UNIVERSITY OF SOUTH CAROLINA

CSCE 774 ROBOTIC SYSTEMS

Path Planning

Ioannis Rekleitis

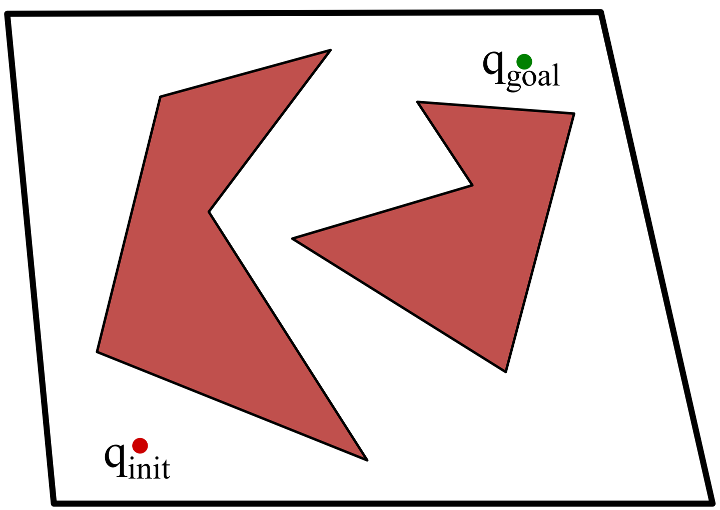
Outline

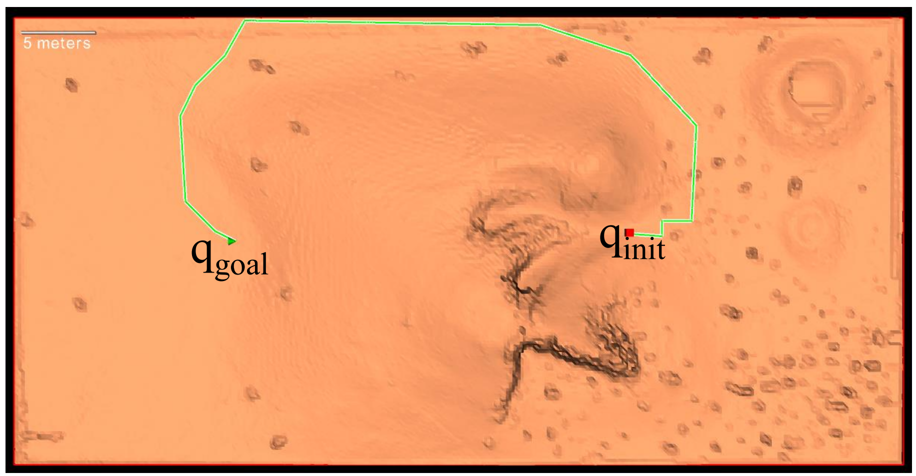
- Path Planning
  - Visibility Graph
  - Potential Fields
  - Bug Algorithms
  - Skeletons/Voronoi Graphs
  - C-Space

Motion Planning

- The ability to go from **A** to **B**
  - Known map – Off-line planning
  - Unknown Environment –Online planning
  - Static/Dynamic Environment

• q_{init} • q_{goal}





Path Planning

World

Robot

Map

Path Planning

World

- Indoor/Outdoor
- 2D/2.5D/3D
- Static/Dynamic
- Known/Unknown
- Abstract (web)

Robot

Map

Path Planning

World

Robot

- Mobile
  - Indoor/Outdoor
  - Walking/Flying/Swimming
- Manipulator
- Humanoid
- Abstract

Map

Path Planning

World

Robot

Map

- Topological
- Metric
- Feature Based
- 1D,2D,2.5D,3D

Path Planning

World

- Indoor/Outdoor
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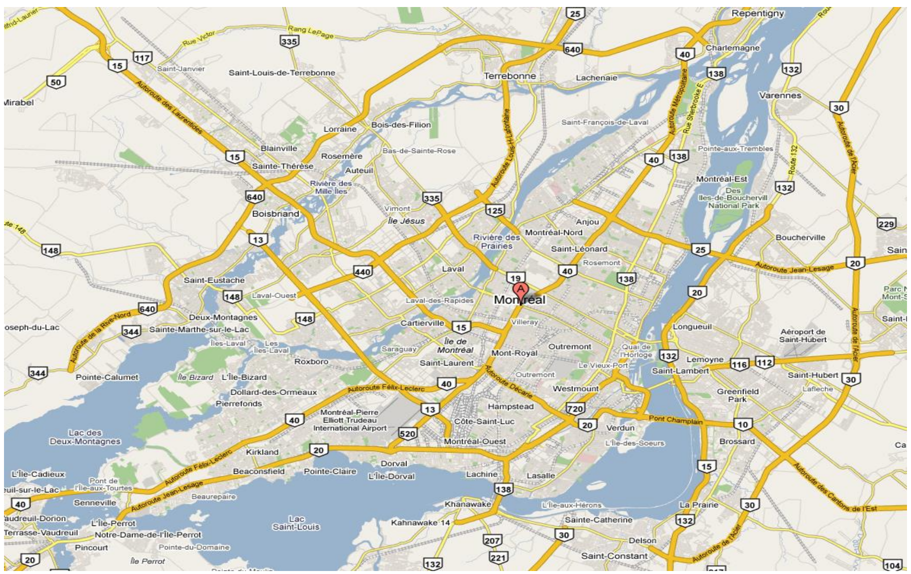
- Abstract

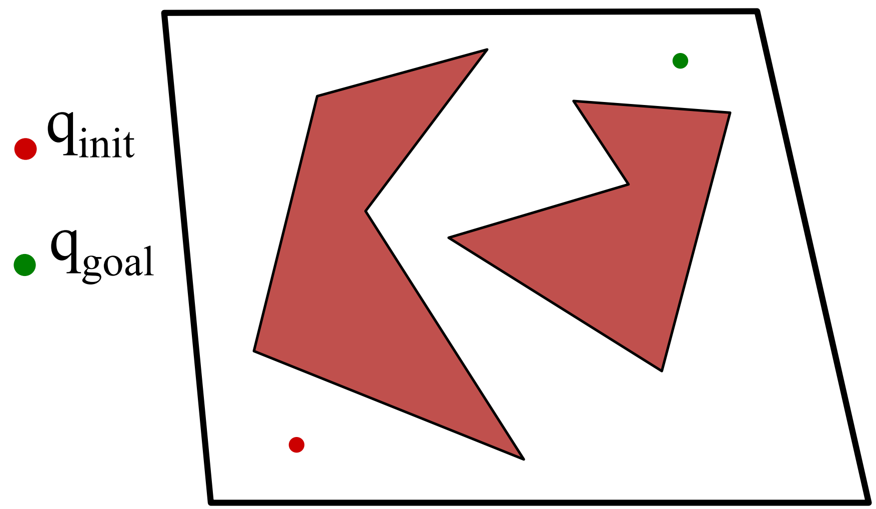
Map

- Topological
- Metric
- Feature Based
- 1D,2D,2.5D,3D

Path Planning: Assumptions

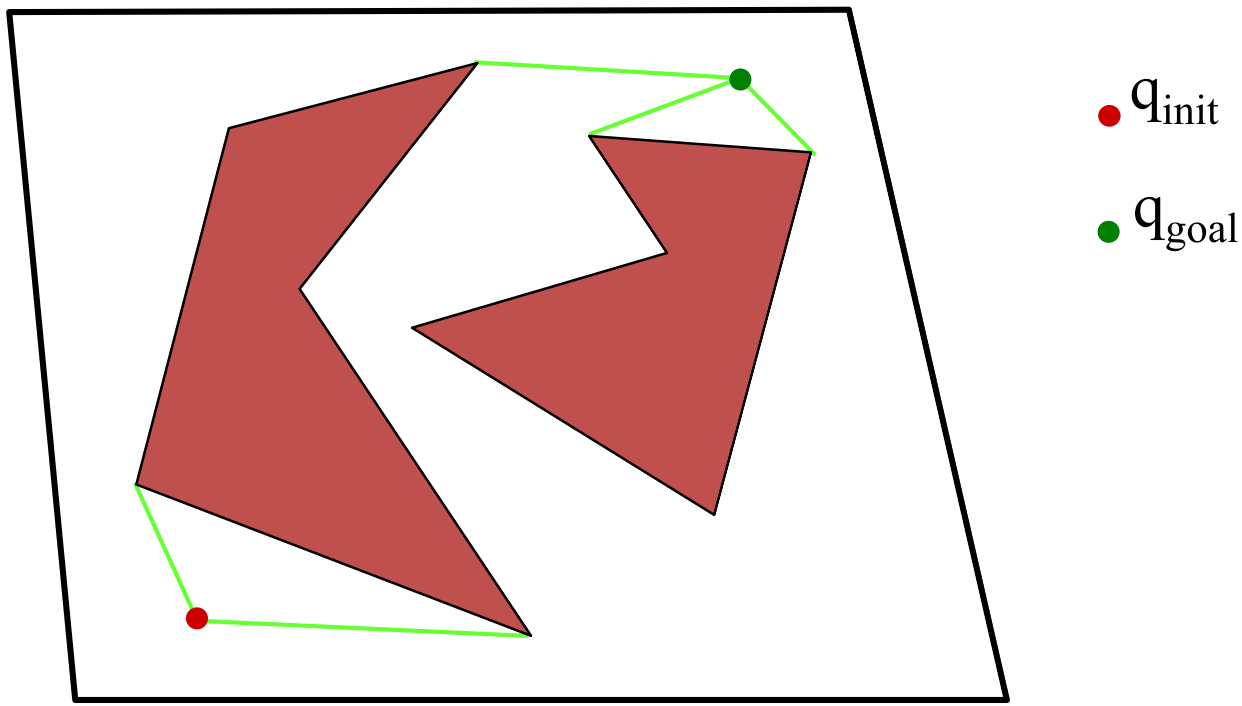
- Known Map
- Roadmaps (Graph representations)
- Polygonal Representation





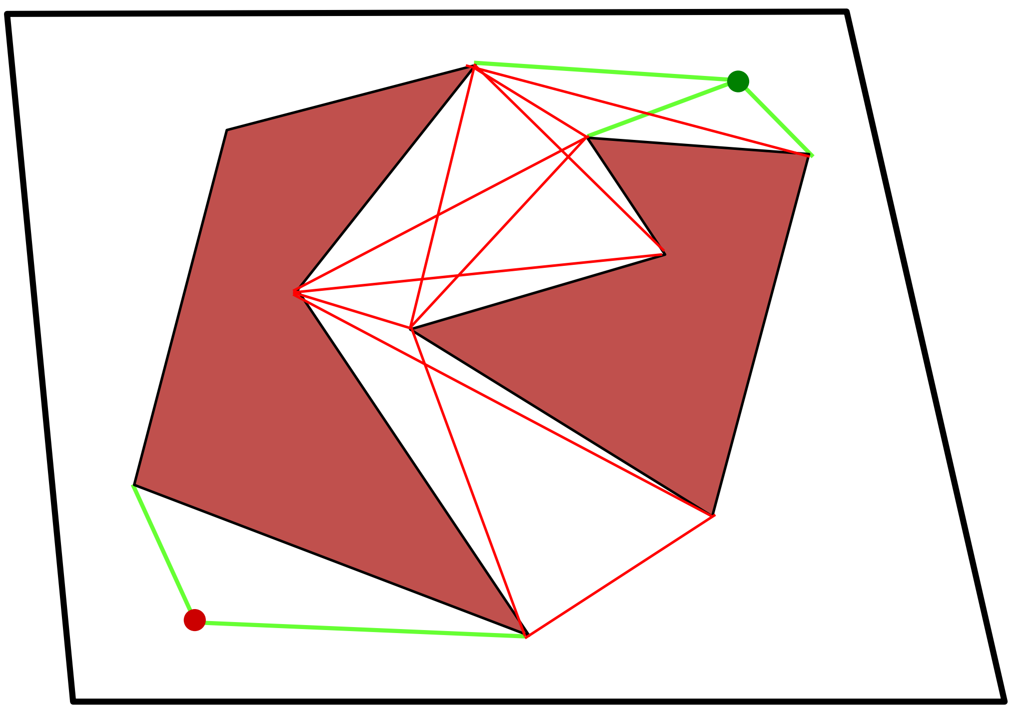
Visibility Graph

- Connect Initial and goal locations with all the visible vertices



Visibility Graph

- Connect initial and goal locations with all the visible vertices
- Connect each obstacle vertex to every visible obstacle vertex

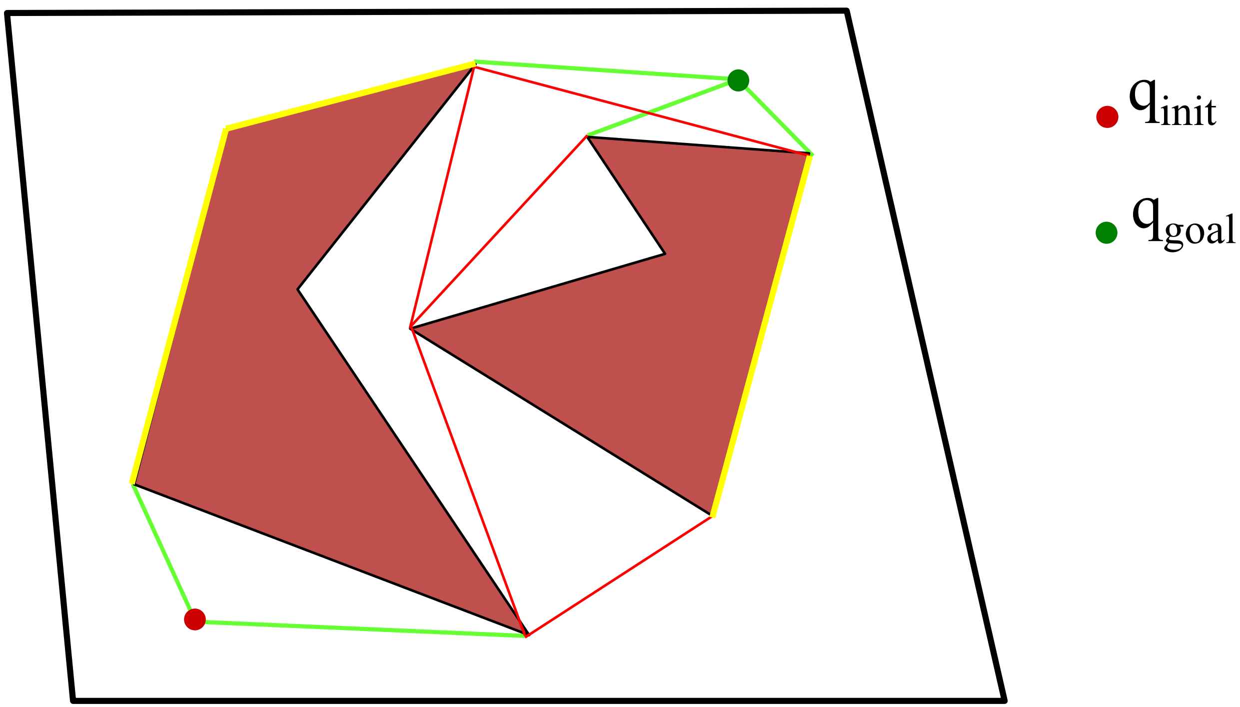


q_{init}

q_{goal}

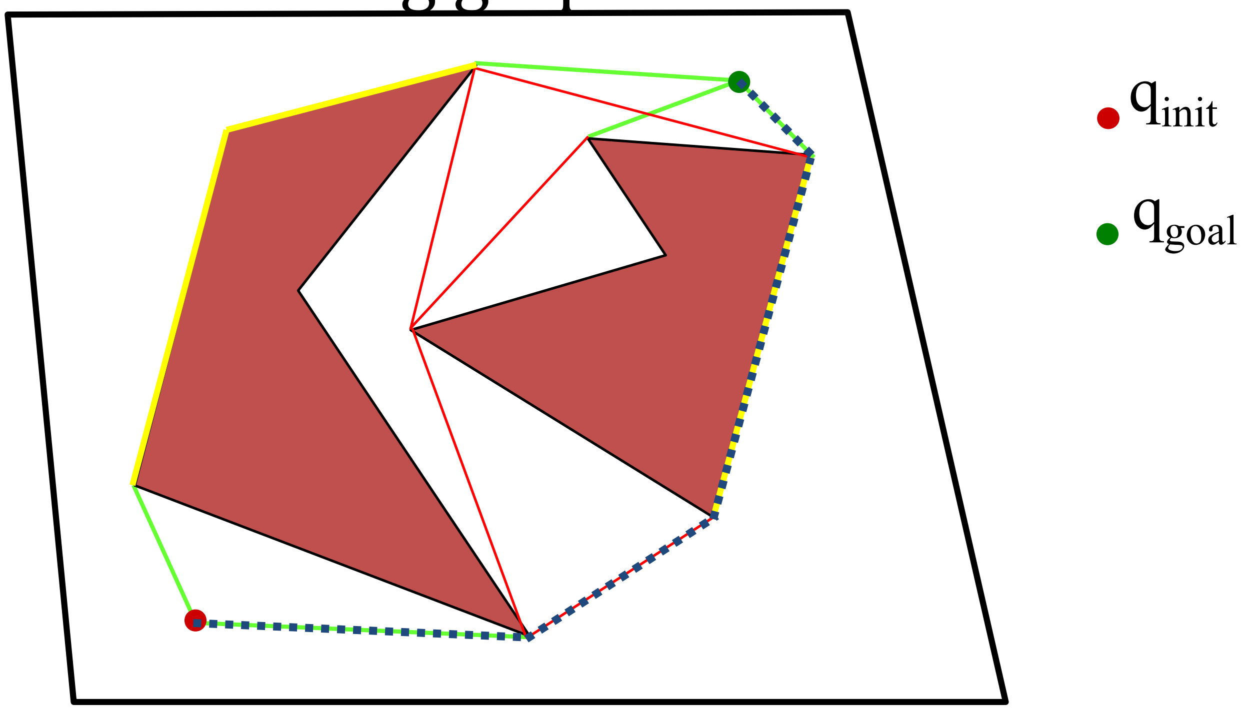
Visibility Graph

- Connect initial and goal locations with all the visible vertices
- Connect each obstacle vertex to every visible obstacle vertex
- Remove edges that intersect the interior of an obstacle



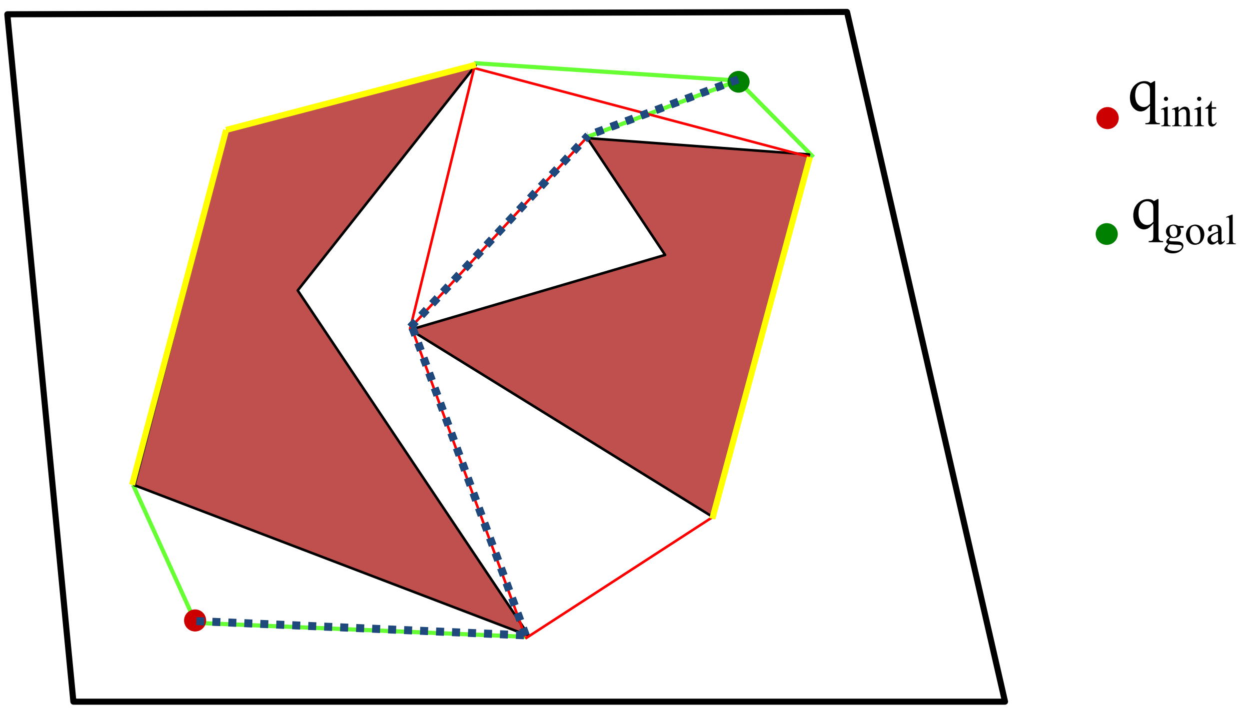
Visibility Graph

- Connect initial and goal locations with all the visible vertices
- Connect each obstacle vertex to every visible obstacle vertex
- Remove edges that intersect the interior of an obstacle
- Plan on the resulting graph



Visibility Graph

- An alternative path
- Alternative name: “Rubber band algorithm”



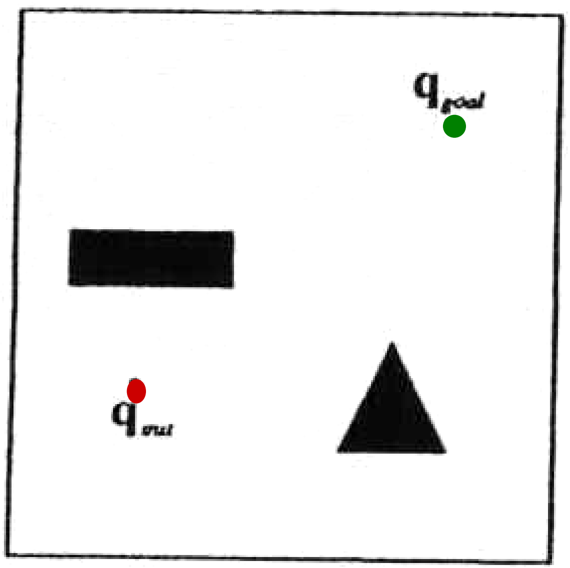
Major Fault

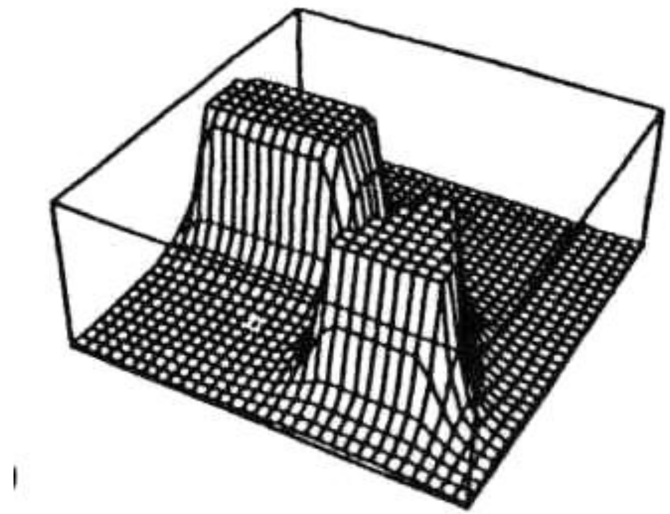
- Point robot
- Path planning like that guarantees to hit the obstacles

Path Planning

Potential Field methods

- compute a repulsive force away from obstacles

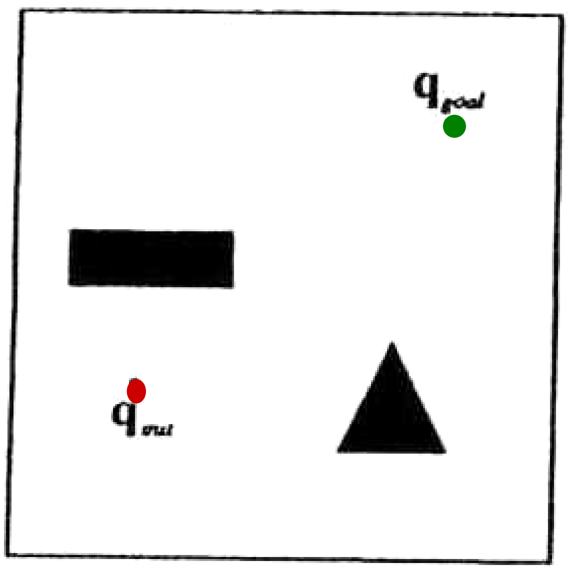


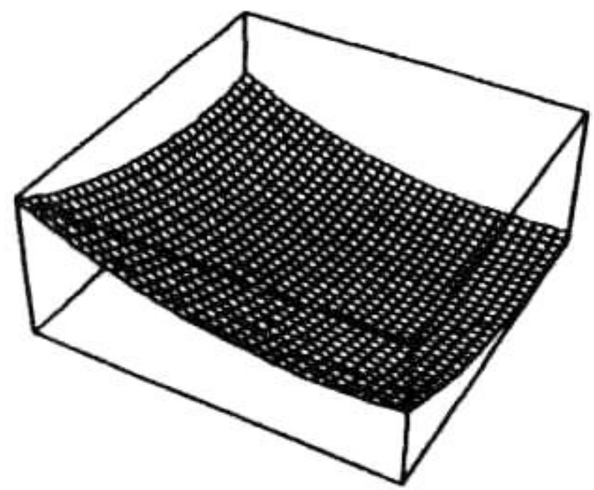


Local techniques

Potential Field methods

- compute a repulsive force away from obstacles
- compute an attractive force toward the goal



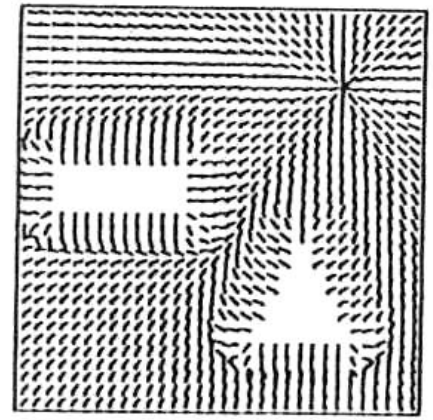


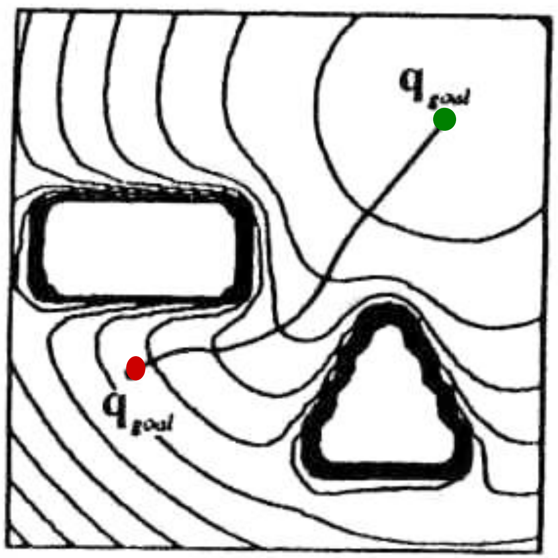
Local techniques

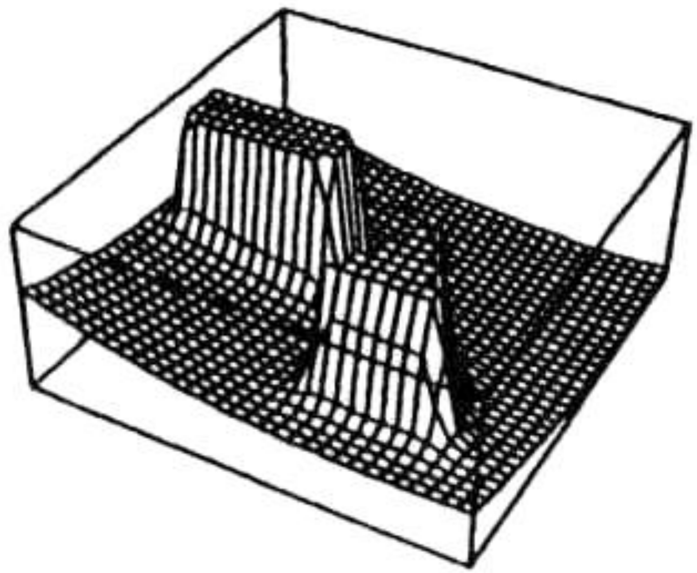
Potential Field methods

- compute a repulsive force away from obstacles
- compute an attractive force toward the goal

→ let the sum of the forces control the robot







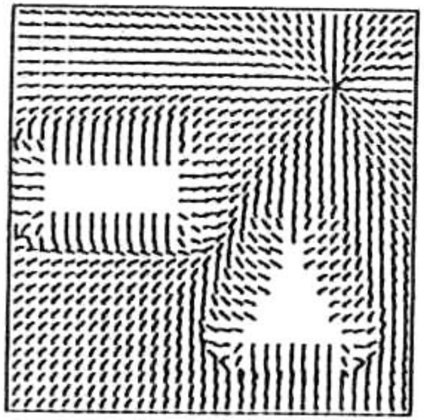
key advantages?

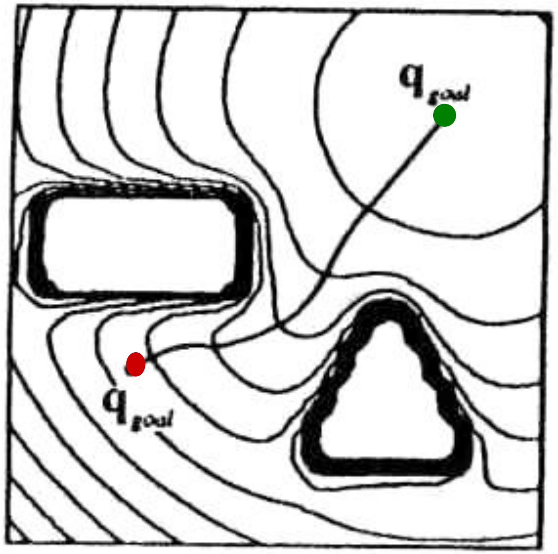
Local techniques

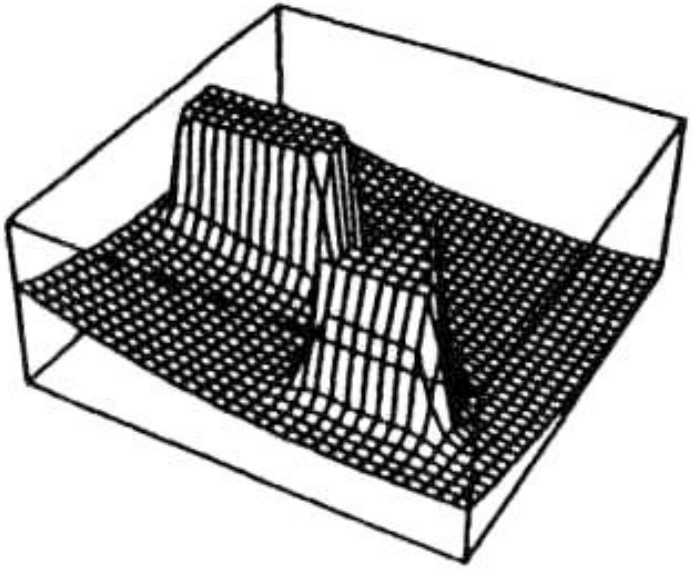
Potential Field methods

- compute a repulsive force away from obstacles
- compute an attractive force toward the goal

→ let the sum of the forces control the robot

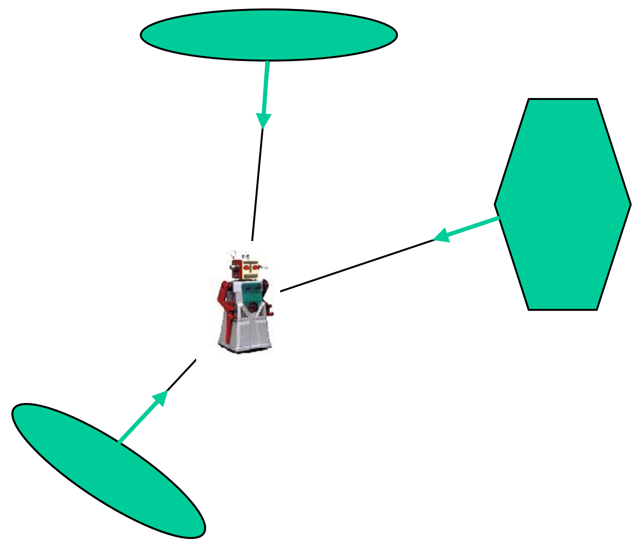






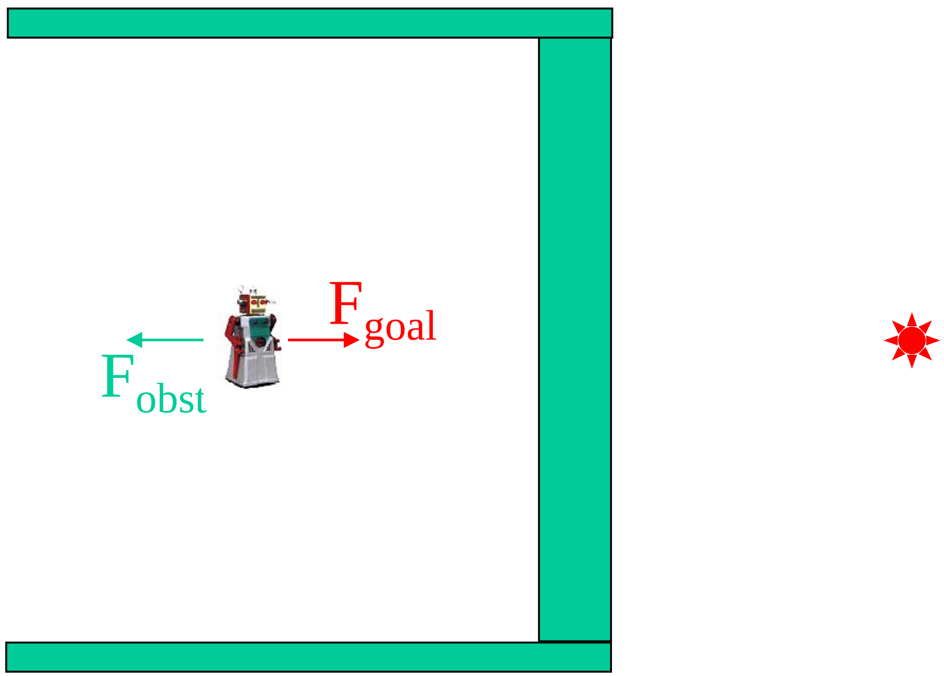
To a large extent, this is computable from sensor readings

Sensor Based Calculations



Major Problem?

Local Minima!

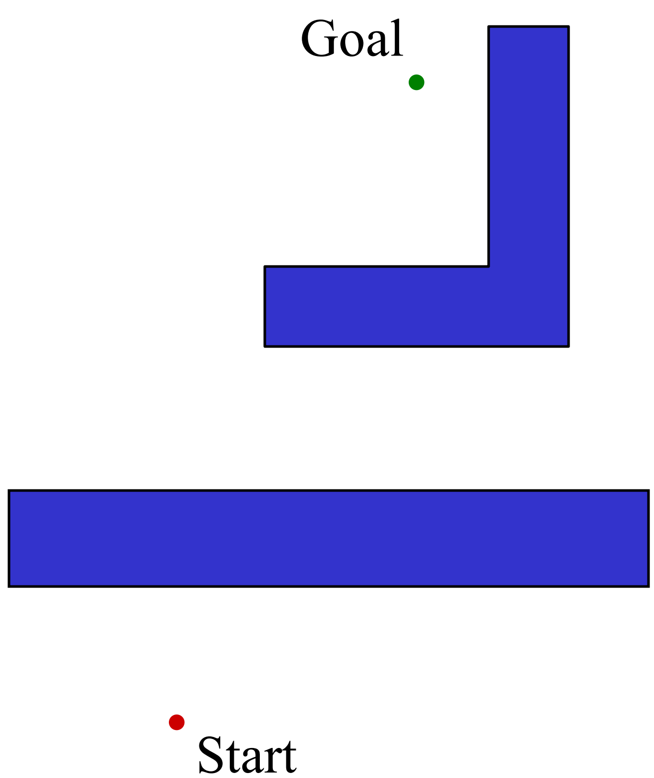


Simulated Annealing

- Every so often add some random force

Limited-knowledge path planning

- Path planning with limited knowledge
  - Insect-inspired “bug” algorithms



- known direction to goal
- otherwise local sensing

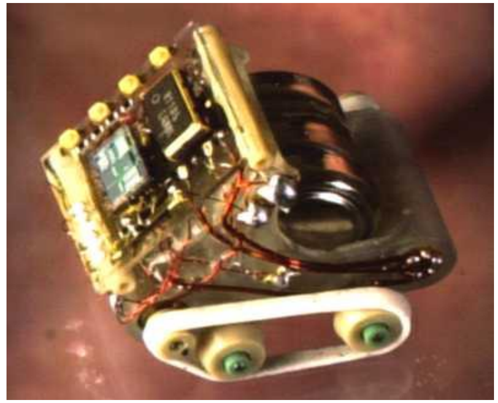
  walls/obstacles encoders

- “reasonable” world

1. finitely many obstacles in any finite disc
2. a line will intersect an obstacle finitely many times

Not truly modeling bugs...

Insects do use several cues for navigation:



neither are the current bug-sized robots

visual landmarks

polarized light

chemical sensing

they're not ears...

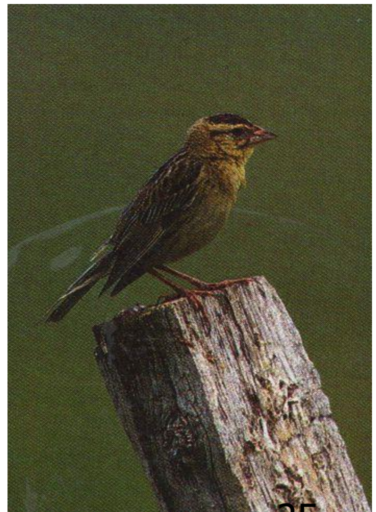
Other animals use information from

magnetic fields

electric currents

temperature

bacteria

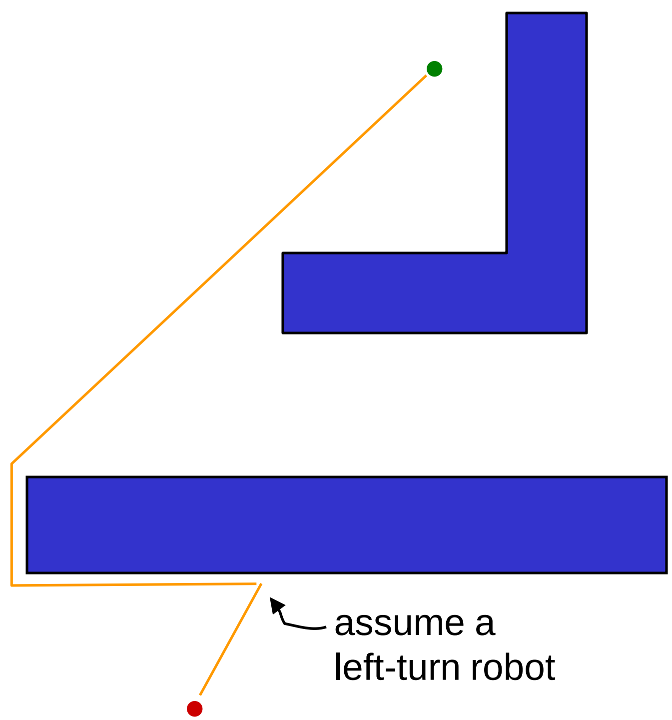


migrating bobolink

Bug Strategy

Insect-inspired "bug" algorithms

- known direction to goal
- otherwise only local sensing
  walls/obstacles encoders



"Bug 0" algorithm

1) head toward goal

2) follow obstacles until you can head toward the goal again

3) continue

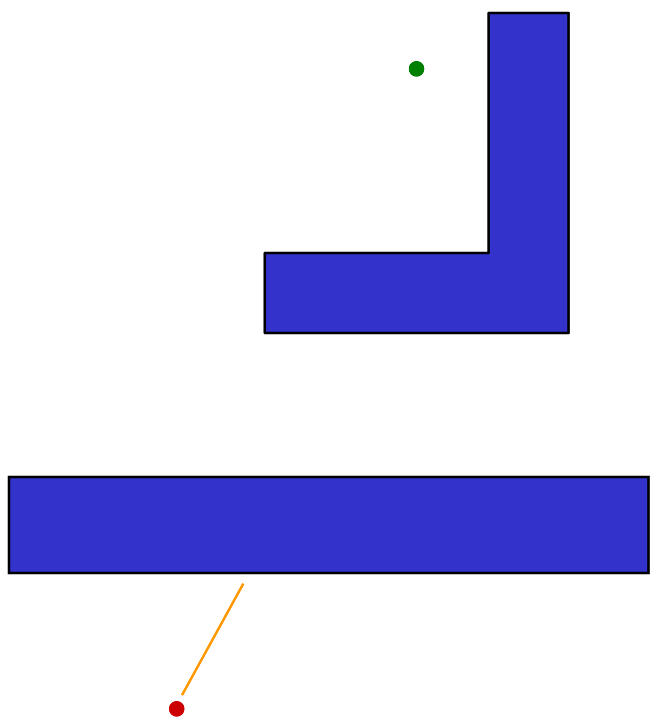
Does It Work?

Bug 1

Insect-inspired "bug" algorithms

- known direction to goal
- otherwise only local sensing

walls/obstacles encoders



"Bug 1" algorithm

1) head toward goal

Bug 1

Insect-inspired "bug" algorithms

- known direction to goal
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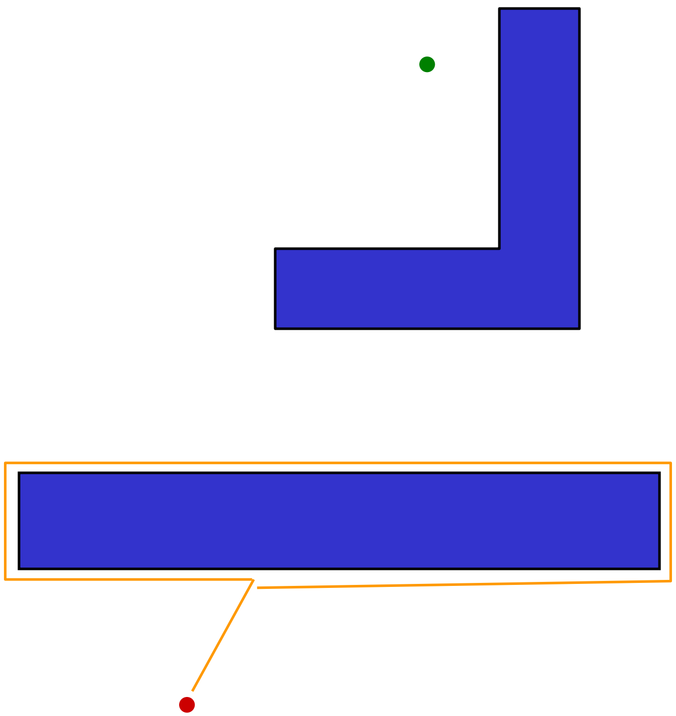
walls/obstacles encoders



"Bug 1" algorithm

1) head toward goal

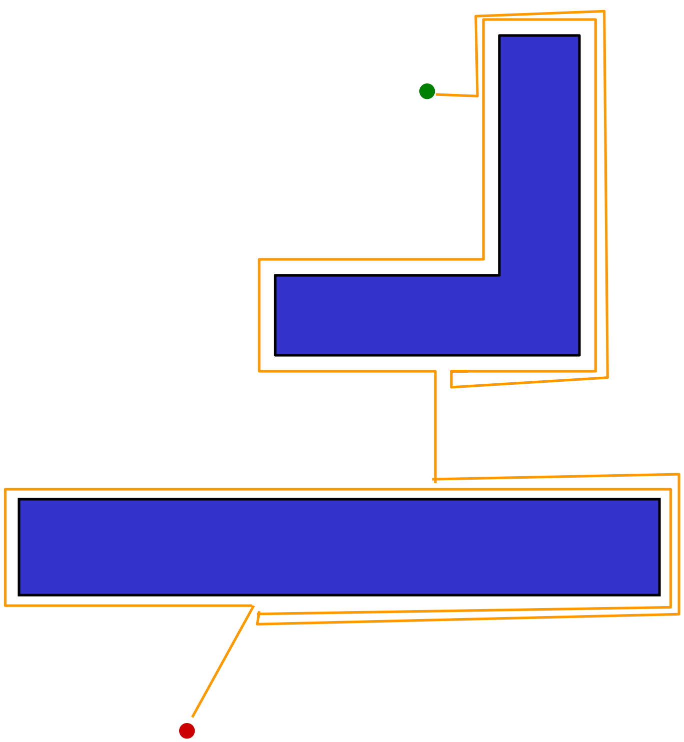
2) if an obstacle is encountered, circumnavigate it *and* remember how close you get to the goal

Bug 1

Insect-inspired "bug" algorithms

- known direction to goal
- otherwise only local sensing

walls/obstacles encoders



"Bug 1" algorithm

1) head toward goal

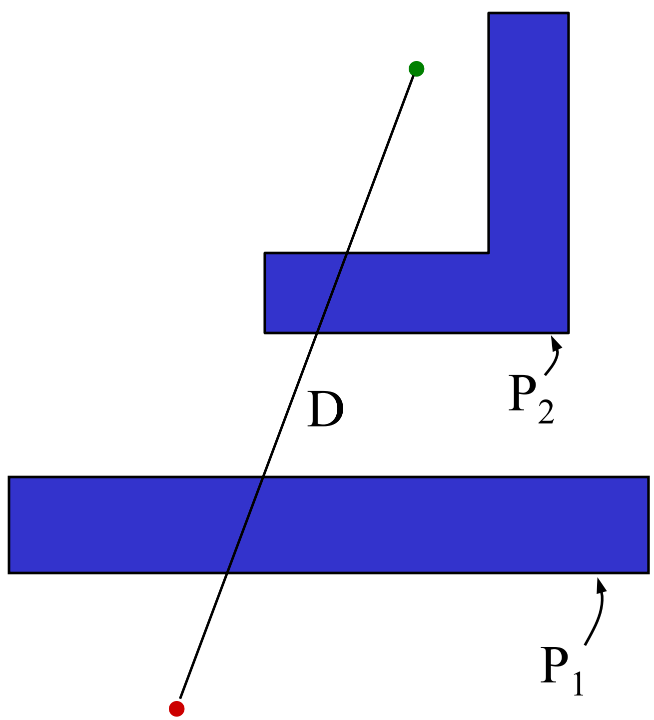
2) if an obstacle is encountered, circumnavigate it *and* remember how close you get to the goal

3) return to that closest point (by wall-following) and continue

Vladimir Lumelsky & Alexander Stepanov Algorithmica 1987

Bug 1 analysis

Distance Traveled



What are bounds on the path length that the robot takes?

Available Information:

D = straight-line distance from start to goal

P_i = perimeter of the *i* th obstacle

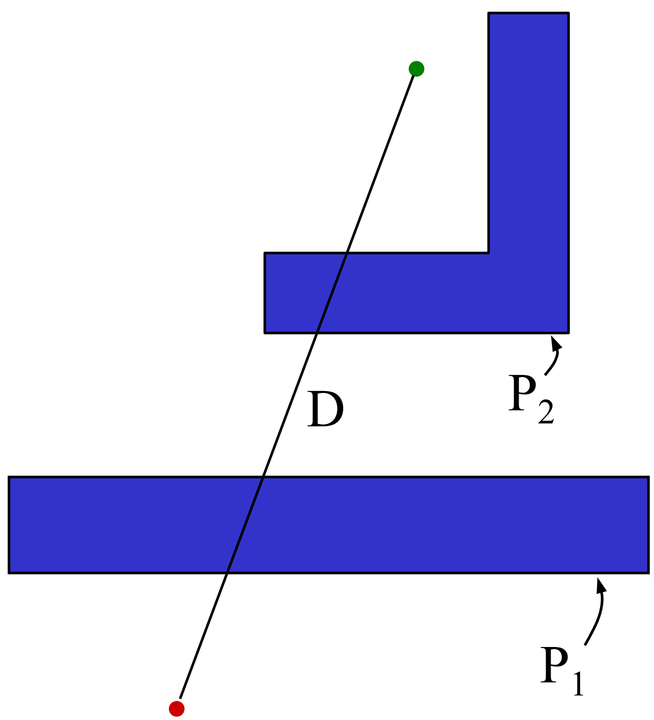
Lower and upper bounds?

Lower bound:

Upper bound:

Bug 1 analysis

Distance Traveled



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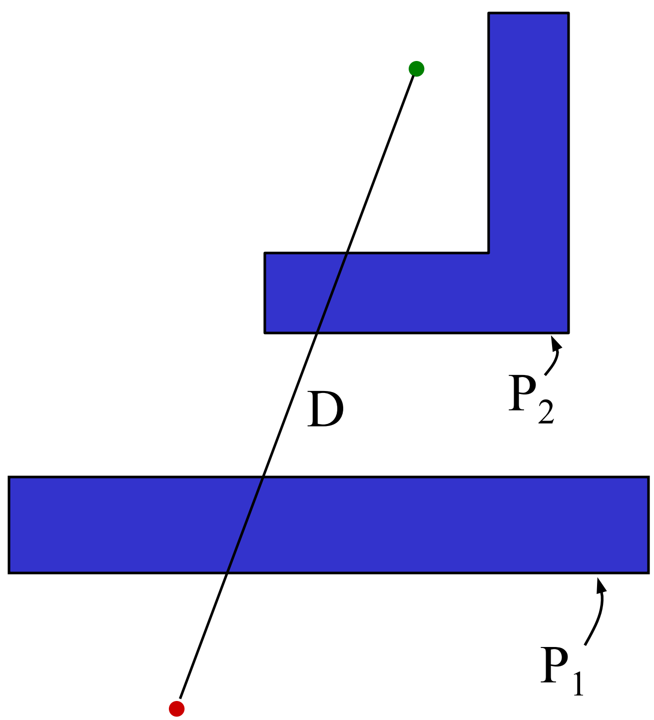
Lower bound: D

Upper bound:

Bug 1 analysis

Distance Traveled



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Available Information:

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Lower and upper bounds?

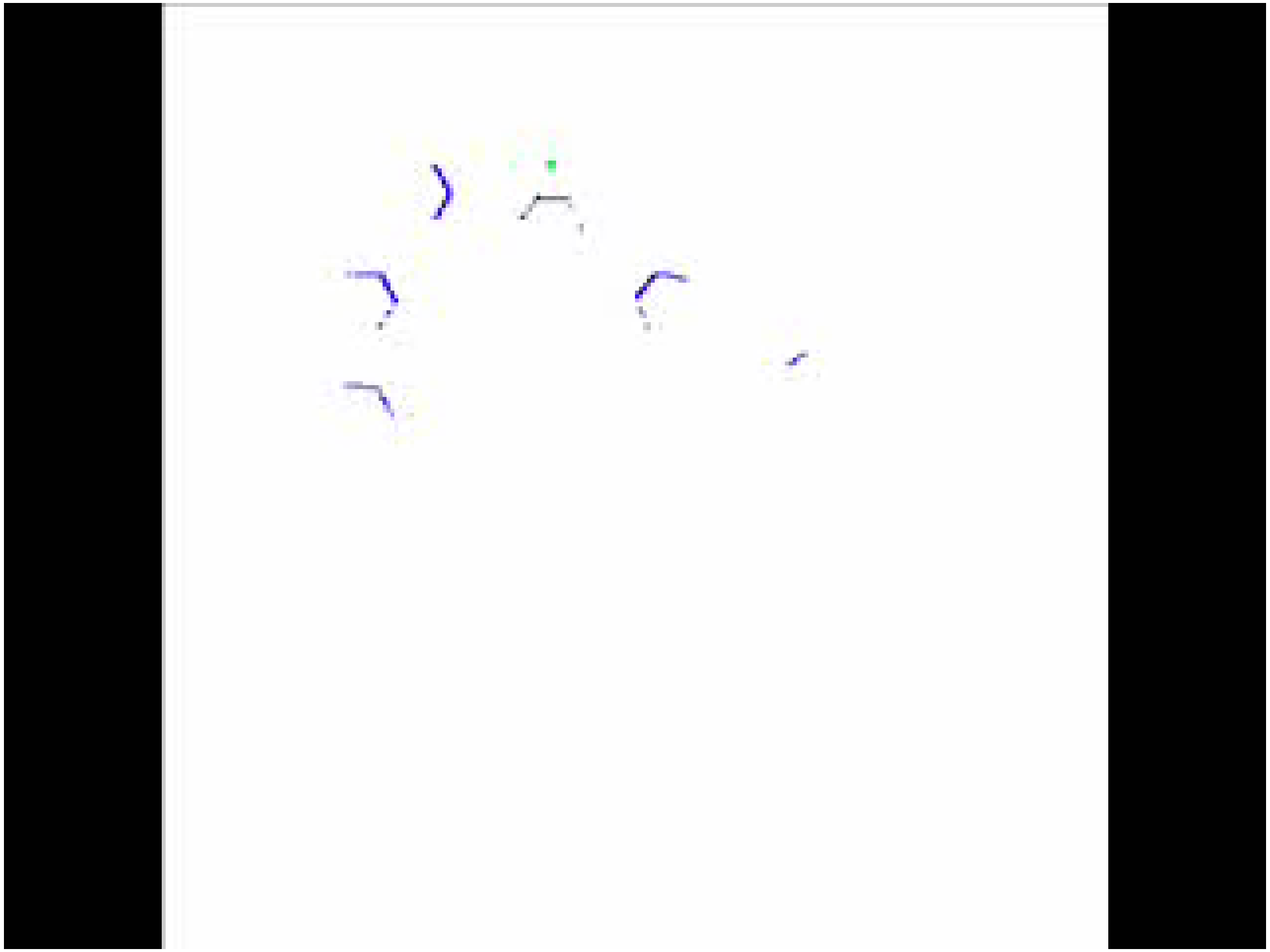
Lower bound: D

Upper bound: $D + 1.5 \sum_i P_i$

How good a bound?

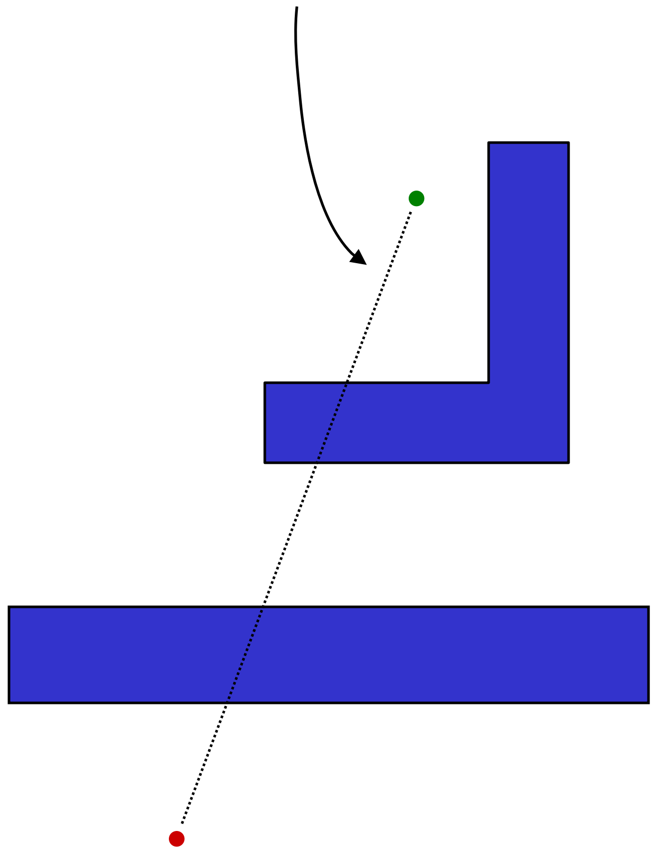
How good an algorithm?

Bug Mapping



A better bug?

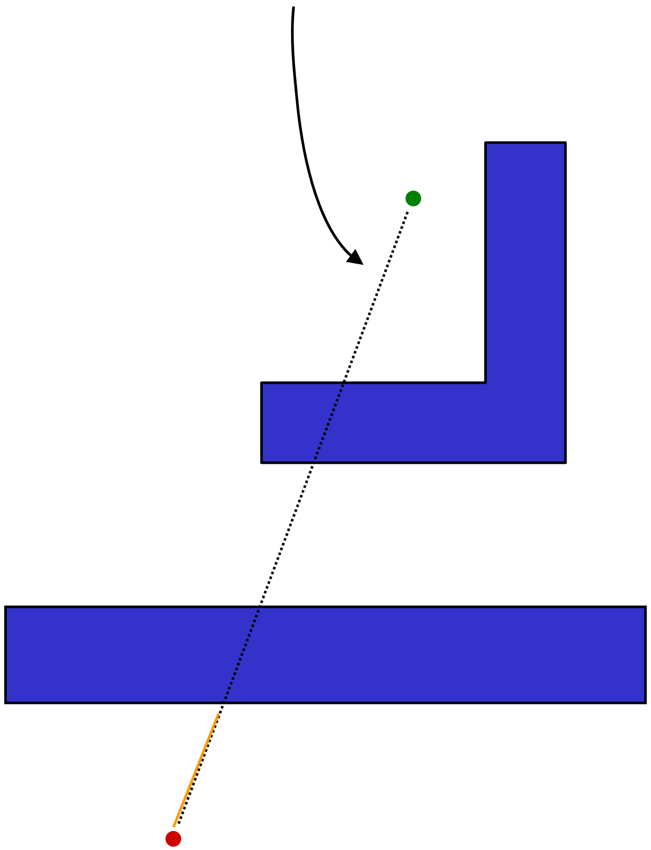
Call the line from the starting point to the goal the ***s-line***



“Bug 2” algorithm

A better bug?

Call the line from the starting point to the goal the ***s-line***

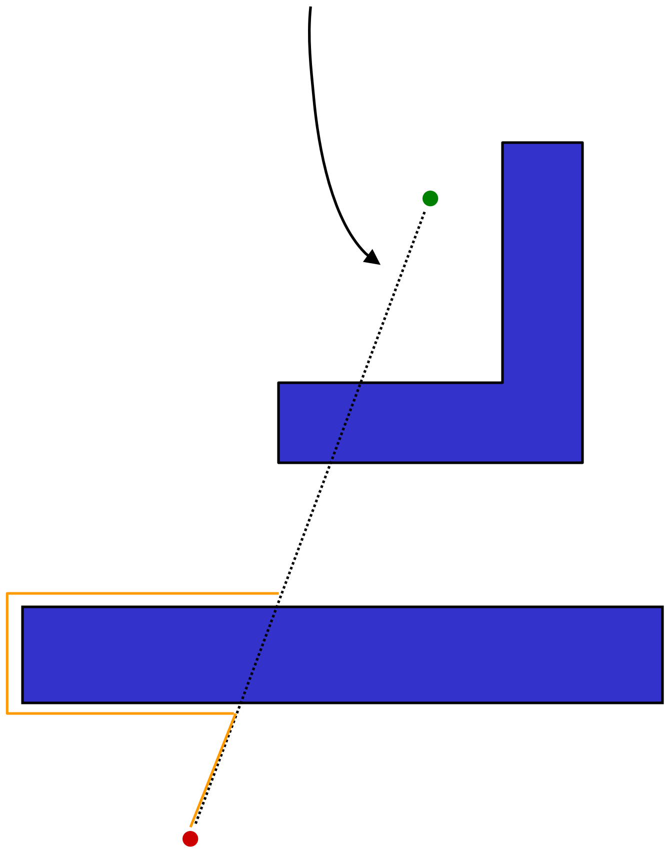


“Bug 2” algorithm

1) head toward goal on the *s-line*

A better bug?

Call the line from the starting point to the goal the ***s-line***

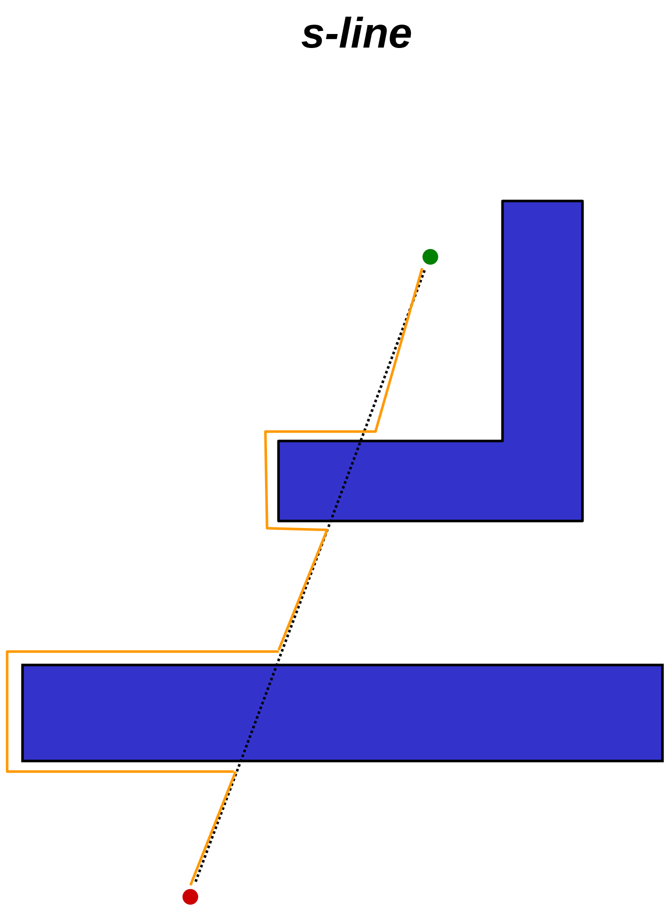


“Bug 2” algorithm

1) head toward goal on the *s-line*

2) if an obstacle is in the way, follow it until encountering the s-line again.

A better bug?



“Bug 2” algorithm

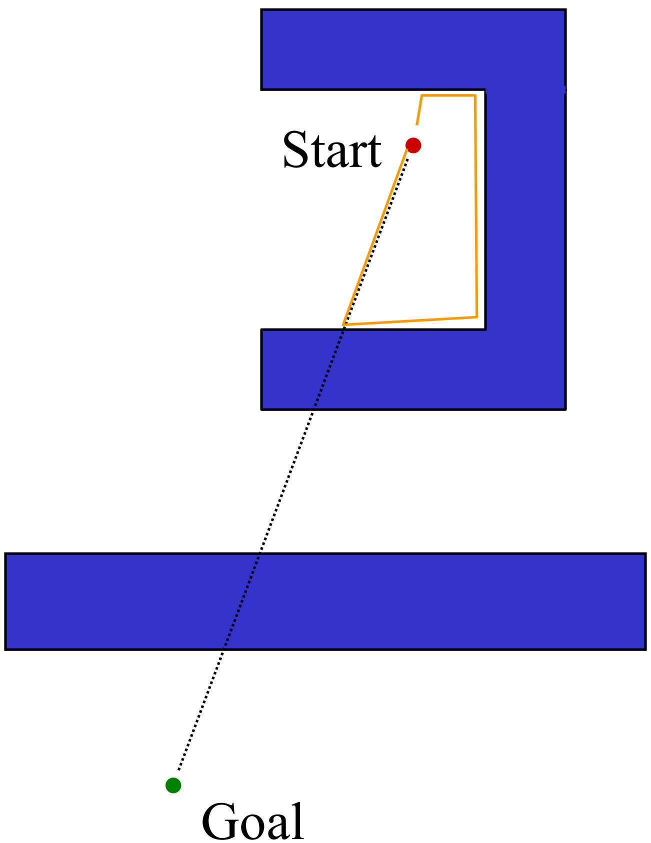
1) head toward goal on the *s-line*

2) if an obstacle is in the way, follow it until encountering the s-line again.

3) Leave the obstacle and continue toward the goal

OK ?

A better bug?



"Bug 2" algorithm

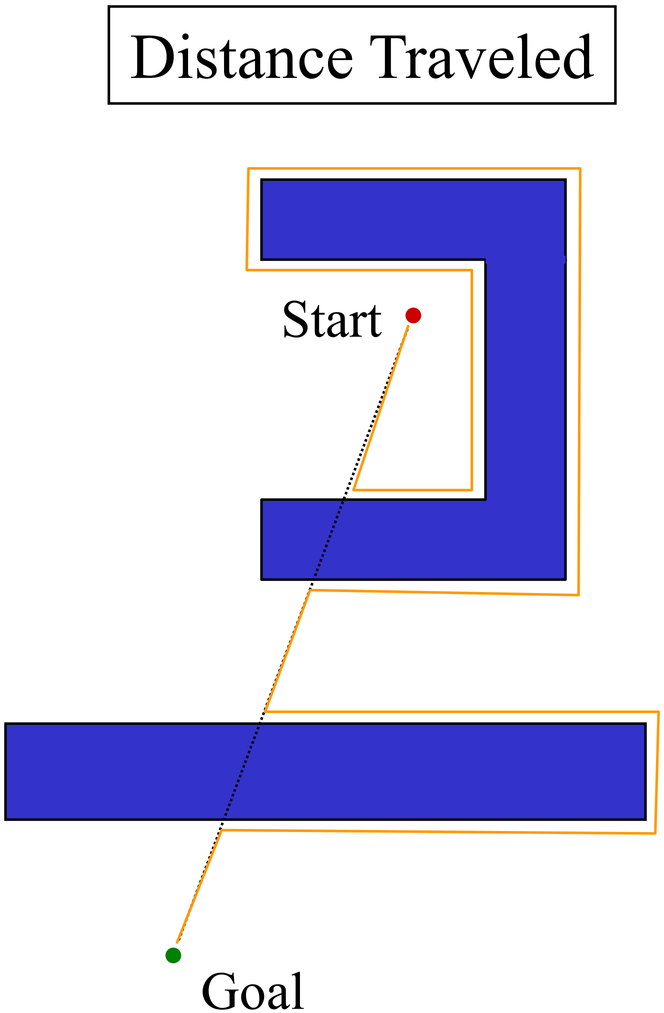
1) head toward goal on the *s-line*

2) if an obstacle is in the way, follow it until encountering the s-line again *closer to the goal*.

3) Leave the obstacle and continue toward the goal

OK ?

Bug 2 analysis



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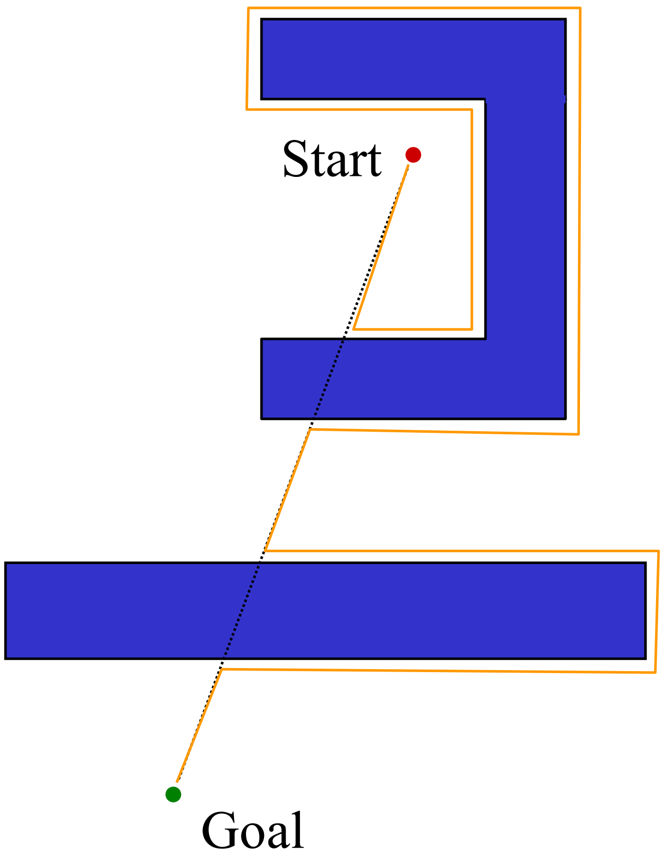
Lower and upper bounds?

Lower bound:

Upper bound:

Bug 2 analysis

Distance Traveled



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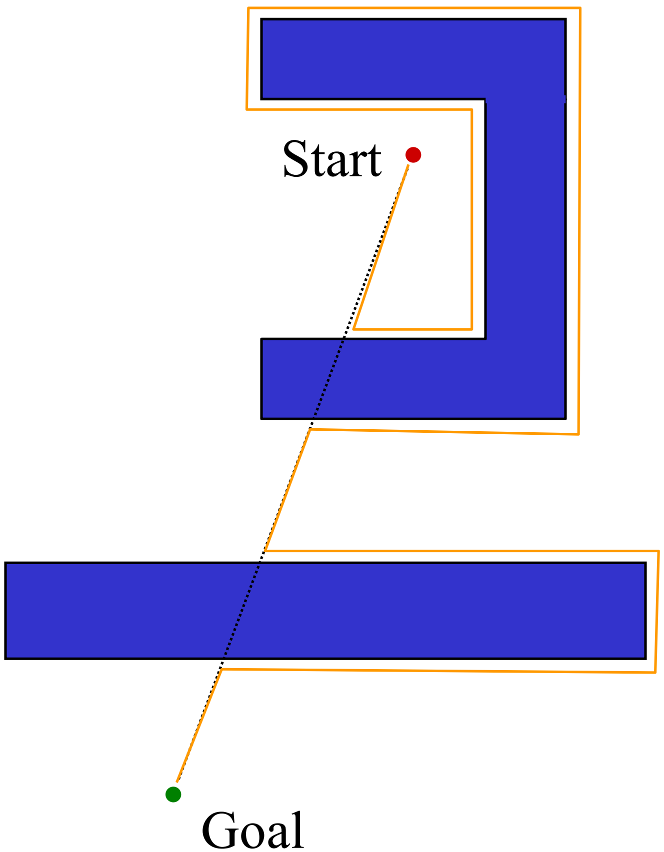
Lower and upper bounds?

Lower bound:

Upper bound:

Bug 2 analysis

Distance Traveled



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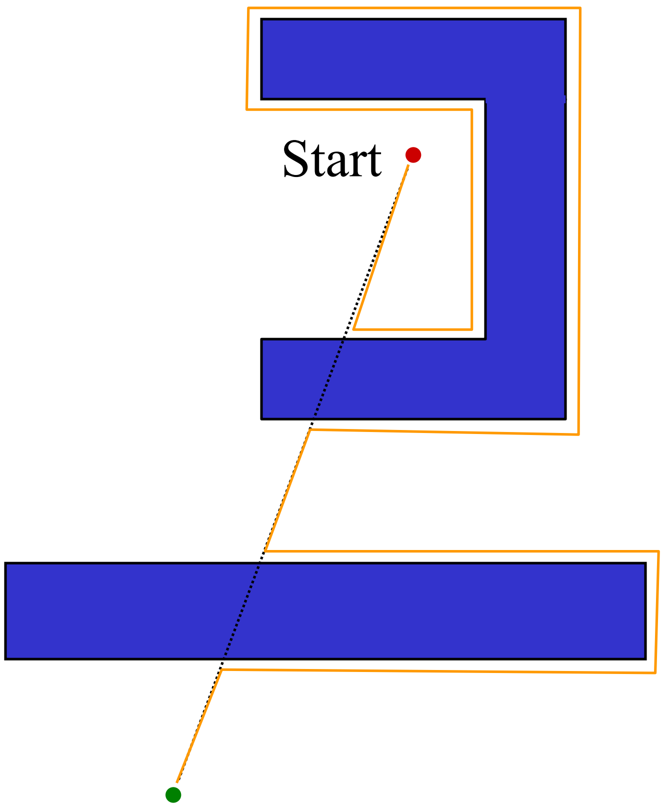
Lower and upper bounds?

Lower bound: D

Upper bound:

Bug 2 analysis

Distance Traveled



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Lower and upper bounds?

Lower bound: D

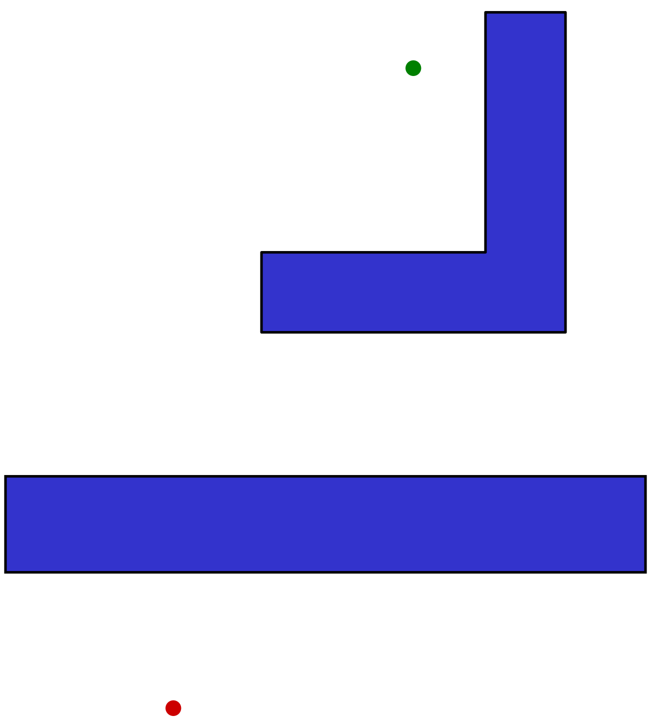
Upper bound: $D + 0.5 \sum_i N_i P_i$

head-to-head comparison

or thorax-to-thorax, perhaps

What are worlds in which Bug 2 does better than Bug 1 (and vice versa) ?

Bug 2 beats Bug 1	Bug 1 beats Bug 2



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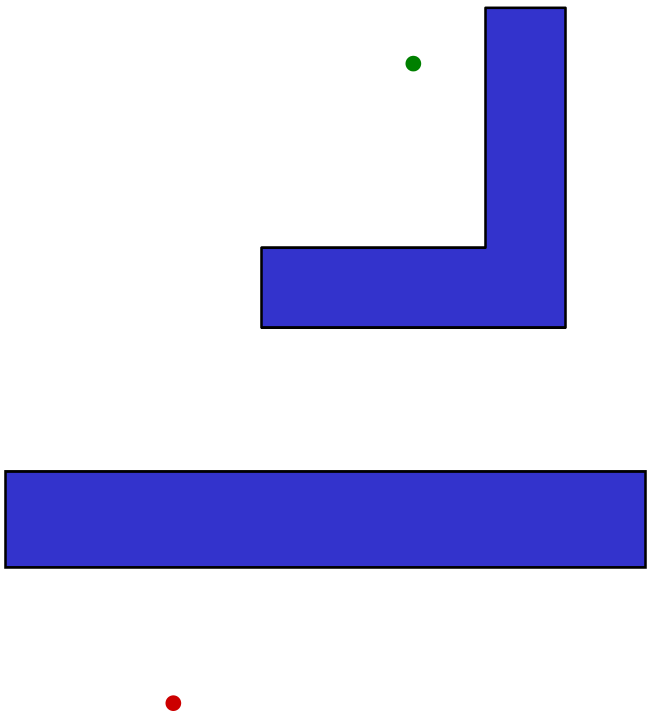
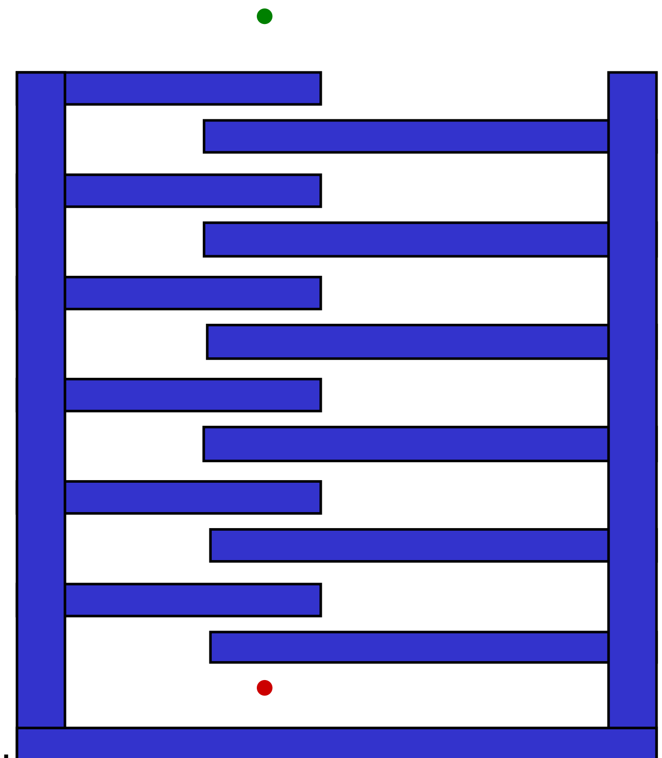
Bug 2 beats Bug 1



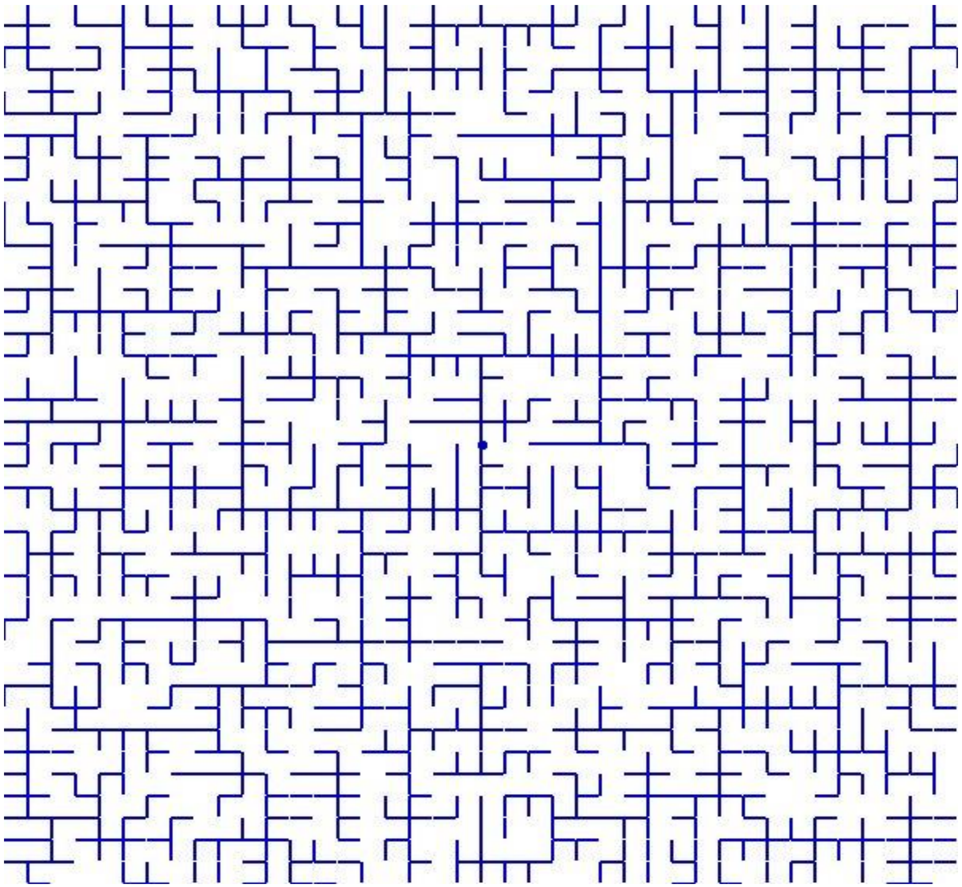
Bug 1 beats Bug 2

"zipper world"



Other bug-like algorithms

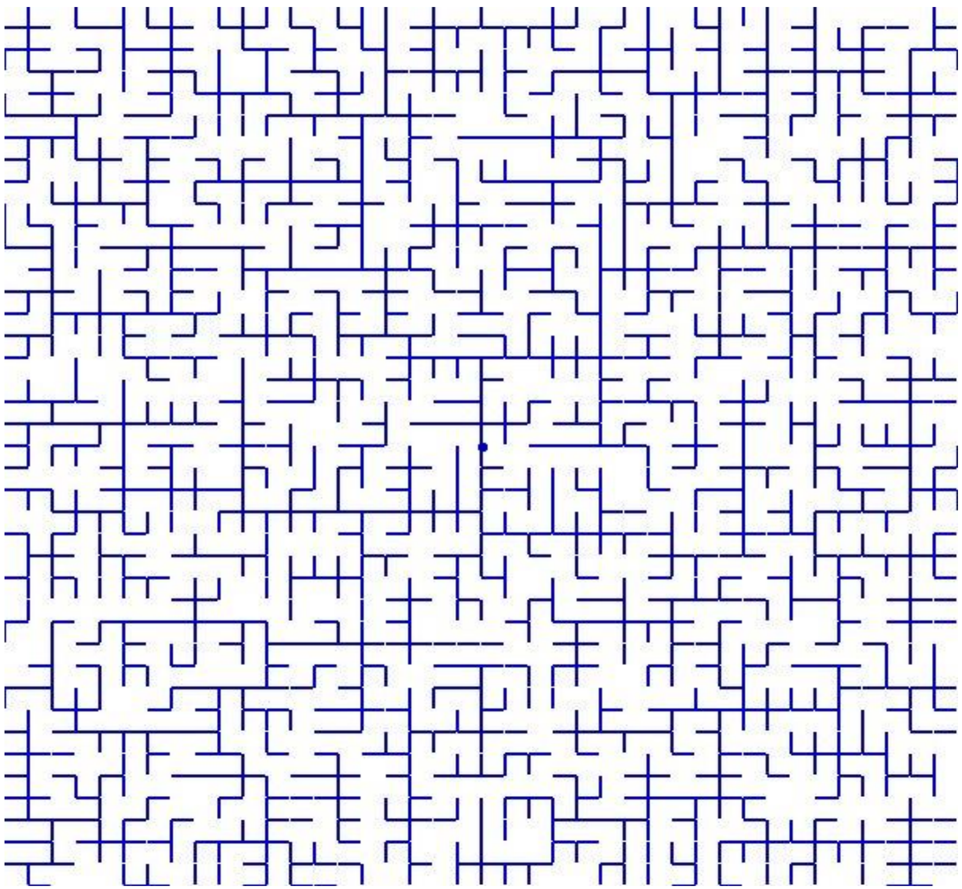
The Pledge maze-solving algorithm



1. Go to a wall
2. Keep the wall on your right
3. Continue until out of the maze

Other bug-like algorithms

The Pledge maze-solving algorithm



discretized RRT

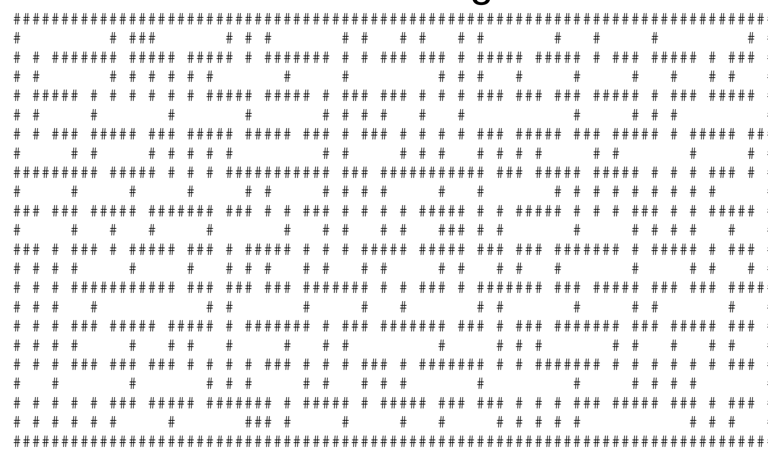
1) Go to a wall

2) Keep the wall on your right

3) Continue until out of the maze

```
int a[1817];main(z,p,q,r){for(p=80;q+p-80;p=2*a[p])
for(z=9;z--;)q=3&(r=time(0)+r*57)/7,q=q?q-1?q-2?1-p%79?-
1:0:p%79-77?1:0:p<1659?79:0:p>158?-
79:0,q?!a[p+q*2]?a[p+=a[p+=q]=q]=q:0:0;for(;q++-
1817;)printf(q%79?"%c":"%c\n"," #"[!a[q-1]]);}
```

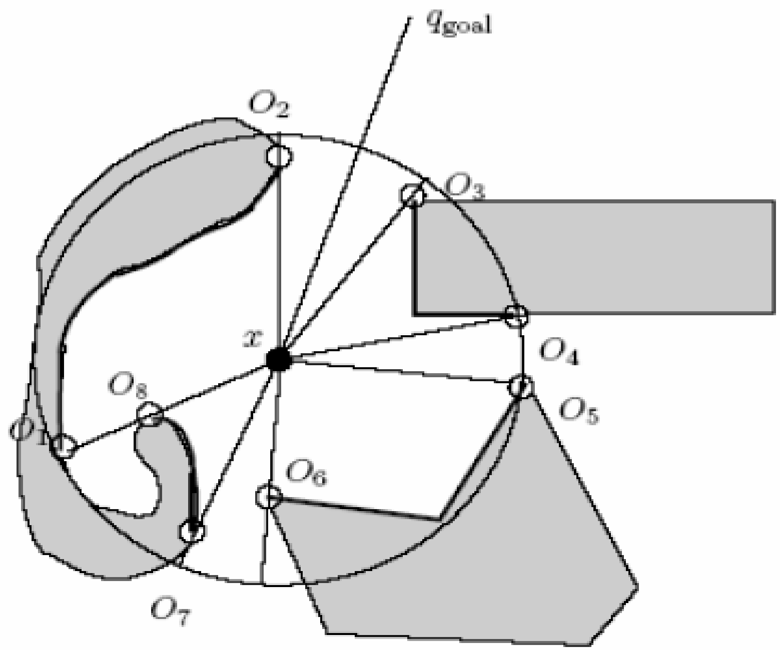
IOCCC random maze generator



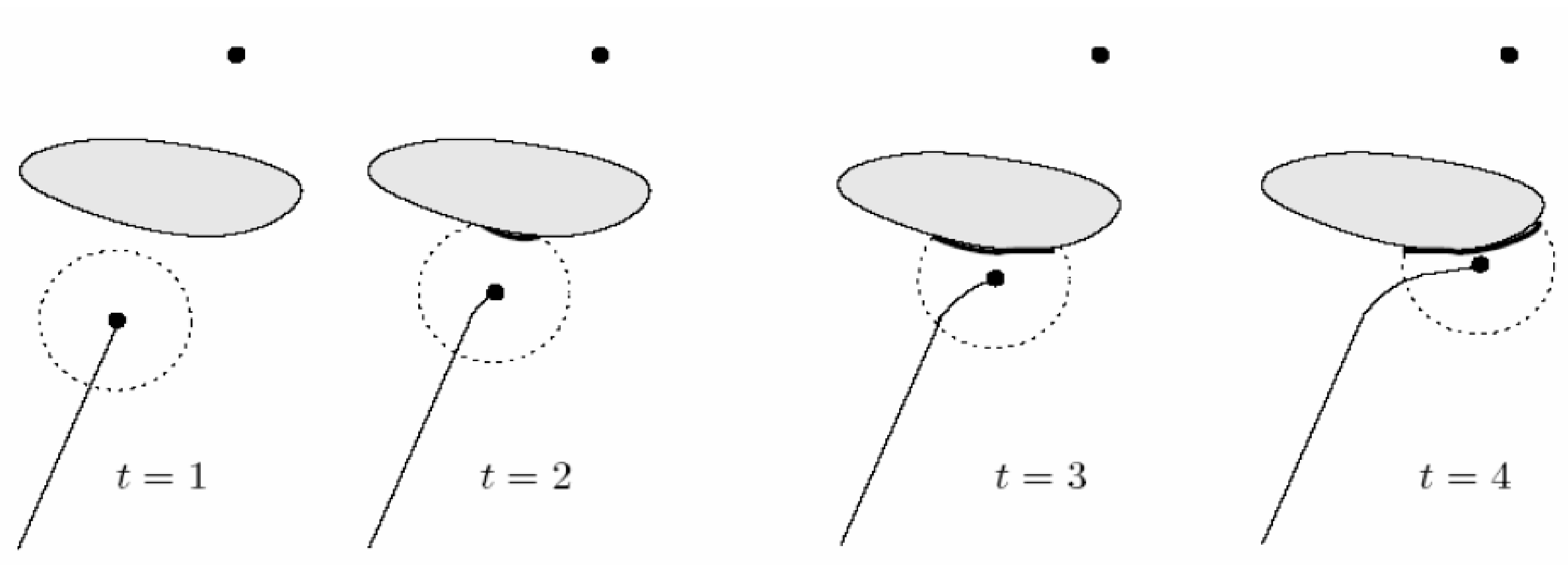
mazes of unusual origin

Tangent Bug

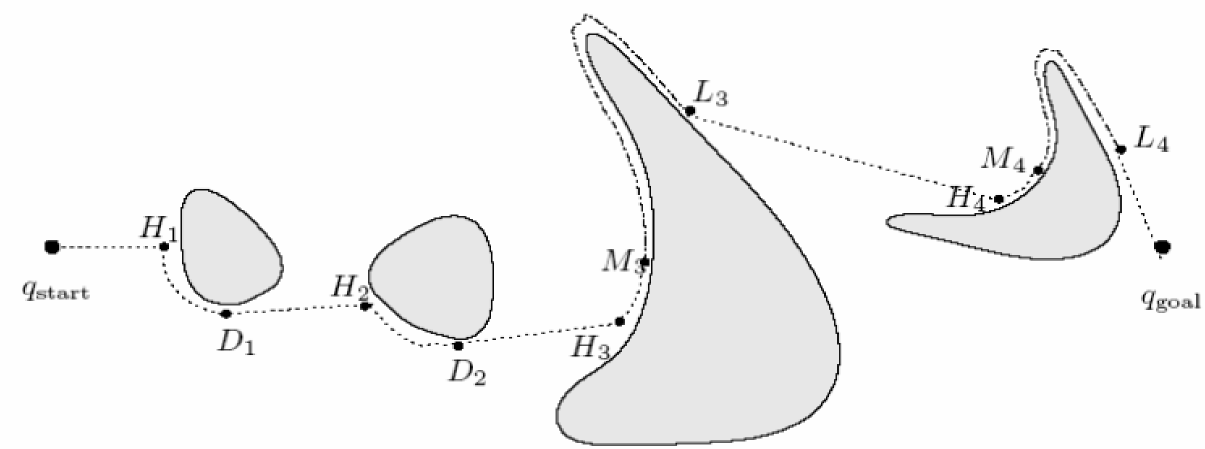
- Limited Range Sensor
- Tangent Bug relies on finding endpoints of finite, continues segments of the obstacles



Tangent Bug

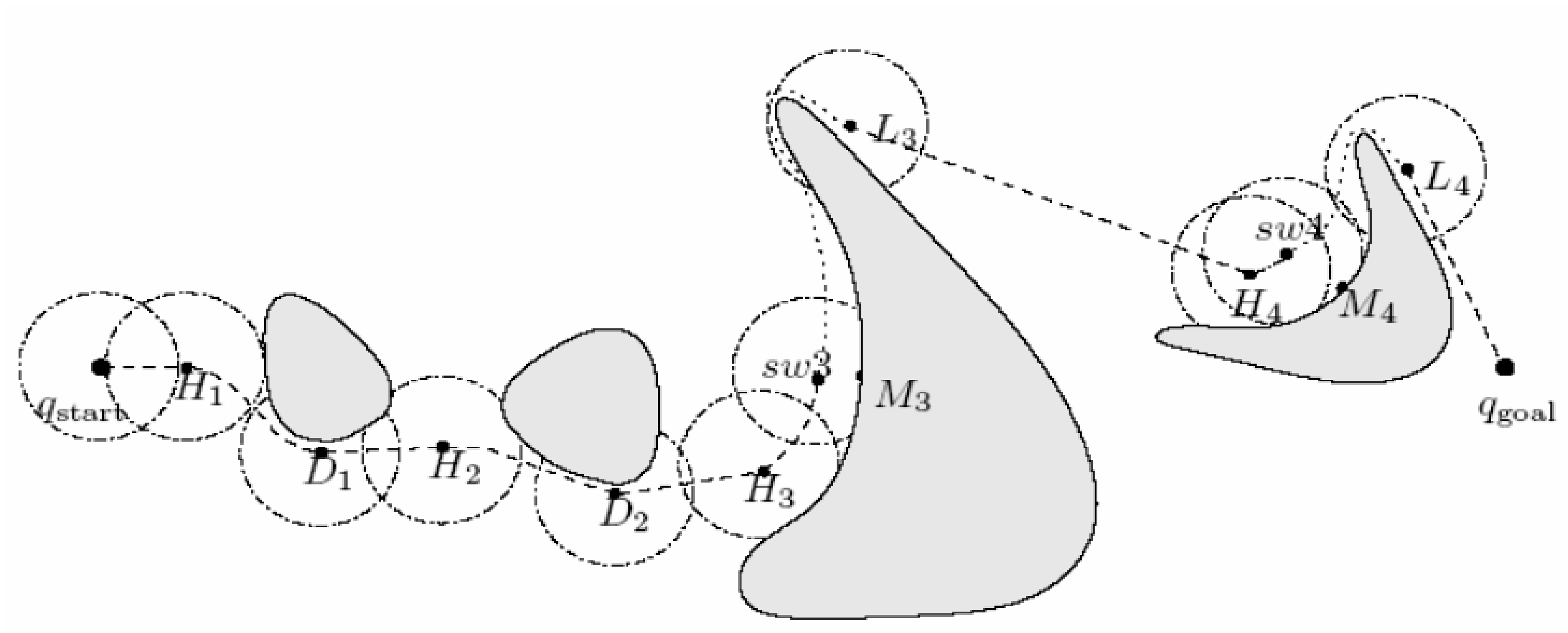


Contact Sensor Tangent Bug

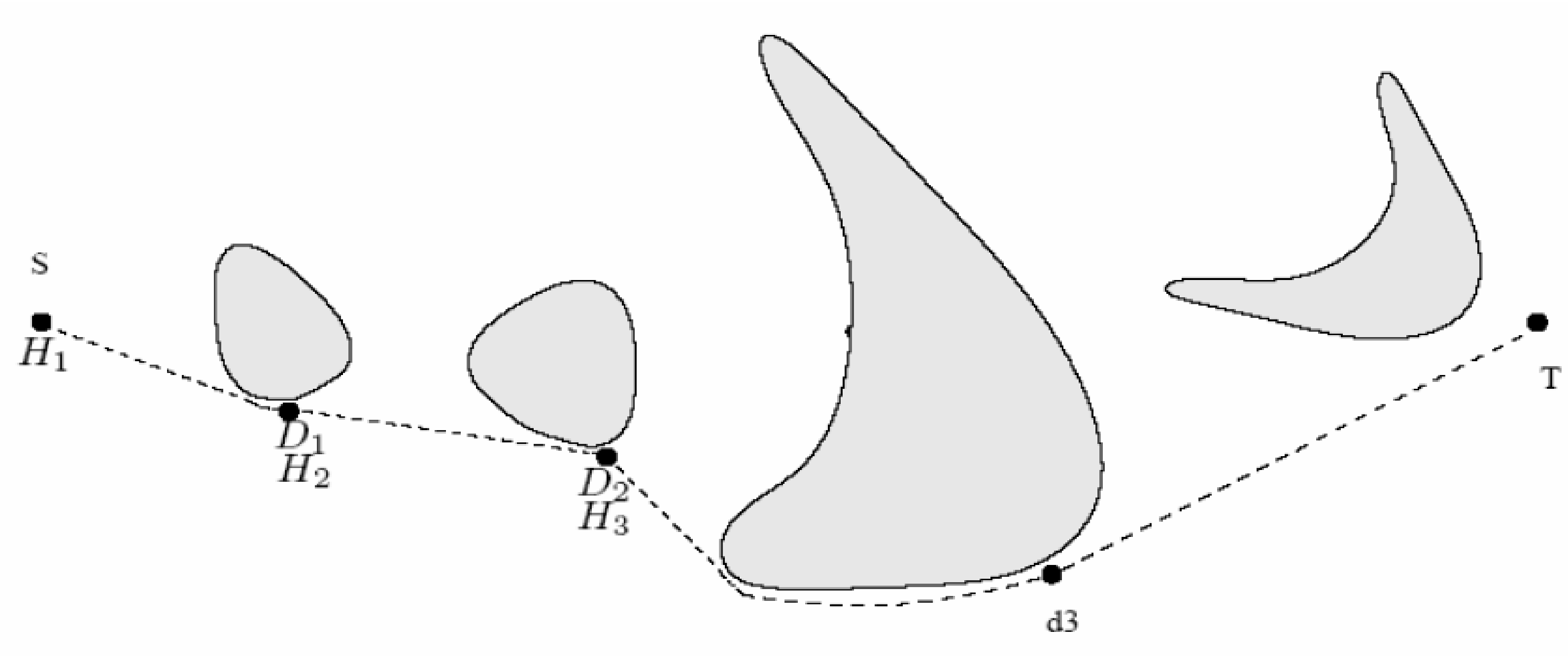


1. Robot moves toward goal until it hits obstacle 1 at H1
2. Pretend there is an infinitely small sensor range and the direction which minimizes the heuristic is to the right
3. Keep following obstacle until robot can go toward obstacle again
4. Same situation with second obstacle
5. At third obstacle, the robot turned left until it could not increase heuristic
6. D_followed is distance between M3 and goal, d_reach is distance between robot and goal because sensing distance is zero

Limited Sensor Range Tangent-Bug

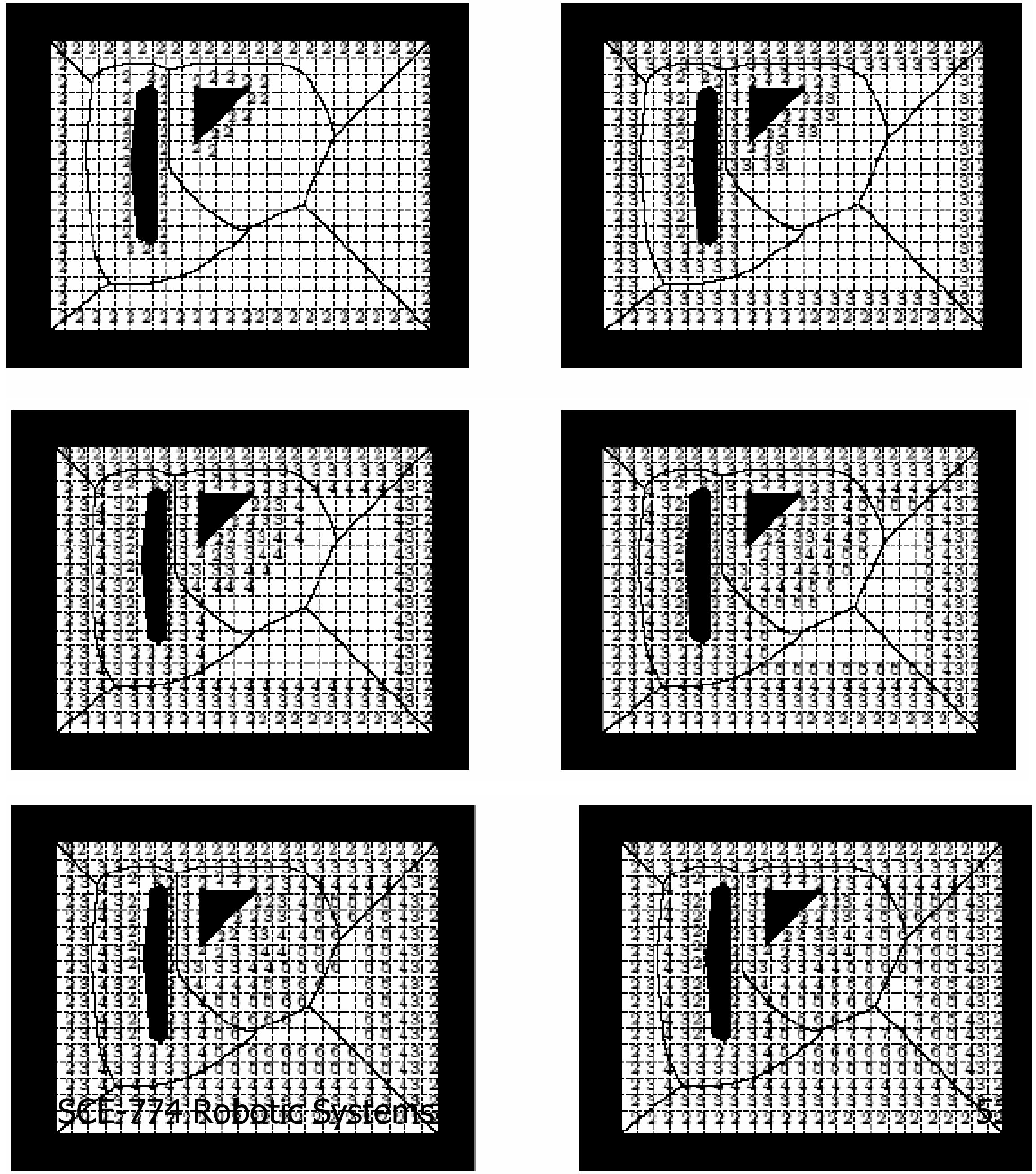


Infinite Sensor Range Tangent Bug



Known Map

Brushfire Transform



The Wavefront Planner: Setup

7	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0
3	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**2**
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

The Wavefront in Action (Part 1)

- Starting with the goal, set all adjacent cells with “0” to the current cell + 1
  - 4-Point Connectivity or 8-Point Connectivity?
  - Your Choice. We’ll use 8-Point Connectivity in our example

7	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0
3	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	**2**
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

The Wavefront in Action (Part 2)

- Now repeat with the modified cells
  - This will be repeated until no 0's are adjacent to cells with values >= 2
- 0's will only remain when regions are unreachable

7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
3	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4
1	0	0	0	0	0	0	0	0	0	0	0	0	0	4	3	3
0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	3	2
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

The Wavefront in Action (Part 3)

- Repeat

7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
3	0	0	0	0	1	1	1	1	1	1	1	1	5	5	5	5
2	0	0	0	0	0	0	0	0	0	0	0	0	5	4	4	4
1	0	0	0	0	0	0	0	0	0	0	0	0	5	4	3	3
0	0	0	0	0	0	0	0	0	0	0	0	0	5	4	3	2
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

The Wavefront in Action (Part 3)

- Repeat

7	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	6	6	6	6
3	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	5	5	5	5
2	0	0	0	0	0	0	0	0	0	0	0	6	5	4	4	4
1	0	0	0	0	0	0	0	0	0	0	0	6	5	4	3	3
0	0	0	0	0	0	0	0	0	0	0	0	6	5	4	3	**2**
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

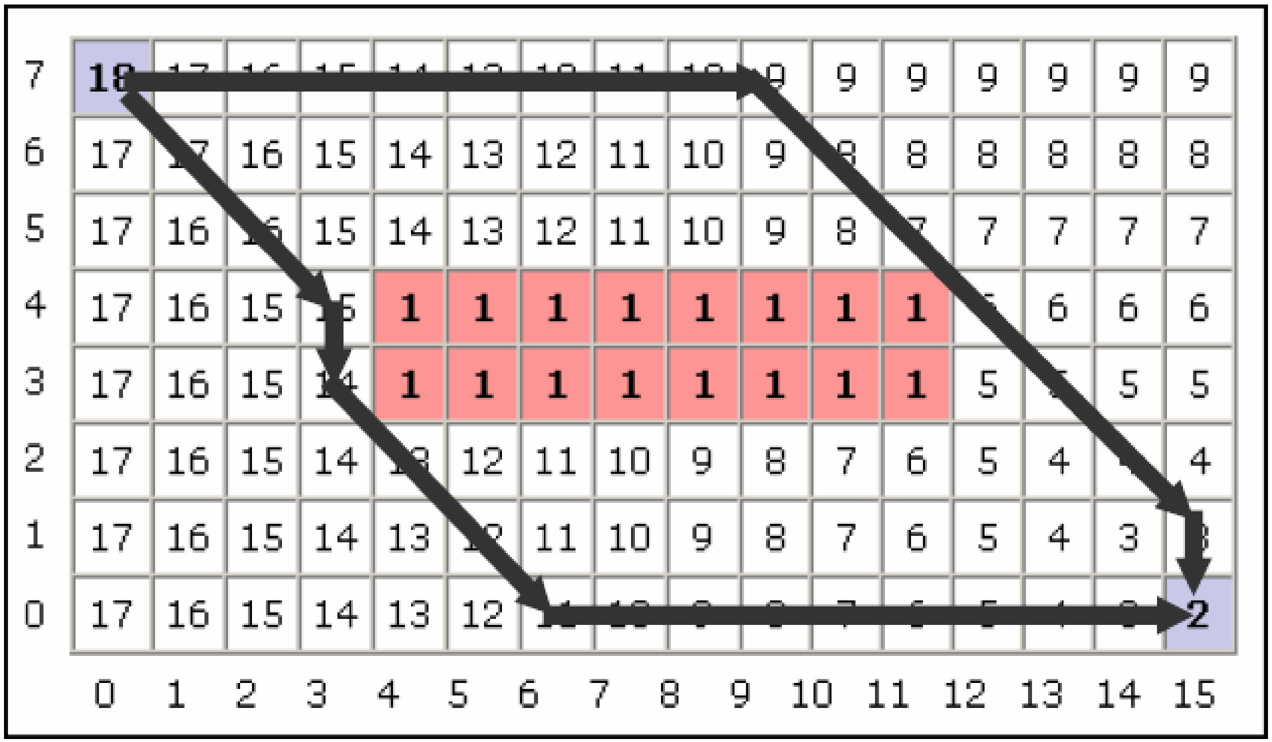
The Wavefront in Action (Part 3)

- Until Done
  - 0’s would only remain in the unreachable areas

7	**18**	17	16	15	14	13	12	11	10	9	9	9	9	9	9	9
6	17	17	16	15	14	13	12	11	10	9	8	8	8	8	8	8
5	17	16	16	15	14	13	12	11	10	9	8	7	7	7	7	7
4	17	16	15	15	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	6	6	6	6
3	17	16	15	14	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	5	5	5	5
2	17	16	15	14	13	12	11	10	9	8	7	6	5	4	4	4
1	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	3
0	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	**2**
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

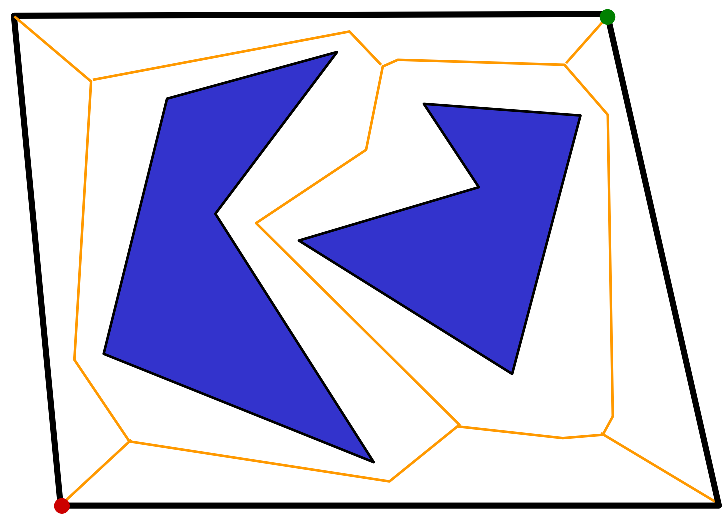
The Wavefront in Action

- To find the shortest path, according to your metric, simply always move toward a cell with a lower number
  - The numbers generated by the Wavefront planner are roughly proportional to their distance from the goal

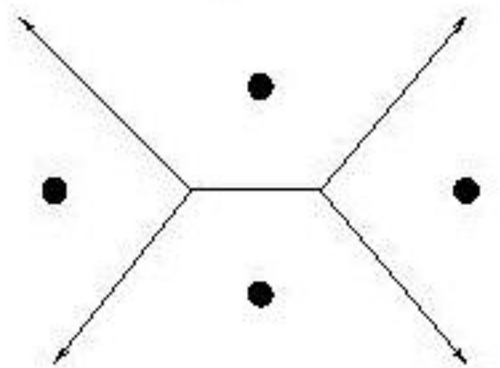


Two possible shortest paths shown

An alternative roadmap



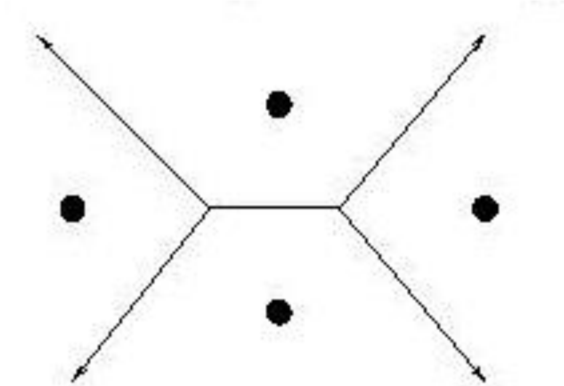
Voronoi diagrams



These line segments make up the **Voronoi diagram** for the four points shown here.

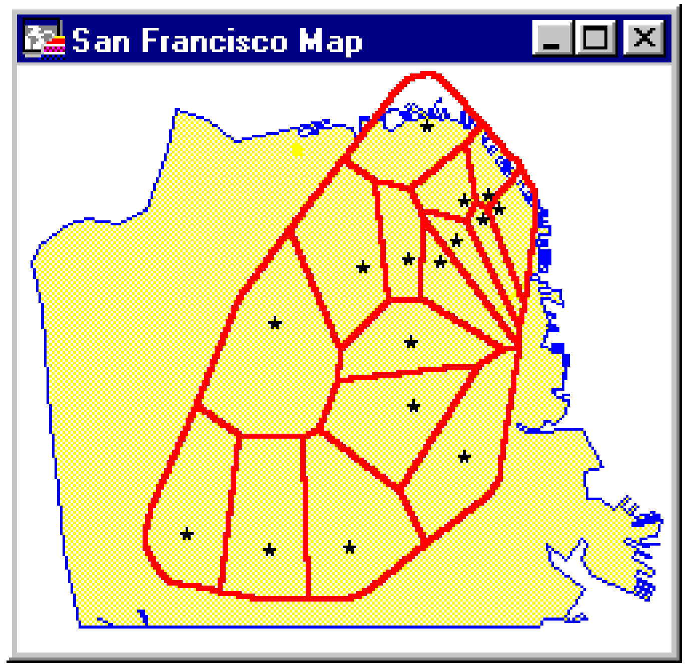
Solves the "Post Office Problem"

Voronoi diagrams



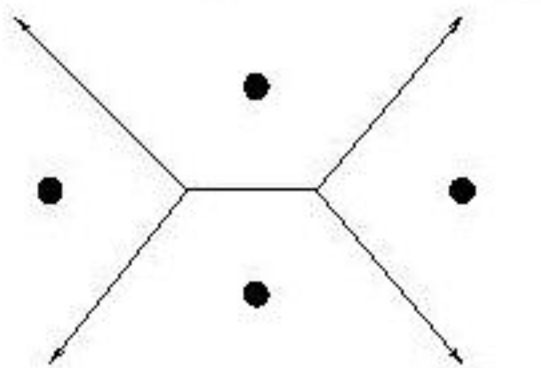
These line segments make up the **Voronoi diagram** for the four points shown here.

Solves the "Post Office Problem"



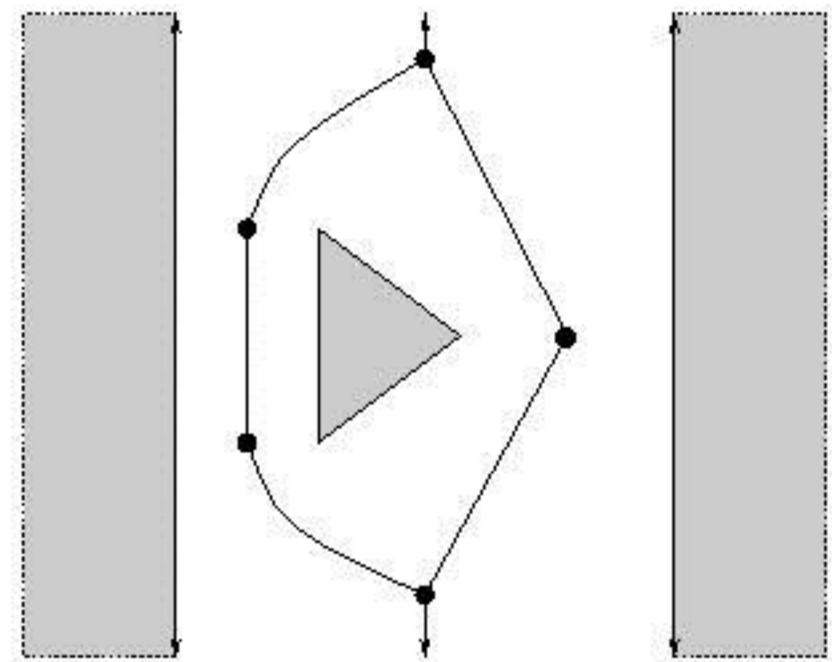
or, perhaps, more important problems...

Voronoi diagrams



“true” Voronoi diagram

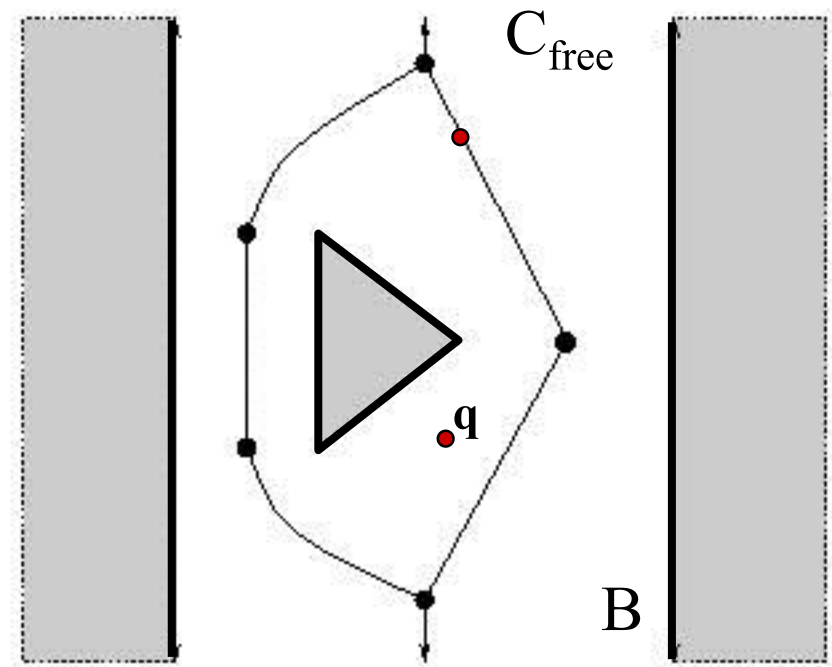
(isolates a set of points)



generalized Voronoi diagram

What is it?

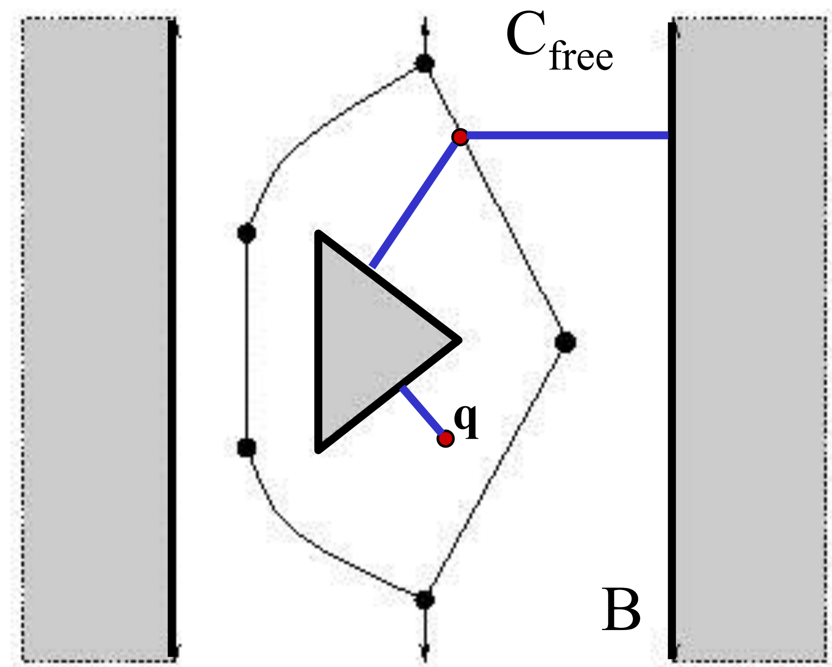
Voronoi diagrams



Let B = the boundary of C_{free} .

Let **q** be a point in C_{free} . (•)

Voronoi diagrams

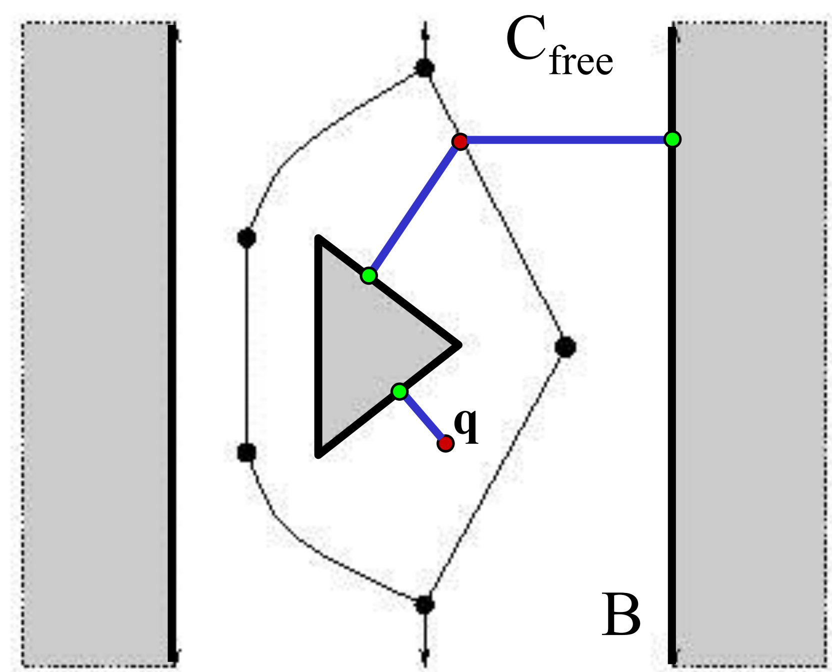


Let B = the boundary of C_{free} .

Let **q** be a point in C_{free} .

Define *clearance*(q) = min { | q - p | }, for all $p \in B$

Voronoi diagrams



Let B = the boundary of C_{free} .

Let **q** be a point in C_{free} .

Define *clearance*(q) = min { | q - p | }, for all p ∈ B

Define *near*(q) = { p ∈ B such that | q - p | = *clearance*(q) }

Voronoi diagrams

Evaluation

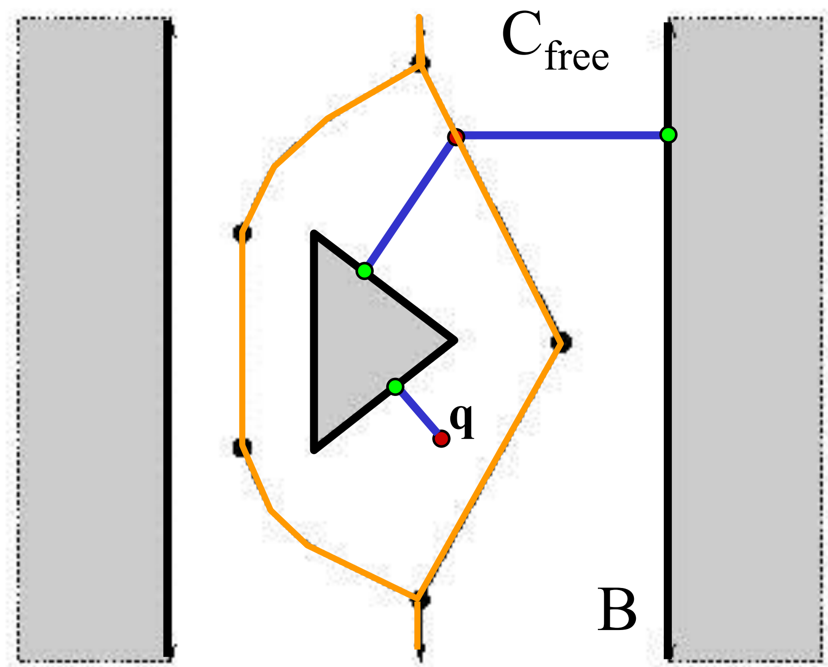
+ maximizes distance from obstacles

+ reduces to graph search

+ can be used in higher-dimensions

- nonoptimal

- real diagrams tend to be noisy



Let B = the boundary of C_{free} .

Let **q** be a point in C_{free} .

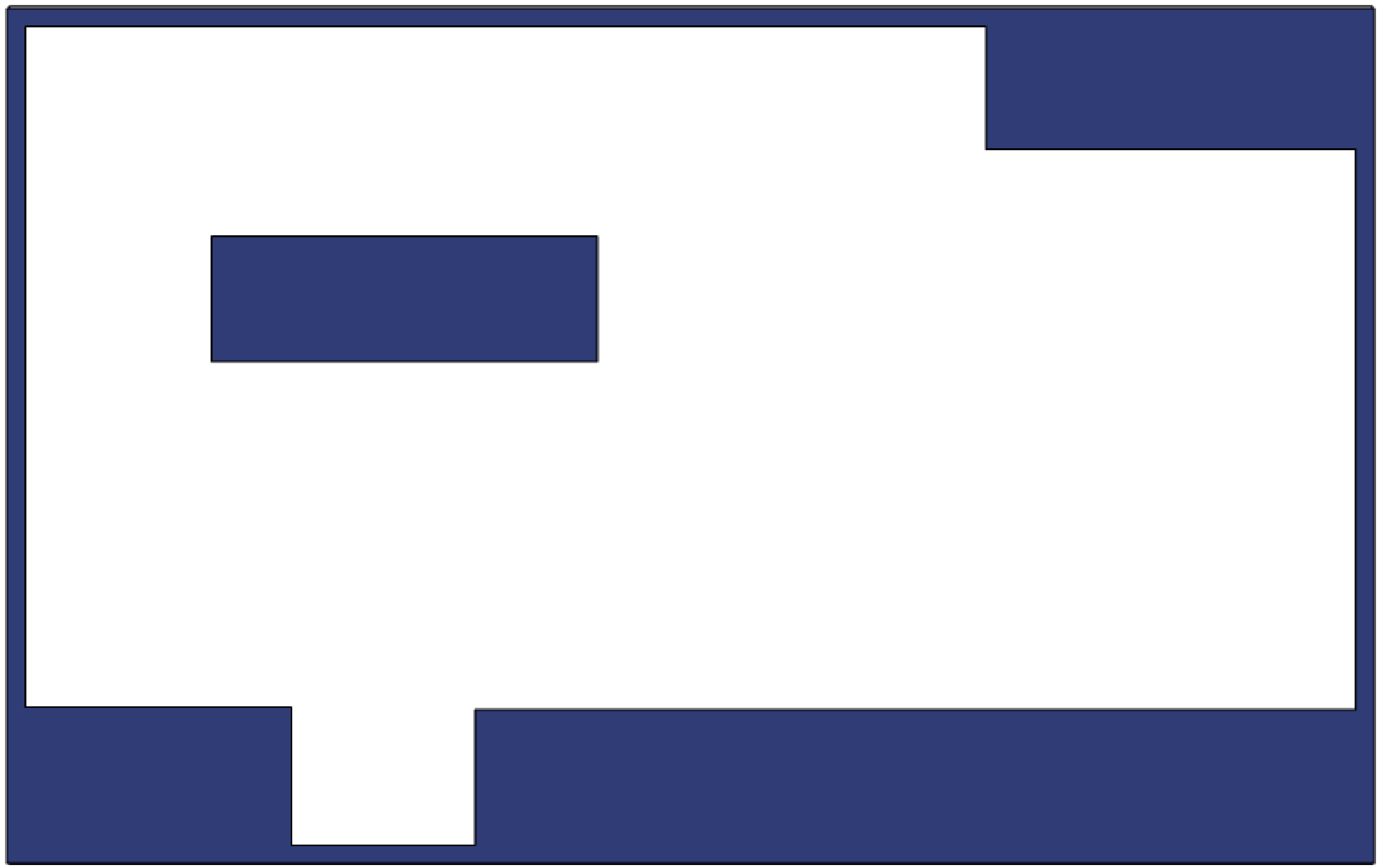
Define *clearance*(q) = min { | q - p | }, for all p ∈ B

Define *near*(q) = { p ∈ B such that | q - p | = *clearance*(q) }

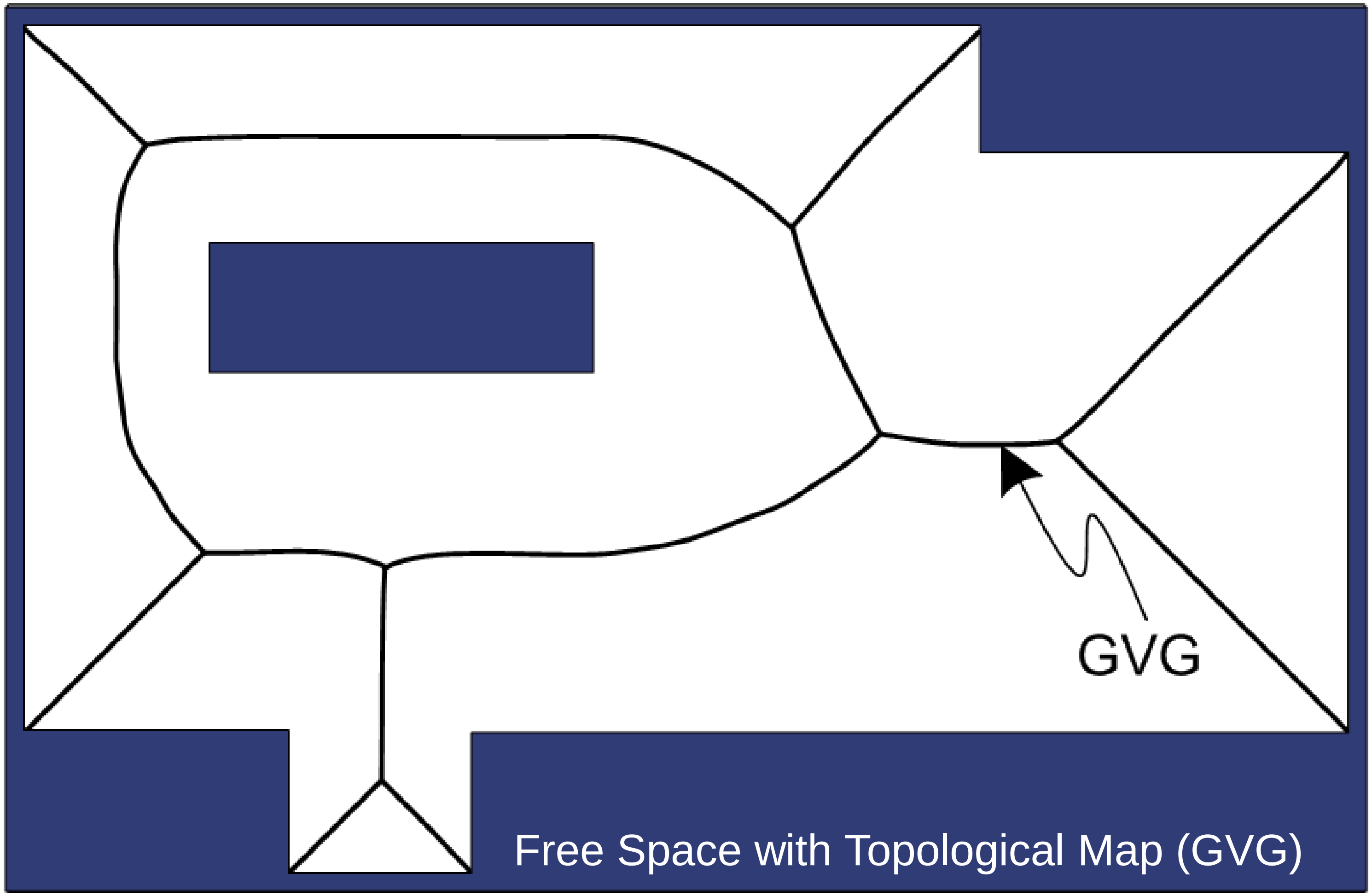
q is in the *Voronoi diagram* of C_{free} if | *near*(q) | > 1

number of set elements

Generalized Voronoi Graph (GVG)

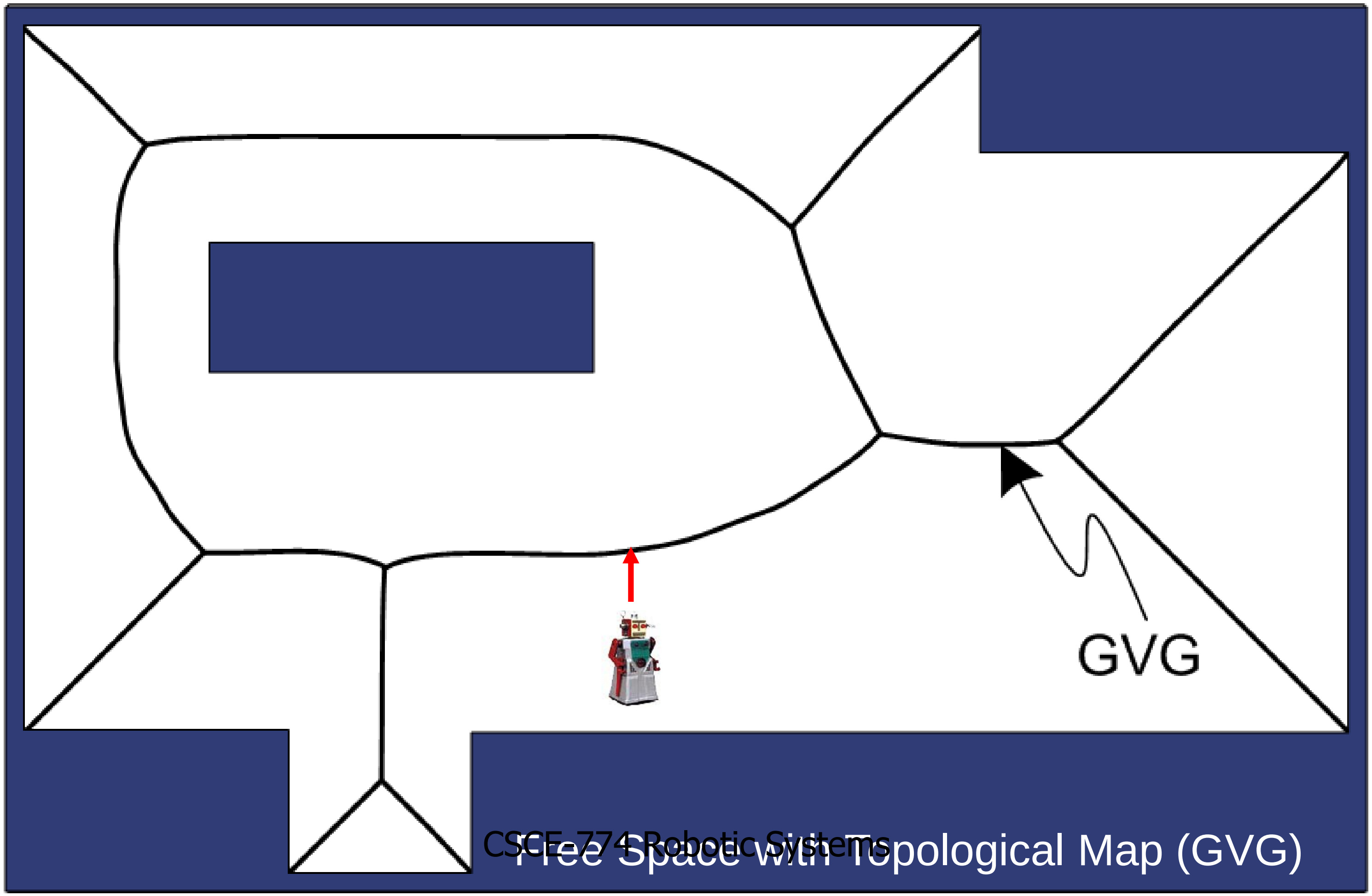


Generalized Voronoi Graph (GVG)



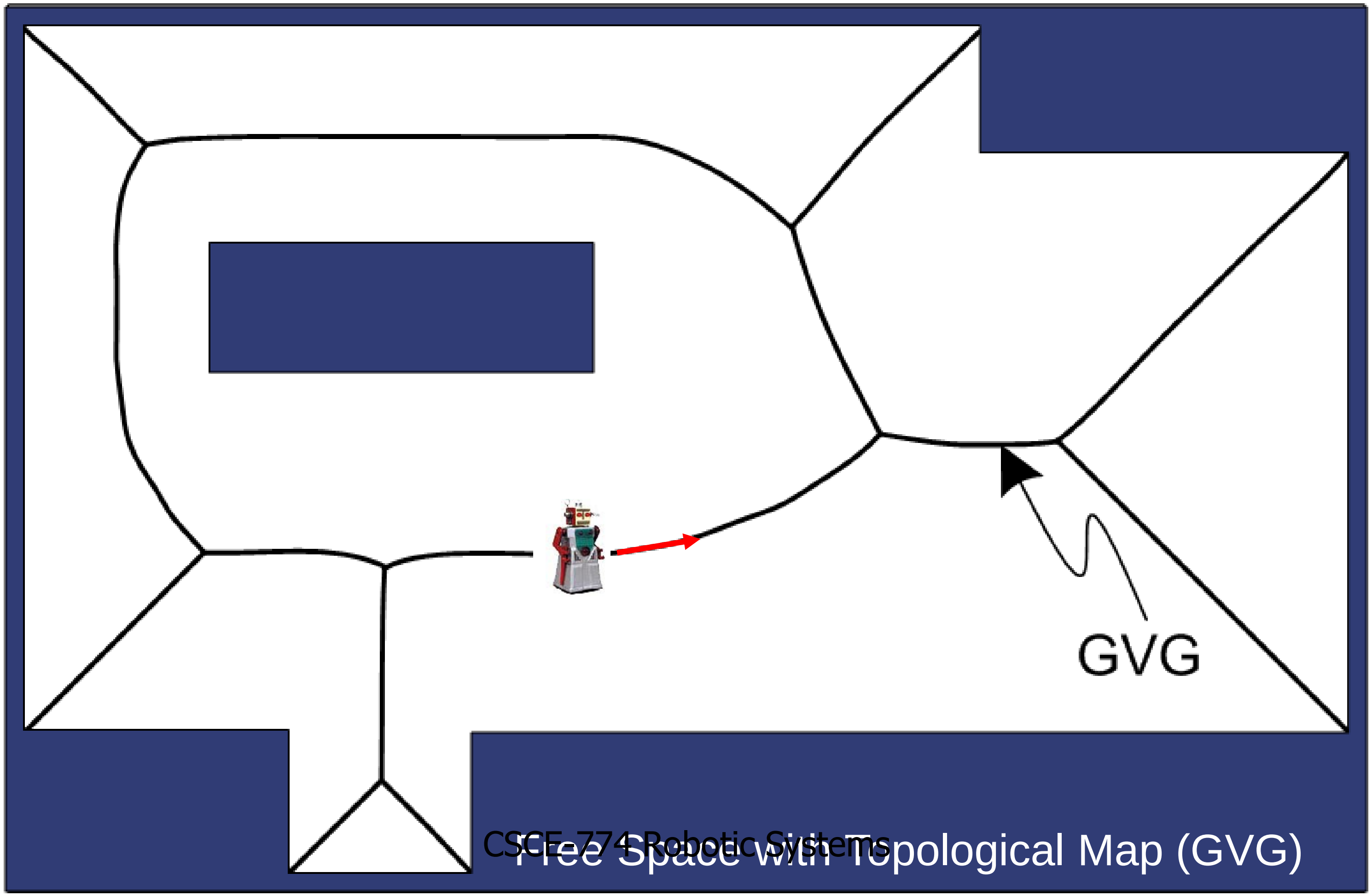
Generalized Voronoi Graph (GVG)

- Access GVG



Generalized Voronoi Graph (GVG)

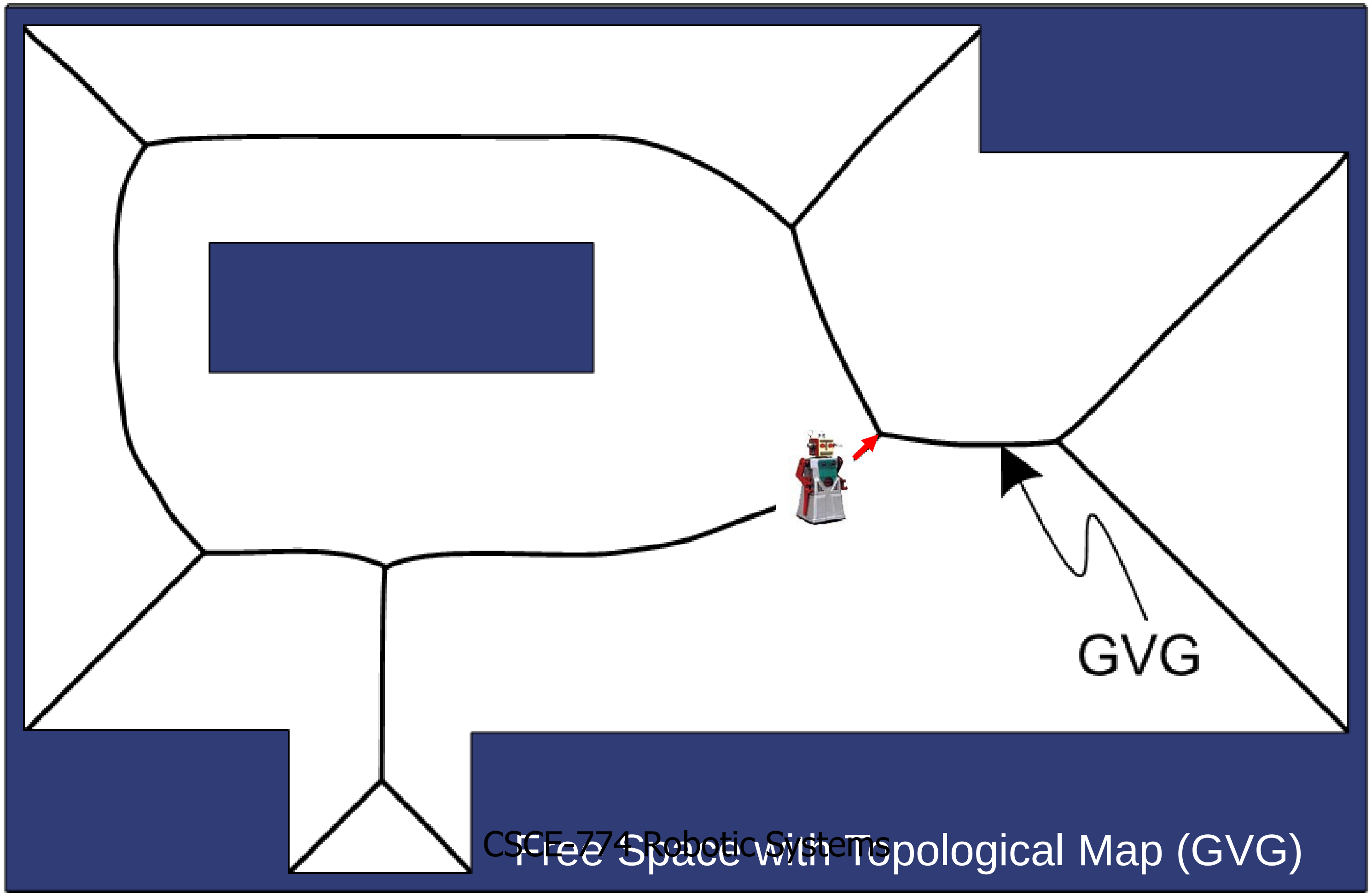
- Access GVG
- Follow Edge



Free Space with Topological Map (GVG)

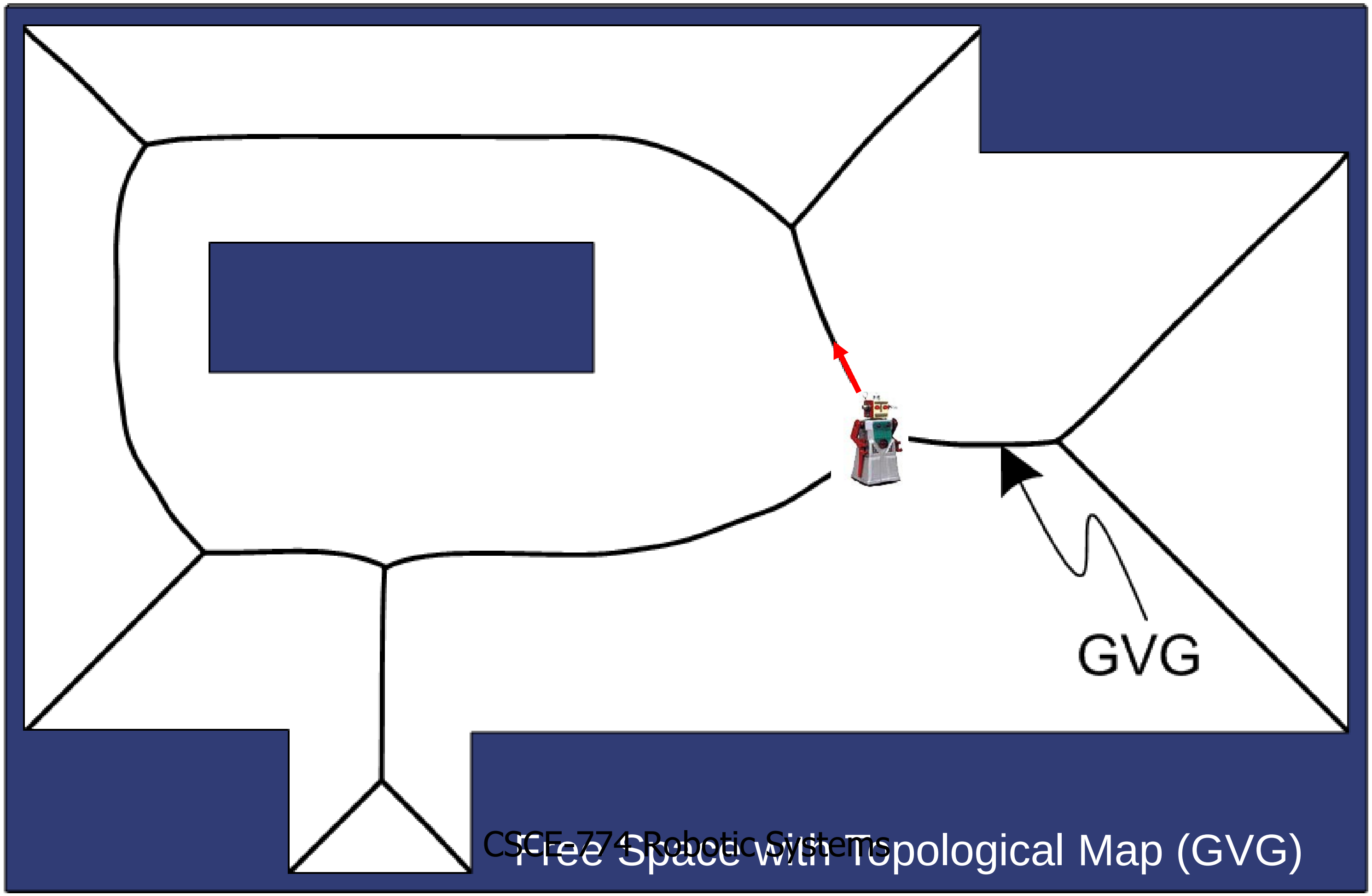
Generalized Voronoi Graph (GVG)

- Access GVG
- Follow Edge
- Home to the MeetPoint



Generalized Voronoi Graph (GVG)

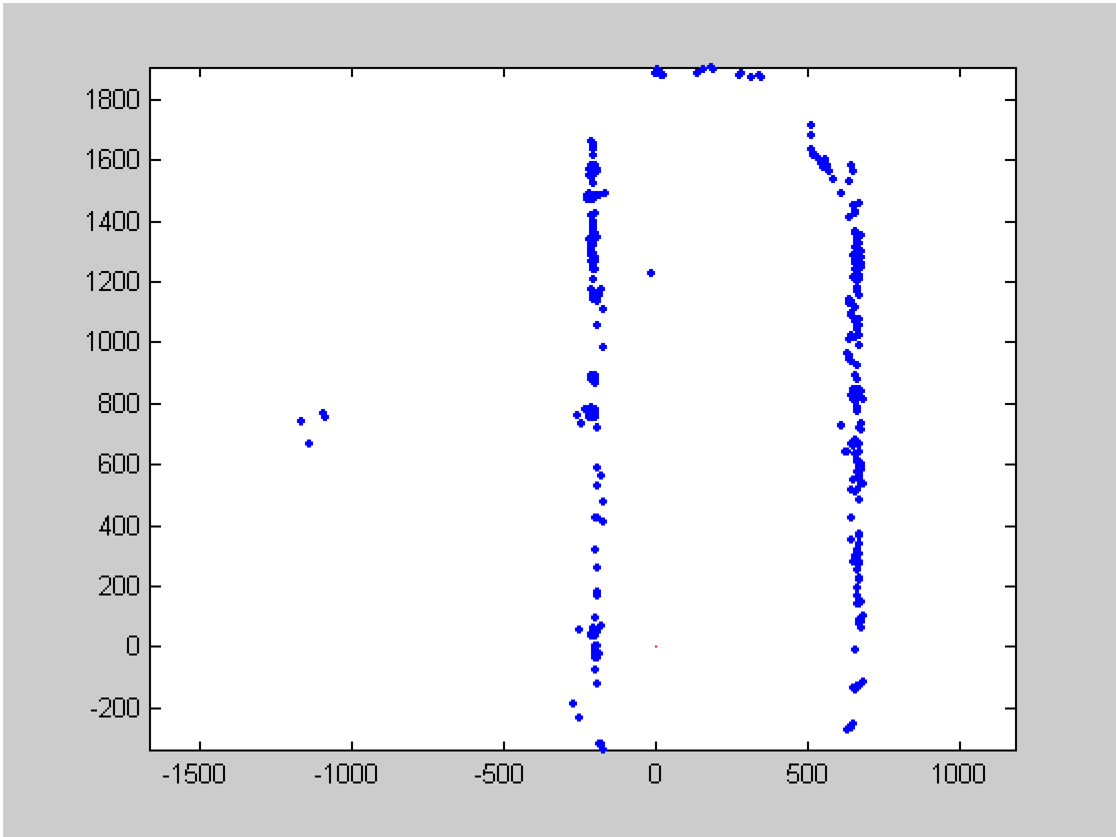
- •Access GVG
- •Follow Edge
- •Home to the MeetPoint
- •Select Edge



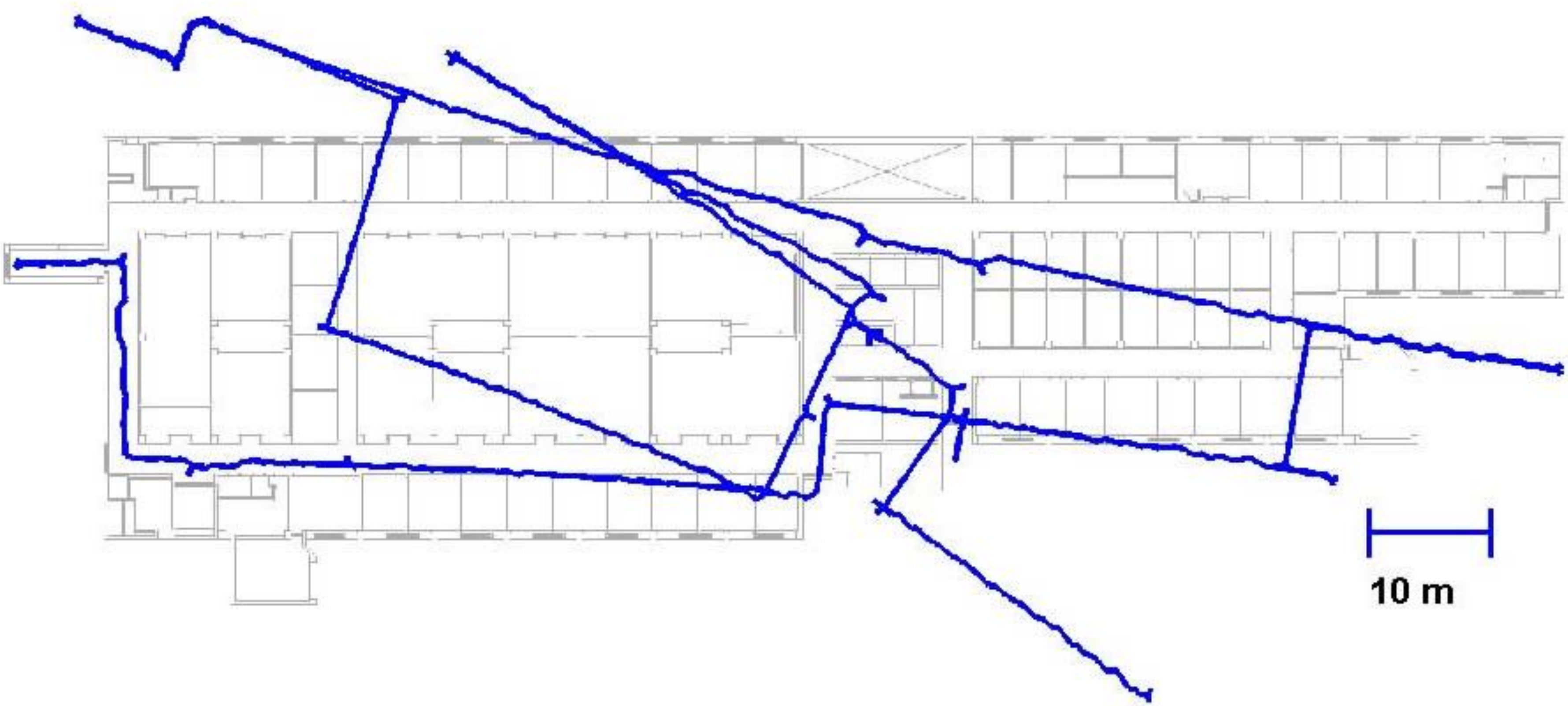
GVG construction using sonar

- Nomadic Scout
- Sonar (GVG navigation)
- Camera with omni-directional mirror (feature detection)
- Onboard 1.2 GHz processor

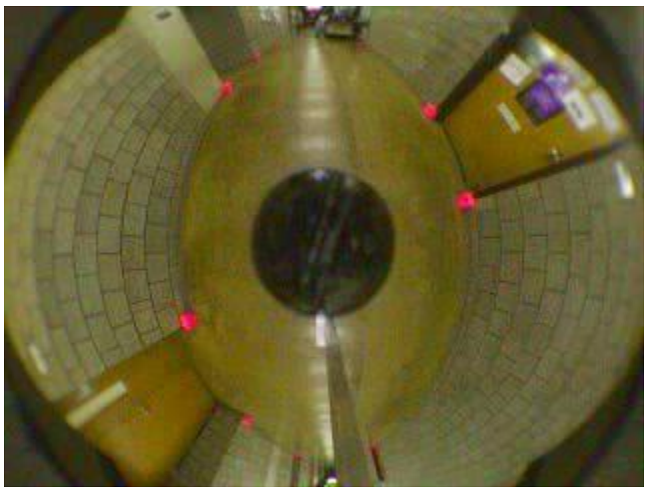
GVG construction using sonar



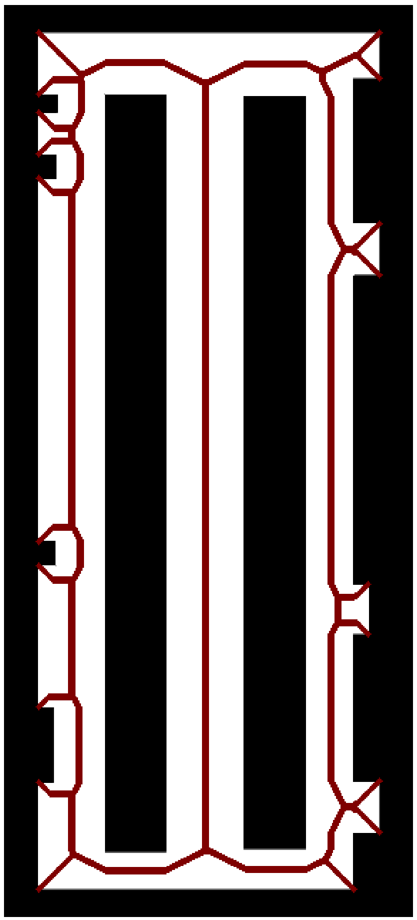
GVG construction using sonar



Slammer in Action



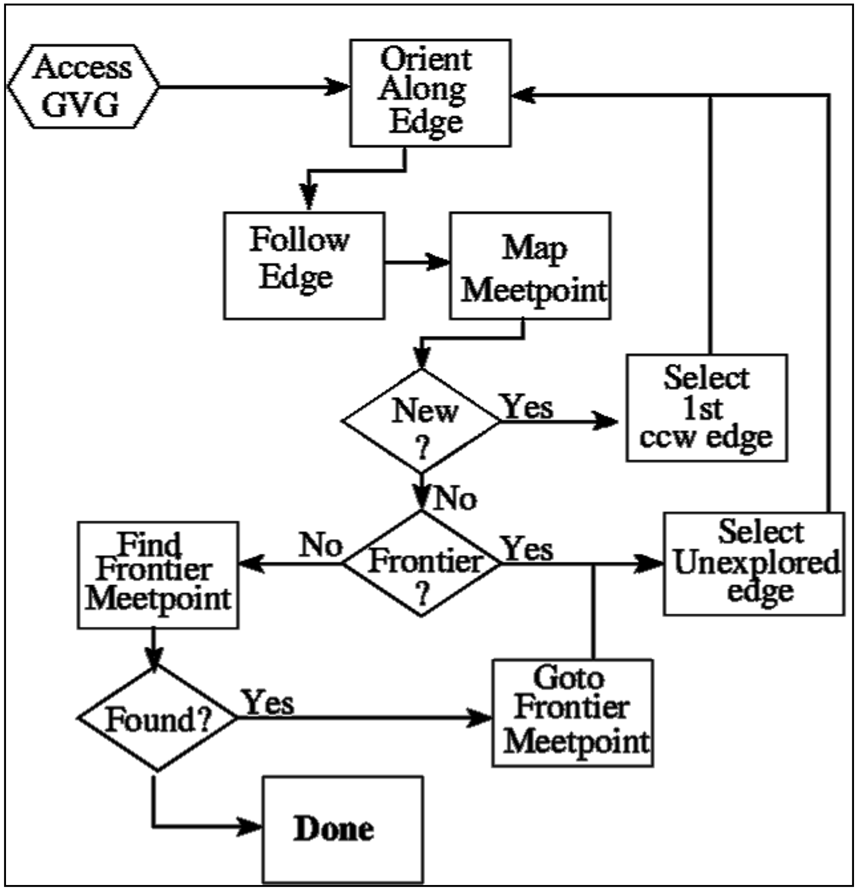
Removing Edges

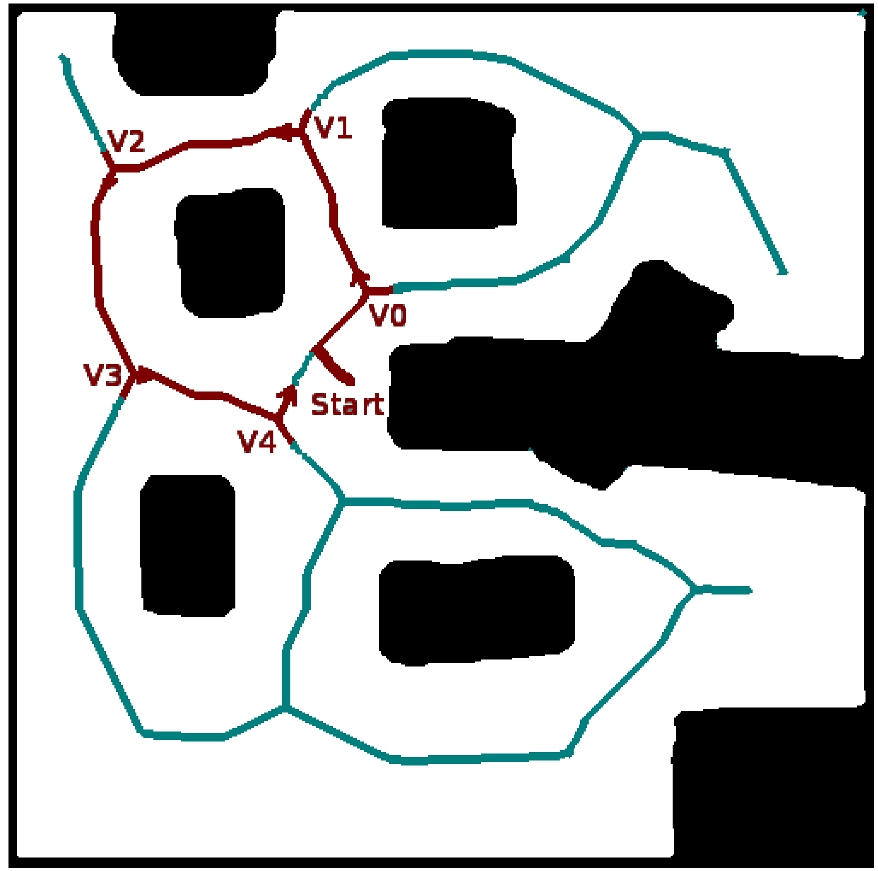


Meetpoint Detection

- 3σ uncertainty ellipse of explored meetpoints
- Meetpoint degree (branching factor)
- Distances to local obstacles
- Relative angle bearings
- Edge signature
  - Edge length
  - Edge Curvature
- Vertex signal

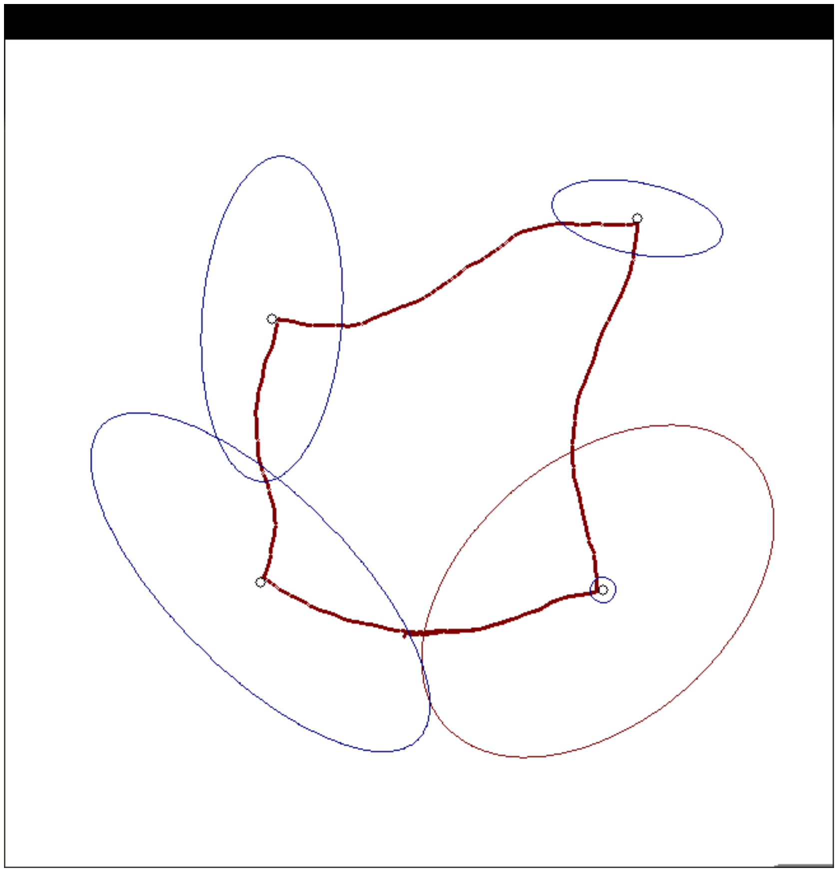
Ear-based Exploration



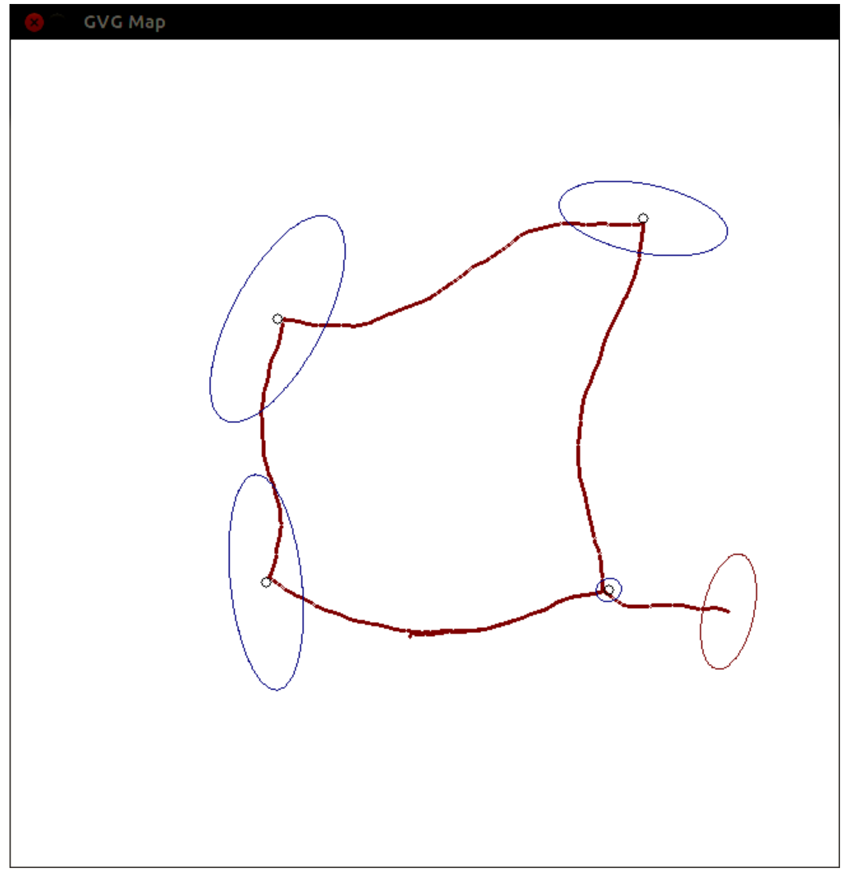


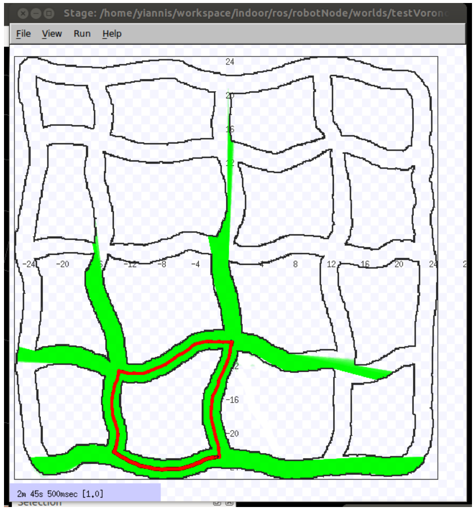
Uncertainty Reduction

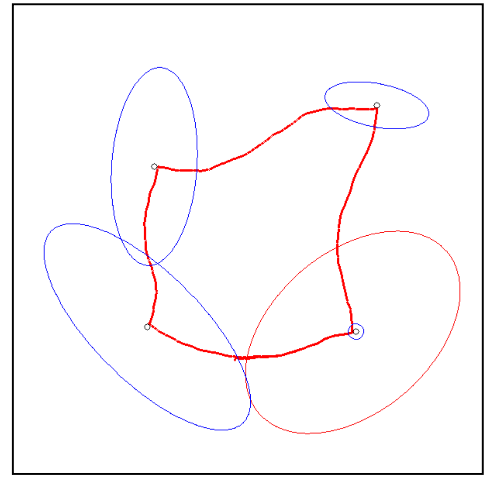
Before Loop-closure



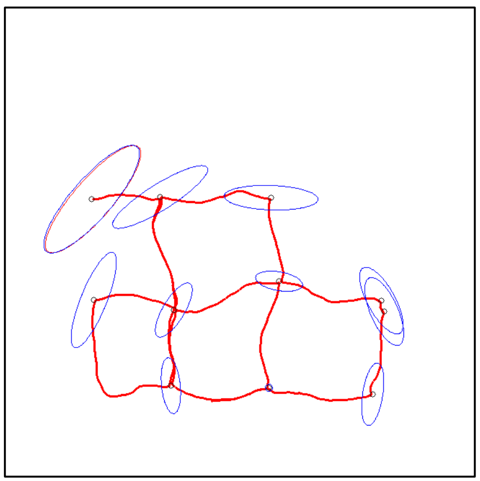
After Loop-closure

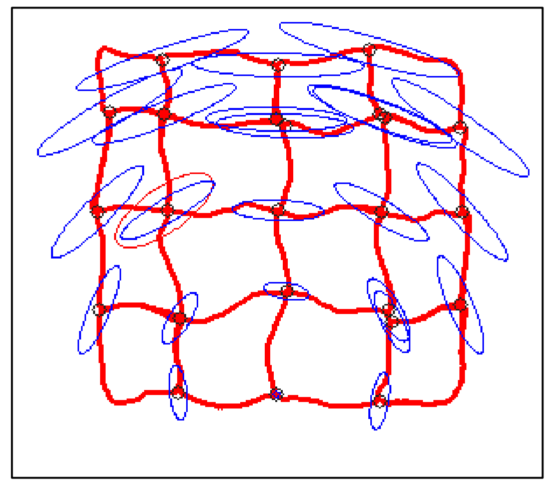






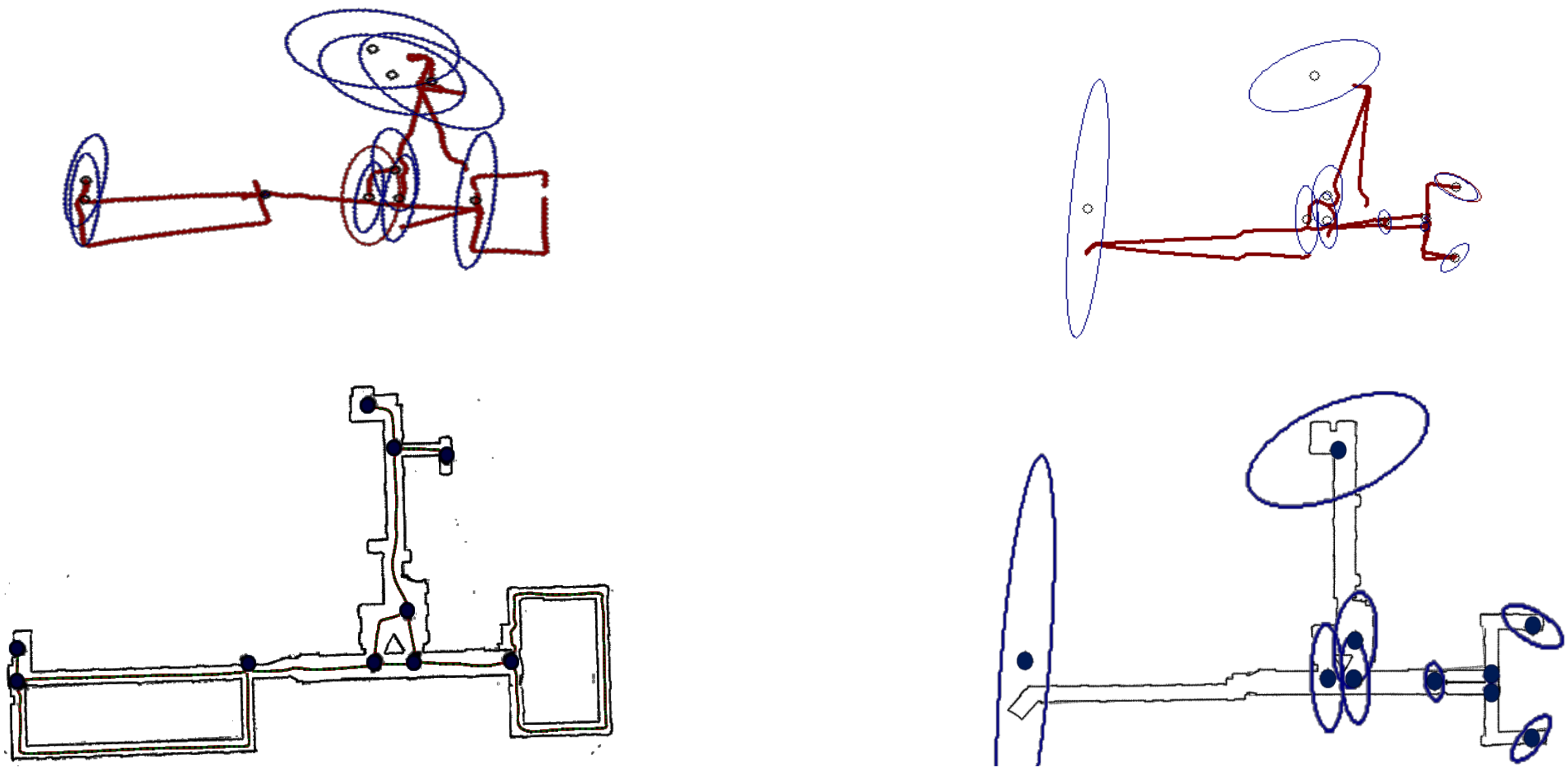
Simulation





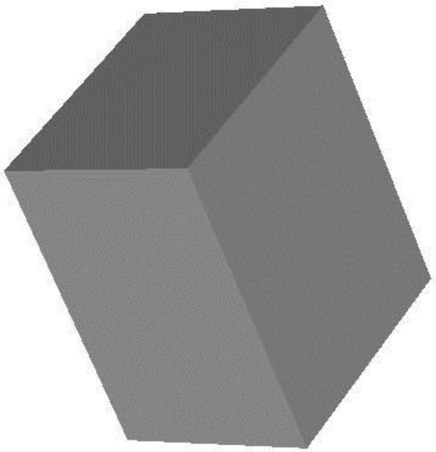
Code available online at https://github.com/QiwenZhang/gvg

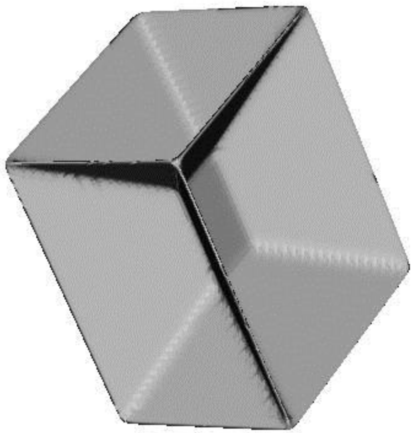
Real Environment

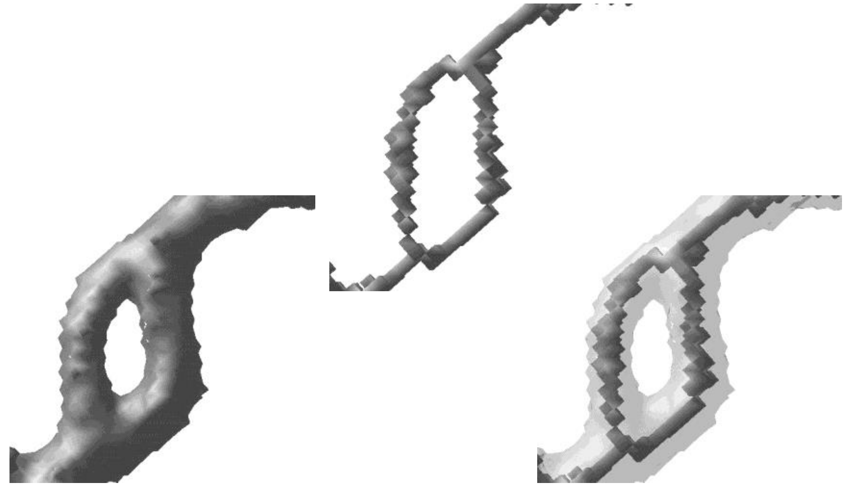


Ear-based Exploration on Hybrid Metric/Topological Maps

Voronoi applications



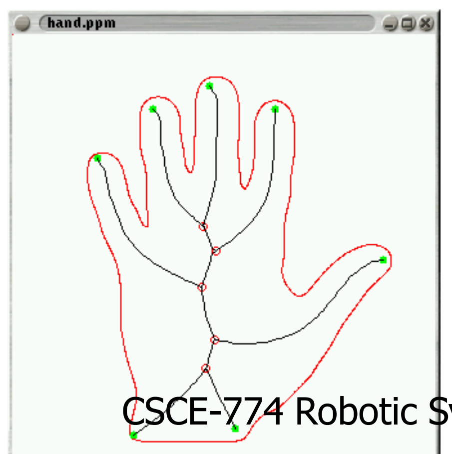


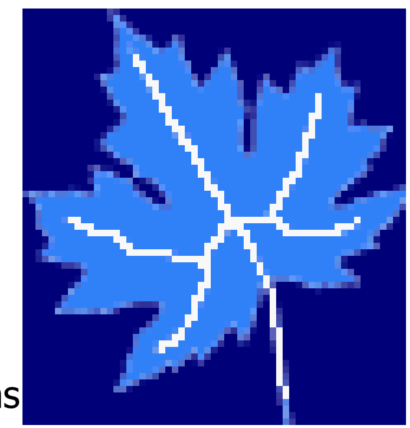


A retraction of a 3d object
== "*medial surface*"

what?

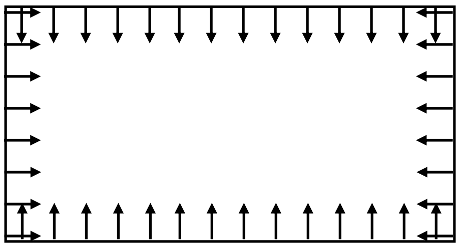
Skeletonizations resulting from constant-speed curve evolution



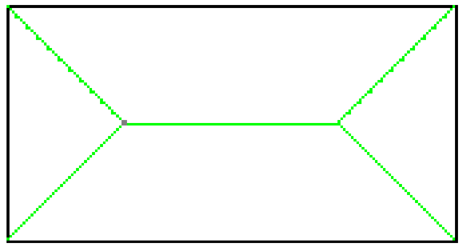


in 2d, it's called a *medial axis*

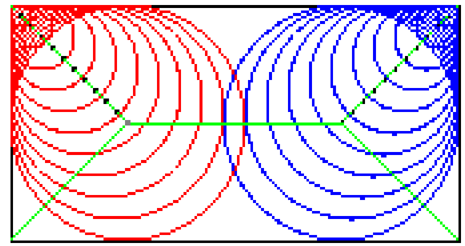
skeleton ⟷ shape



curve evolution



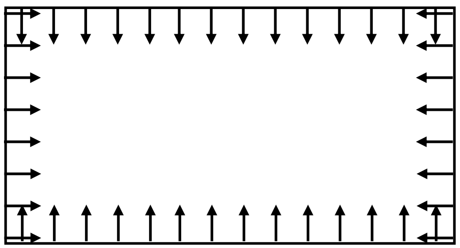
where wavefronts collide



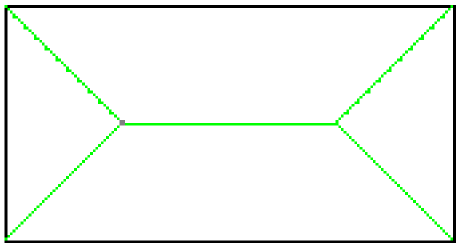
centers of maximal disks

again reduces a 2d (or higher) problem to a question about graphs...

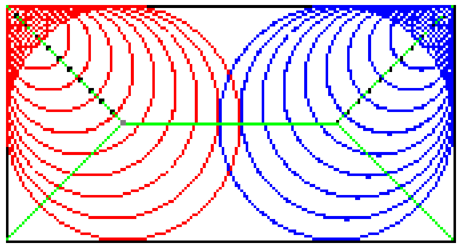
skeleton ⟷ shape



curve evolution

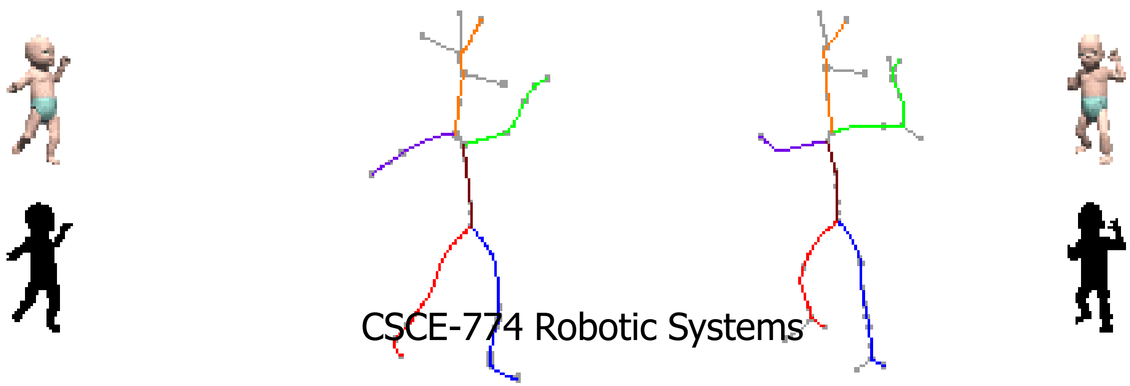


where wavefronts collide

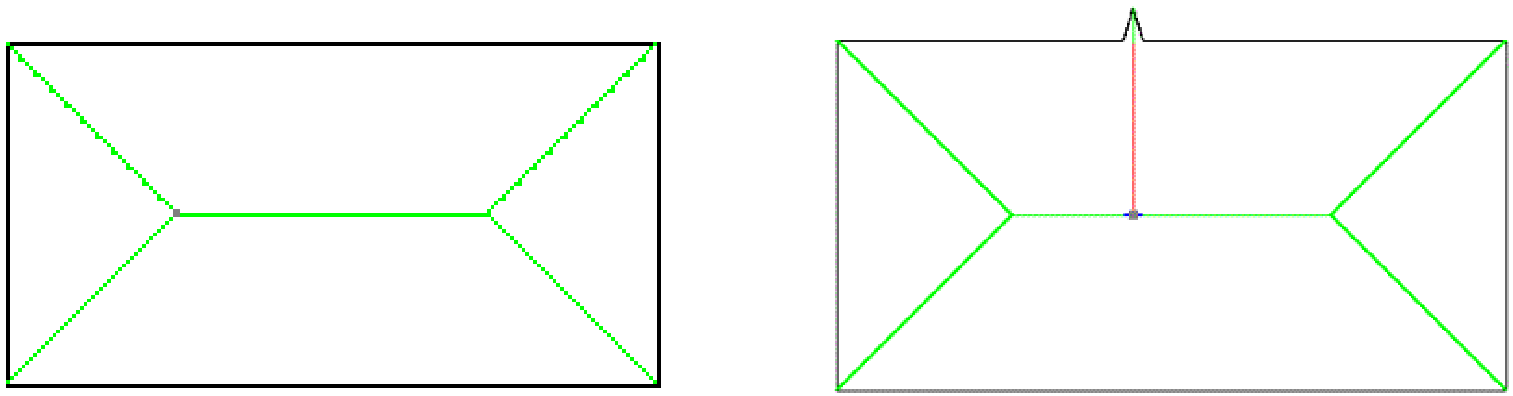


centers of maximal disks

again reduces a 2d (or higher) problem to a question about graphs...



Problems

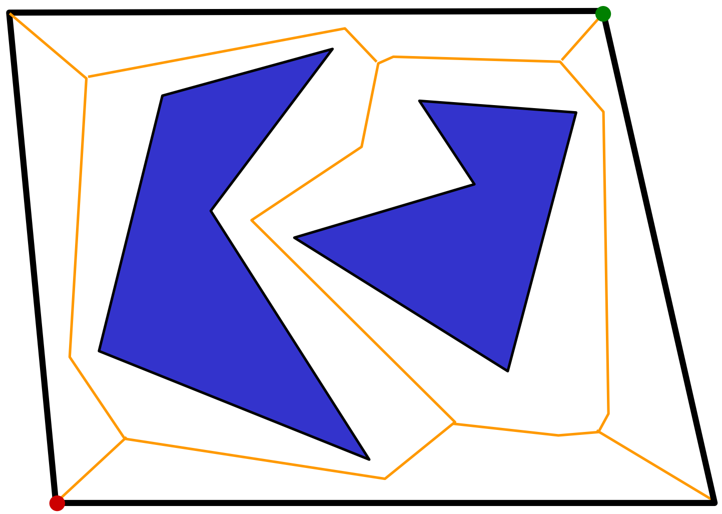


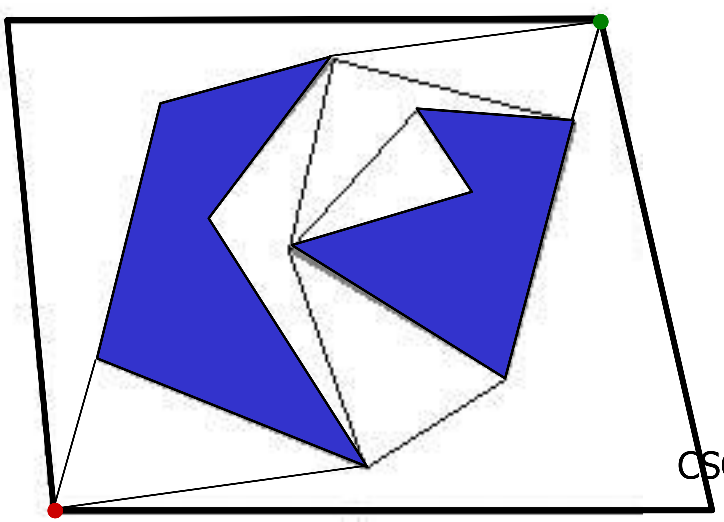
The skeleton is sensitive to small changes in the object's boundary.

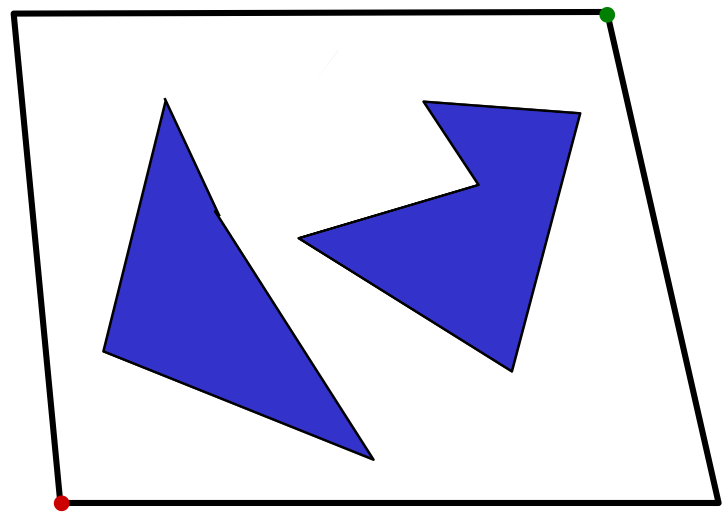
- graph isomorphism (and lots of other graph questions) : NP-complete

Roadmap problems

If an obstacle decides to roll away... (or wasn't there to begin with)







recomputing in less than O(N²) time?