CSCE 590 INTRODUCTION TO IMAGE PROCESSING

Denoising

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Image Degradation and Restoration

http://fireoracleproductions.com/services__samples

Google image

Image degradation due to

- noise in transmission
- imperfect image acquisition
    - environmental condition
    - quality of sensor

Properties of Noise

- Spatial properties
  - Spatially periodic noise
  - Spatially independent noise
- Frequency properties
  - White noise – noise containing all frequencies within a bandwidth

Image Restoration with Additive Noise

$$g(x,y) = f(x,y) + \eta(x,y)$$
$$G(u,v) = F(u,v) + N(u,v)$$

Noise models:

- Impulse noise: pepper and salt
- Continuous noise model:
  - Gaussian, Rayleigh, Gamma, Exponential, Uniform

Image Restoration with Additive Noise

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$$G(u,v) = F(u,v) + N(u,v)$$

Noise models:

- Impulse noise: pepper and salt
- Continuous noise model:
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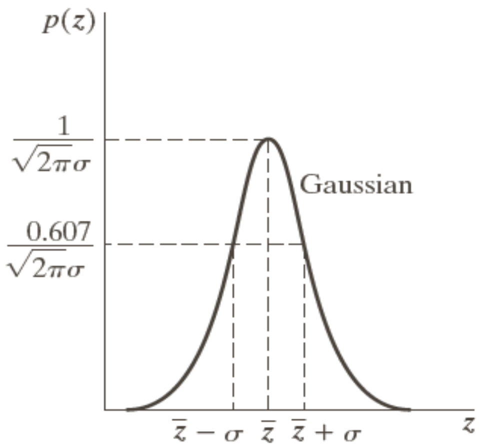
Some Important Noise Model

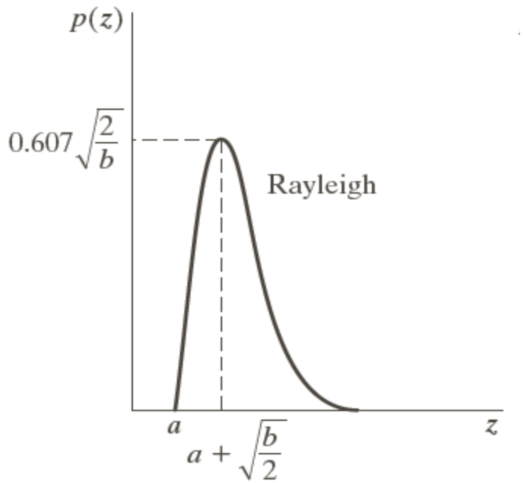


$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\bar{z})^2}{2\sigma^2}}$$

- Due to electronic circuit
- Due to the image sensor
  - poor illumination
  - high temperature

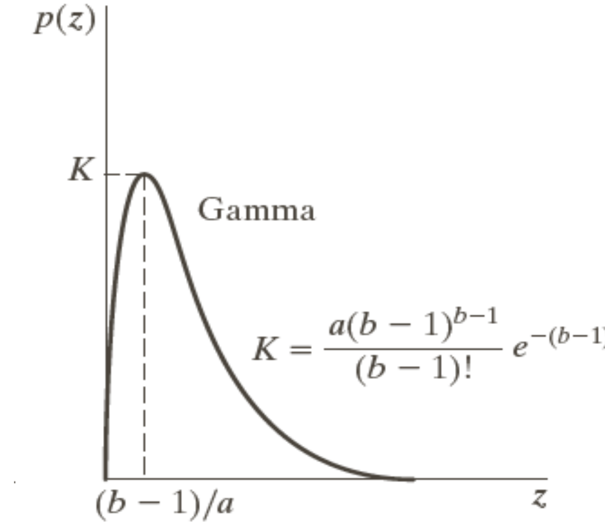
http://www.gergltd.com/cse486/project2/

Some Important Noise Model



$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b}} & z \ge a \\ 0 & z < a \end{cases}$$

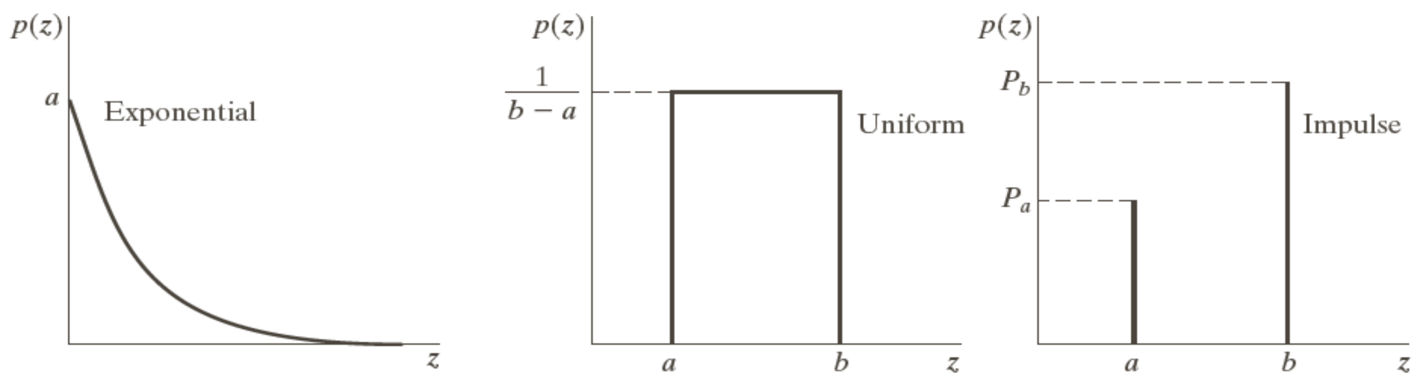
Rayleigh noise
- range imaging
- Background model for Magnetic Resonance Imaging (MRI) images



$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & z \ge 0 \\ 0 & z < 0 \end{cases}$$

Gamma noise
- laser imaging

Some Important Noise Model



$$P(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$$P(z) = \begin{cases} \dfrac{1}{b-a} & a \leq z \leq b \\ 0 & otherwise \end{cases}$$

$$P(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{Otherwise} \end{cases}$$

Exponential noise
- laser imaging

Uniform noise

Impulse noise
- salt and pepper noise
- A/D converter error
- bit error in transmission

An Example

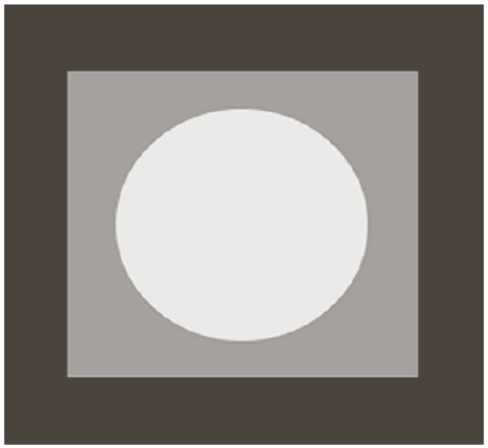


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

What is its histogram?

An Example (cont.)

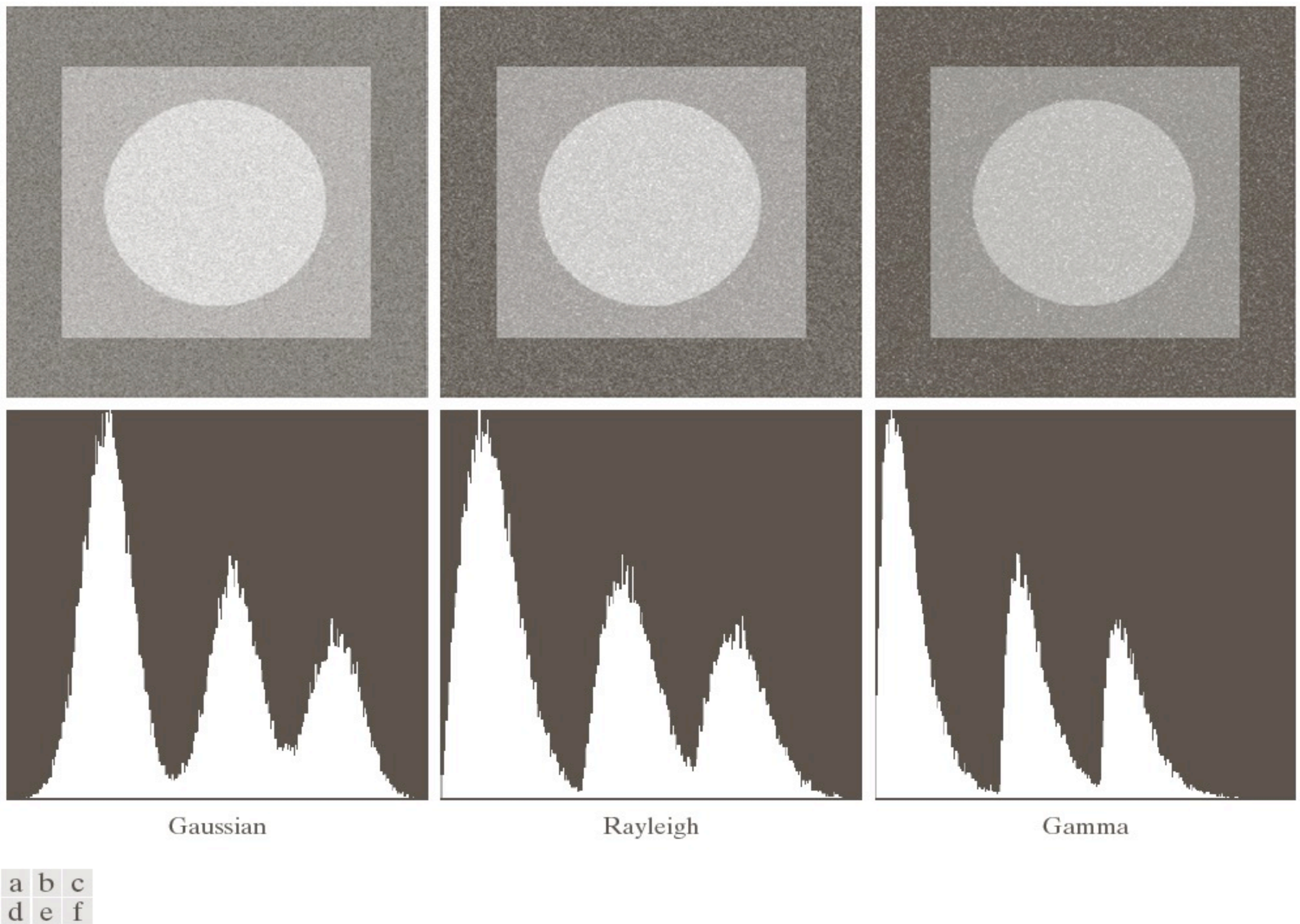


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Slides courtesy of Prof. Yan Tong

An Example (cont.)

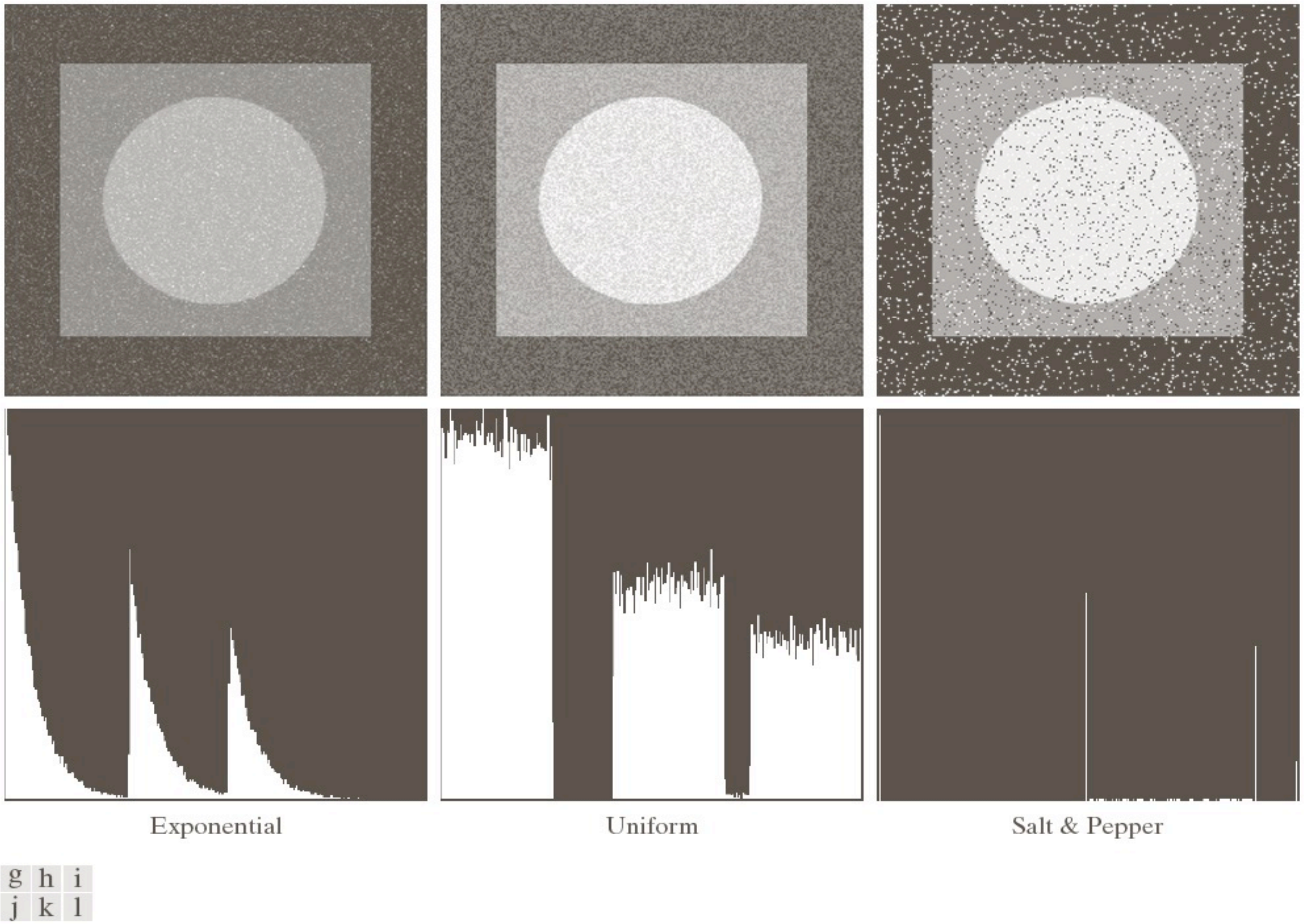
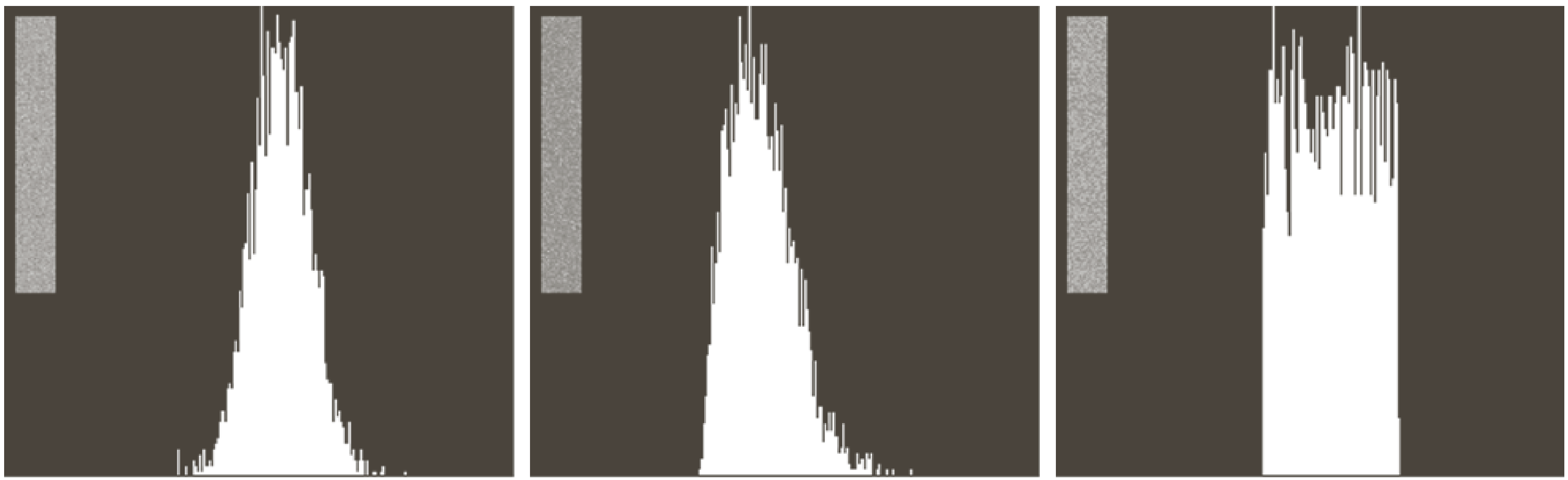


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Estimation of Noise Parameters



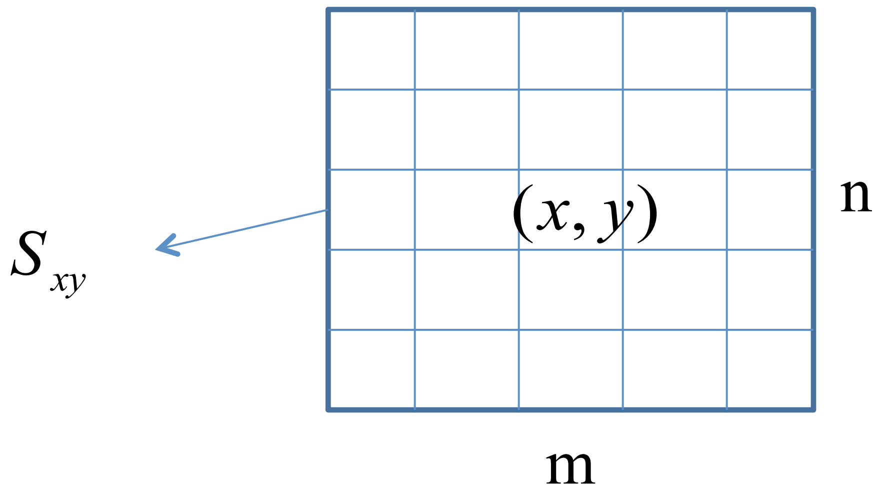
a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Take a small stripe of the background, do statistics for the mean and variance.

Slides courtesy of Prof. Yan Tong

Mean Filters for Continuous Noise Models



Arithmetic Mean Filter: a linear filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

Slides courtesy of Prof. Yan Tong

Non-linear Mean Filters

Geometric Mean Filter

Harmonic Mean Filter

Contraharmonic Mean Filter

Slides courtesy of Prof. Yan Tong

Non-linear Mean Filters

Geometric Mean Filter

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Harmonic Mean Filter

$$\hat{f}(x,y) = \frac{1}{\frac{1}{mn} \sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Work well for

- Continuous noise
- Salt noise

Fail for the pepper noise

Contraharmonic Mean Filter

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

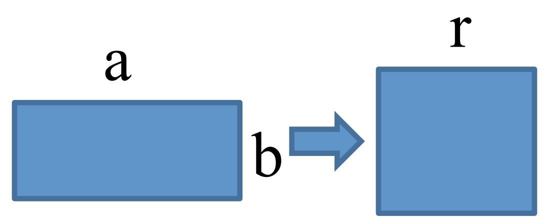
Q is the order of the filter

- positive Q removes pepper noise
- negative Q removes salt noise
- Special cases: Q=0, Q=-1

Mean Filter

Geometric Mean Filter

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$



- Removing the salt noise
- Fail in the pepper noise

An Example

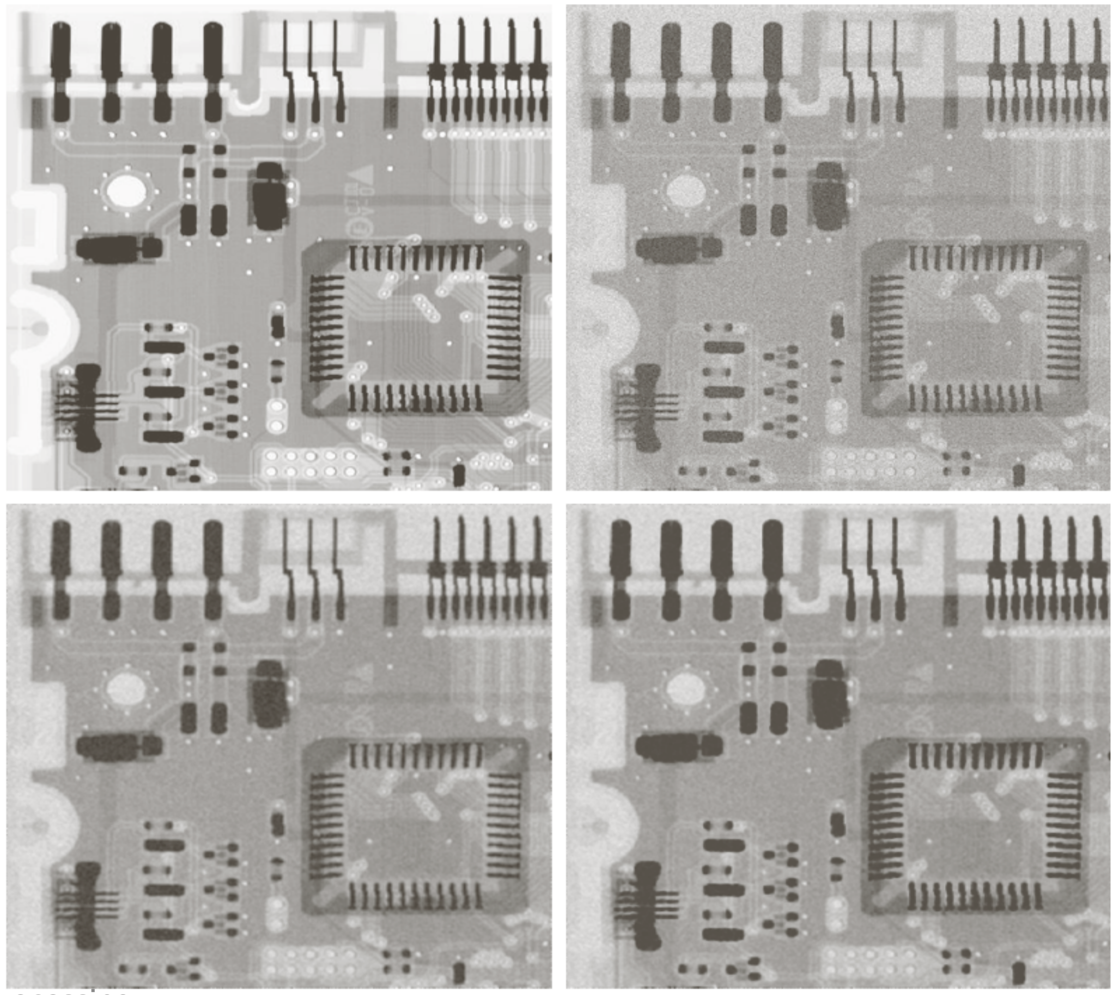


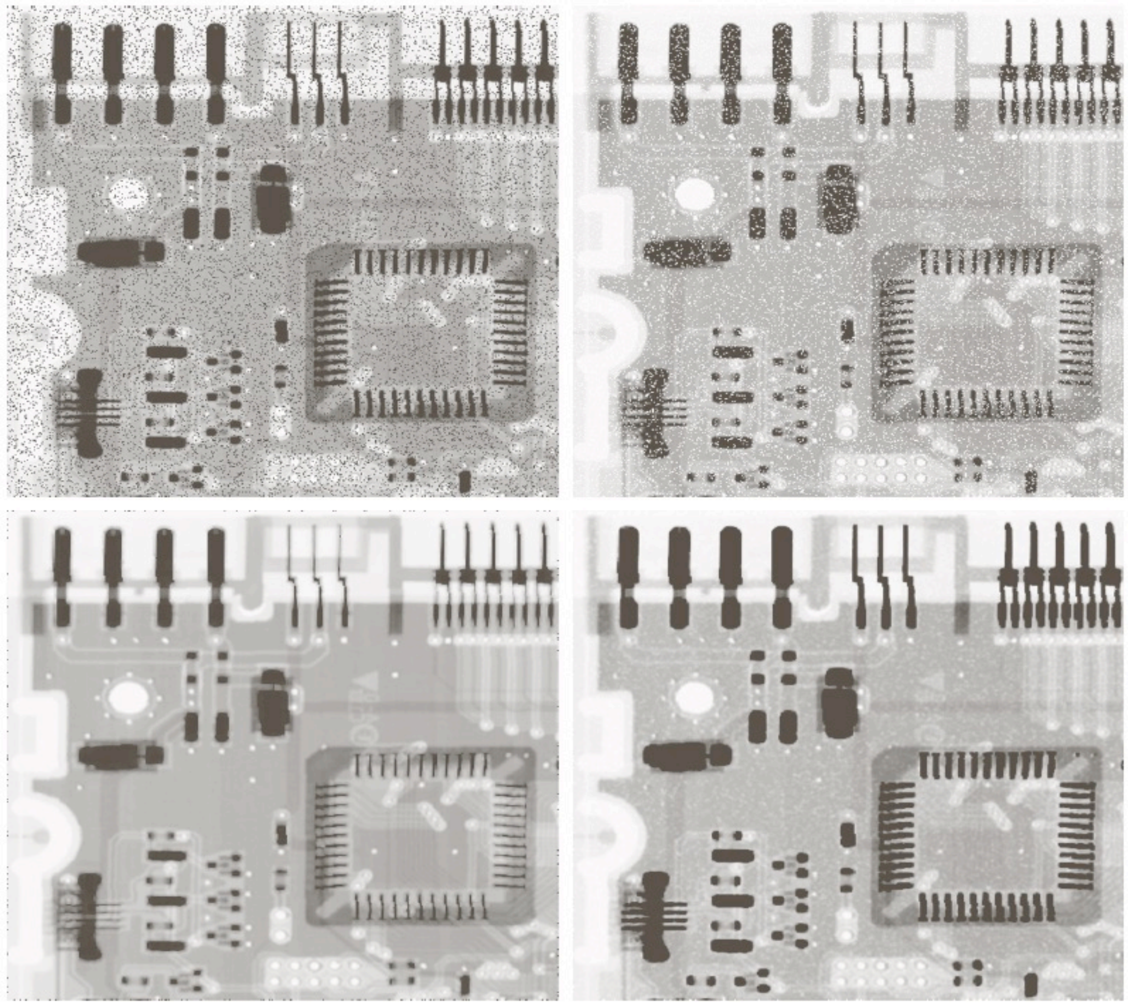
FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

An Example

Pepper noise

Salt noise

Q=1.5



Q=-1.5

a b
c d

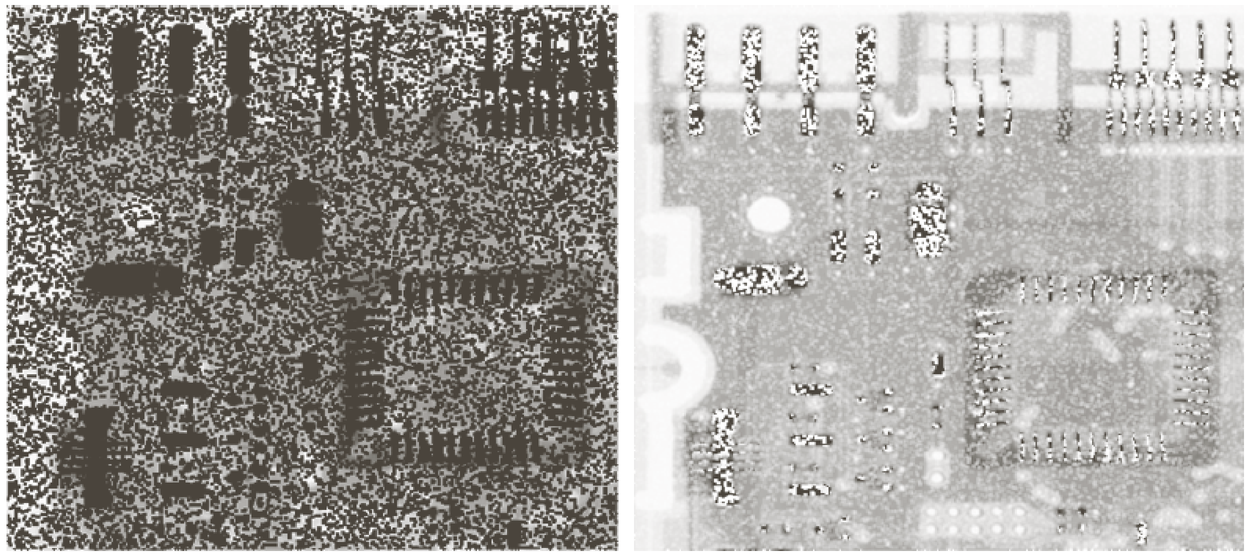
FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3 × 3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Slides courtesy of Prof. Yan Tong

A Failed Case with Wrong Sign of Contraharmonic Filter

a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.



Slides courtesy of Prof. Yan Tong

Order-Statistic Filters -- Median Filter

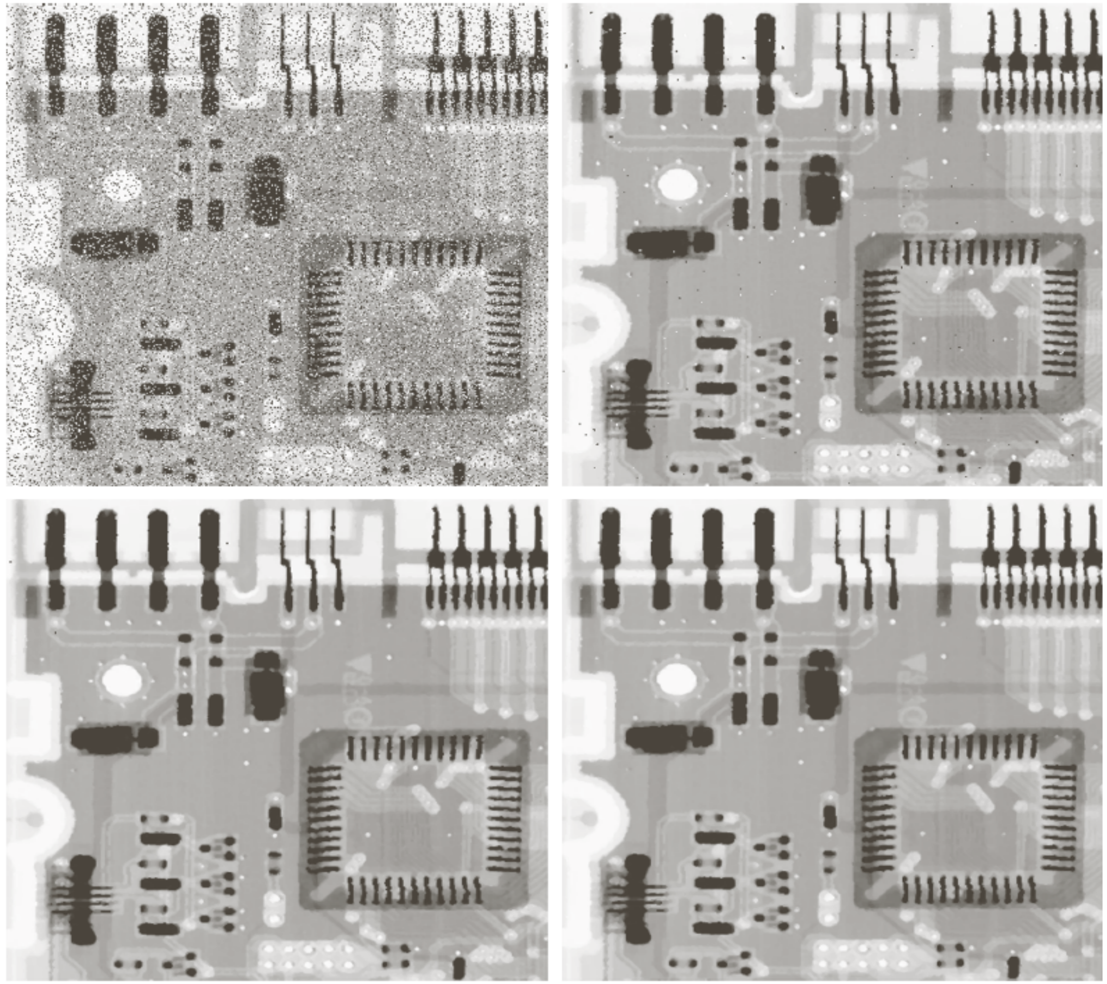
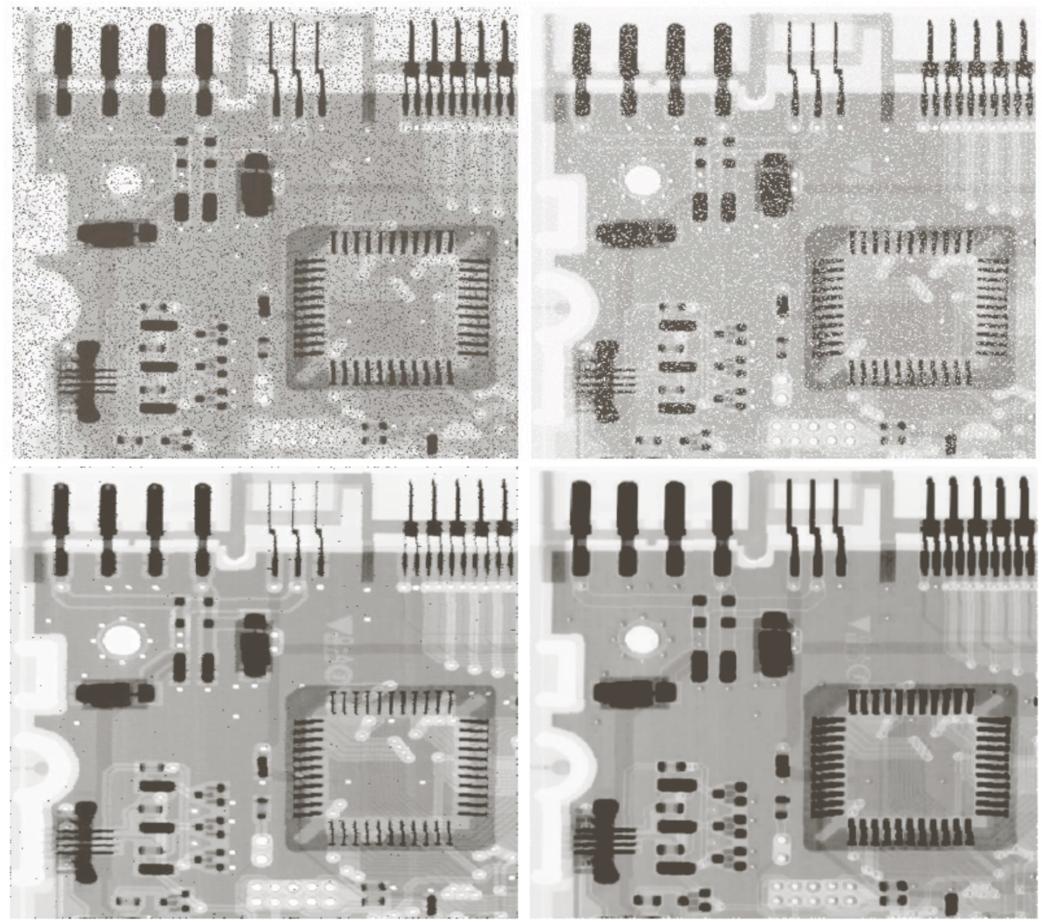


FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

Repeating median filter will remove most of the noise while increase image blurring

Slides courtesy of Prof. Yan Tong

Order-Statistic Filters -- Max/Min Filters



a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.

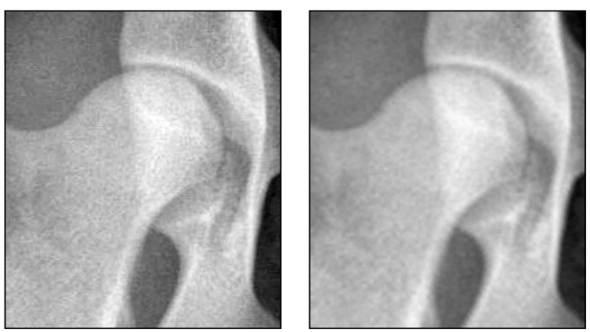
- Find the extreme points
- Remove the targeting impulse noise

Slides courtesy of Prof. Yan Tong

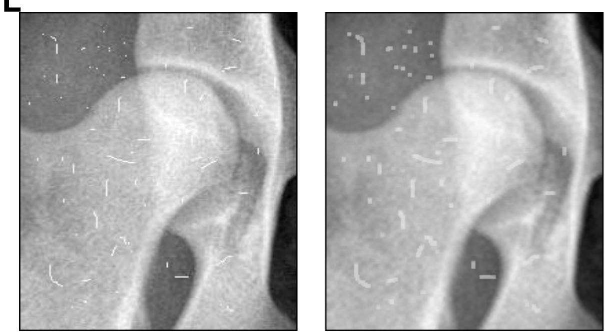
Order-Statistic Filters

- Midpoint filter
  - Combine order statistics and averaging
  - Works best for randomly distributed noise, like Gaussian or uniform noise
  - Not suitable for impulse noise and blur the boundary

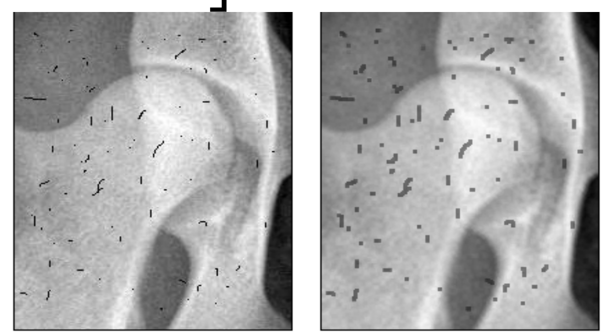
$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$



Random noise



Salt noise



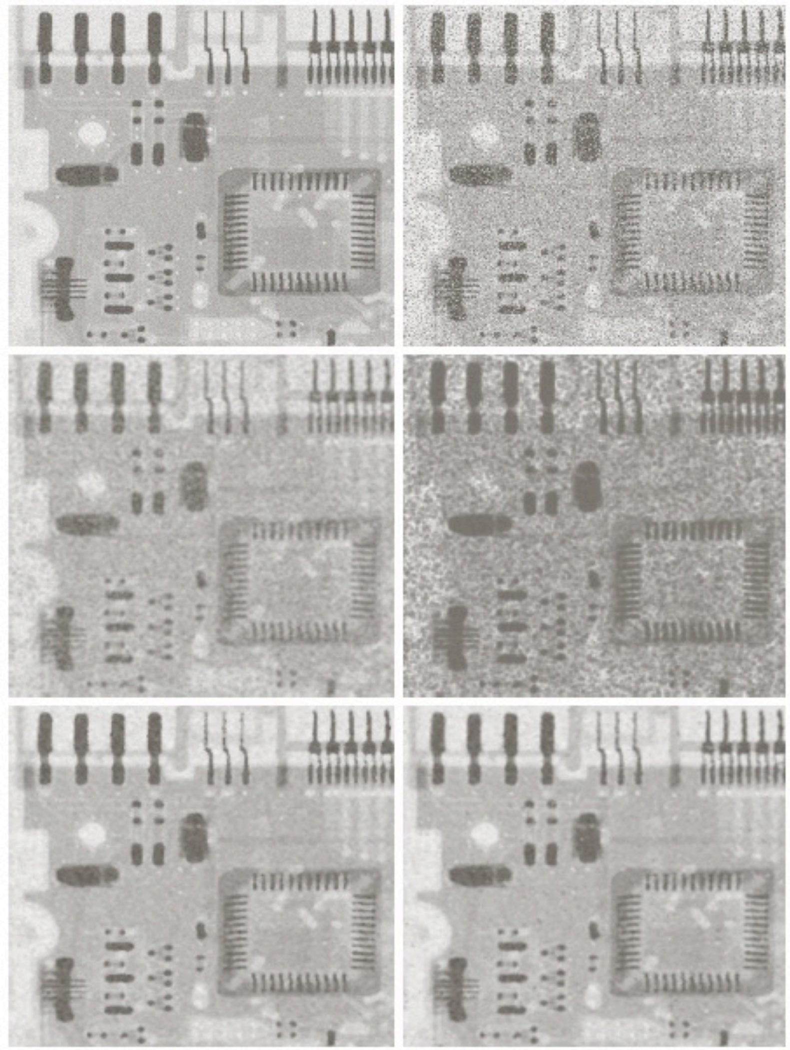
Pepper noise

http://www.digimizer.com/manual/m-image-filtermid.php

Order-Statistic Filters

- Alpha-trimmed mean filter
  - Delete d/2 lowest and d/2 highest intensity values
  - A balance between arithmetic mean filter and median filter $\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$
  - Suitable for combined salt-and-pepper and Gaussian noise

Example



a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5 × 5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

Adaptive Filters

- Adaptive local noise reduction filter
- Key elements:
  - the intensity value $g(x, y)$
  - the variance of the noise σ_η^2
  - the local mean of the neighborhood m_L
  - the local variance of the neighborhood σ_L^2

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x, y) - m_L\right]$$

- Properties:
  - If $\sigma_\eta^2 = 0$, $\hat{f}(x, y) = g(x, y)$ → ideal case
  - If $\sigma_\eta^2 / \sigma_L^2$ is low, preserve the edge information $\hat{f}(x, y) \approx g(x, y)$

Examples

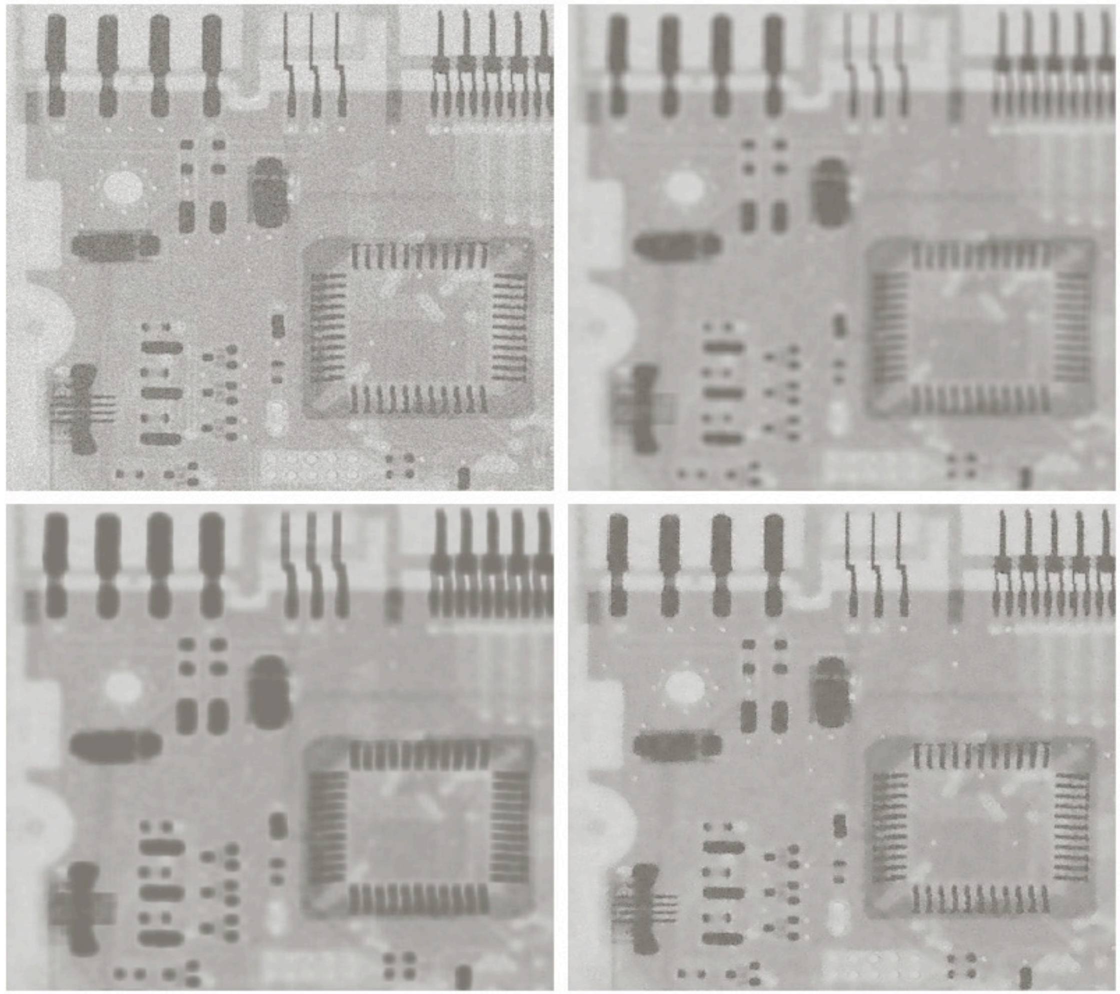


FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.

Slides courtesy of Prof. Yan Tong

Adaptive Median Filter

- Stage A: check if the median value is an extreme value

$A1 = z_{\text{med}} - z_{\text{min}}$

$A2 = z_{\text{med}} - z_{\text{max}}$

If $A1 > 0$ AND $A2 < 0$, go to stage B

Else increase the window size

If window size $\leq S_{\text{max}}$ repeat stage A

Else output z_{med}

Goal 1: remove salt-and-pepper noise with higher probability

Goal 2: smoothing the noise other than impulses

Goal 3: reduce distortion

- Stage B: check if the center pixel is an extreme value

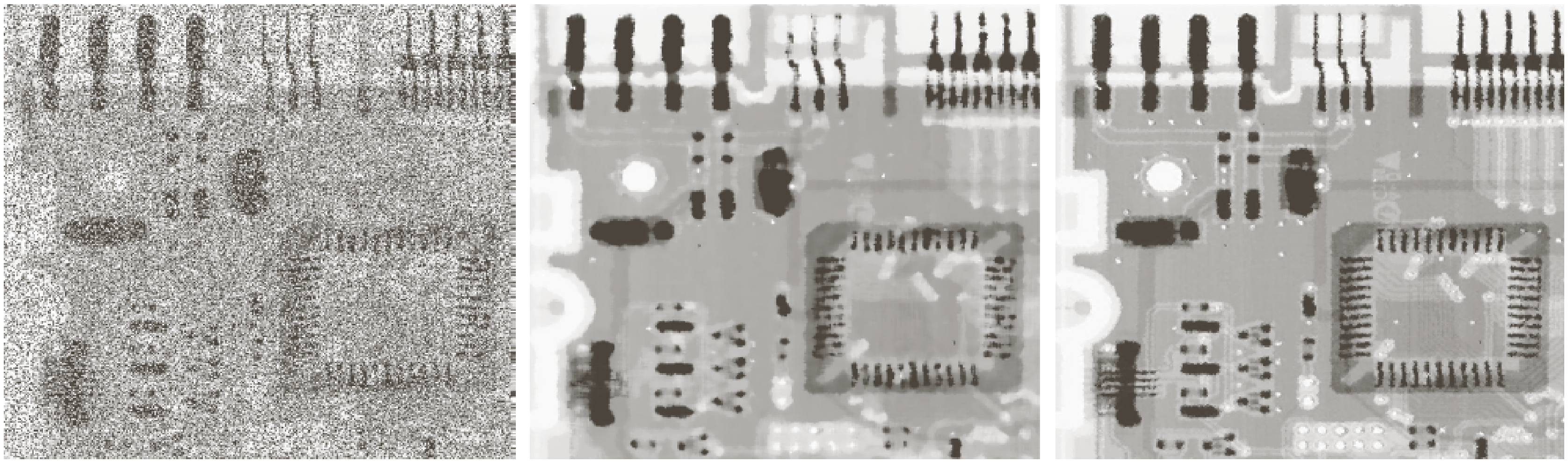
$B1 = z_{xy} - z_{\text{min}}$

$B2 = z_{xy} - z_{\text{max}}$

If $B1 > 0$ AND $B2 < 0$, output z_{xy}

Else output z_{med}

Adaptive Median Filter -- Example



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Slides courtesy of Prof. Yan Tong

Mean Filter

Harmonic Mean Filter

$$\hat{f}(x,y) = \frac{1}{\frac{1}{mn}\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

- Removing the salt noise
- Fail in the pepper noise

Mean Filter

Contraharmonic Mean Filter

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q is the order of the filter

- positive Q removes pepper noise
- negative Q removes salt noise
- Special cases: Q=0, Q=-1

Slides courtesy of Prof. Yan Tong

Periodical Noise

a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

Image is corrupted by a set of sinusoidal noise of different frequencies

Estimate the Degradation Function

- Observation
- Experimentation
- Mathematical modeling

Estimate Degradation Function - Observation

Assumptions:

- **The degradation function is linear and position-invariant**
- **No other knowledge about the degradation function**

Estimation by image observation:

- **Extract a subimage with strong signal** → Higher signal-to-noise ratio
- **Perform restoration on the subimage**

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

degradation function in the subimage ← $H_s(u,v)$

$G_s(u,v)$ → Observed subimage

$\hat{F}_s(u,v)$ → Restored subimage

$H_s(u,v)$ ⇓ $H(u,v)$

Application: restoring old pictures

Estimate Degradation Function - Experimentation

Assumptions:

- A similar equipment is available
- Change the system setting can achieve similar degraded images

$$H(u,v) = \frac{G(u,v)}{A}$$

$G(u,v)$ → Observed image

A → Impulse signal

a b

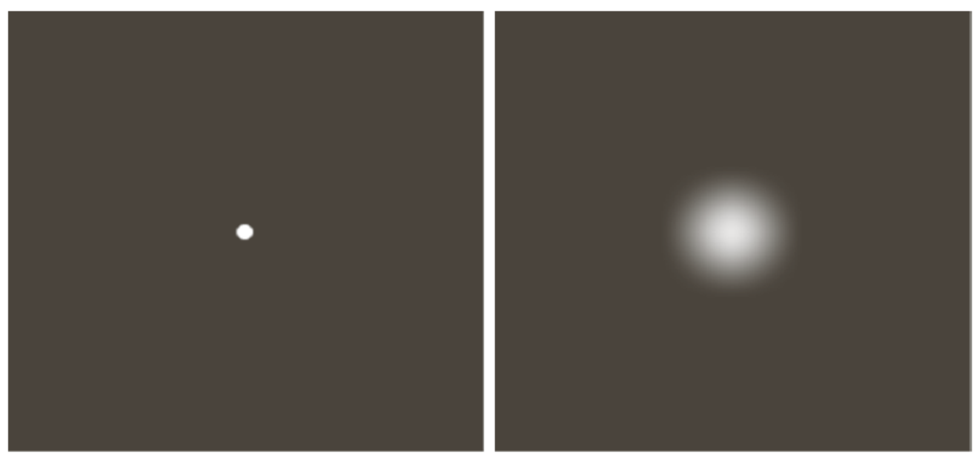


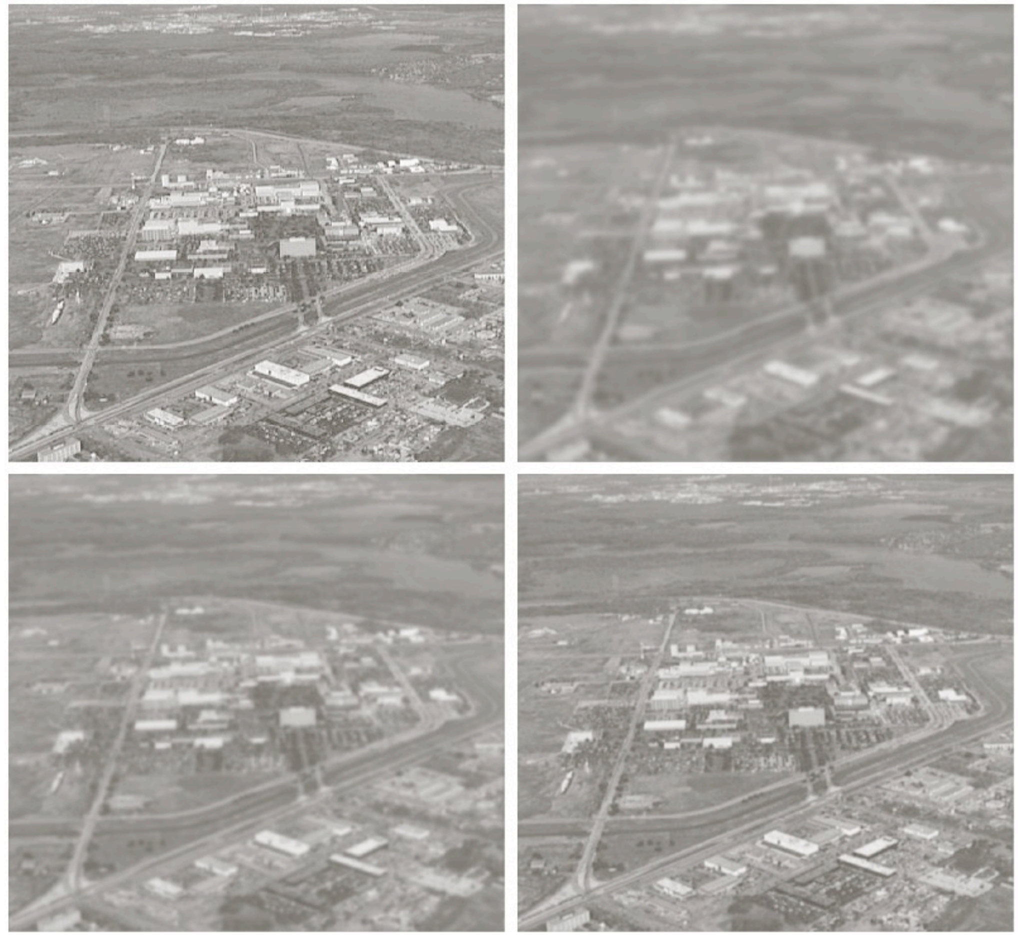
FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

Slides courtesy of Prof. Yan Tong

Estimation by Modeling

a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)



Modeling the atmospheric turbulence

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

Slides courtesy of Prof. Yan Tong

Estimation by Modeling – Motion Blur

Constant velocity along x and y direction:

$$x_0(t) = at/T \qquad y_0(t) = bt/T$$

a b

FIGURE 5.26
(a) Original image.
(b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Estimation by Modeling – Cont.

An example of motion blur

$$g(x,y)=\int_0^T f[x-x_0(t),y-y_0(t)]dt$$

Motion in both x and y direction during acquisition

Estimation by Modeling – Cont.

An example of motion blur

$$G(u,v) = F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

$$\Rightarrow \quad H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

Estimation by Modeling – Example

Constant velocity along x and y direction:

$$x_0(t) = at/T \qquad y_0(t) = bt/T$$

What is H(u,v)?

$$H(u,v) = T\frac{\sin[\pi(ua+vb)]}{\pi(ua+vb)}e^{-j\pi(ua+vb)}$$

Questions?