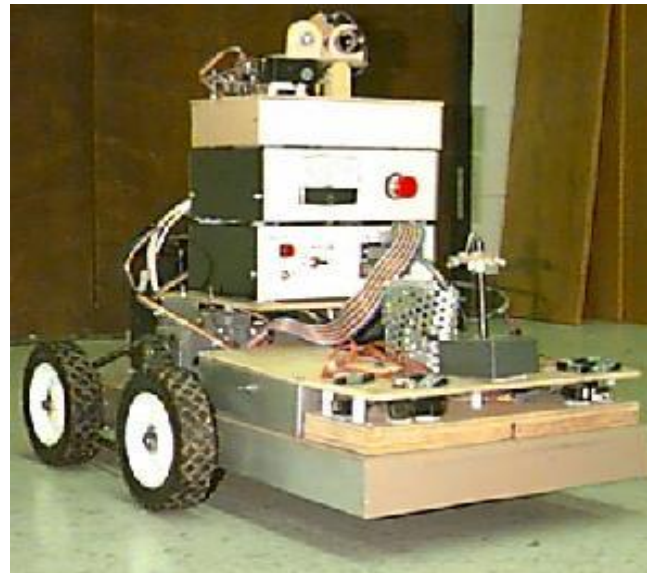


CSCE 774: Robotic Systems



Motivation

Lawn Mowing



Motivation

Vacuum Cleaning



Robotic Coverage

- More than 5 million Roombas sold!
- Automated Car Painting



Roomba Costumes



Coverage

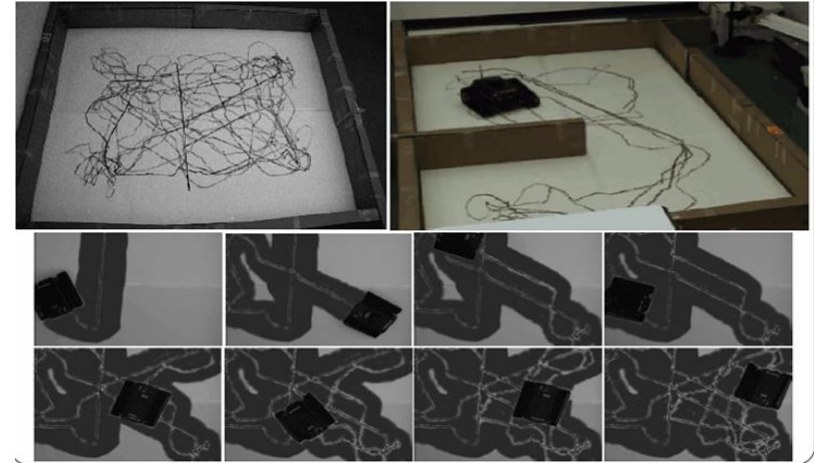
- First Distinction
 - Deterministic **Demining**
 - Random **Vacuum Cleaning**
- Second Distinction
 - Complete
 - No Guarantee
- Third Distinction
 - Known Environment
 - Unknown Environment



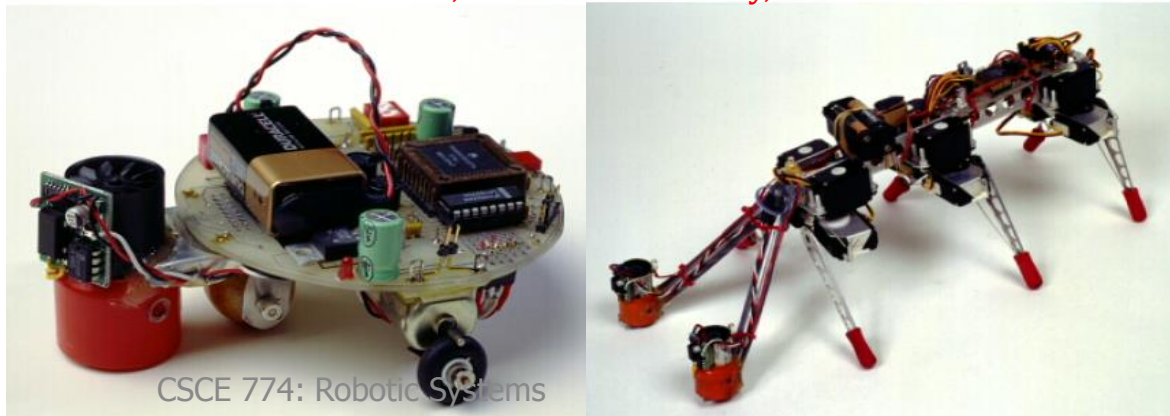
Non-Deterministic Coverage

- Complete Random Walk
- Ant Robotics
 - Leave trail
 - Bias the behavior towards or away from the trails

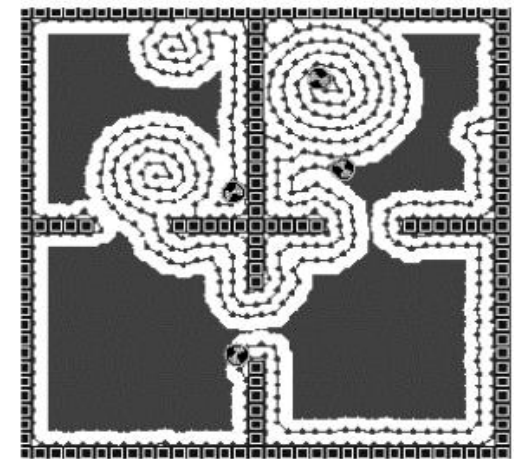
S. Koenig Ant Robotics, terrain coverage



Andrew Russell, Monash University, Australia



CSCSE 774: Robotic Systems



Ant Robotics: I. Wagner, IBM & Technion



Deterministic Coverage

- Complete Algorithm
- Guarantees Complete Coverage



Cell-Decomposition Methods

Two families of methods:

- **Exact cell decomposition**

The free space F is represented by a collection of non-overlapping cells whose union is exactly F

Examples: trapezoidal and cylindrical decompositions





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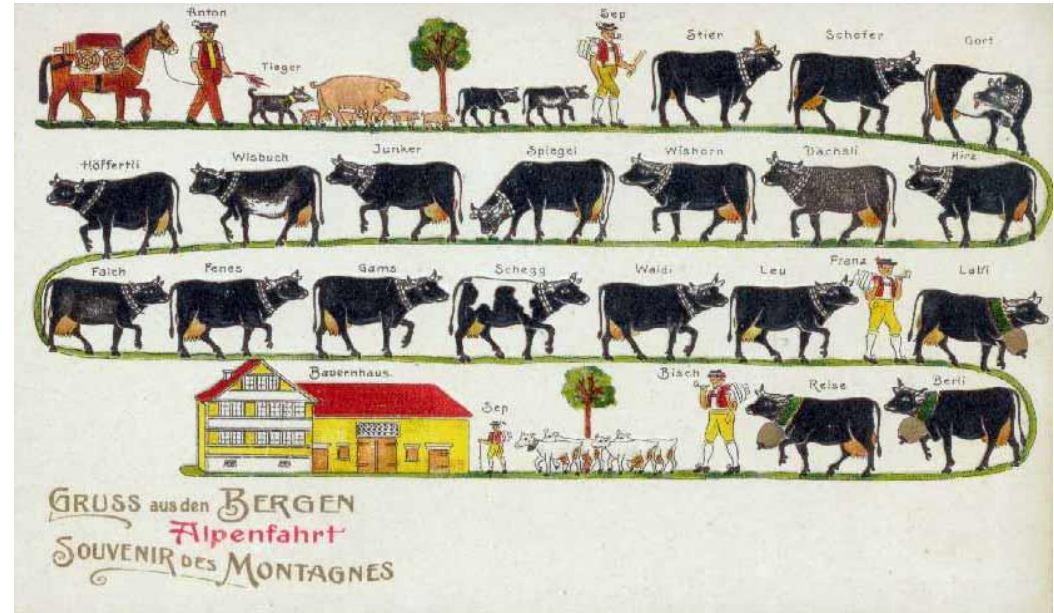
BOUSTROPHEDON CELLULAR DECOMPOSITION

The way of the Ox!



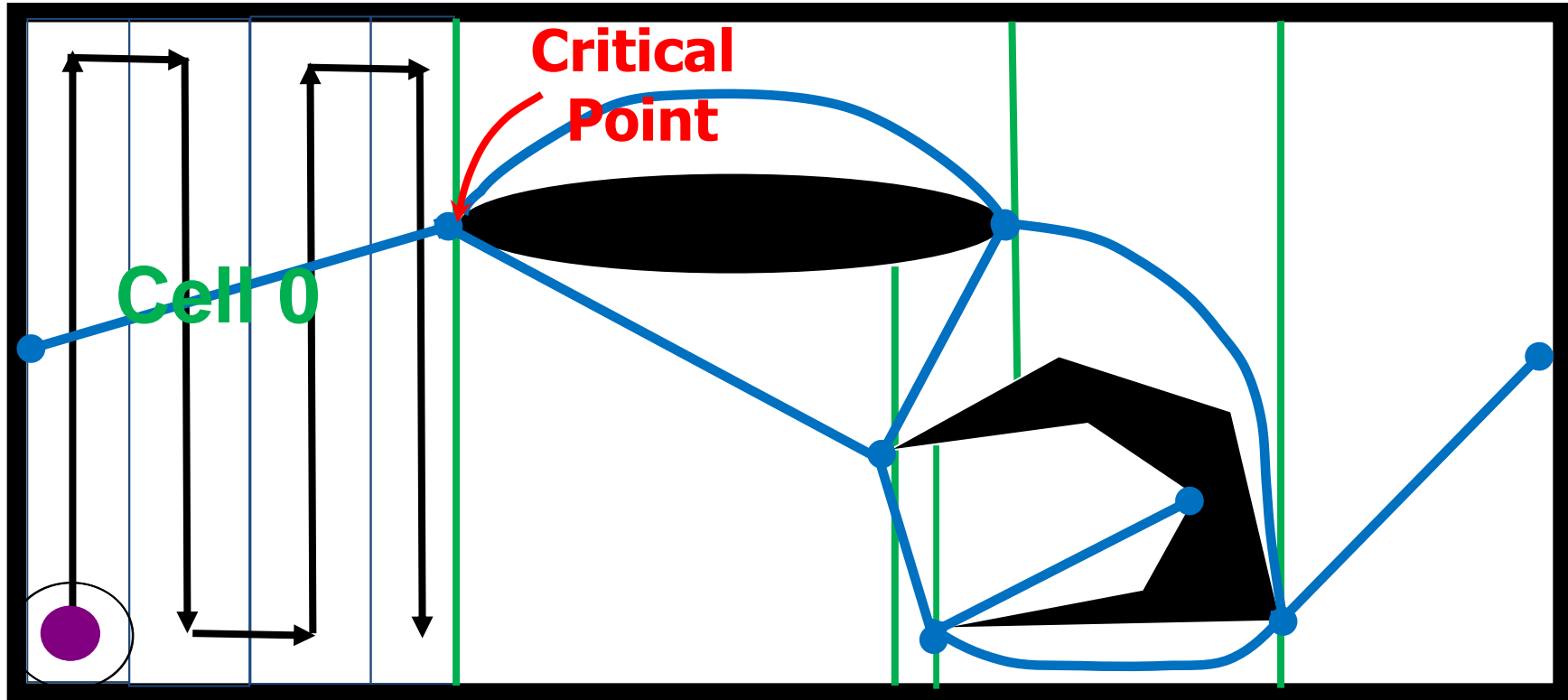
Single Robot Coverage

- Deterministic algorithm
- Guarantee of completeness
- Sensor based
- Unknown Environment



- Seed spreader algorithm: Lumelsky et al, “Dynamic path planning in sensor-based terrain acquisition”, IEEE Transactions on Robotics and Automation, August 1990.
- Boustrophedon algorithm: Choset and Pignon, “Coverage path planning: The boustrophedon cellular decomposition”, International Conference on Field and Service Robotics,1997.

Single Robot Coverage



→
Direction of Coverage

—
Cellular Decomposition

—•—•—
Reeb graph
Vertices: Critical Points
Edges: Cells



Critical Points

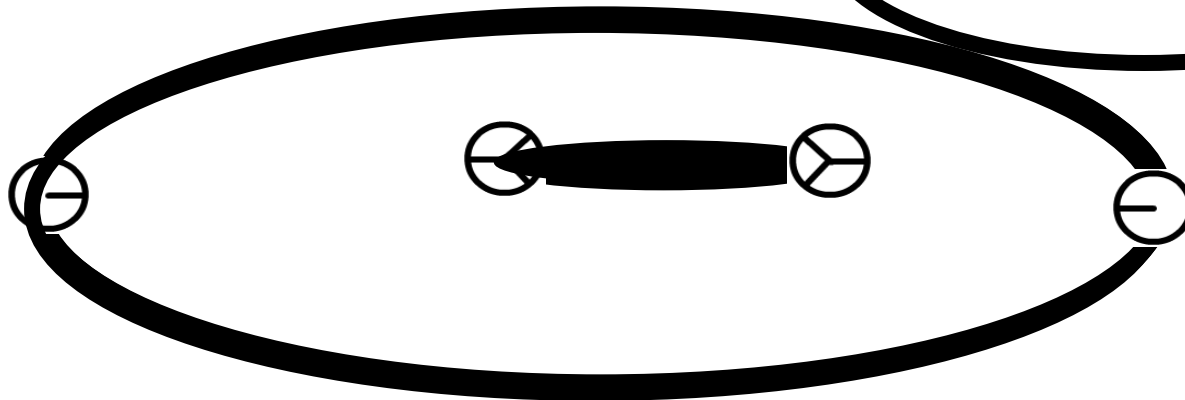
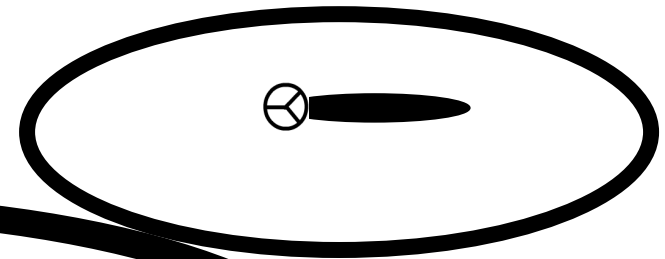
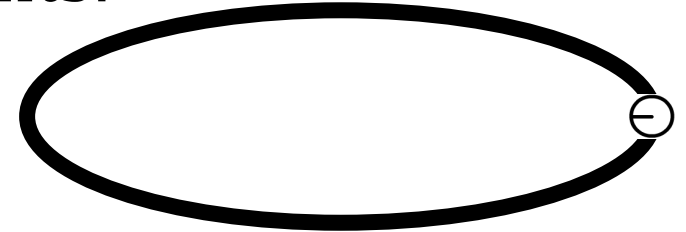
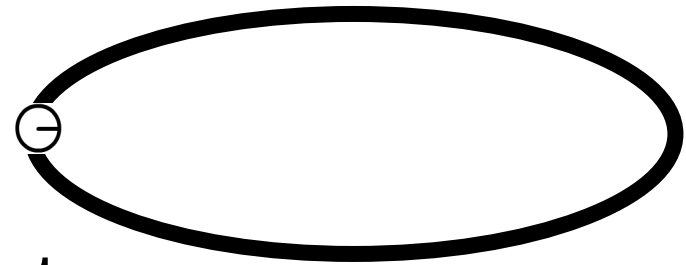
- There are four types of critical points:

⊖ Forward Concave critical point

⊖ Reverse Concave critical point

⊕ Reverse Convex critical point

⊕ Forward Convex critical point



Direction of Coverage



Optimal Coverage

- Find an order for traversing the Reeb graph such that the robot would not go through a cell more times than necessary

Solution

- Use the Chinese Postman Problem



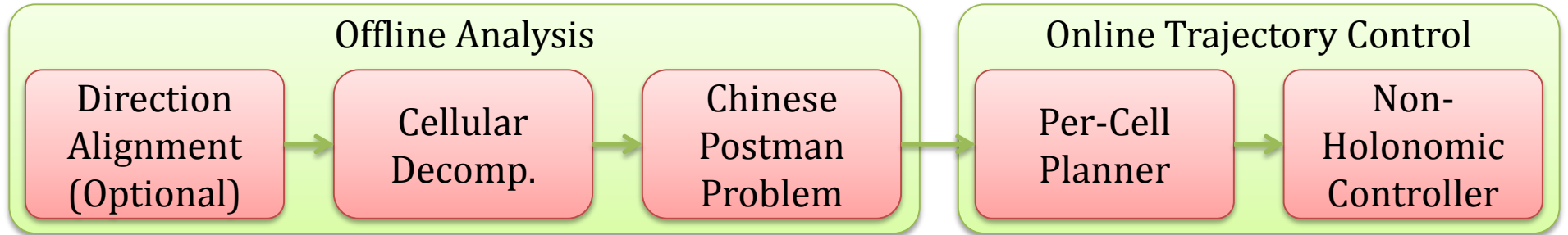
Chinese Postman Problem

- The Chinese postman problem (CPP), is to find a shortest closed path that visits every edge of a (connected) undirected graph. When the graph has an Eulerian circuit (a closed walk that covers every edge once), that circuit is an optimal solution.

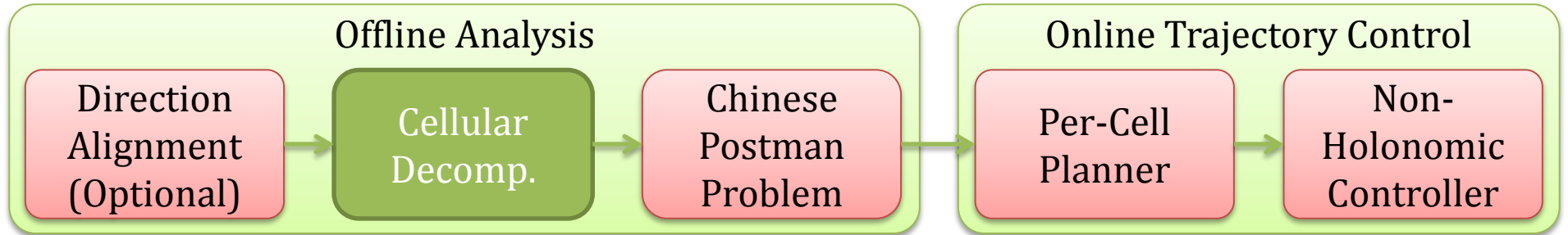
See: J. Edmonds and E.L. Johnson, Matching Euler tours and the Chinese postman problem, Math. Program. (1973).



Offline Analysis Algorithm



Offline Analysis Algorithm



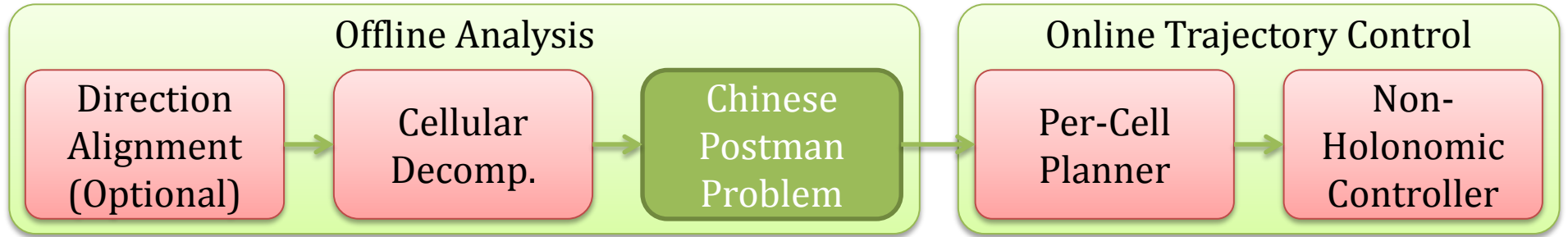
- Input: binary map separating obstacle from free space
- Boustrophedon Cellular Decomposition (BCD)



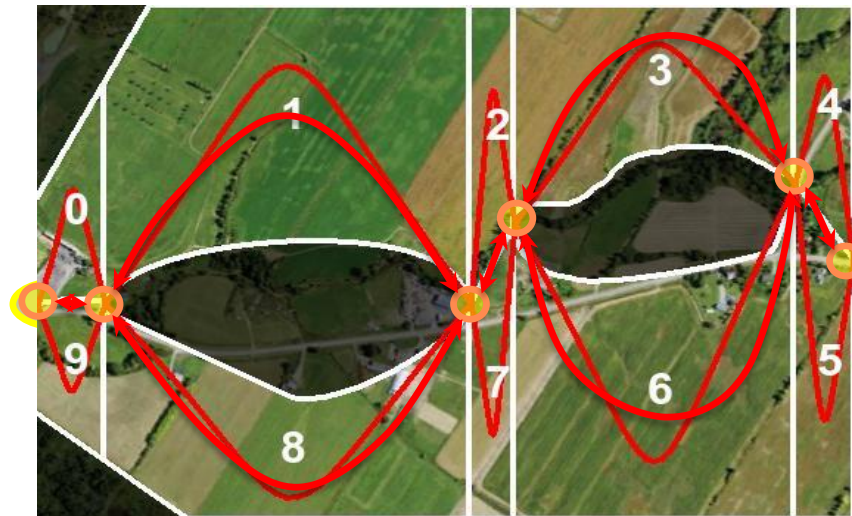
○ : intersections = vertices

↔ : cells = edges

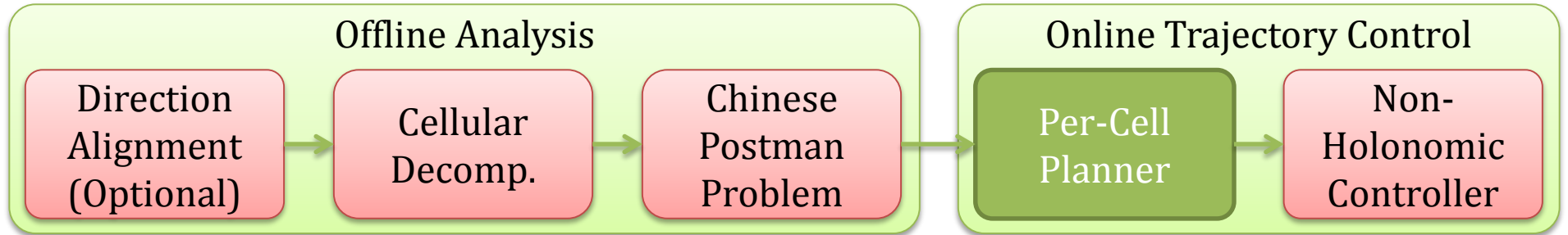
Offline Analysis Algorithm (cont.)



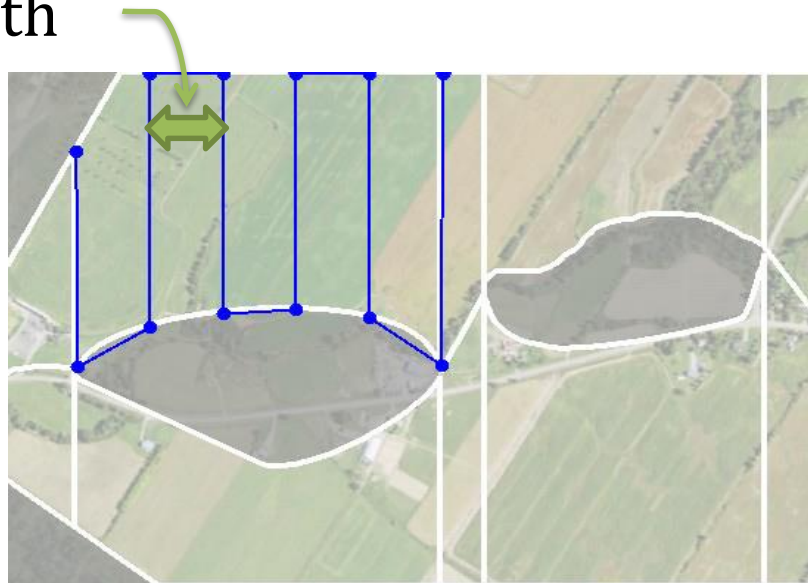
- Chinese Postman Problem
 - Eulerian circuit, i.e. *single* traversal through all cells (edges)



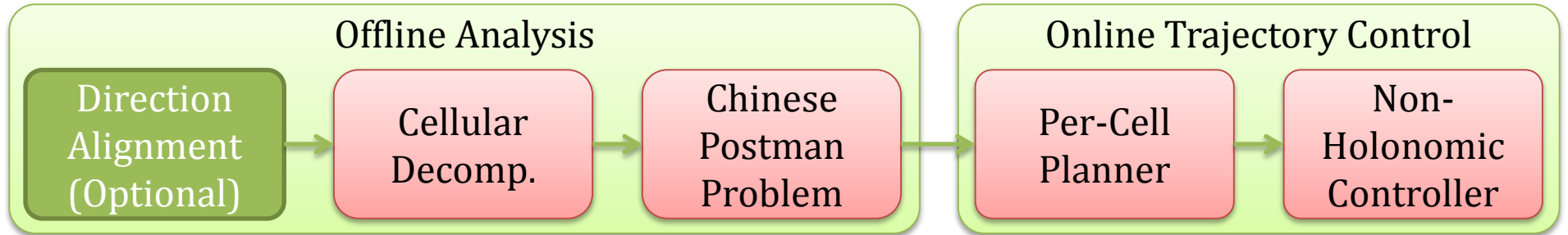
Per-Cell Coverage Planner



- Seed Spreader: piecewise linear sweep lines
- Footprint width



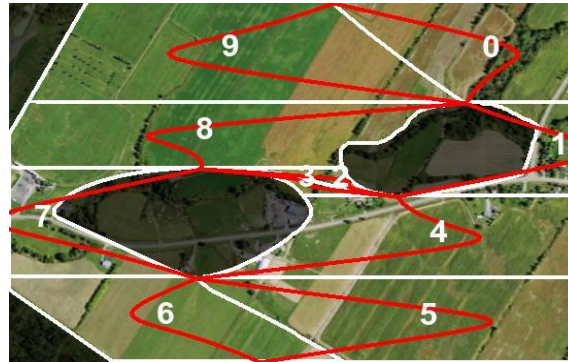
Coverage Direction Alignment



- Static alignment methods



Default



Obstacle Boundaries

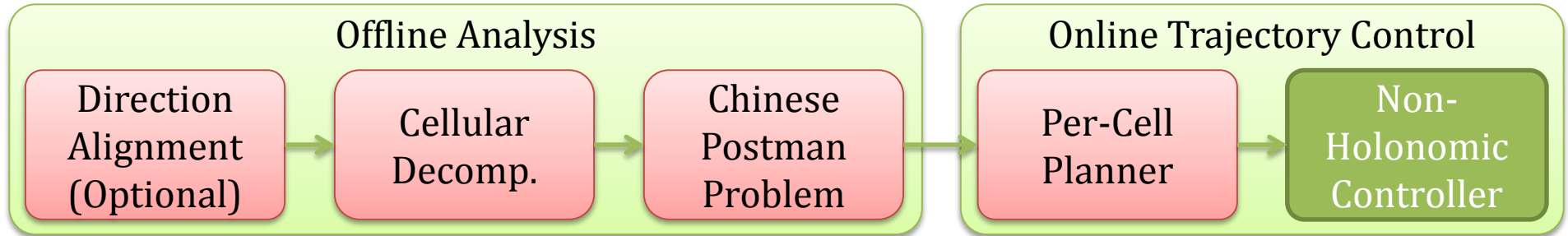


Free Space Distribution

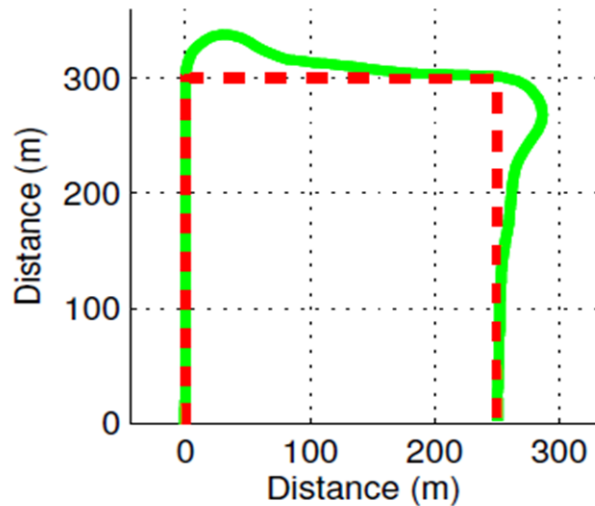
- Alignment with average wind heading (pre-flight)



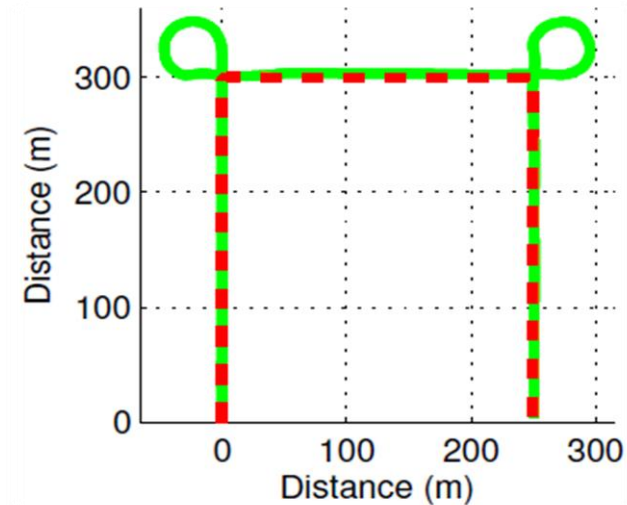
Non-Holonomic Robot Controller



- Turning strategies



Greedy Waypoint
Controller

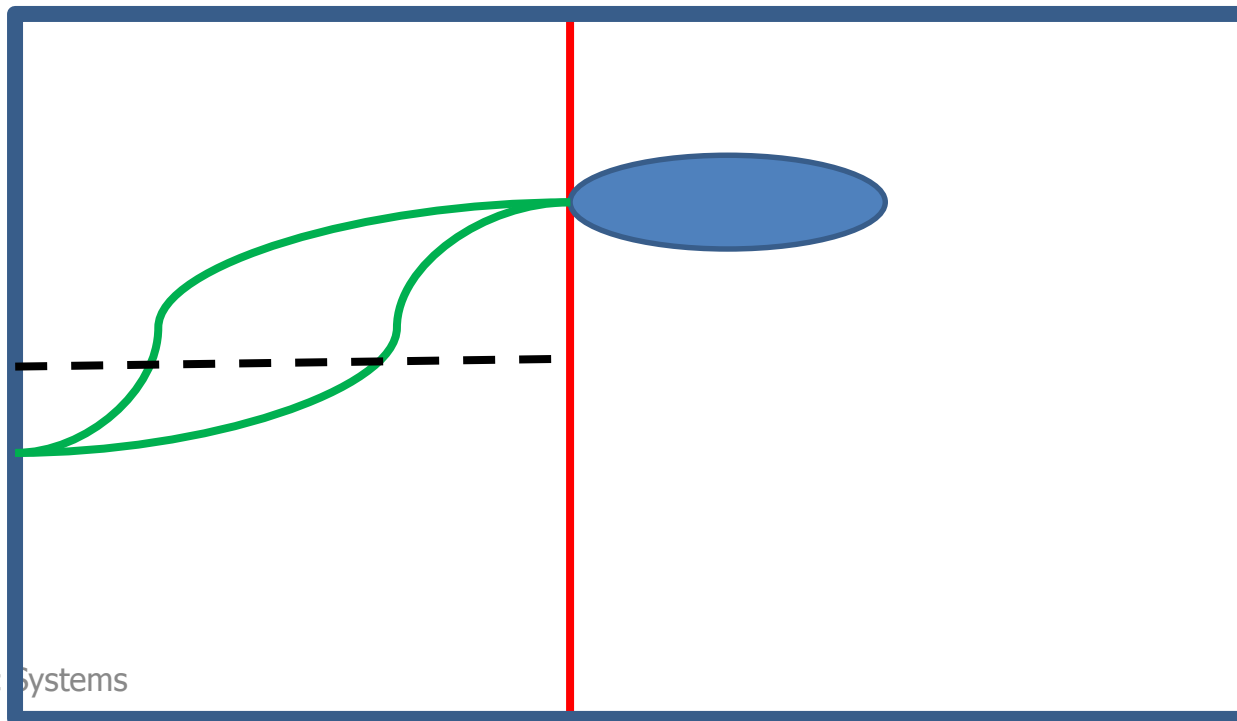


Curlicue Controller



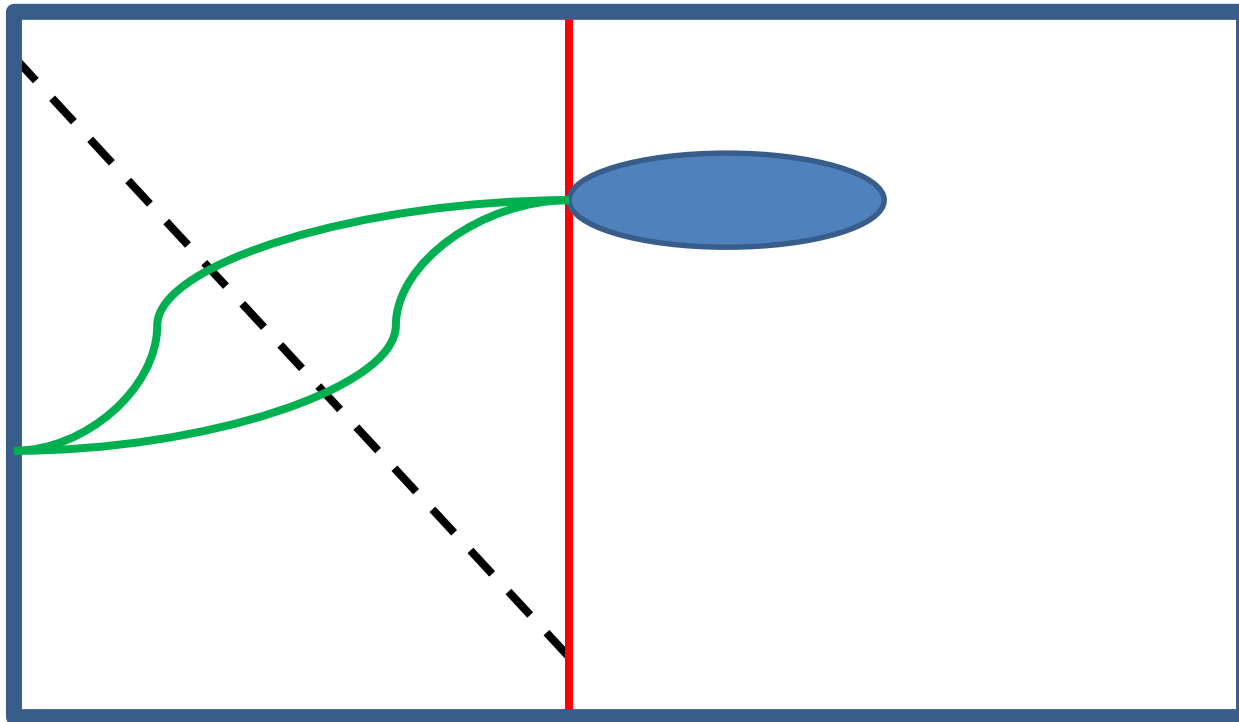
Chinese Postman Problem

- The solution of the CPP guarantees that no edge is doubled more than once
- That means some cells have to be traversed twice
- Cells that have to be traversed/covered are divided in half



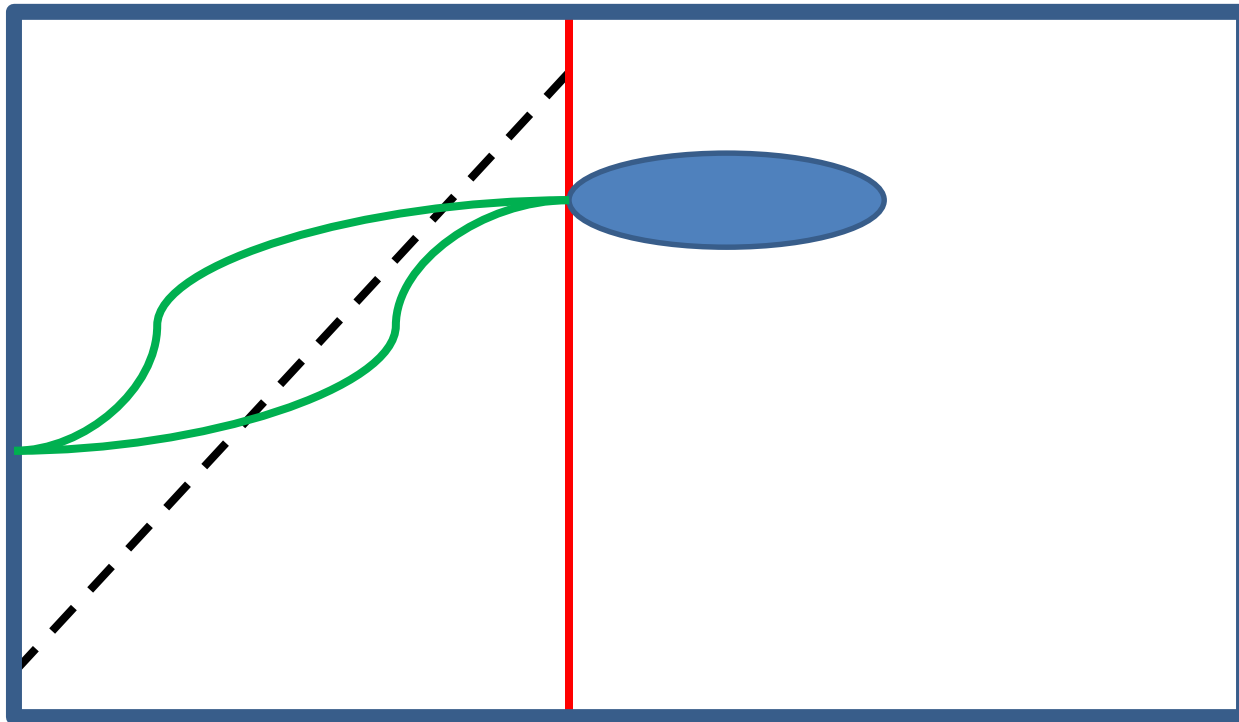
Double Coverage of a Single Cell

- By dividing the cell diagonally we control the beginning and end of the coverage



Double Coverage of a Single Cell

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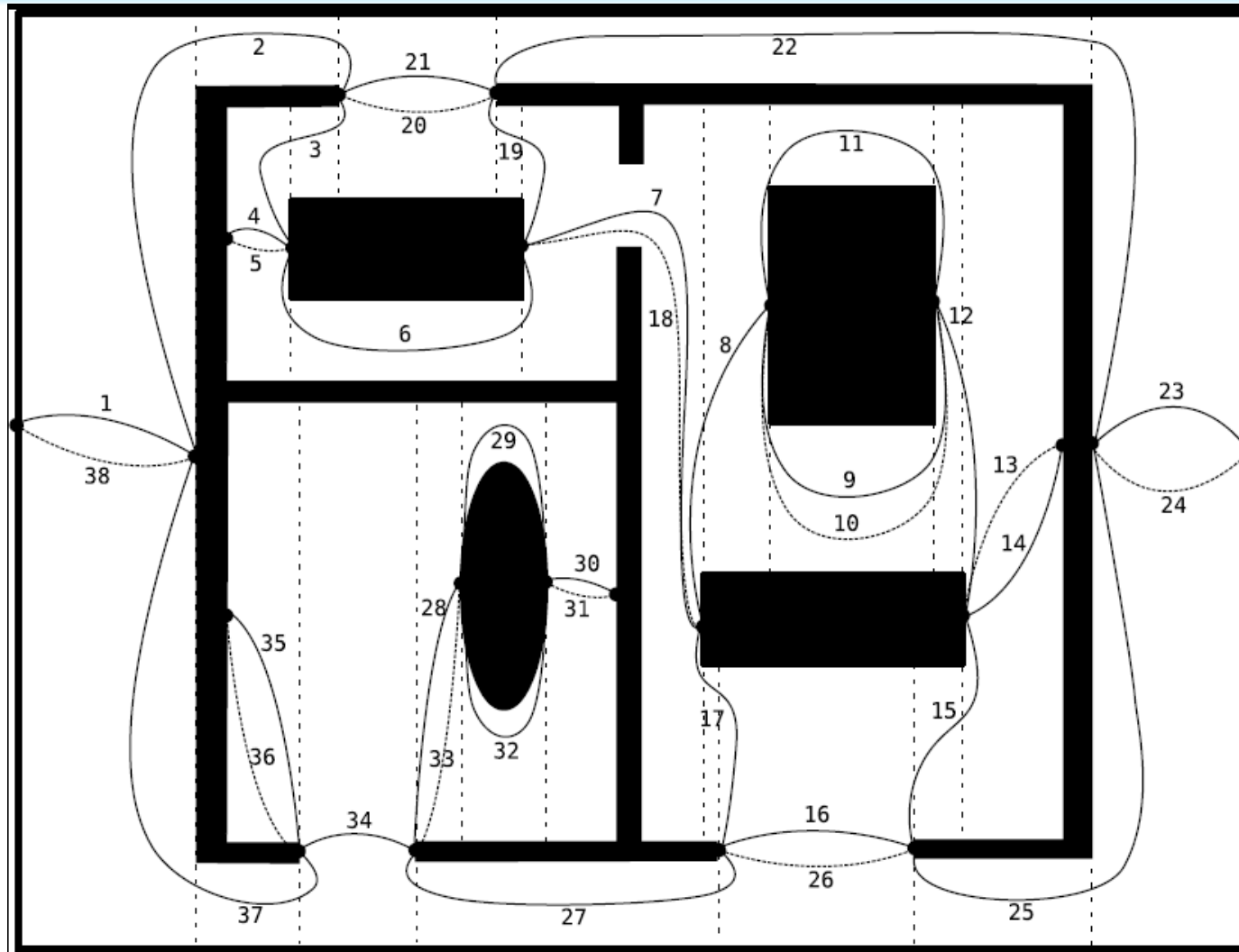


Optimal Coverage Algorithm

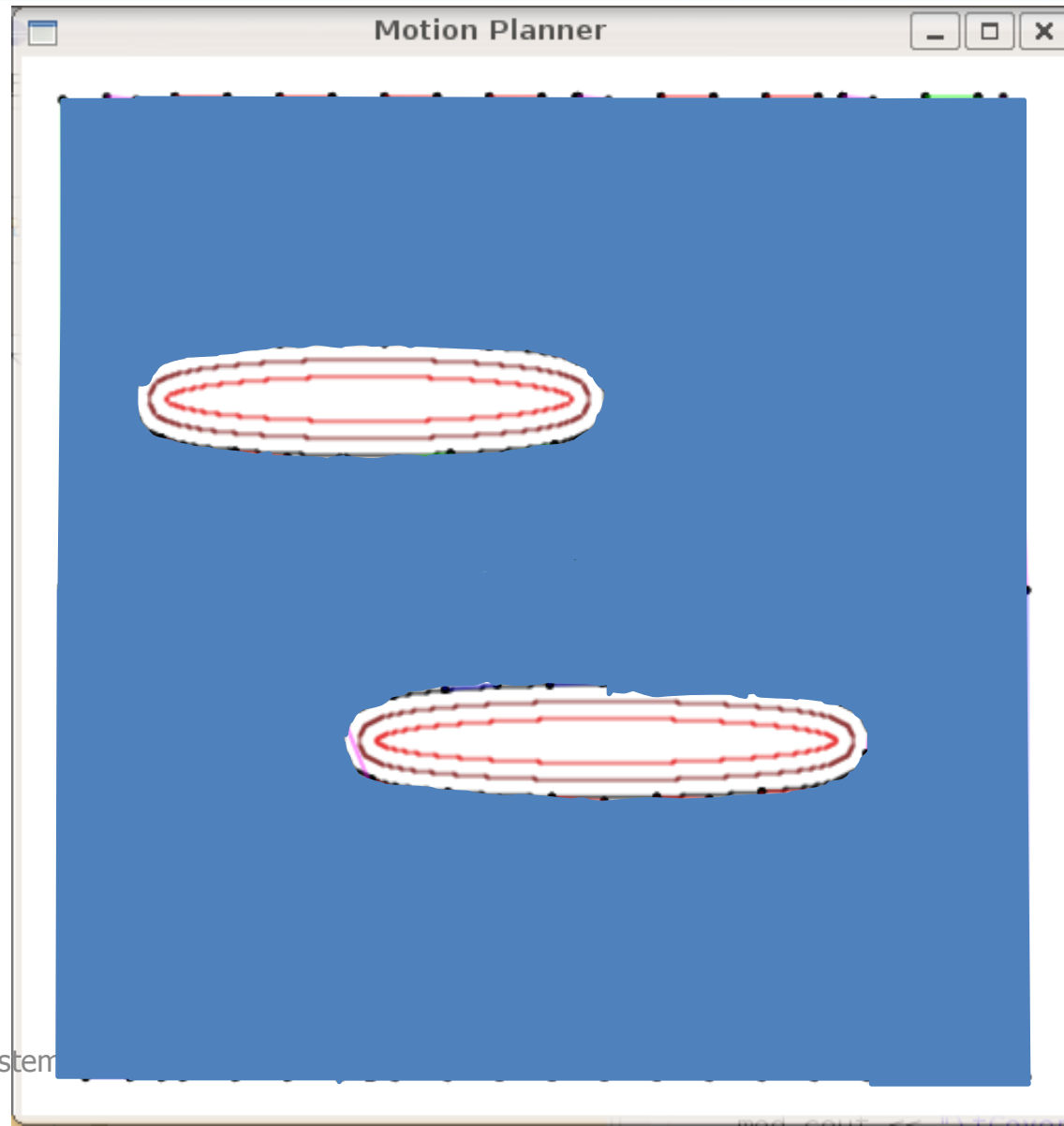
- Given a known environment:
 - Calculate the Boustrophedon decomposition
 - Construct the Reeb graph
 - Use the Reeb graph as input to the Chinese Postman Problem (CPP)
 - Use the solution of the CPP to find a minimum cost cycle traversing every edge of the Reeb graph
 - For every doubled edge divide the corresponding cell in half
 - Traverse the Reeb graph by covering each cell in order



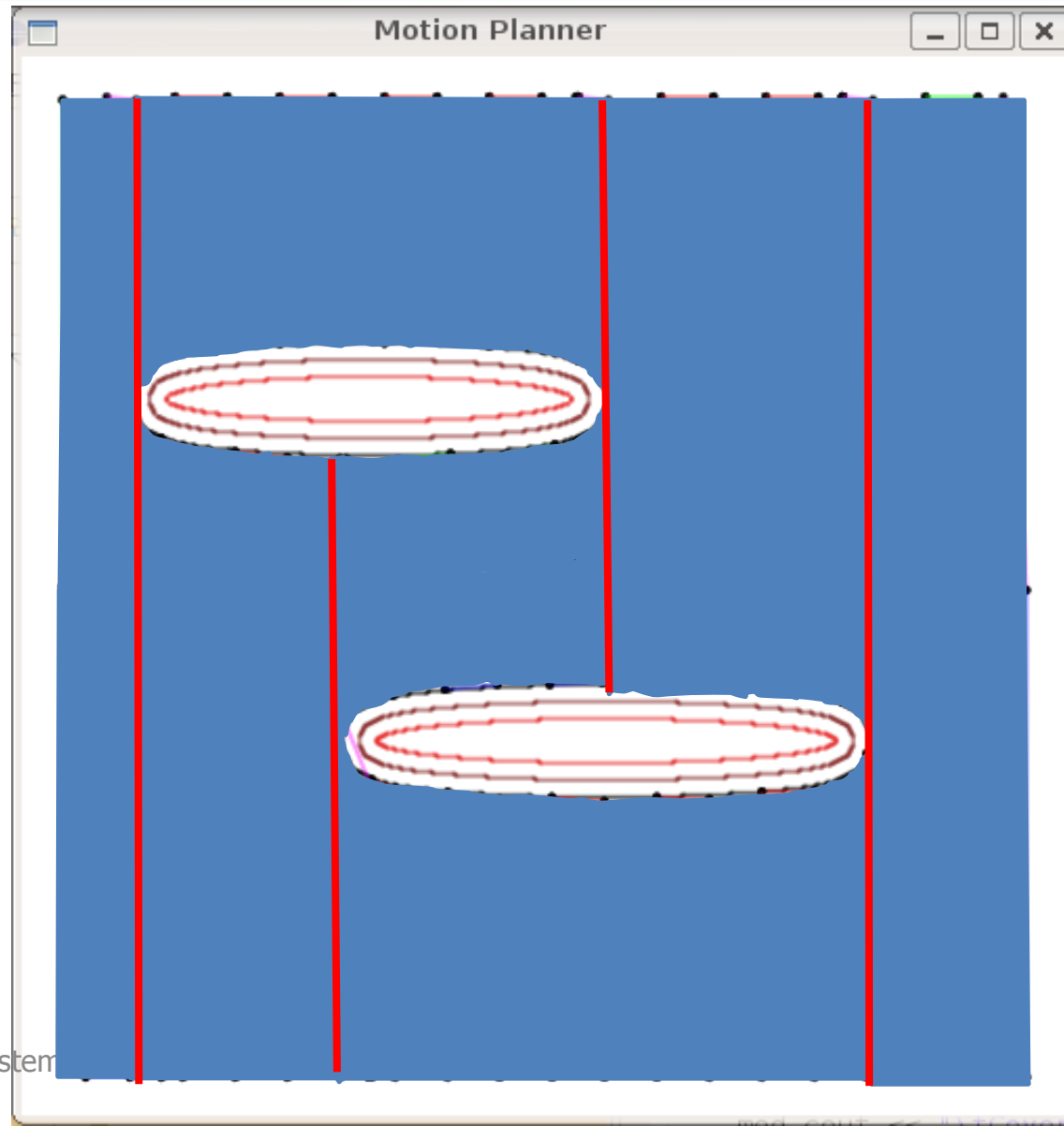
Traversal order of the Reeb graph



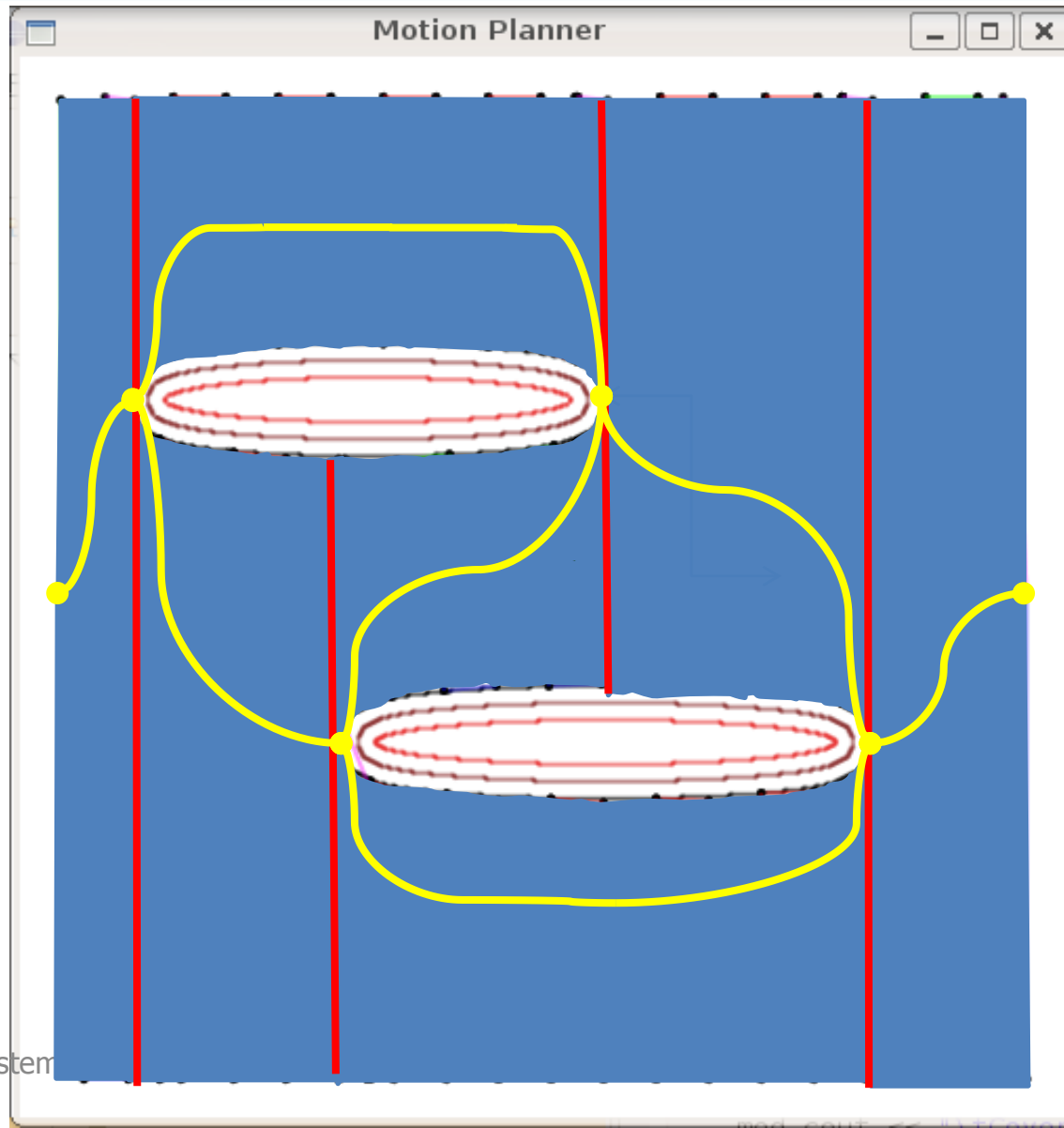
Example



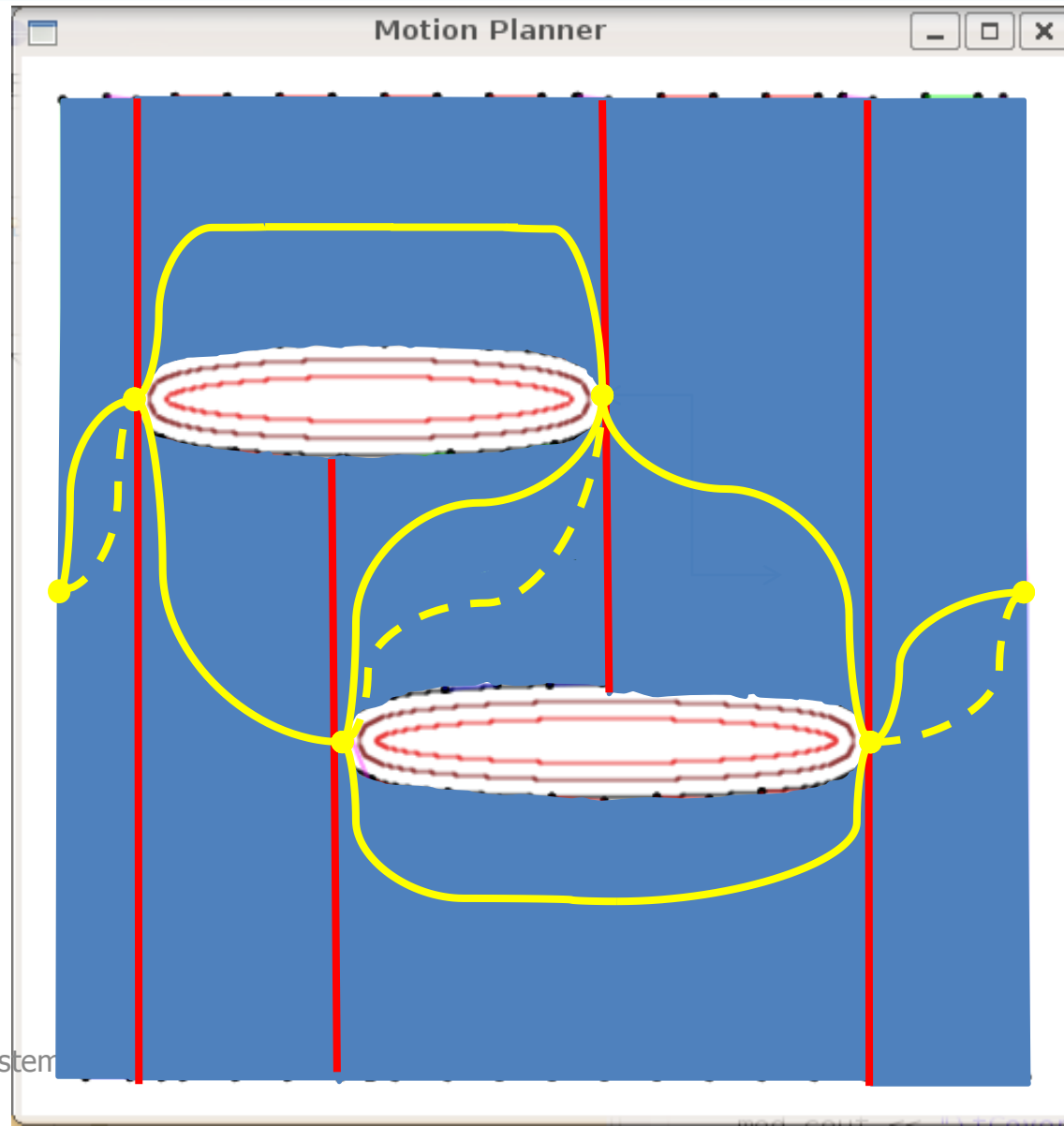
Example: Boustrophedon Decomposition



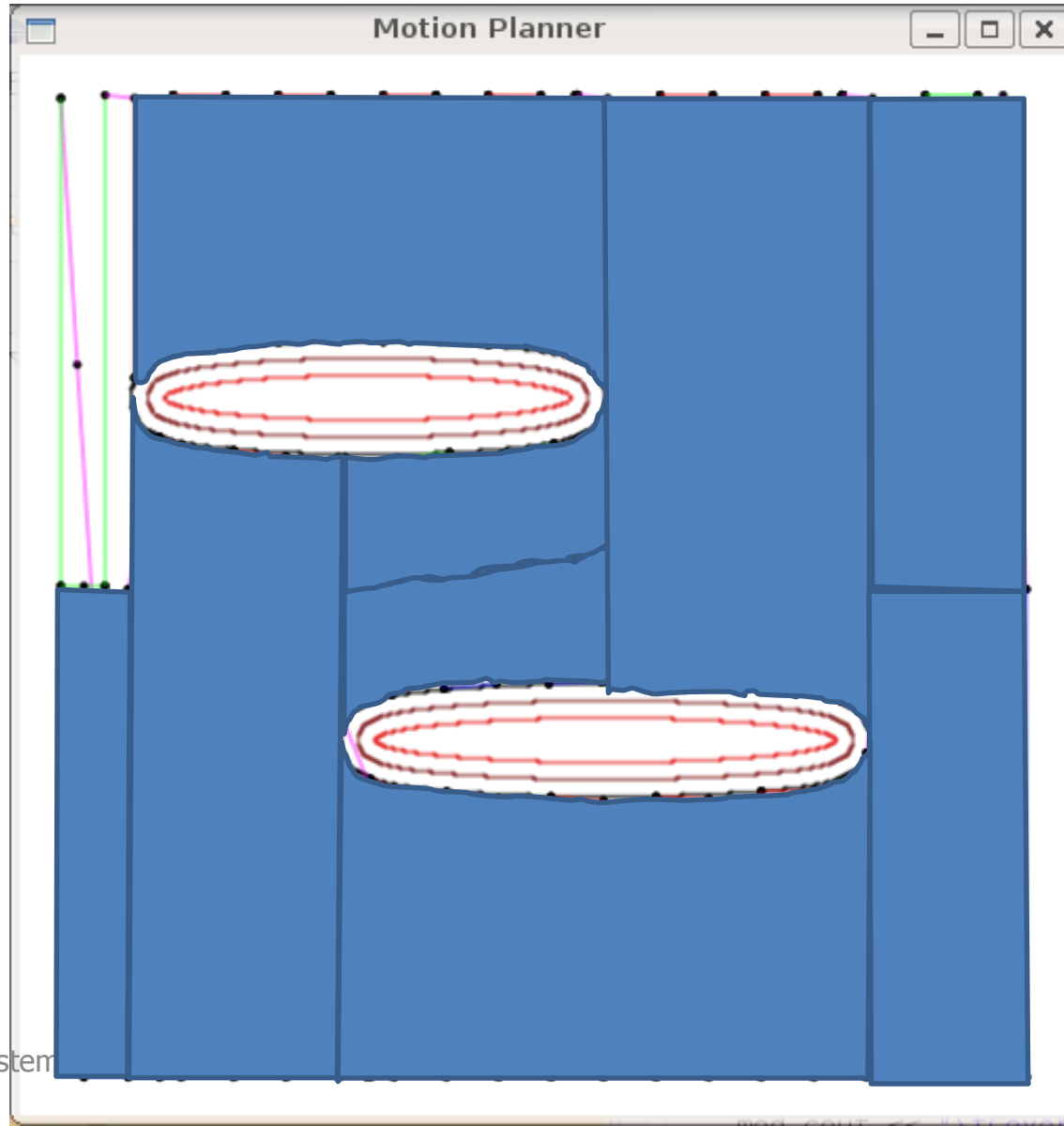
Example: Reeb Graph



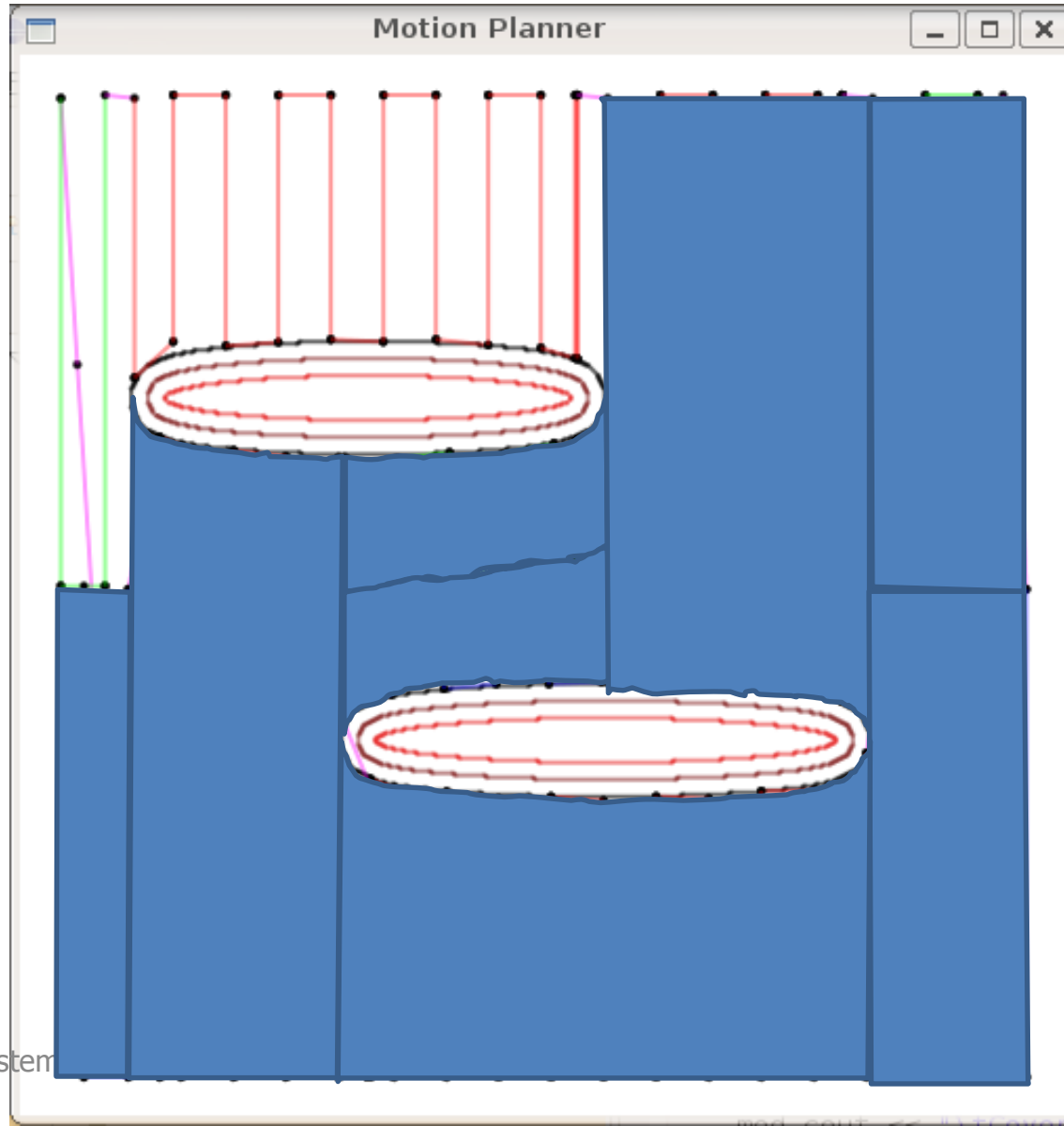
Example: CPP solution



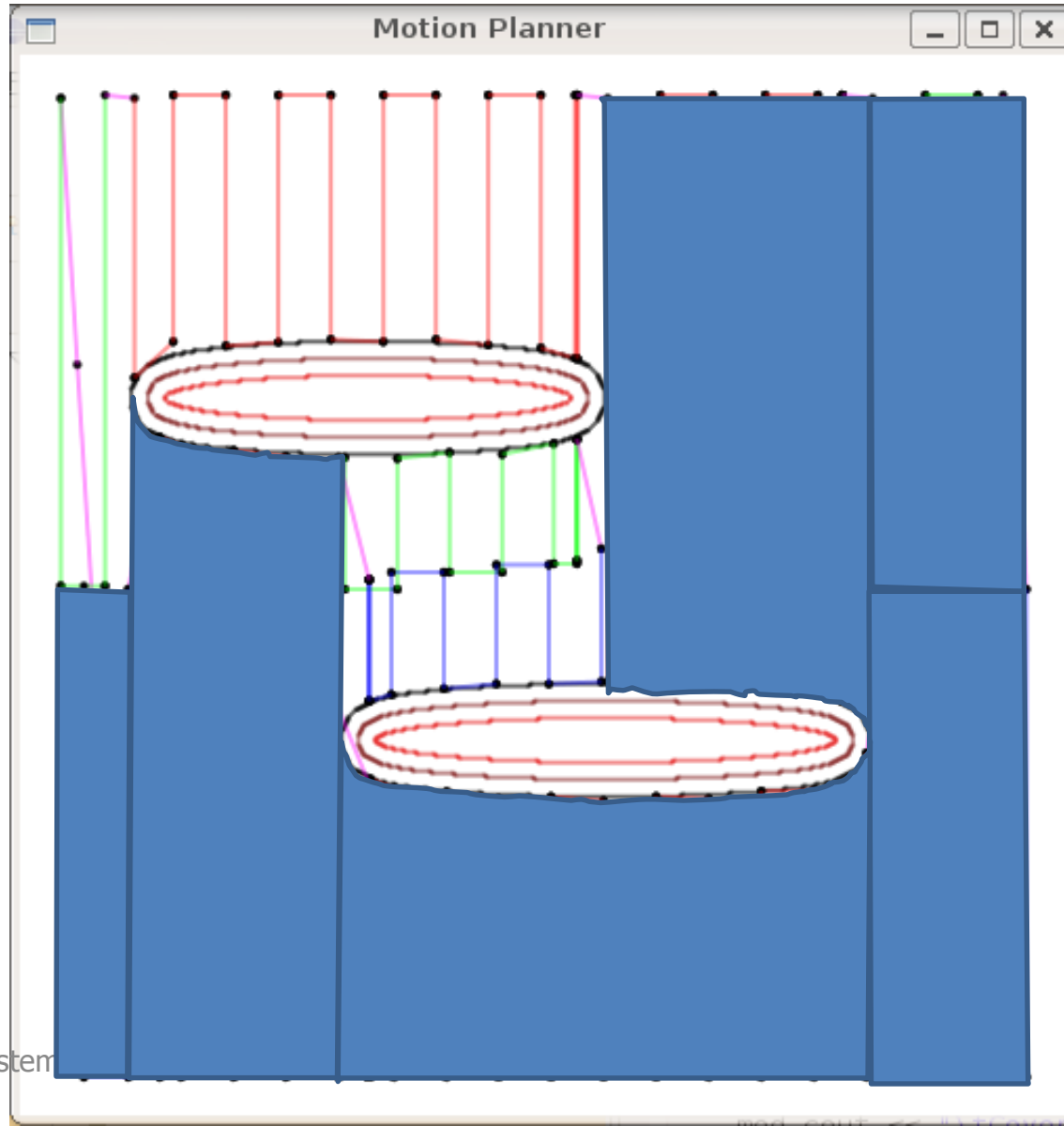
Example



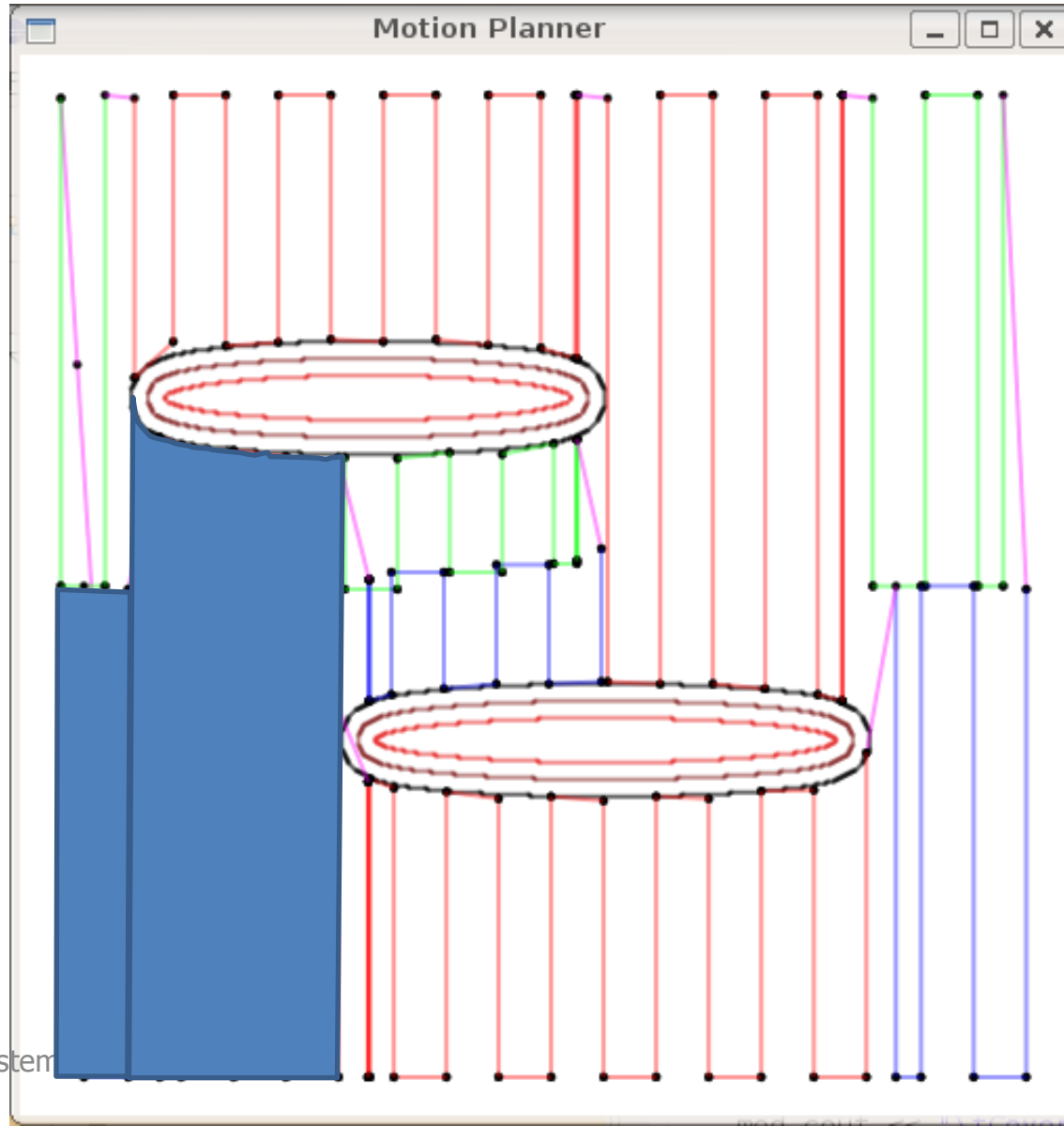
Example



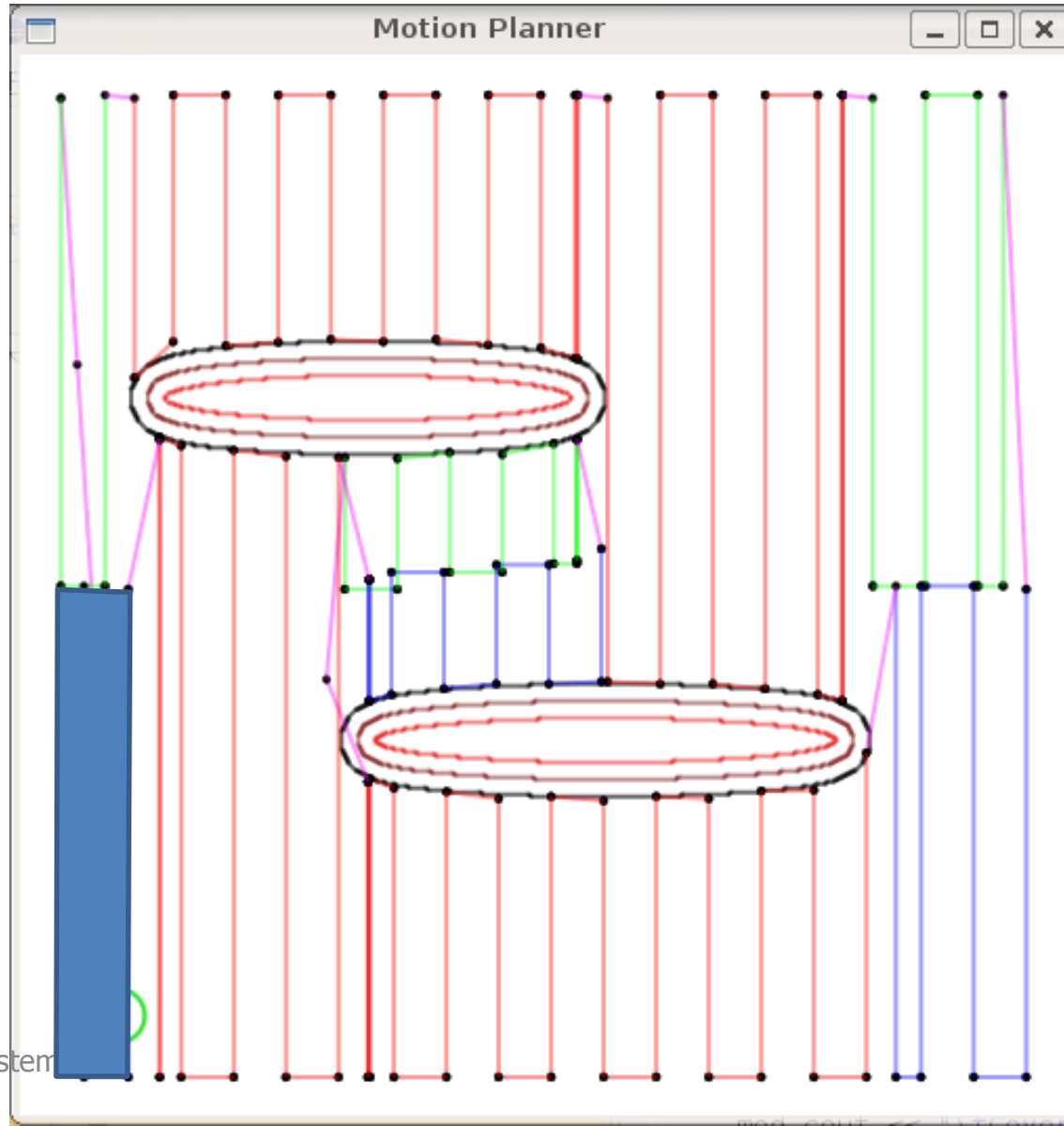
Example



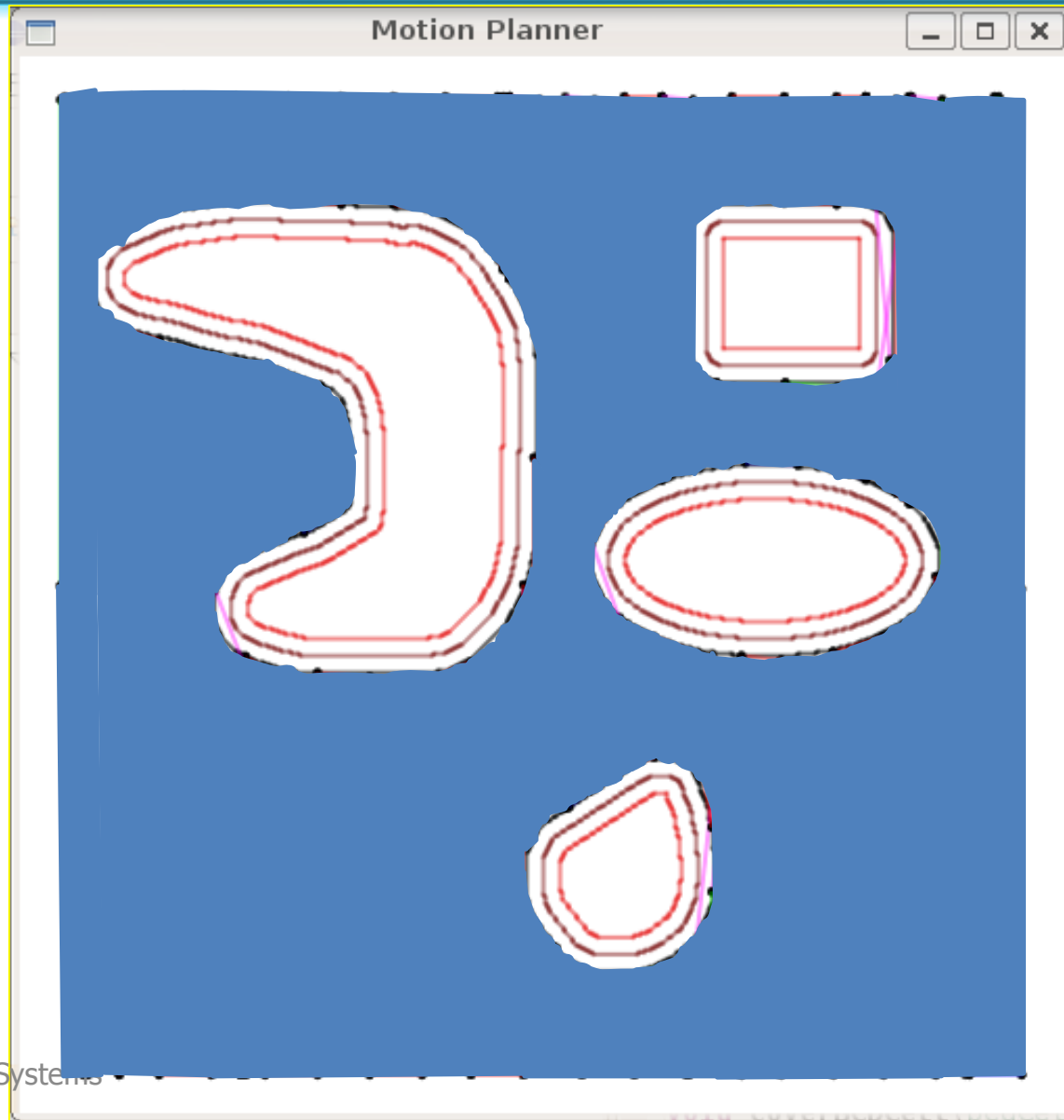
Example



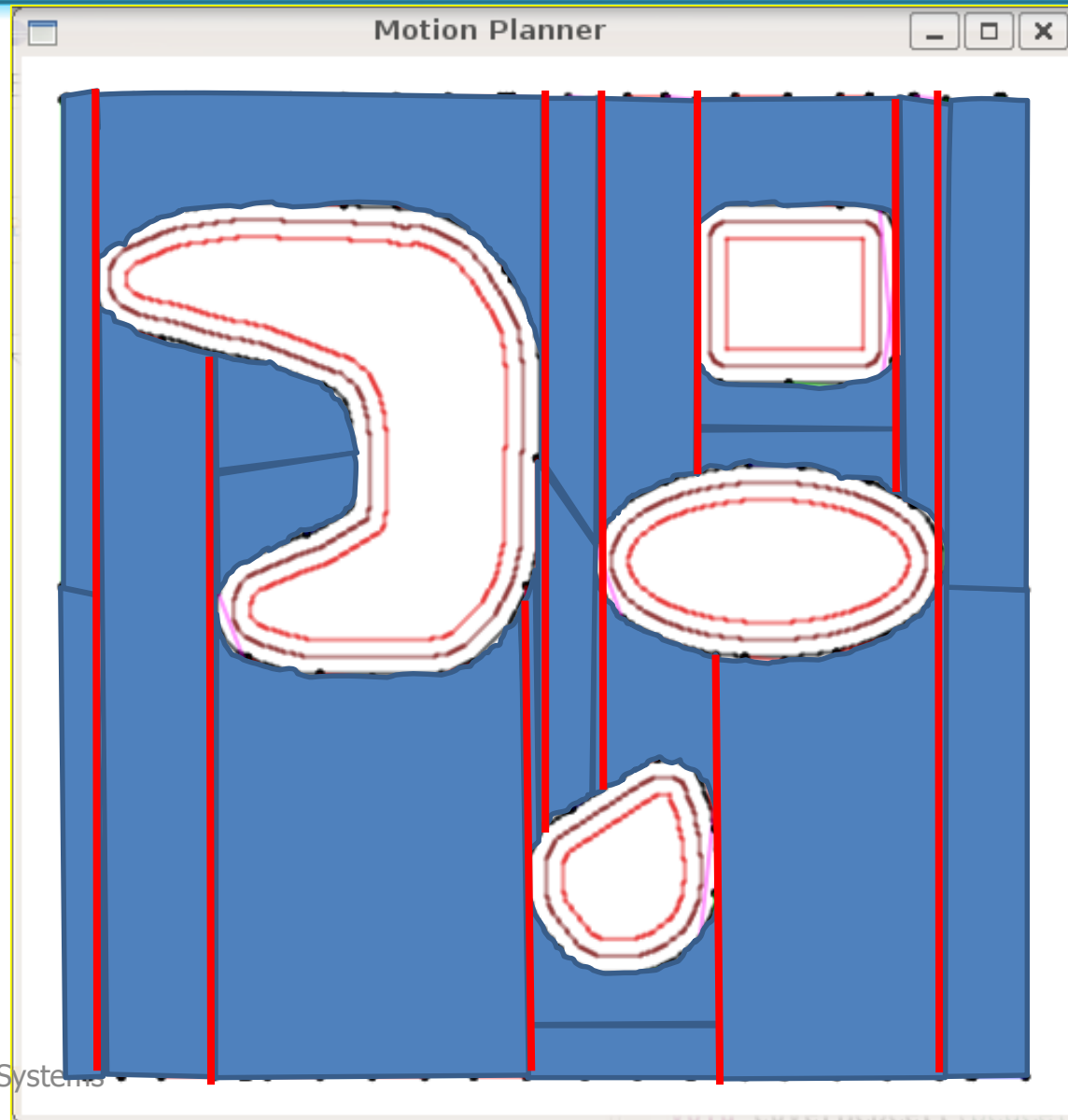
Example



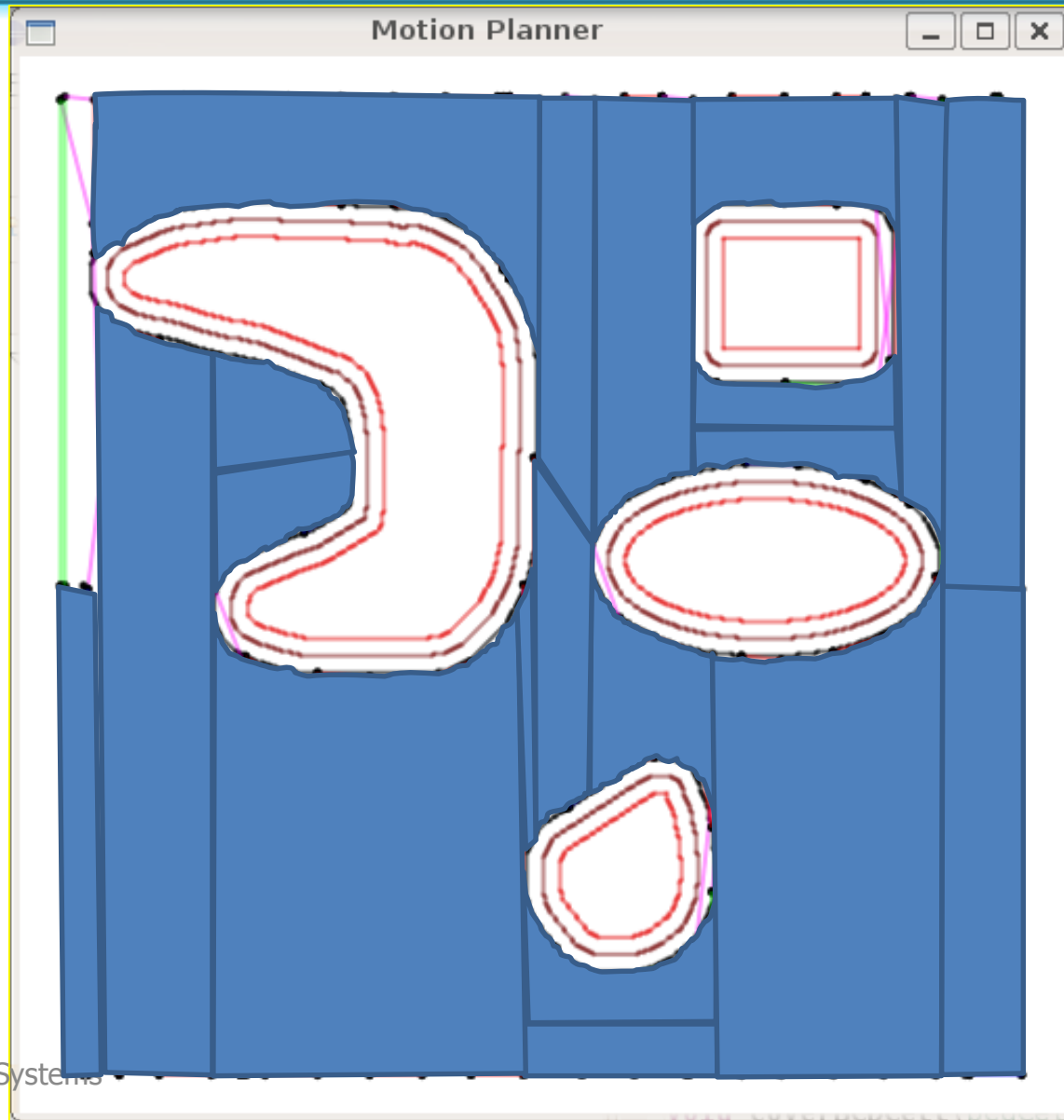
Example 2



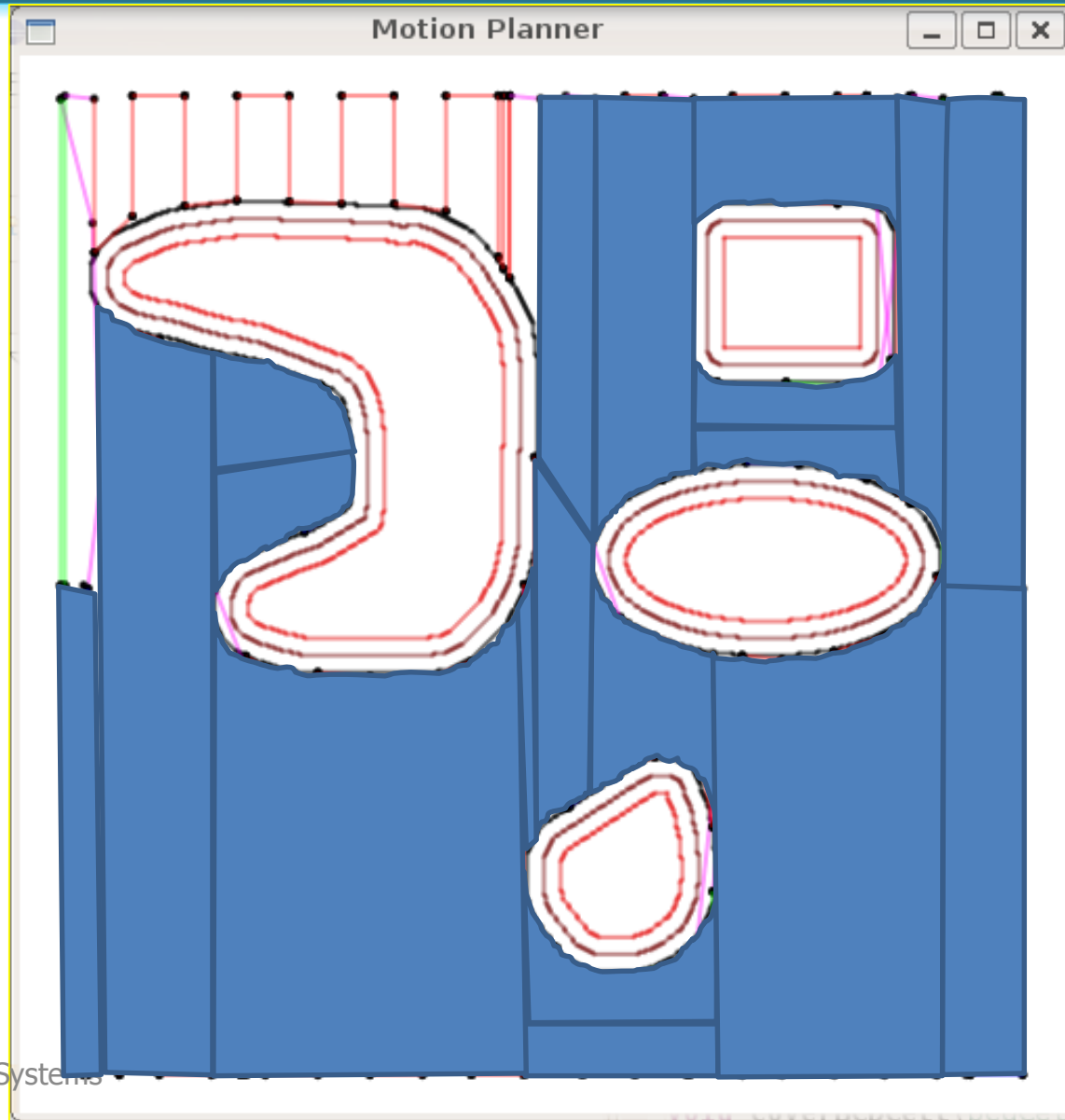
Example 2 Boustrophedon Decomp.



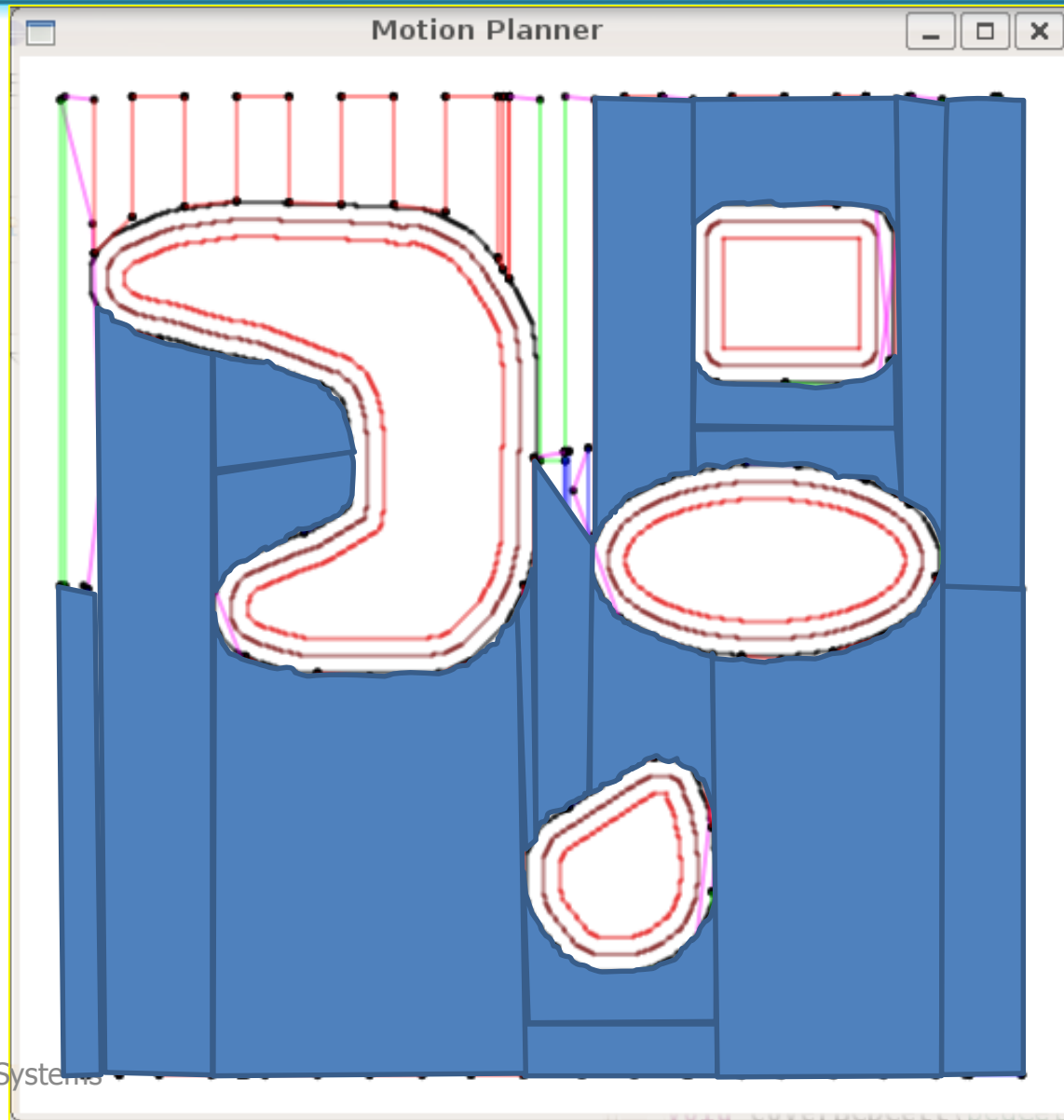
Example 2



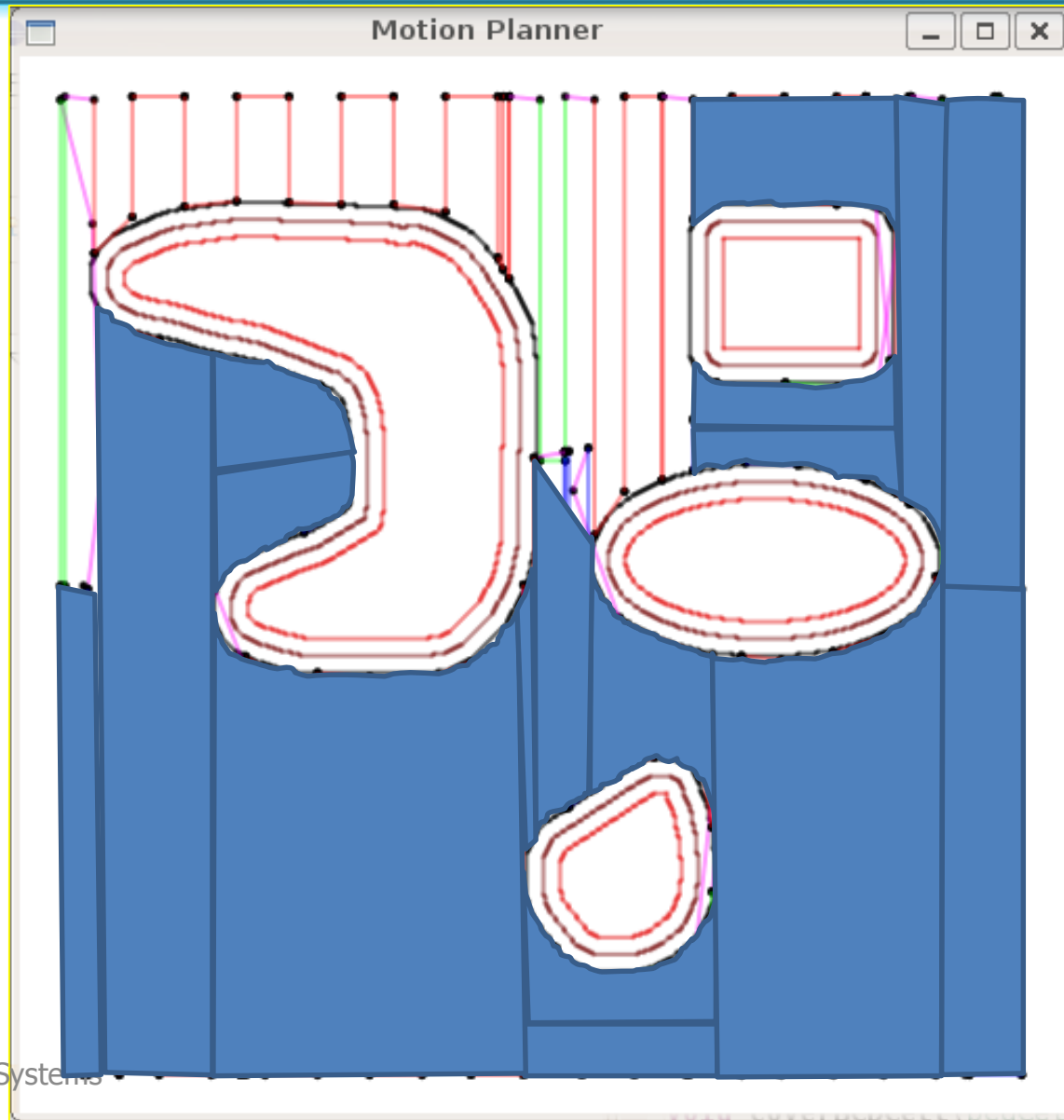
Example 2



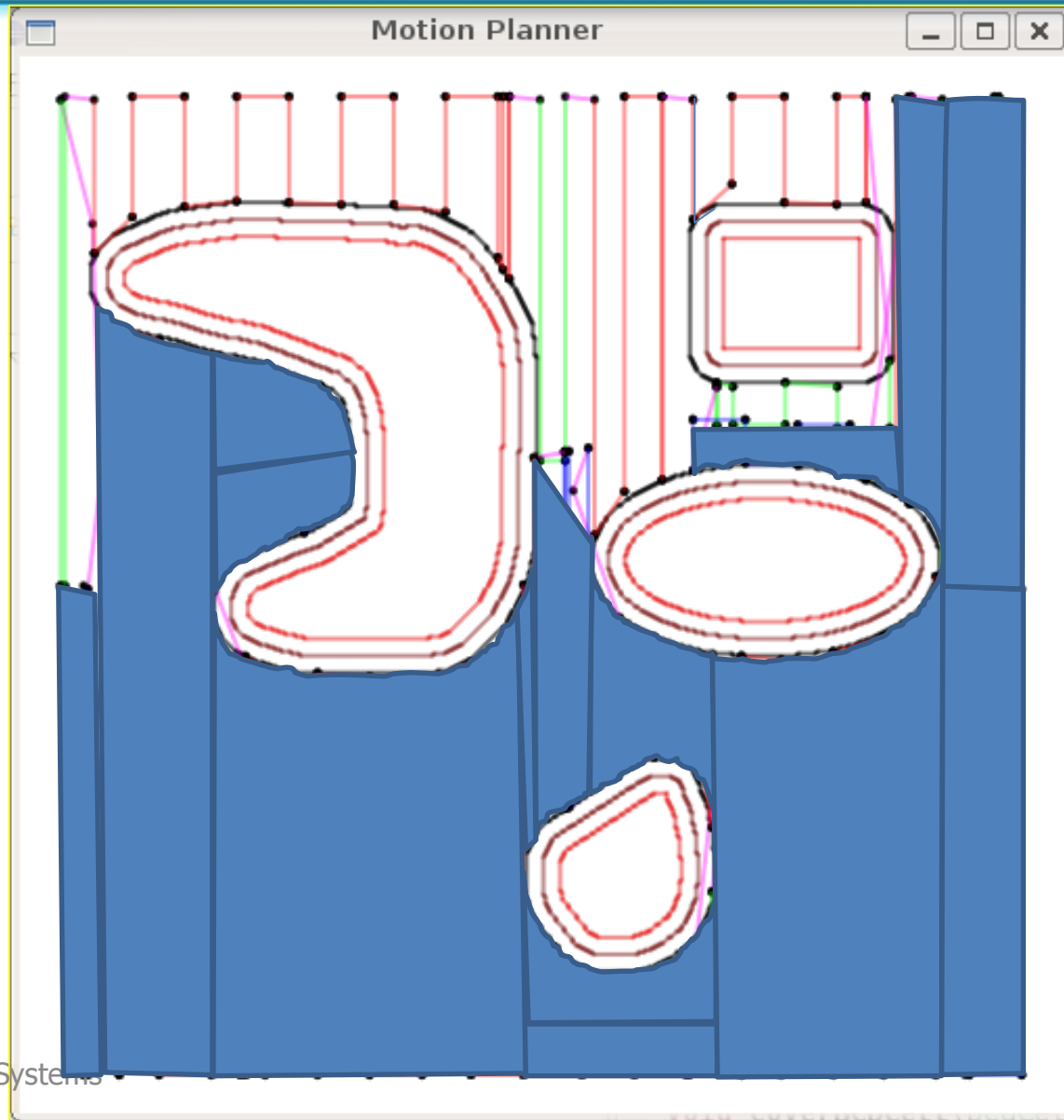
Example 2



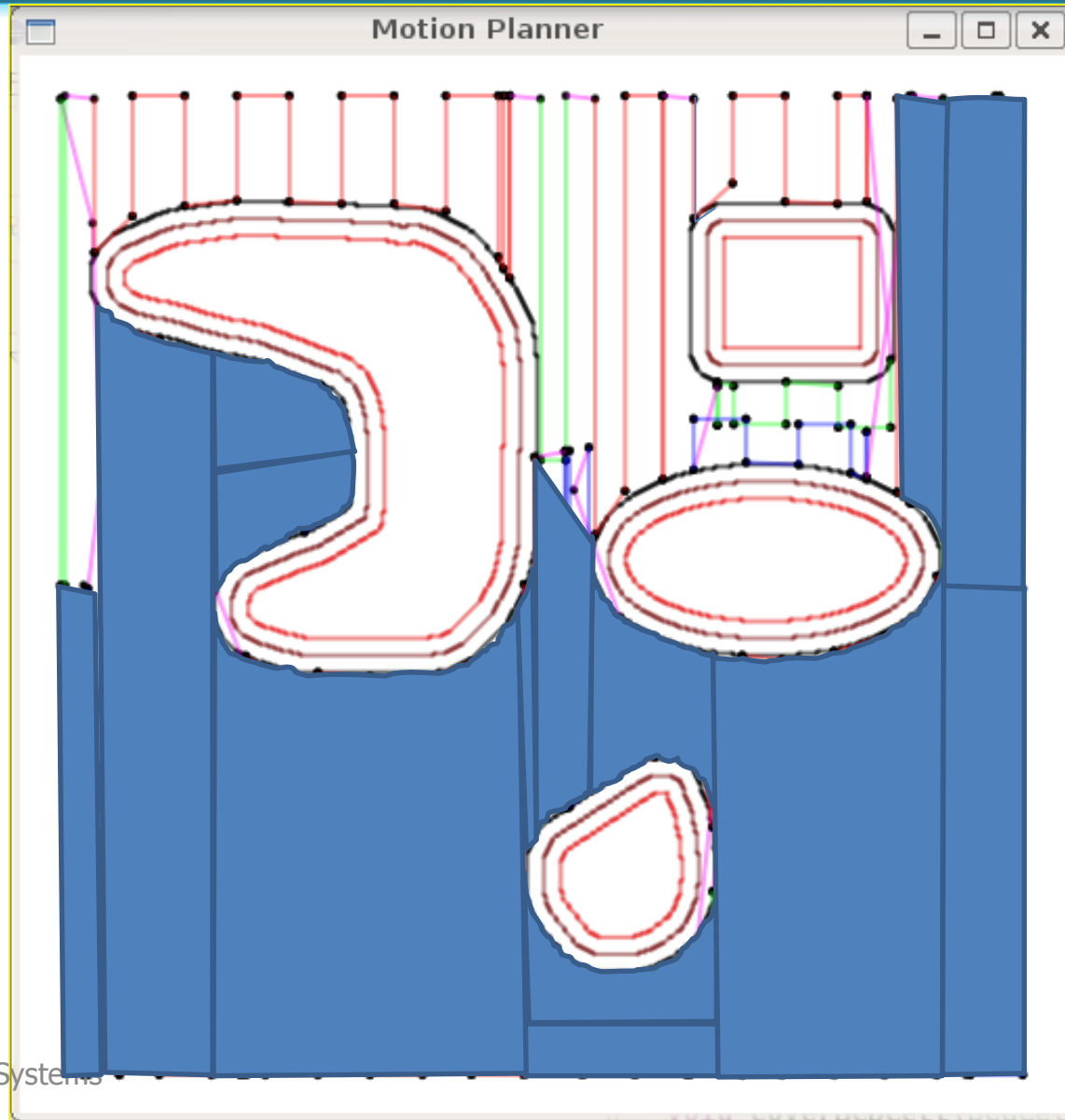
Example 2



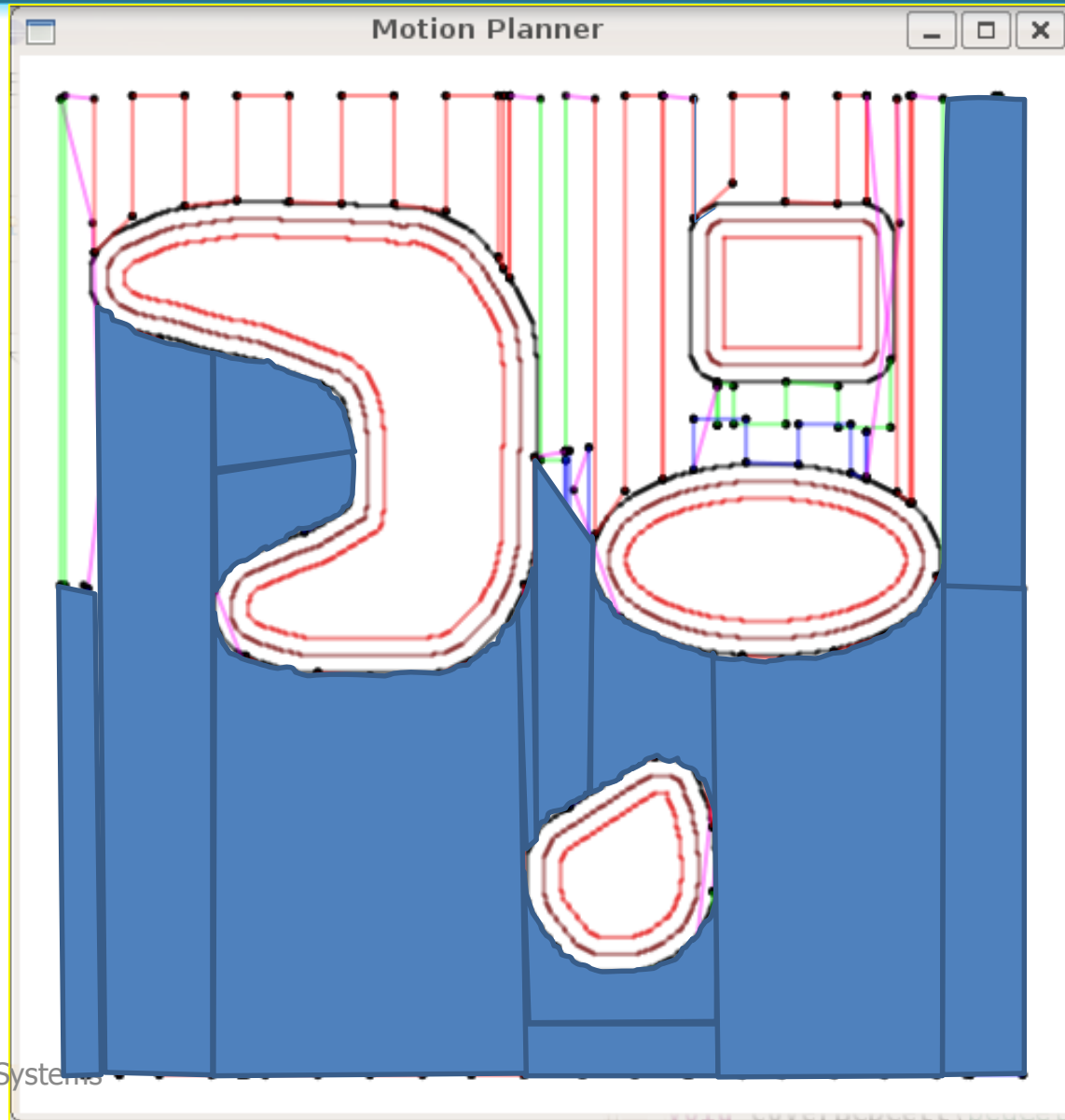
Example 2



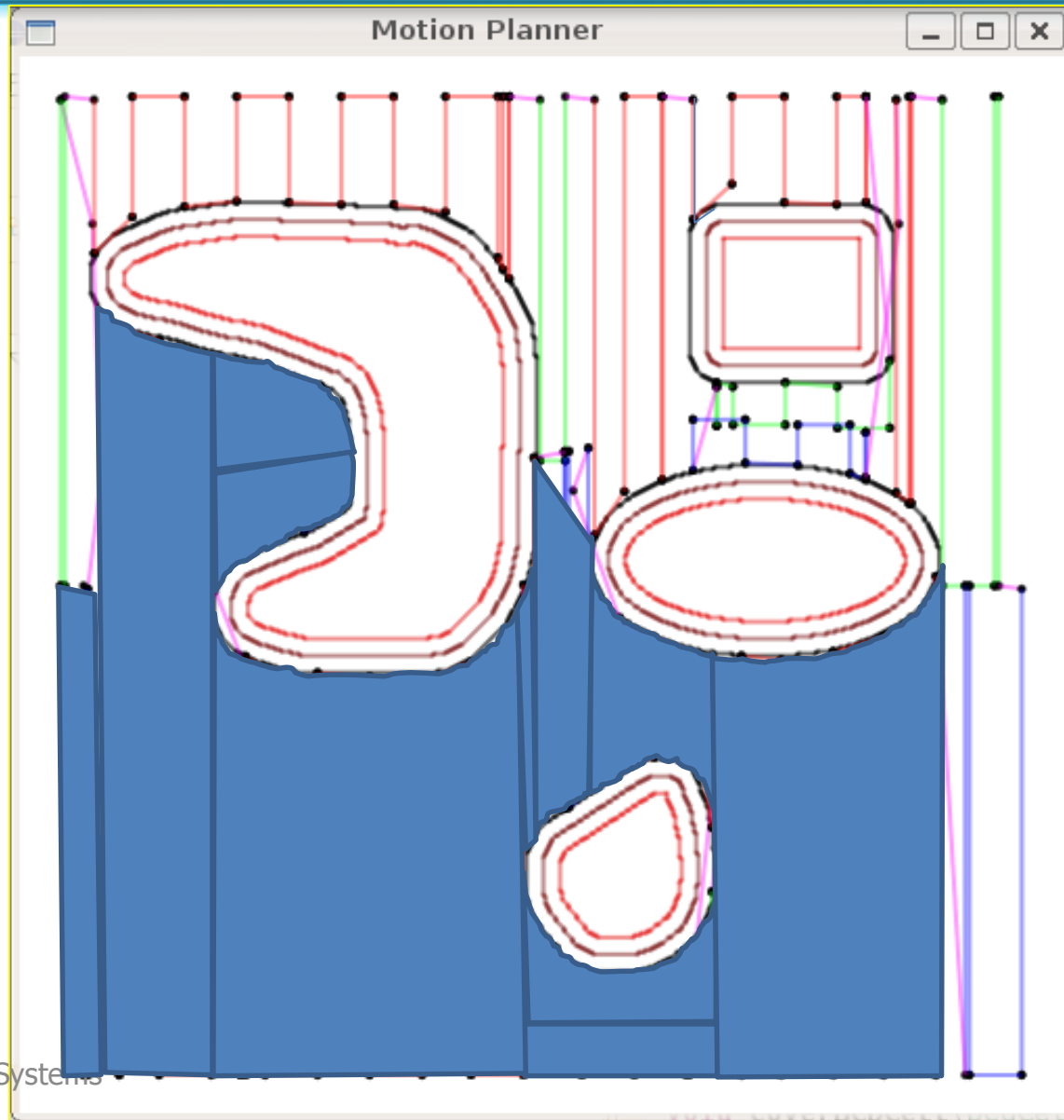
Example 2



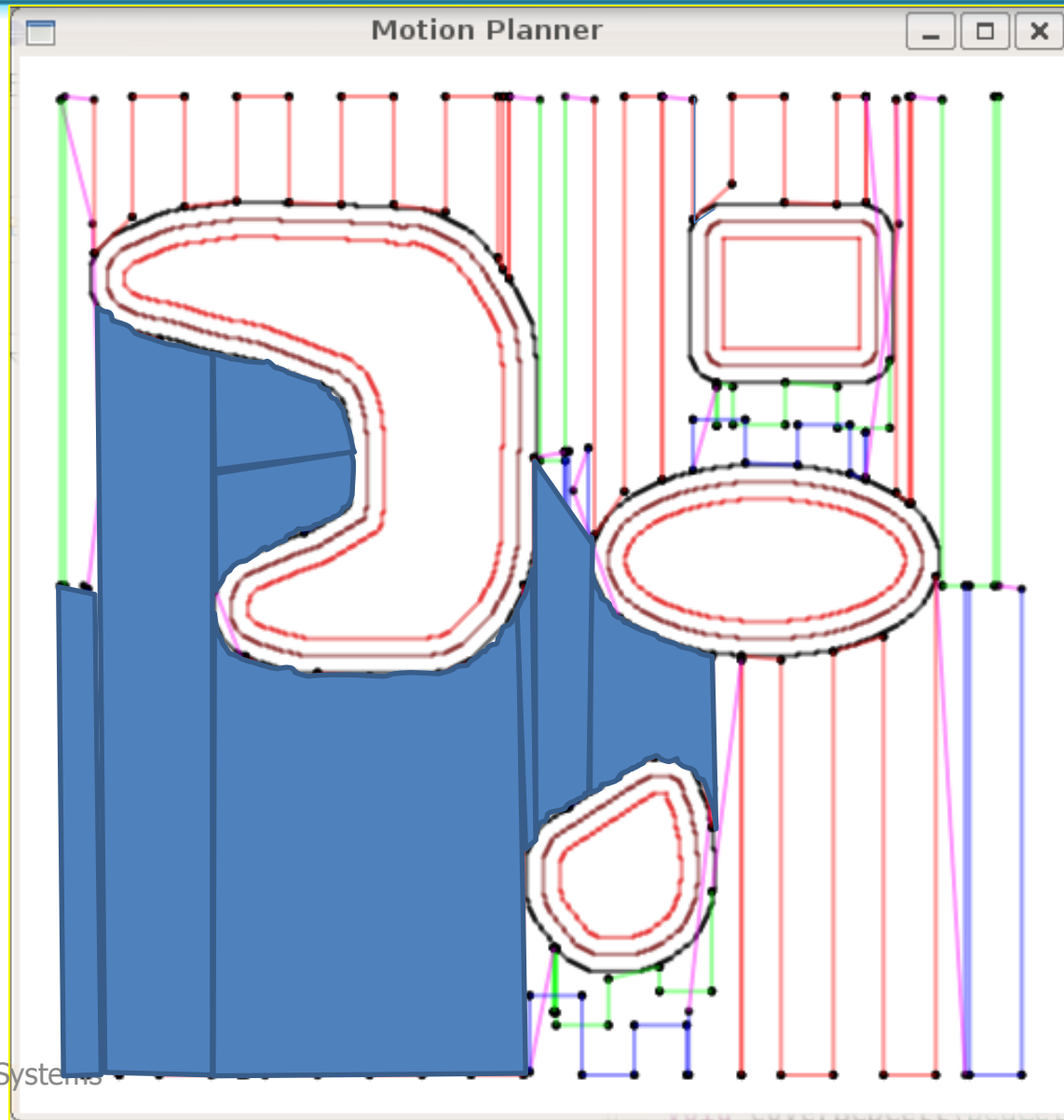
Example 2



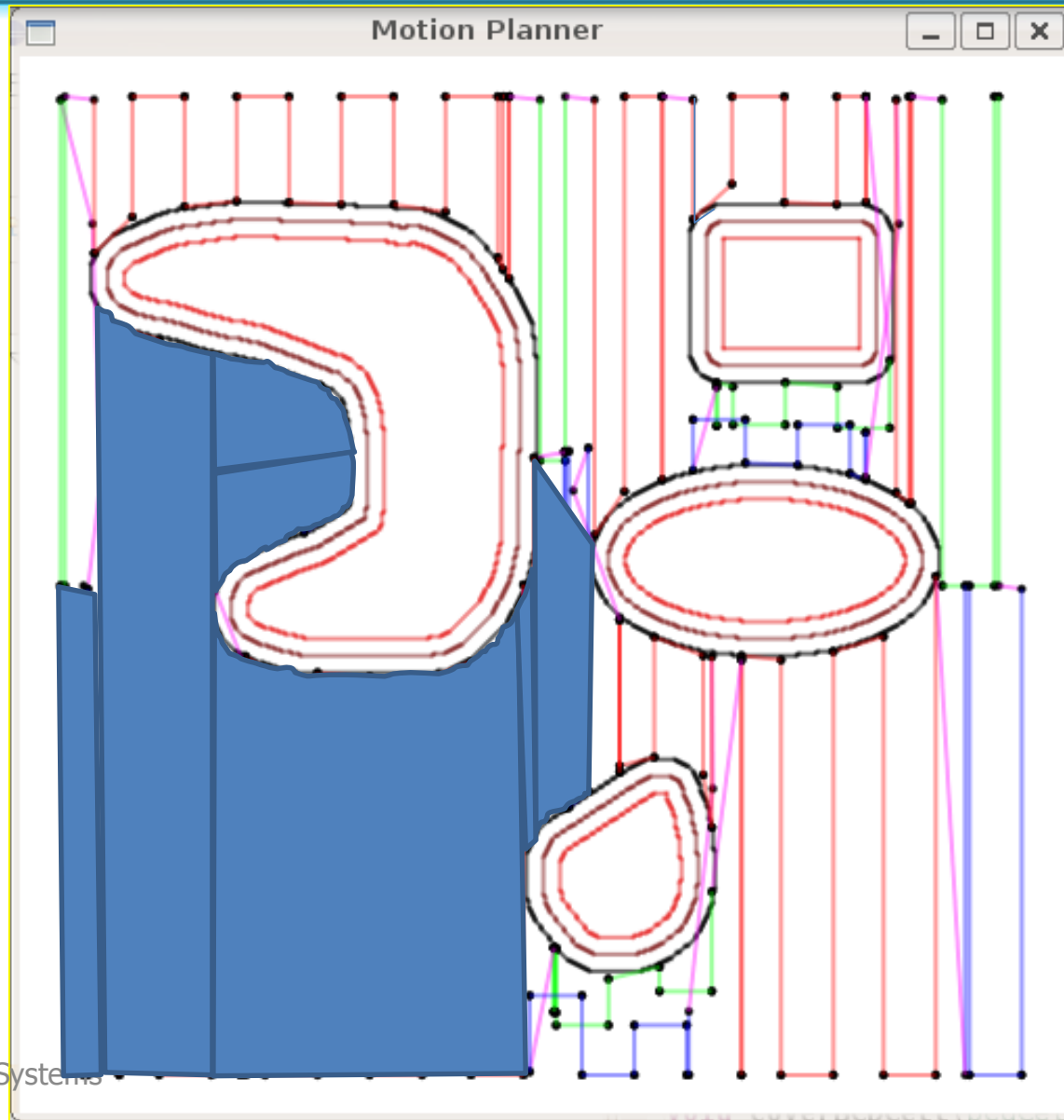
Example 2



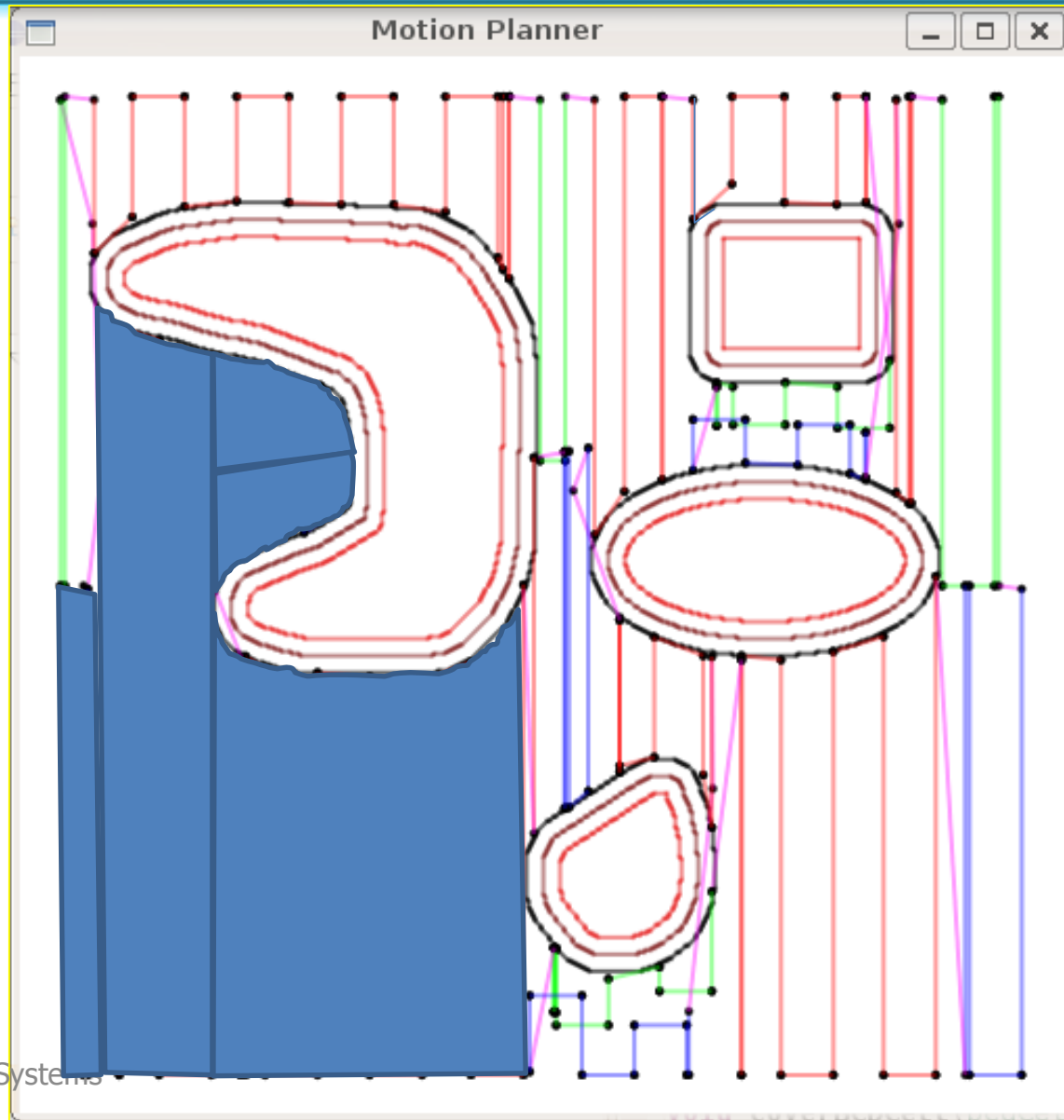
Example 2



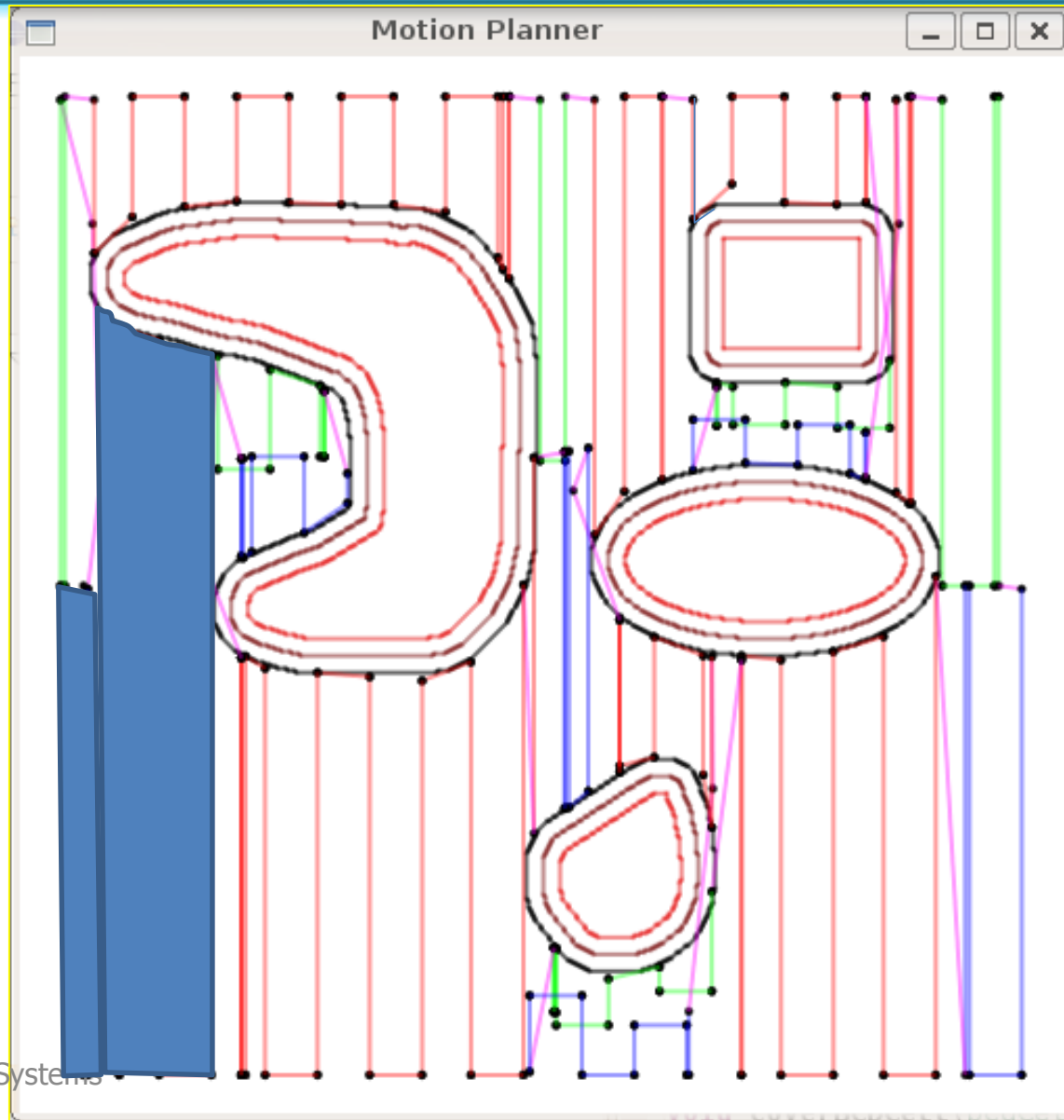
Example 2



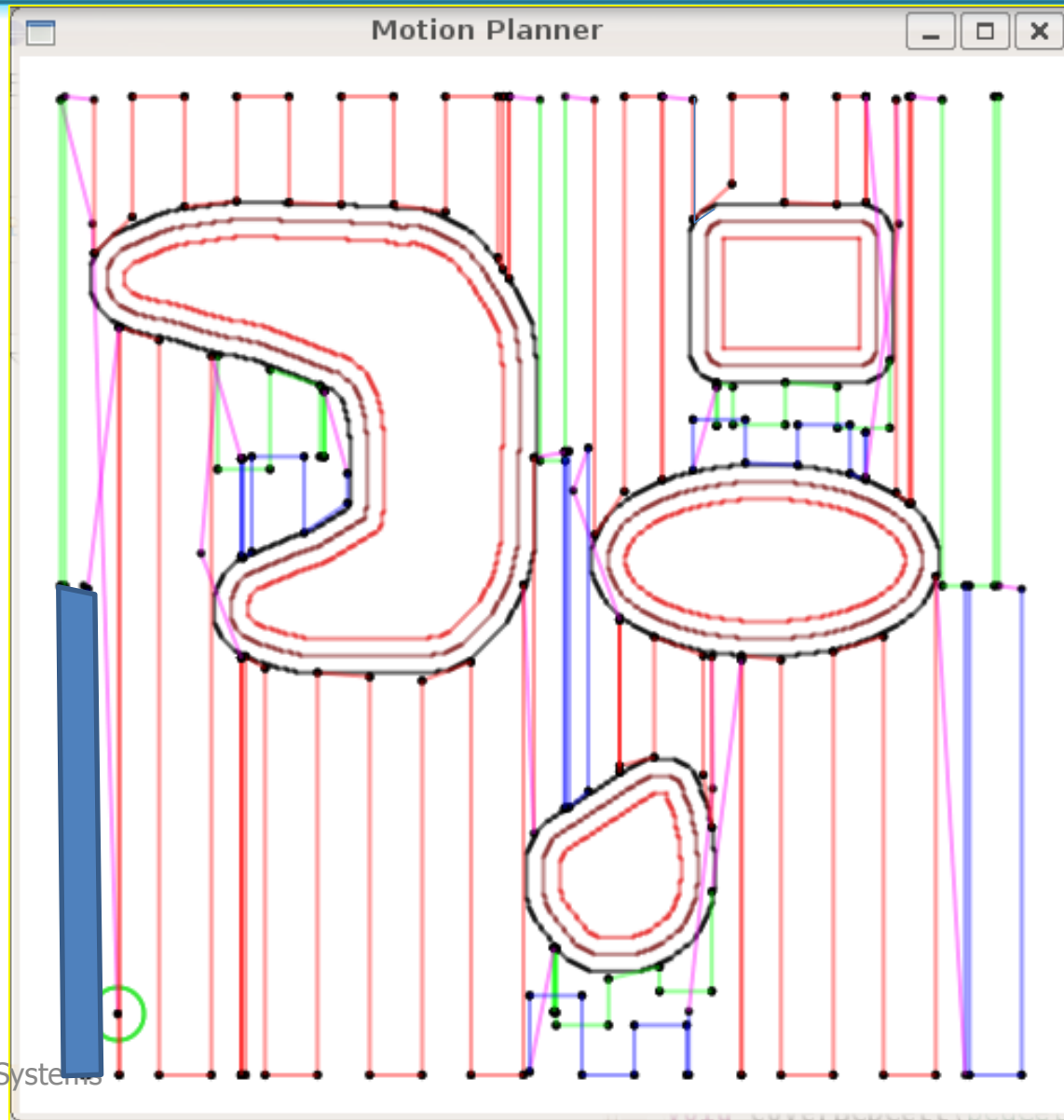
Example 2



Example 2



Example 2



UAV-Optimal Coverage



UAV-Optimal Coverage

- UAVs non-holonomic constraints require special trajectory planning
- 120 Km of flight during coverage

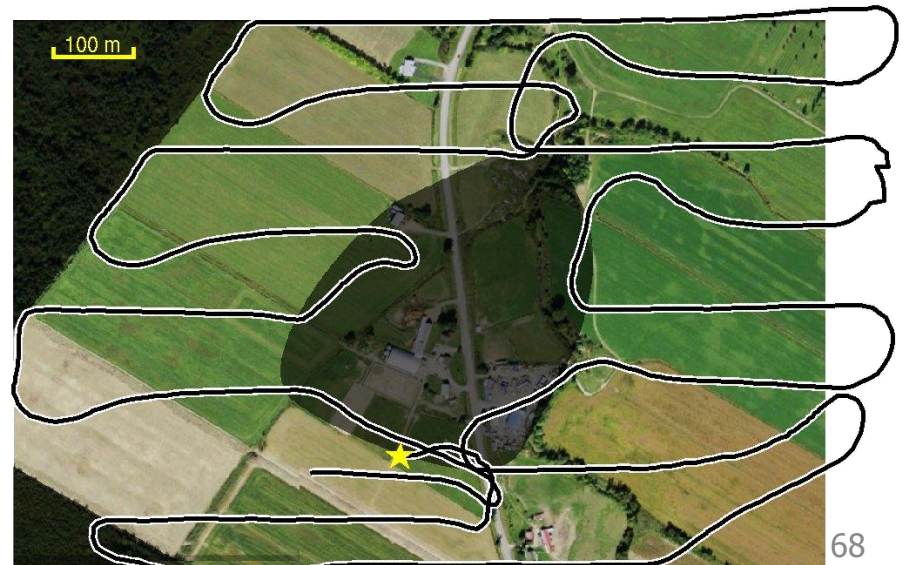
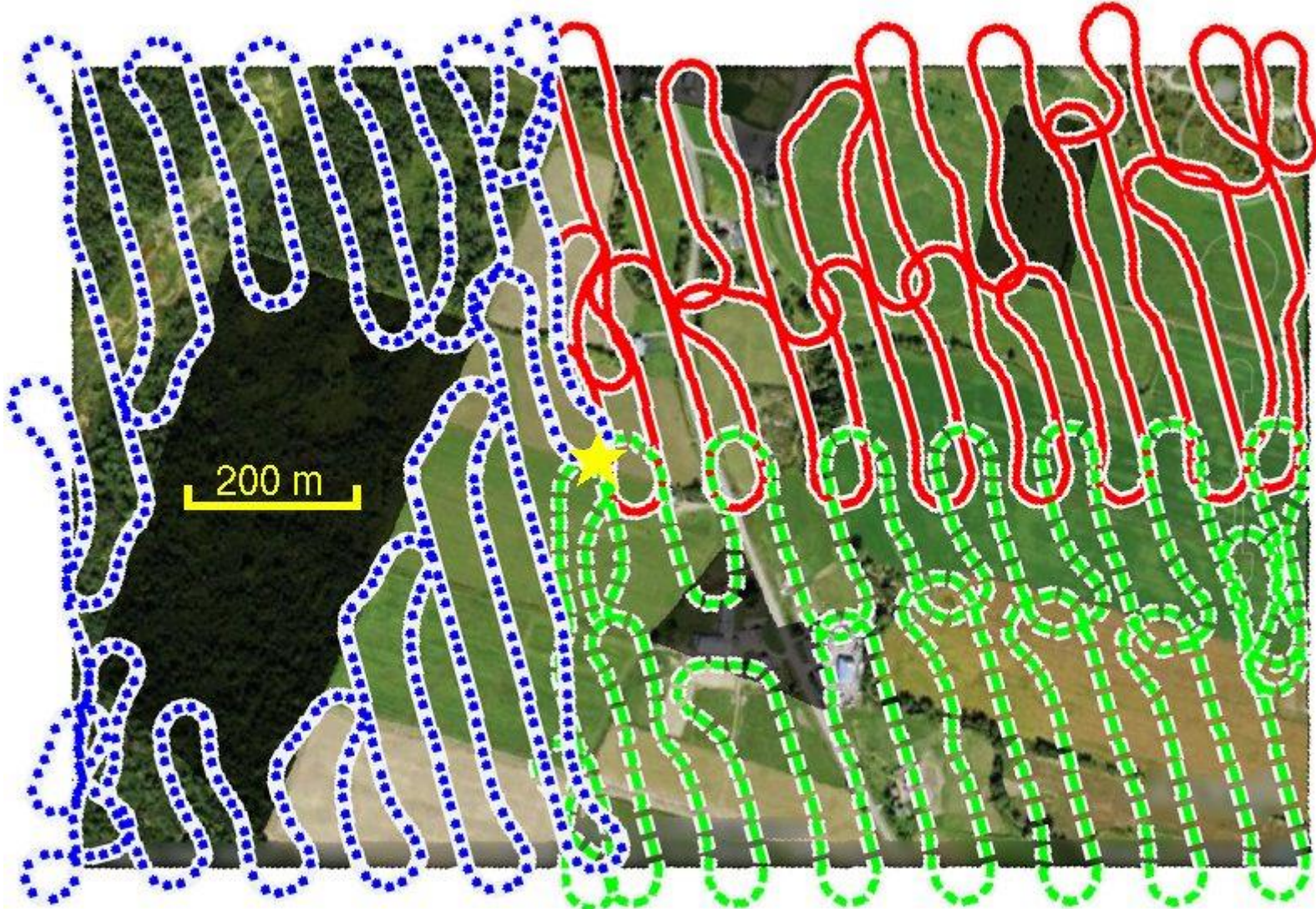


Image Mosaic



Multi-UAV



Video at ICRA 2011

Complete Optimal Terrain Coverage using an Unmanned Aerial Vehicle

Anqi Xu
Chatavut Viriyasuthee
Ioannis Rekleitis

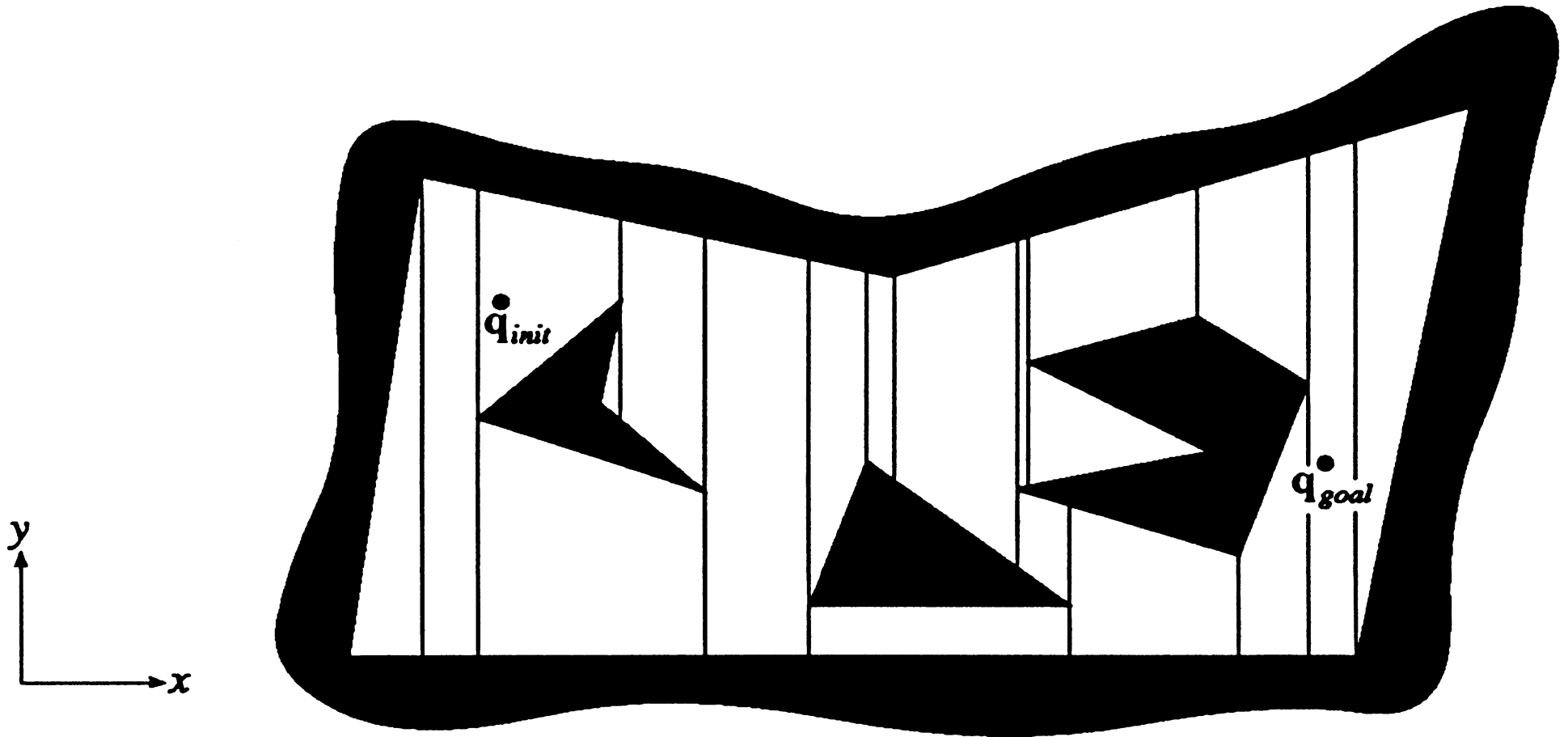


Cell decomposition for Path Planning

- Decompose the free space into **simple** cells and represent the connectivity of the free space by the adjacency graph of these cells

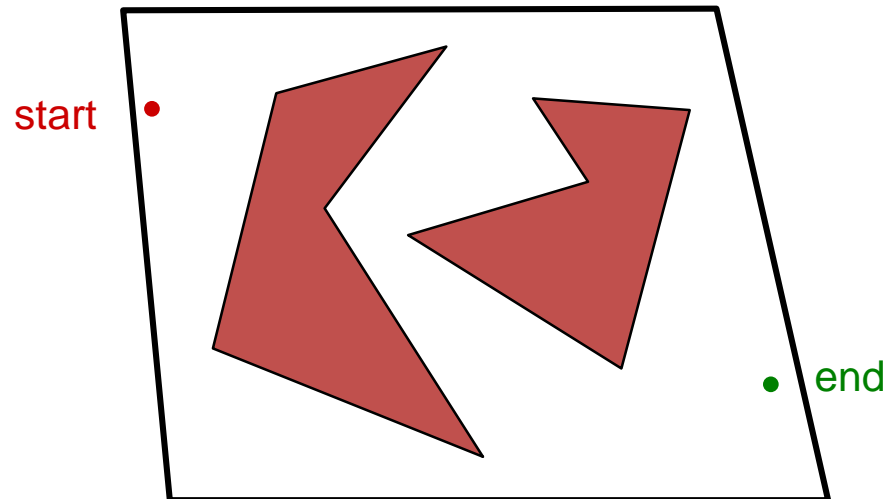


Trapezoidal decomposition



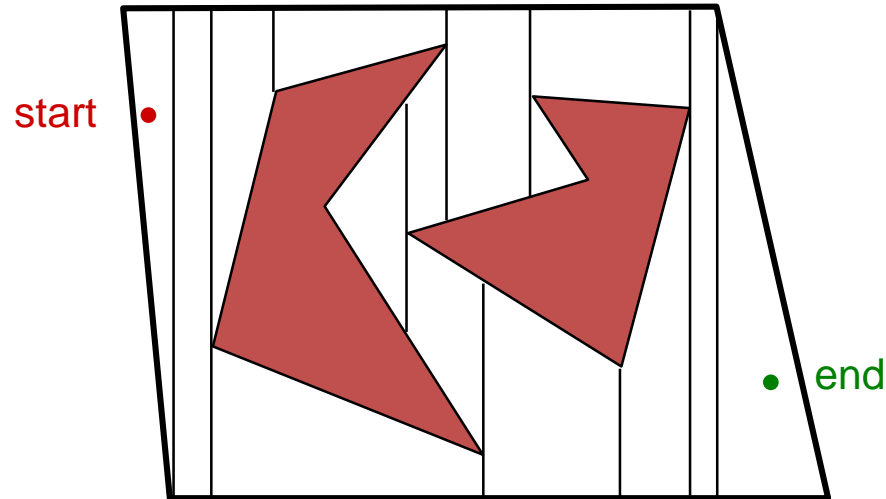
Spatial decompositions

- Dividing free space into pieces and using those...



Spatial decompositions

- Dividing free space into pieces and using those...



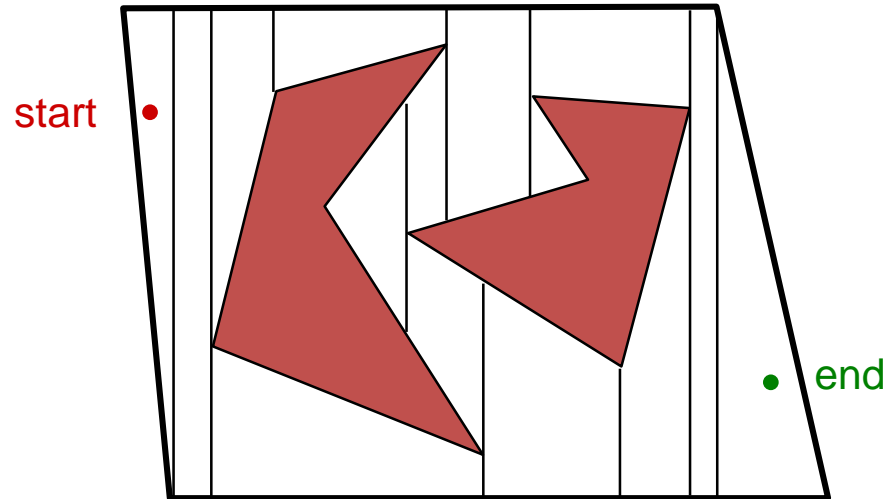
Exact cell decomposition
sweepline algorithm

Running time?



Spatial decompositions

- Dividing free space into pieces and using those...



Exact cell decomposition
sweepline algorithm

Running time?

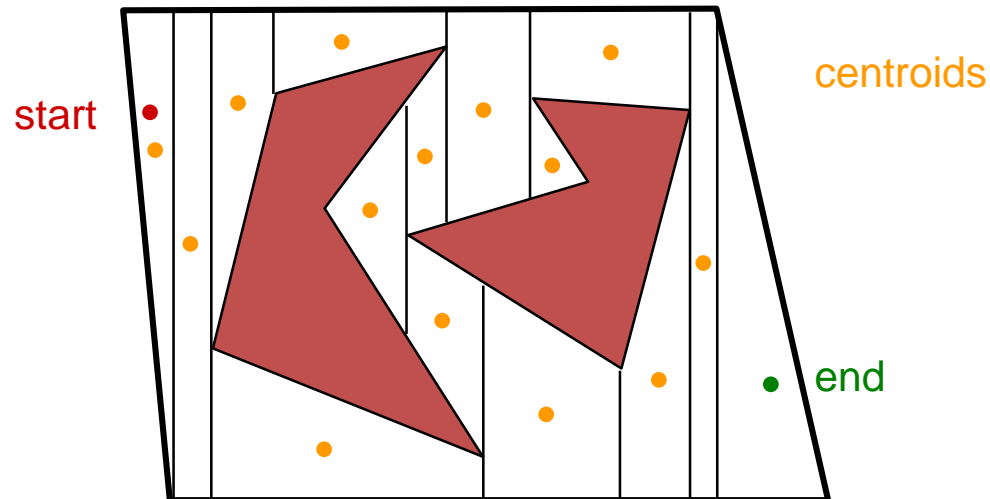
Path?

$O(N \log(N))$



Spatial decompositions

- Dividing free space into pieces and using those...



Exact cell decomposition
sweepline algorithm

Running time?

$O(N \log(N))$

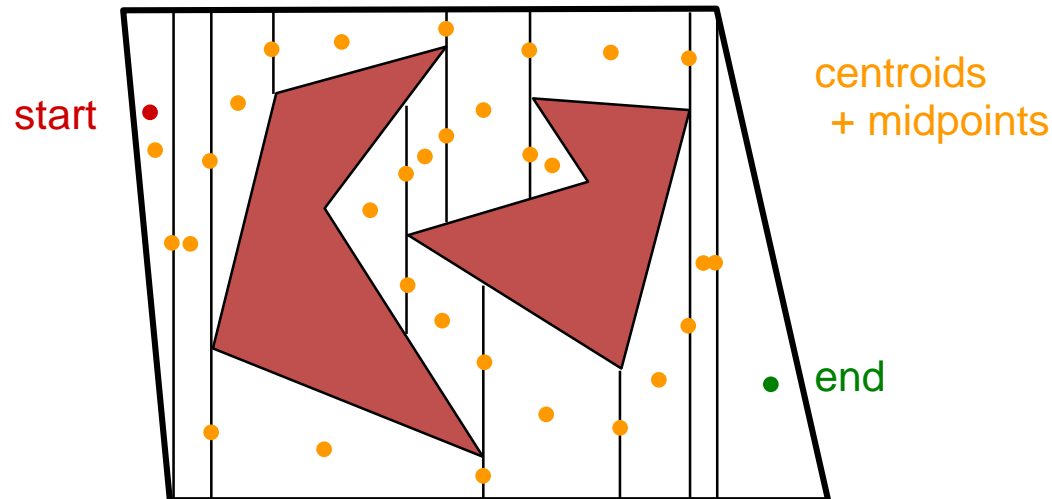
Path?

via centroids



Spatial decompositions

- Dividing free space into pieces and using those...



Exact cell decomposition
sweepline algorithm

Running time?

$O(N \log(N))$

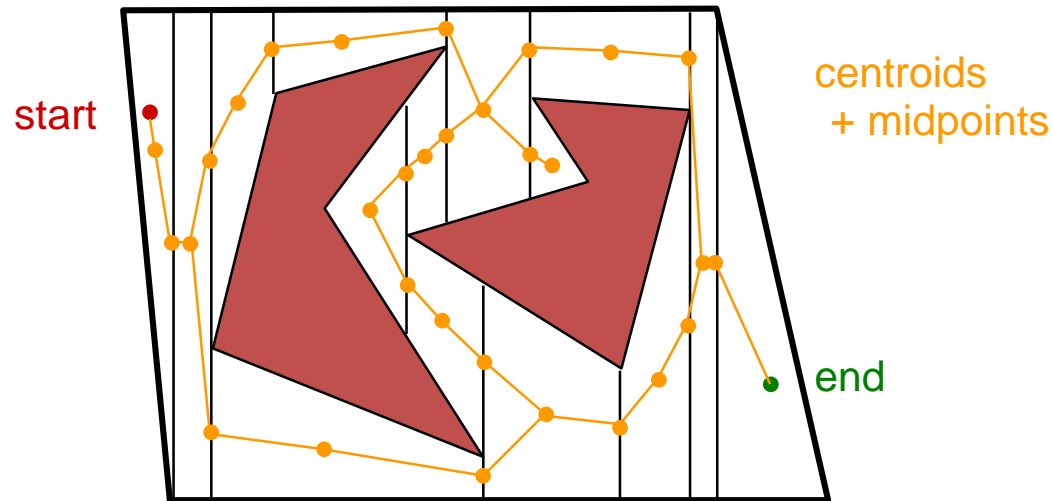
Path?

via centroids
+ edge midpoints



Spatial decompositions

- Dividing free space into pieces and using those...



Exact cell decomposition
sweepline algorithm

Running time?

$O(N \log(N))$

Path?

via centroids
+ edge midpoints
+ graph search

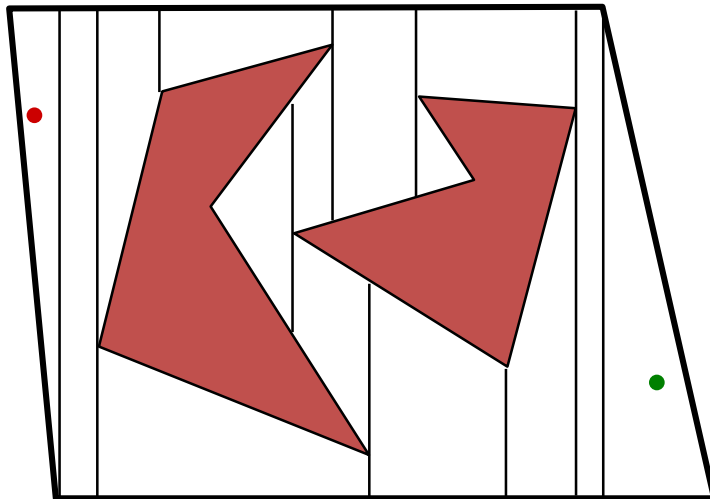
Why?

why else?

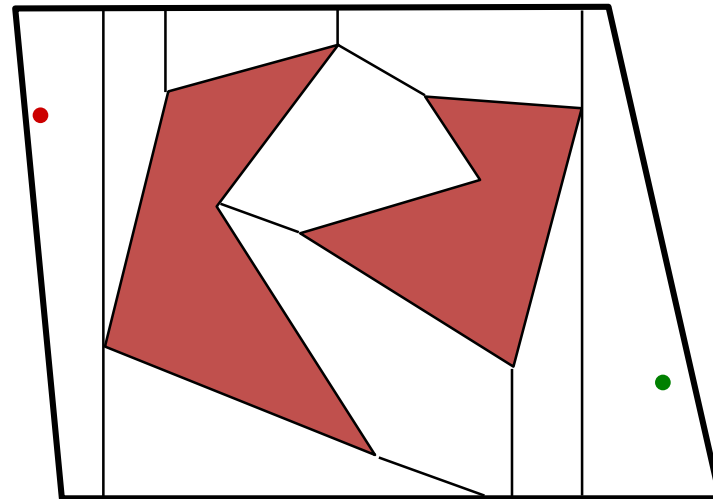


Optimality

- Obtaining the *minimum* number of convex cells is NP-complete.



15 cells



9 cells

Trapezoidal decomposition is exact and complete, but not optimal -- even among convex subdivisions.

Cell-Decomposition Methods

Two families of methods:

- Exact cell decomposition
- Approximate cell decomposition

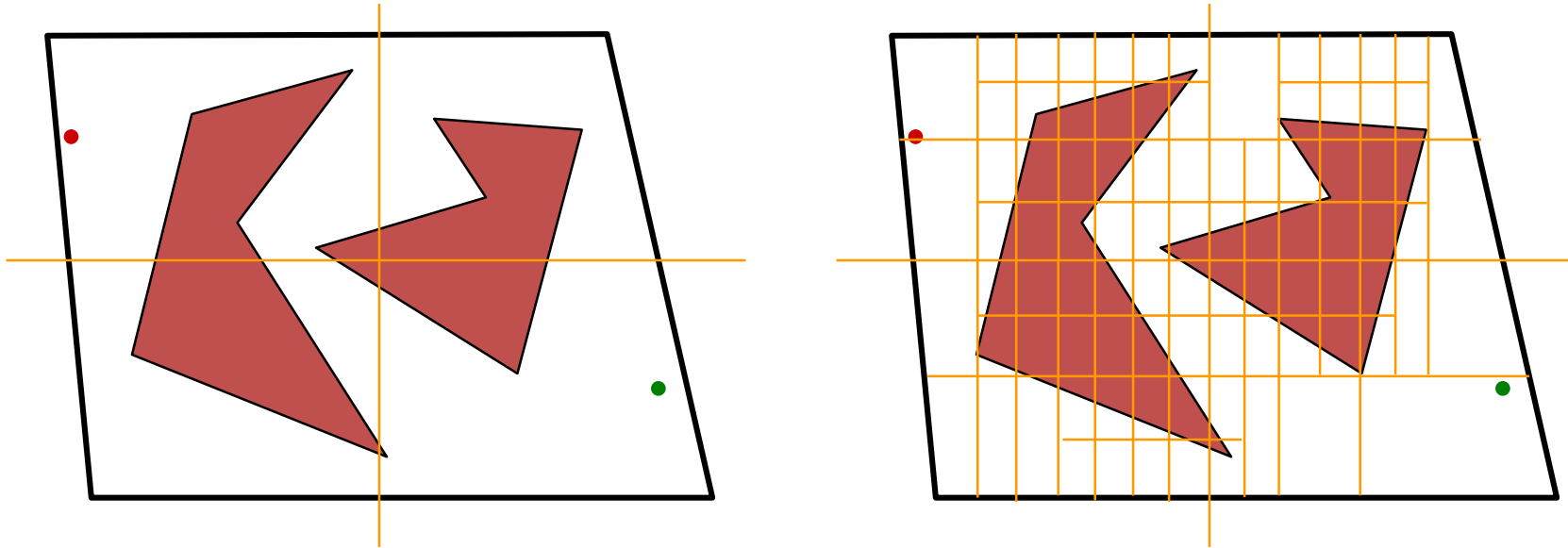
F is represented by a collection of non-overlapping cells whose union is contained in F

Examples: quadtree, octree, 2^n -tree



further decomposing...

- Approximate cell decomposition

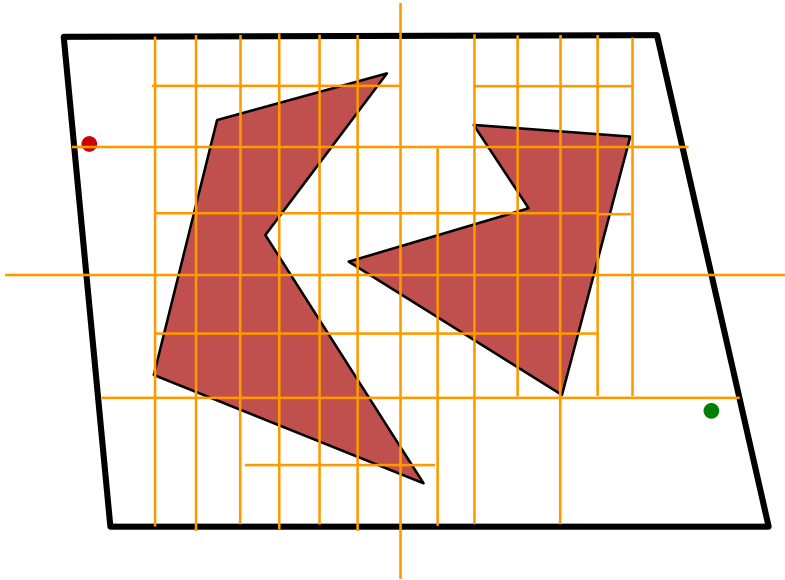


Quadtree:

recursively subdivides each *mixed* obstacle/free (sub)region into four quarters...

further decomposing...

- Approximate cell decomposition

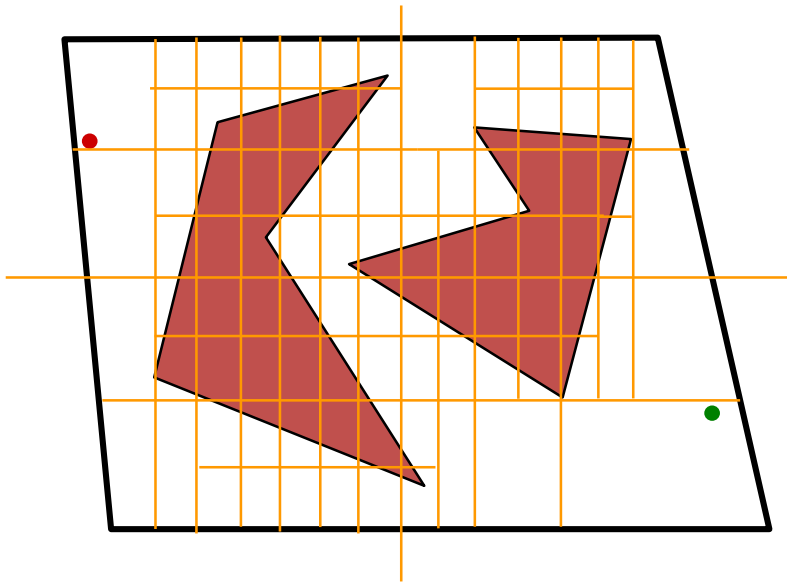


Quadtree:

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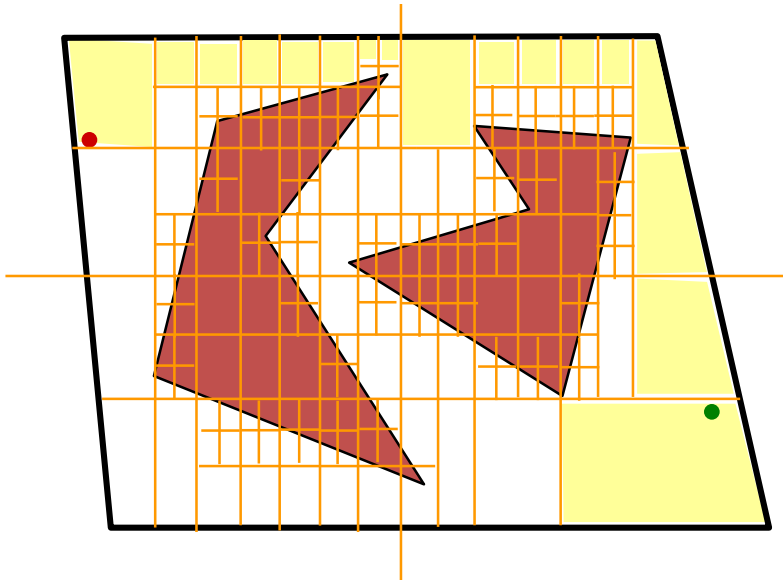


Quadtree:

recursively subdivides each *mixed* obstacle/free (sub)region into four quarters...

further decomposing...

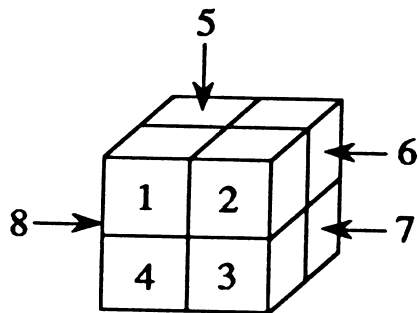
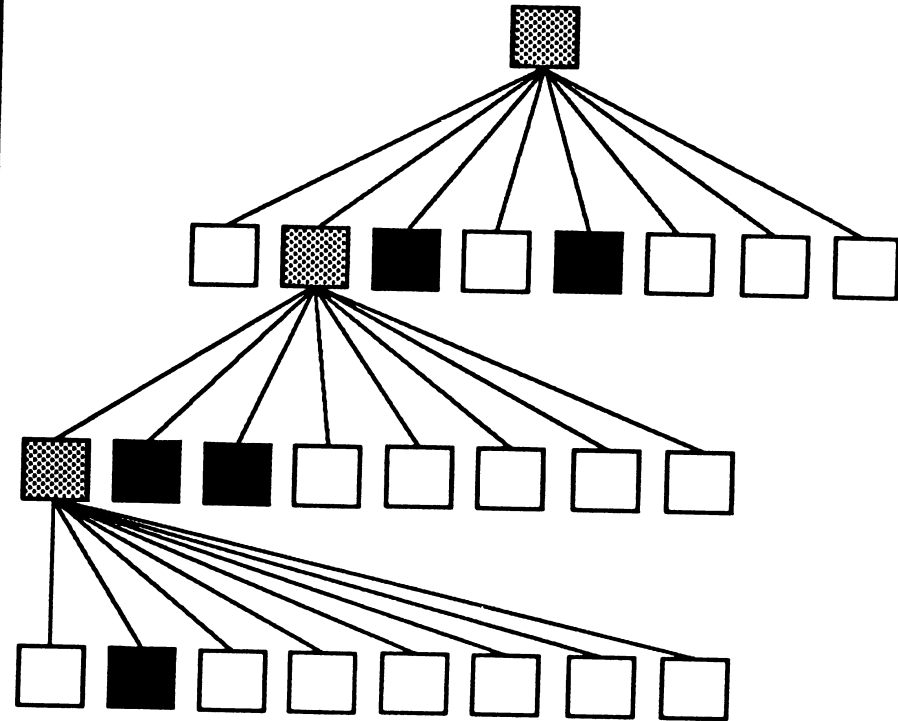
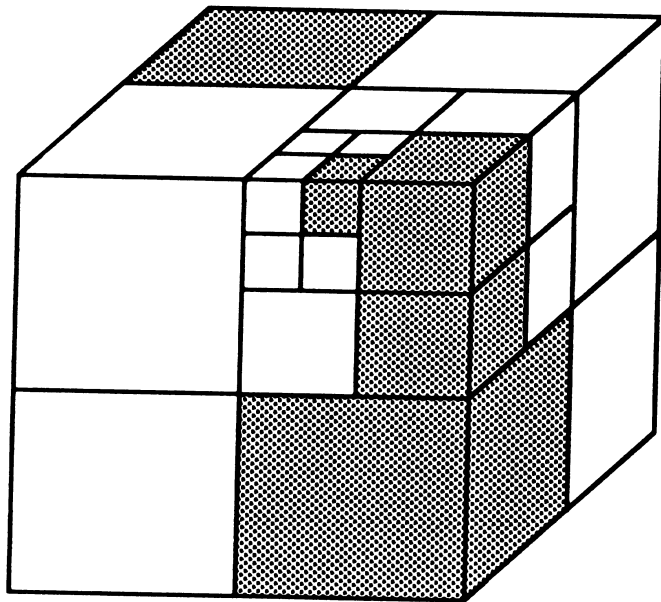
- Approximate cell decomposition



Quadtree

Again, use a graph-search algorithm to find a path from the start to goal

Octree Decomposition



 EMPTY cell  MIXED cell  FULL cell