



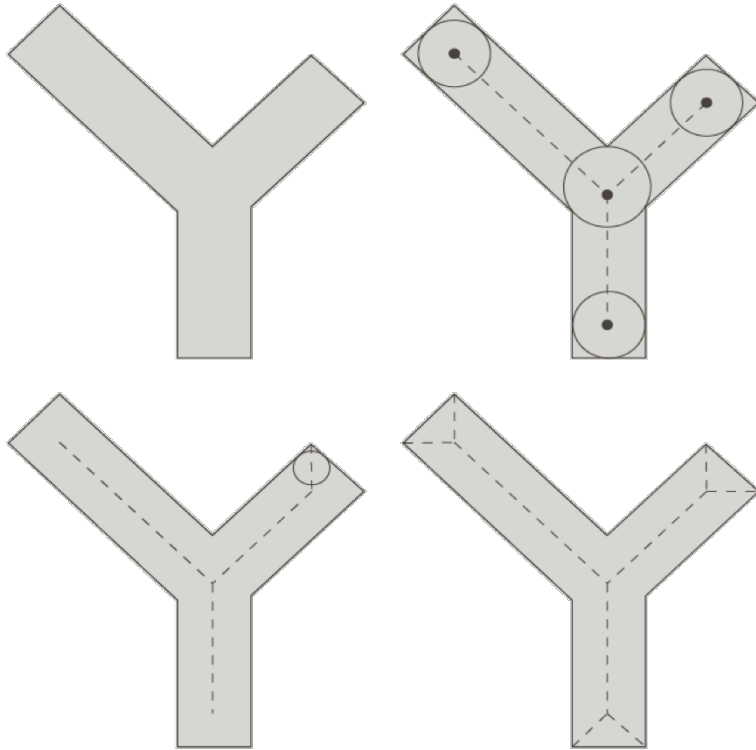
UNIVERSITY OF  
SOUTH CAROLINA

# CSCE 590 INTRODUCTION TO IMAGE PROCESSING

## Segmentation

# Morphological Skeleton

- Skeleton (Defined by centers of maximal disks):

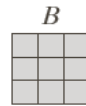


If a point  $\mathbf{Z}$  belongs to the skeleton of  $\mathbf{A}$ , we can find a maximal disk that entirely lies in  $\mathbf{A}$  and touches the boundary of  $\mathbf{A}$  at no less than two positions.

- Applications: an abstract shape representation for high level image understanding, e.g. Optical Character Recognition (OCR)



# Morphological Skeleton



The  $k^{\text{th}}$  erosion

$$S \downarrow k (A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$K = \max\{k \mid (A \ominus kB) \neq \emptyset\}$$

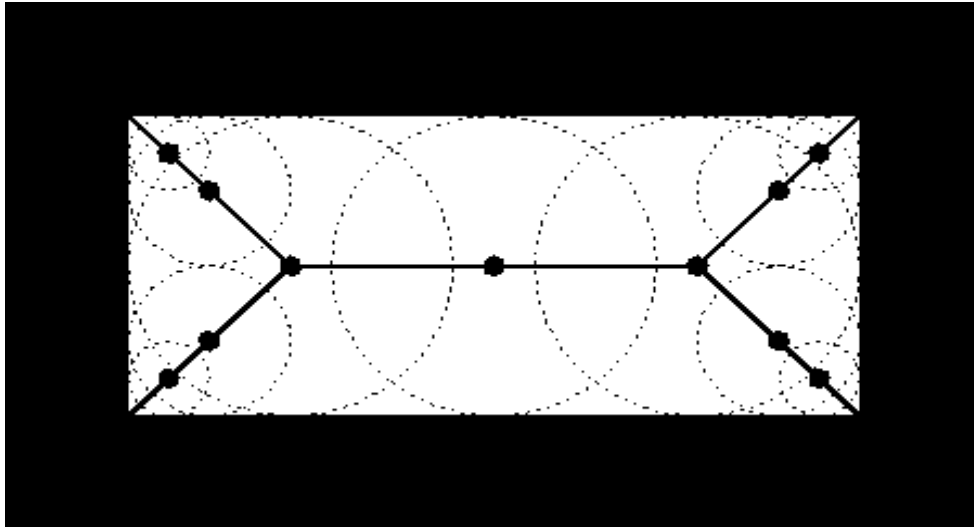
Reconstruct **A** from its skeleton

$$A = \bigcup_{k=0}^K (S \downarrow k (A) \oplus kB)$$

$k$	$A \ominus kB$	$(A \ominus kB) \circ B$	$S_k(A)$	$\bigcup_{k=0}^K S_k(A)$
0				
1				
2				



# Examples



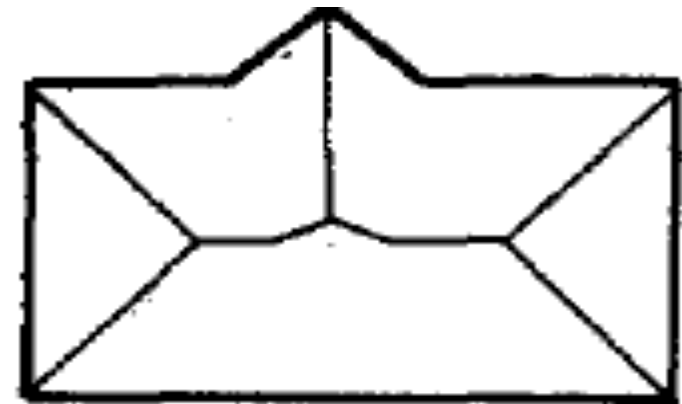
<http://homepages.inf.ed.ac.uk/rbf/HIPR2/skeleton.htm>



Potential issues with skeleton?

Sensitive to noise

Maragos and Schafer, "Morphological Skeleton Representation and Coding of Binary Images", IEEE Trans. on Acoustics, Speech, and Signal Processing, Vol. 34, No. 5, 1986.



# Image Segmentation

A process that partitions  $R$  into subregions  $R_1, R_2, \dots, R_n$



# Image Segmentation – Applications

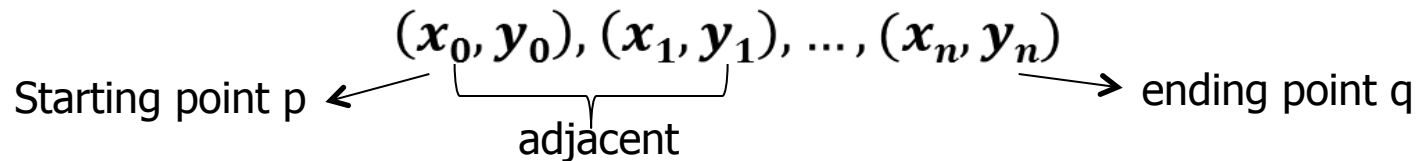
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- Object localization
- Object recognition
- Specially important for medical imaging



# Brief Review of Connectivity

- Path from p to q: a sequence of distinct pixels with coordinates



- p and q are *connected*: if there is a path from p to q in S
- *Connected component*: all the pixels in S connected to p
- *Connected set*: S has only one connected component
- R is a region if R is a connected set
- $R_i$  and  $R_j$  are adjacent if  $R_i \cup R_j$  is a connected set



## Image Segmentation

$$(a) \bigcup_{i=1}^n R_i = R \quad \forall i=1 \wedge n \quad R \setminus i = R$$

(b)  $R_i$  is a connected set,  $i = 1, \dots, n$

$$(c) R_i \cap R_j = \phi, \forall i \neq j$$

$$(d) Q(R_i) = TRUE$$

(e)  $Q(R_i \cup R_j) = FALSE$  for adjacent regions  $R_i$  and  $R_j$

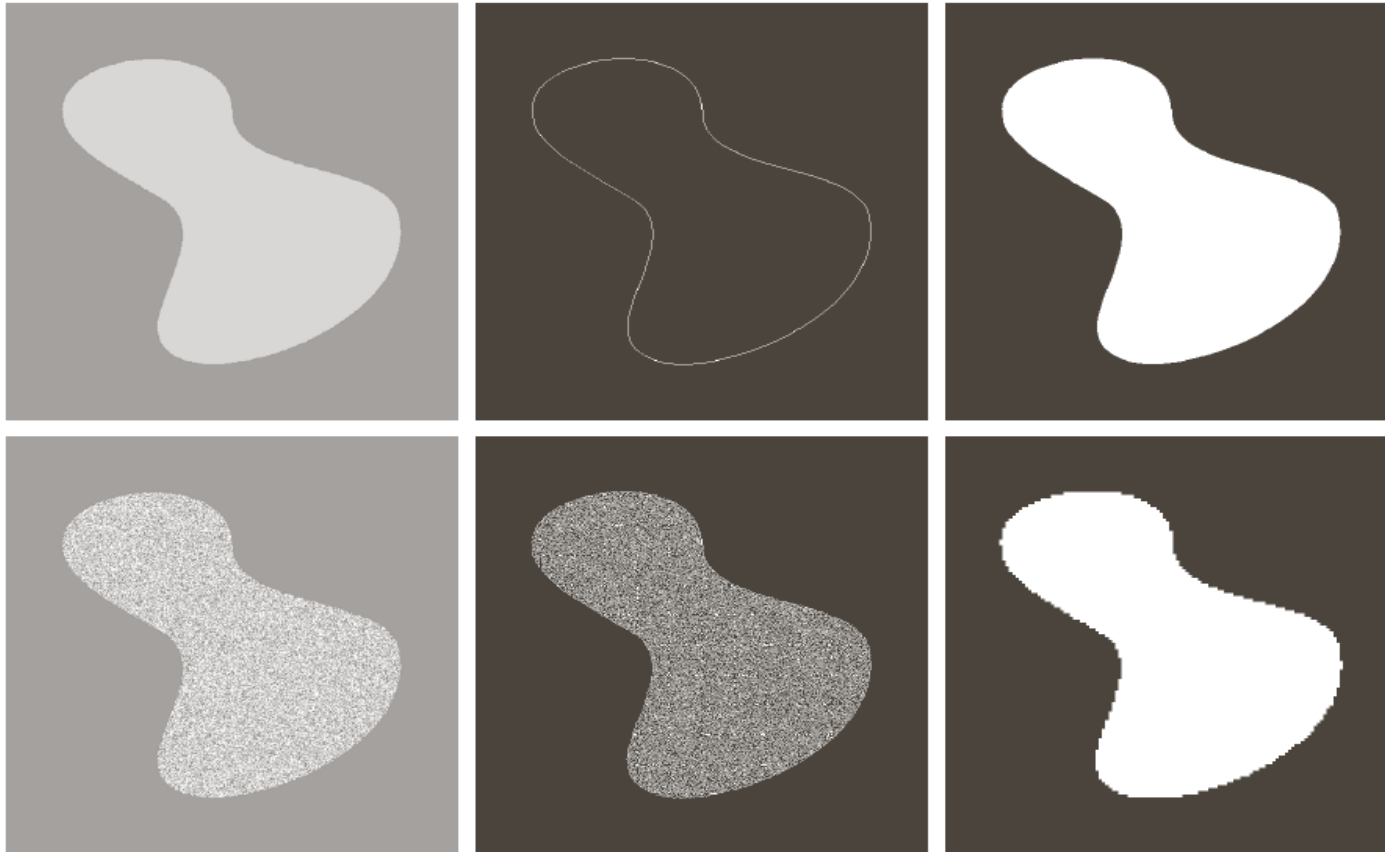
Two categories based on intensity properties:

- Discontinuity – edge-based algorithms
- Similarity – region-based algorithms





# Edge-based and Region-based Segmentation



a b c  
d e f

**FIGURE 10.1** (a) Image containing a region of constant intensity. (b) Image showing the boundary of the inner region, obtained from intensity discontinuities. (c) Result of segmenting the image into two regions. (d) Image containing a textured region. (e) Result of edge computations. Note the large number of small edges that are connected to the original boundary, making it difficult to find a unique boundary using only edge information. (f) Result of segmentation based on region properties.

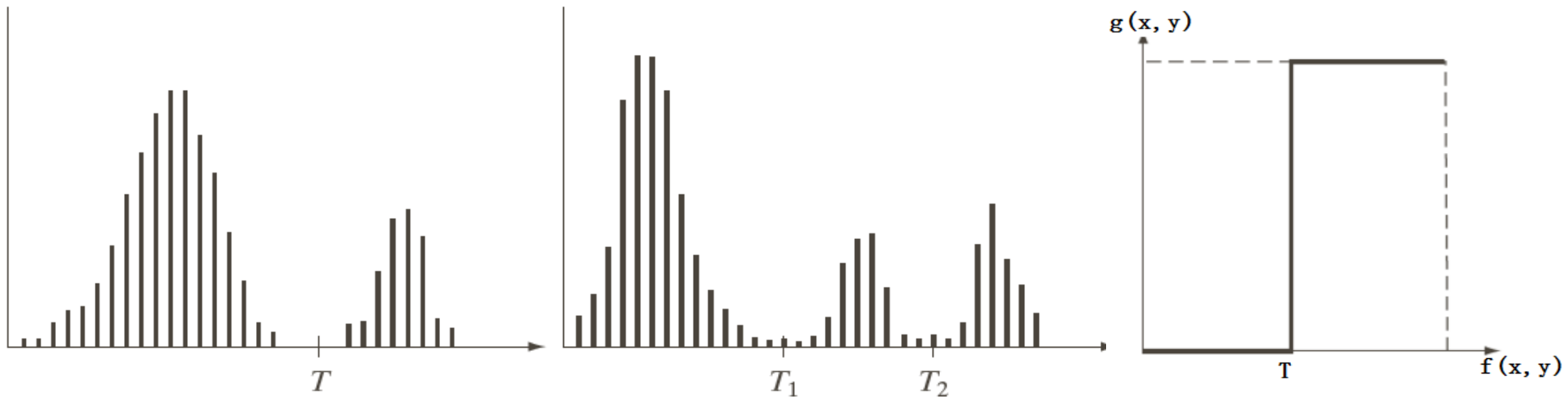
# Edge-based Segmentation

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- Thresholding is required to get edge pixels and realize edge-based segmentation
- Boundary detection (edge linking) is still a hot research topic in image processing and computer vision
- Incorporate domain knowledge:
  - Psychology rules on the boundary: smooth, convex, symmetry, closed, complete, etc
  - Template shape information: hand, stomach, lip, etc
  - Appearance information: region-based texture, intensity/color, etc.



# Edge-based Segmentation -- Intensity Thresholding



**Object/background segmentation:**

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) > T \\ 0 & \text{if } f(x, y) \leq T \end{cases}$$

- A constant  $T$  - global thresholding
- A variable  $T$  - local/regional thresholding; adaptive thresholding
- Multiple  $T$  - multiple thresholding

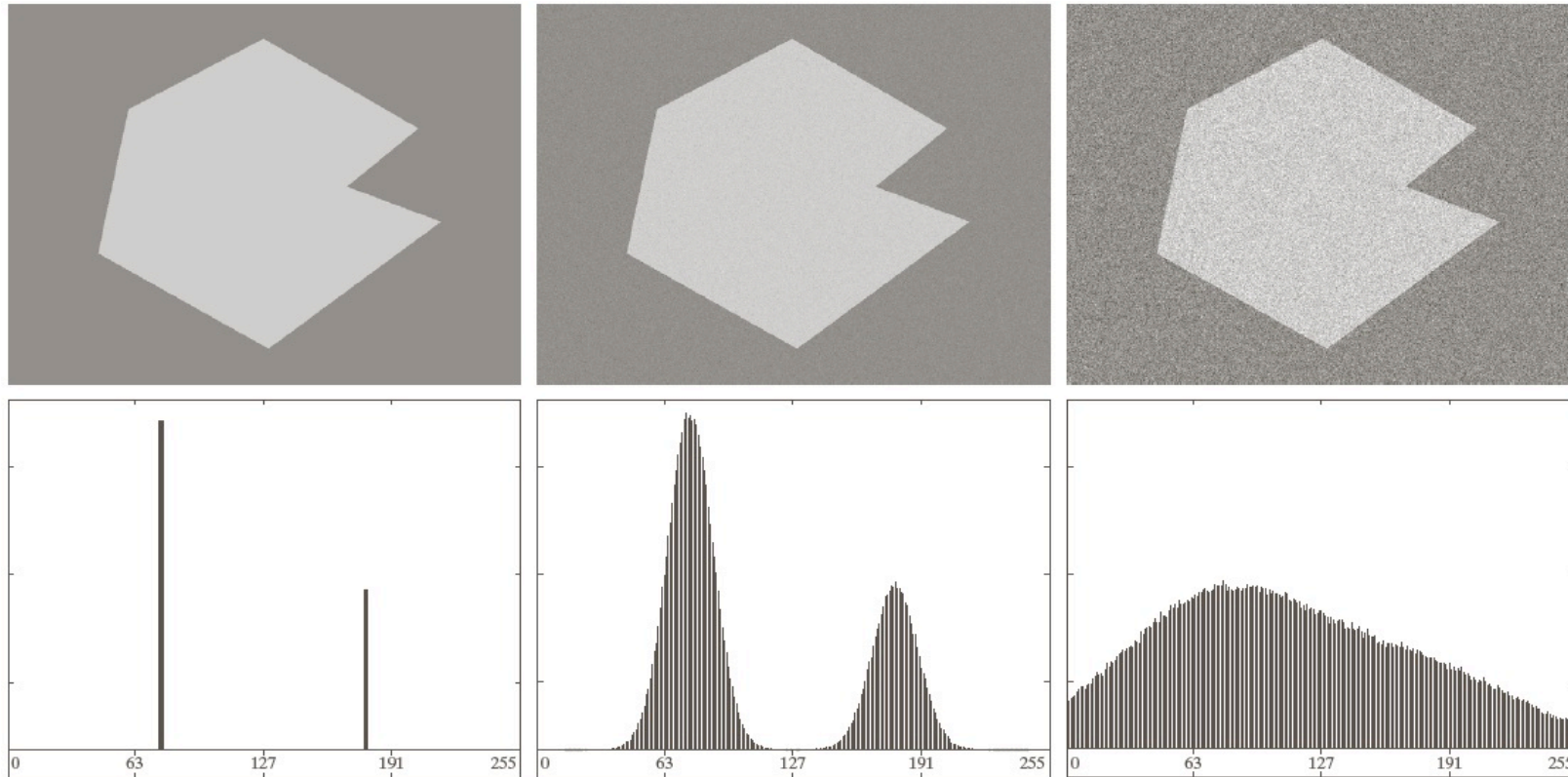


## Key Factors Affect Thresholding

- Separation between peaks
- Noise level
- Relative sizes of objects and background
- Uniformity of the illumination source
- Uniformity of the reflectance of the image



# The Role of Noise in Image Thresholding

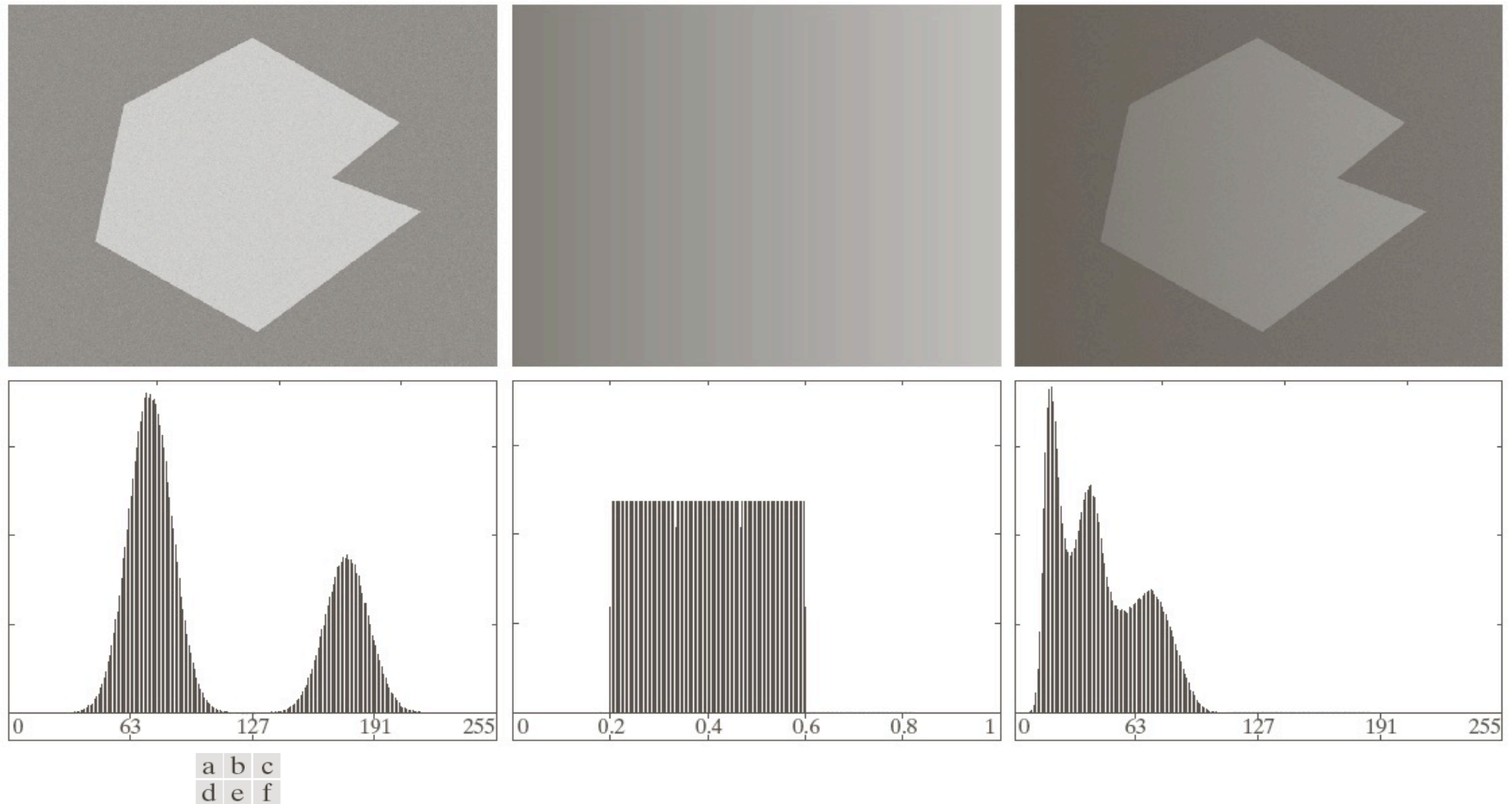


a b c  
d e f

**FIGURE 10.36** (a) Noiseless 8-bit image. (b) Image with additive Gaussian noise of mean 0 and standard deviation of 10 intensity levels. (c) Image with additive Gaussian noise of mean 0 and standard deviation of 50 intensity levels. (d)–(f) Corresponding histograms.



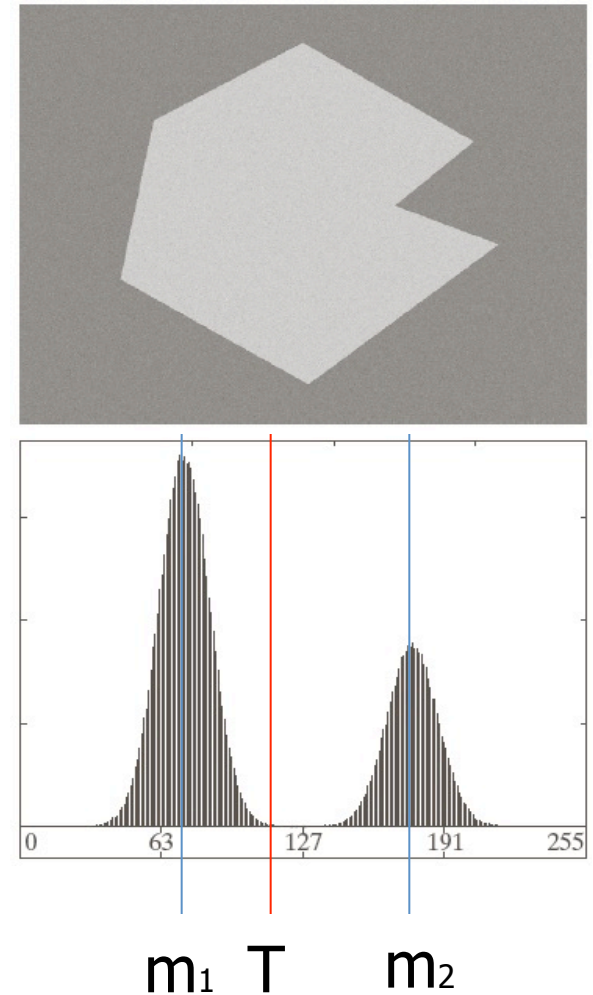
# The Role of Illumination in Thresholding



**FIGURE 10.37** (a) Noisy image. (b) Intensity ramp in the range  $[0.2, 0.6]$ . (c) Product of (a) and (b). (d)–(f) Corresponding histograms.

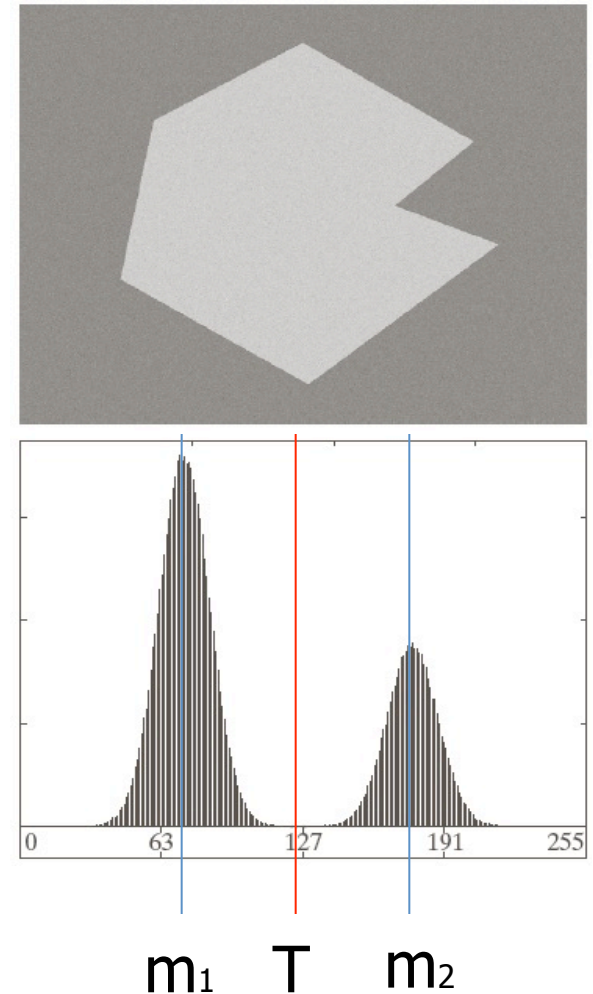
# How to Pick the Threshold

1. Select an initial estimate for the global threshold,  $T$ .
2. Segment the image using  $T$  by producing two groups of pixels
3. Compute the mean of these two groups of pixels, say  $m_1$  and  $m_2$ .
4. Update the threshold  $T=(m_1 +m_2)/2$
5. Repeat Steps 2 through 4 until convergence



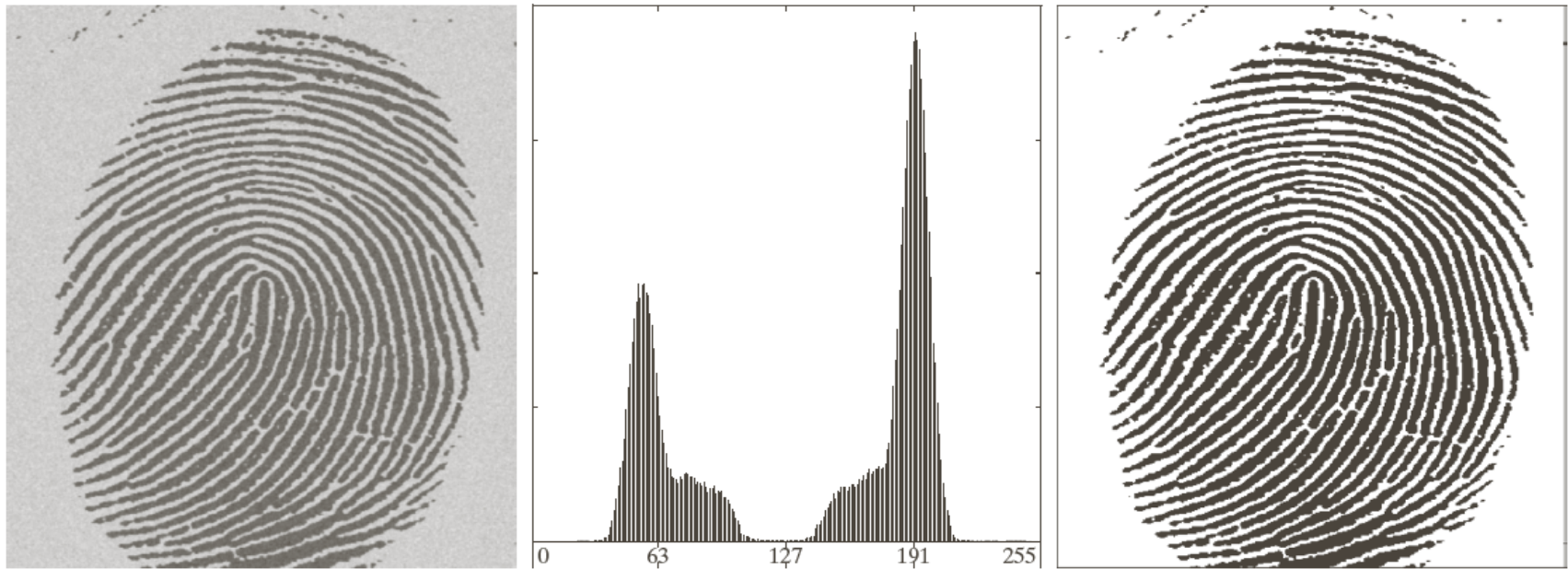
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5. Repeat Steps 2 through 4 until convergence





# An Example



a b c

**FIGURE 10.38** (a) Noisy fingerprint. (b) Histogram. (c) Segmented result using a global threshold (the border was added for clarity). (Original courtesy of the National Institute of Standards and Technology.)



# Image Segmentation

$$(a) \bigcup_{i=1}^n R_i = R$$

(b)  $R_i$  is a connected set,  $i = 1, \dots, n$

$$(c) R_i \cap R_j = \phi, \forall i \neq j$$

$$(d) Q(R_i) = TRUE$$

(e)  $Q(R_i \cup R_j) = FALSE$  for adjacent regions  $R_i$  and  $R_j$

Two categories based on intensity properties:

- Discontinuity – edge-based algorithms
- Similarity – region-based algorithms



# Region-Based Segmentation

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- Region growing
- Region splitting and merging



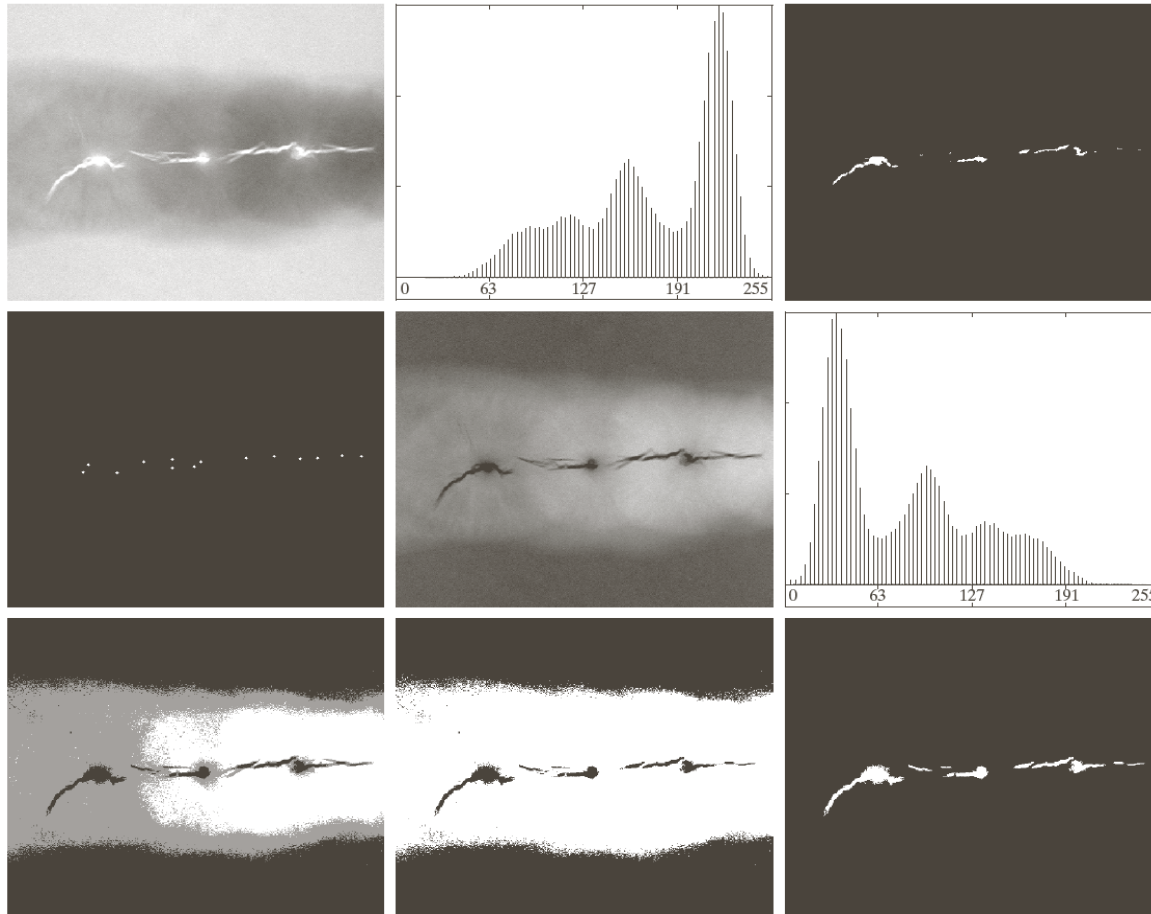
# Region Growing Algorithm

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- A procedure that groups pixels or subregions into larger regions based on predefined criteria for growth
- Start with a set of “seed” points and grow regions by appending neighboring pixels that satisfy the given criteria
  - Connectivity
  - Stopping rules
    - Local criteria: intensity values, textures, color
    - Prior knowledge: size and shape of the object



# An Example



$$Q = \begin{cases} \blacksquare & \text{True if } |I \downarrow \text{seed} - I(x,y)| \leq T \\ \square & \text{False otherwise} \end{cases}$$

**FIGURE 10.51** (a) X-ray image of a defective weld. (b) Histogram. (c) Initial seed image. (d) Final seed image (the points were enlarged for clarity). (e) Absolute value of the difference between (a) and (c). (f) Histogram of (e). (g) Difference image thresholded using dual thresholds. (h) Difference image thresholded with the smallest of the dual thresholds. (i) Segmentation result obtained by region growing. (Original image courtesy of X-TEK Systems, Ltd.)

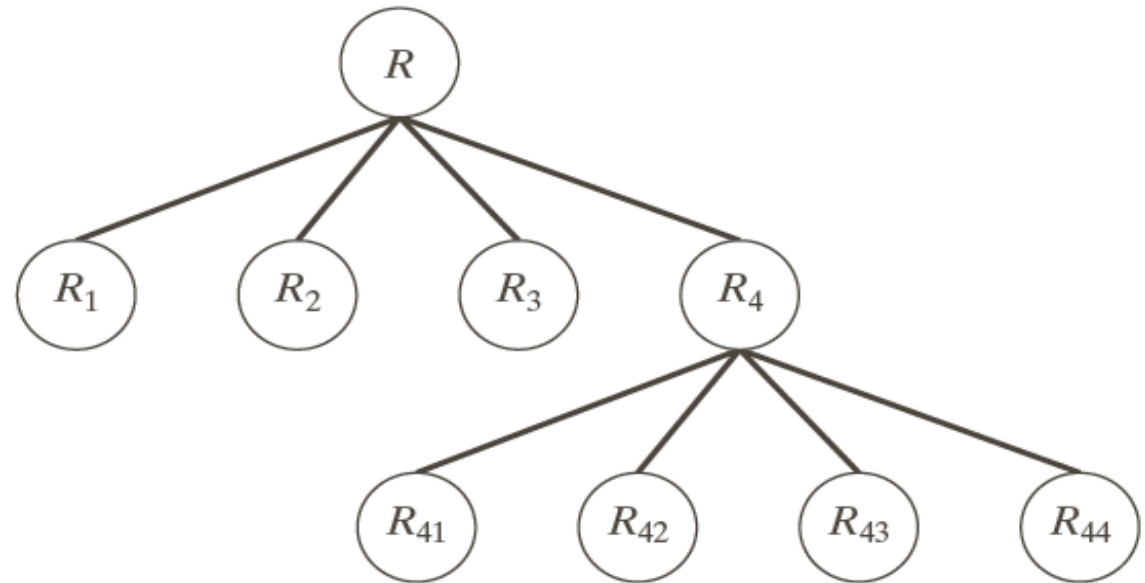
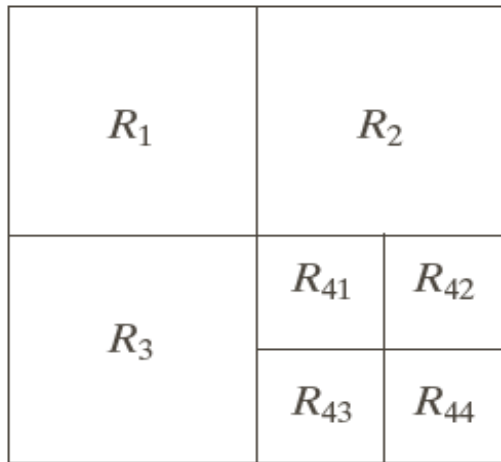


# Region-Splitting and Merging Algorithm

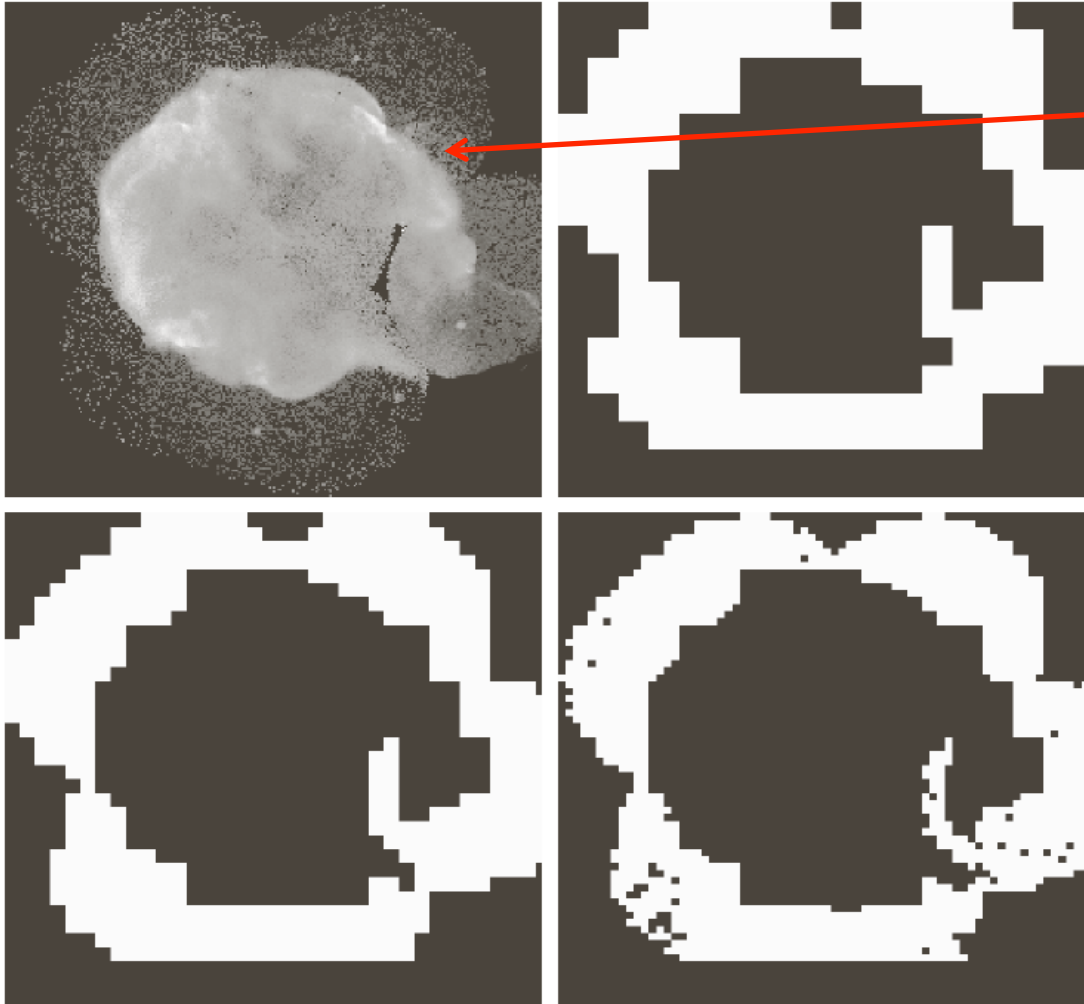
- Step 1: Keep splitting the region while  $Q(R_i) = FALSE$  and  $R_i > \min Size$
- Step 2: Merge the subregions while  $Q(R_i \cup R_j) = TRUE$

a b

**FIGURE 10.52**  
(a) Partitioned image.  
(b) Corresponding quadtree.  $R$  represents the entire image region.



# An Example



a b  
c d

**FIGURE 10.53**  
 (a) Image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of limiting the smallest allowed quadregion to sizes of  $32 \times 32$ ,  $16 \times 16$ , and  $8 \times 8$  pixels, respectively. (Original image courtesy of NASA.)

Extract the outer ring

$$Q = \begin{cases} TRUE & \text{if } \sigma > a \ \& \ 0 < m < b \\ FALSE & \text{otherwise} \end{cases}$$

# Advanced Approaches for Image Segmentation

- General-purpose image segmentation is far from well solved
- It is still a research problem that is being investigated by many researchers
- Image segmentation by K-means clustering
- Image segmentation with Graphic Models (MRF, CRF, etc)
- Semantic Segmentation with Deep Learning
- <http://blog.qlure.ai/notes/semantic-segmentation-deep-learning-review>





# Morphological Image Processing – Techniques to Improve Image Segmentation

- Objective: Extract image components for representation and description of region shape including
  - Boundaries
  - Skeletons
  - Convex hull
- Applications:
  - Edge detection
  - Blob/connected component detection



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

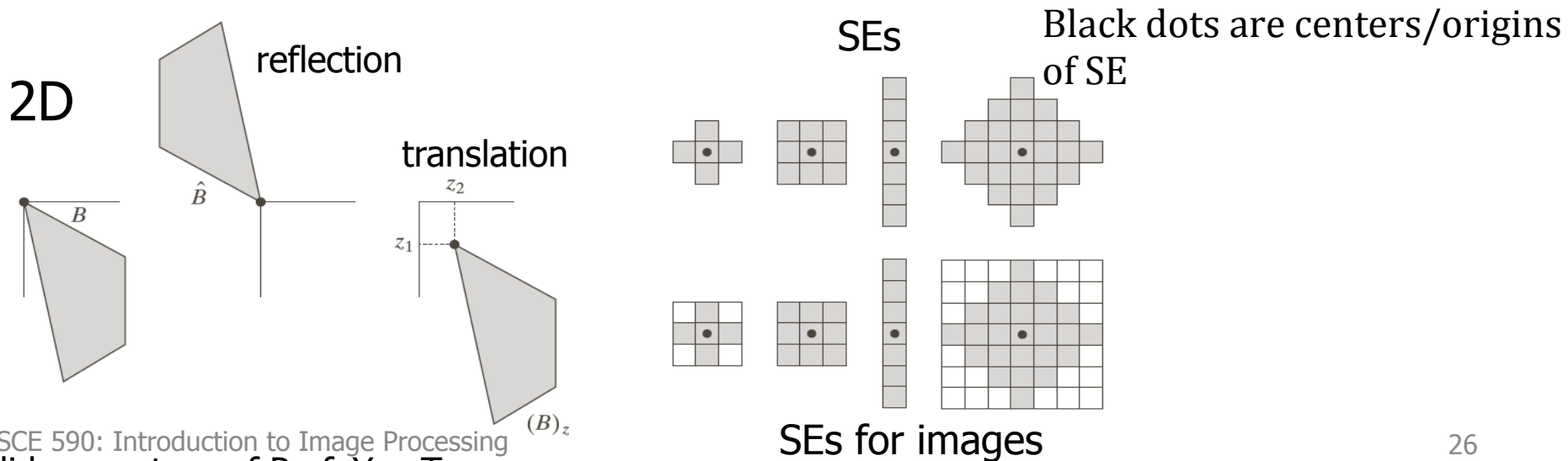


Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



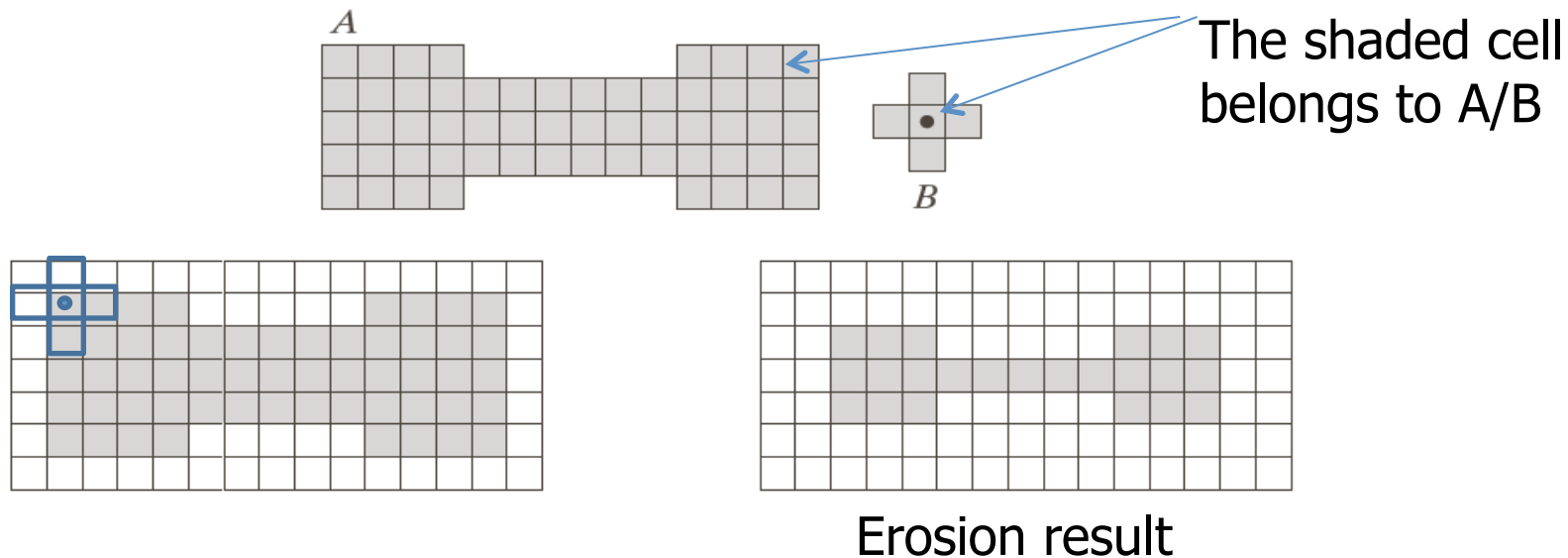
# Basic Concepts

- Union, intersection, complement, difference
- Set reflection
- Set translation  $\hat{B} = \{w \mid w = -b, b \in B\}$  -- move the center/  
origin of  $B$  by  $z$  pixels  
 $(B)_z = \{c \mid c = b + z, b \in B\}$
- Structure elements (SEs): small sets/subimages used in morphology



# An Example of Morphology

- Create a new set by running B over A so that the origin of B visits every element of A.
- An example of erosion: If B is completely contained in A for each operation, the new element is a member of the new set.



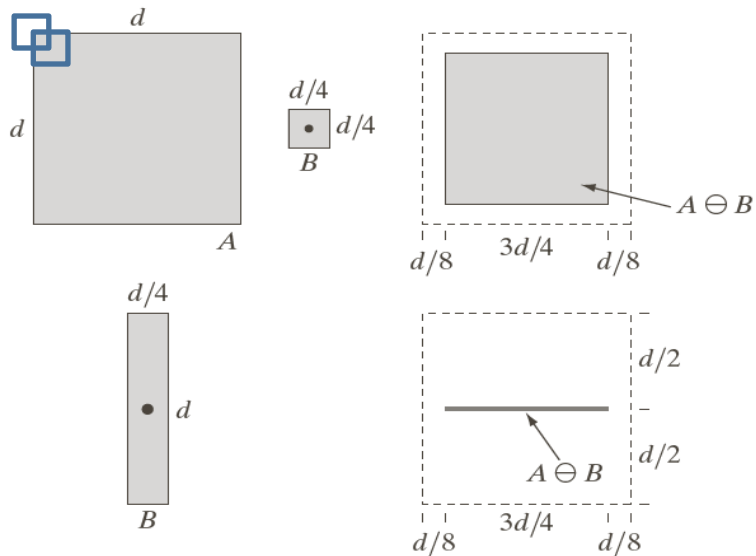
# Common Morphological Operations

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- Two basic operations
- Erosion
- Dilation
- Other operations
- Opening/closing
- Hit-or-Miss transform
- Thinning/thickening
- Hole filling

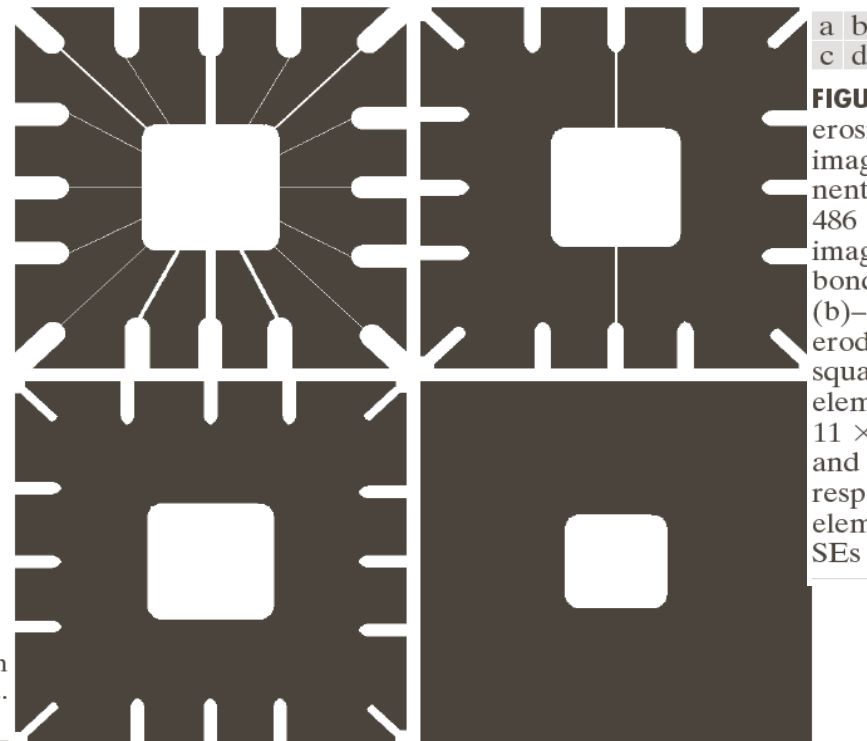


# Erosion



a	b	c
d	e	

**FIGURE 9.4** (a) Set  $A$ . (b) Square structuring element,  $B$ . (c) Erosion of  $A$  by  $B$ , shown shaded. (d) Elongated structuring element. (e) Erosion of  $A$  by  $B$  using this element. The dotted border in (c) and (e) is the boundary of set  $A$ , shown only for reference.



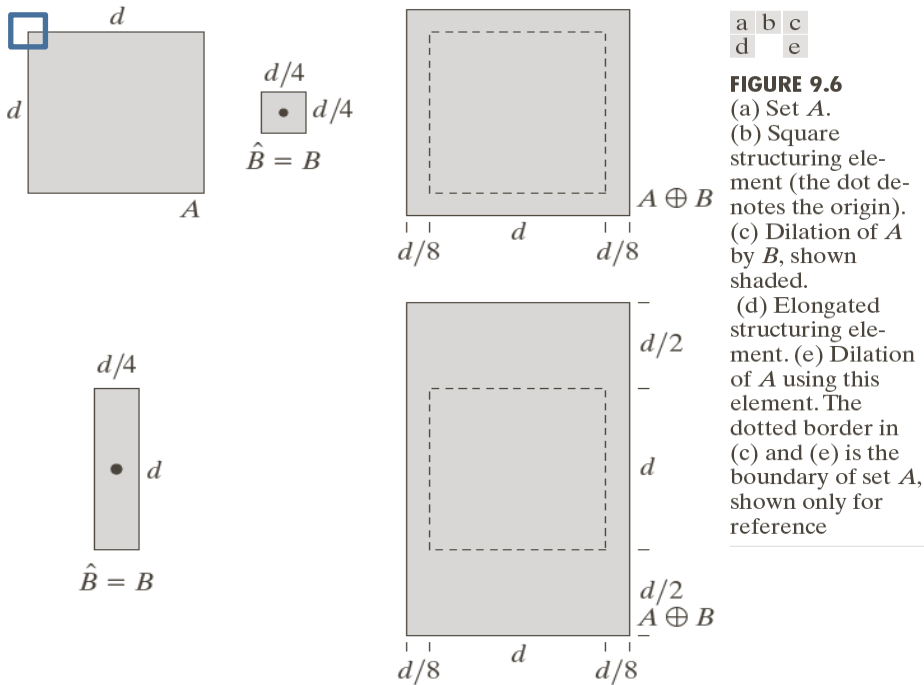
**FIGURE 9.5** Using erosion to remove image components. (a)  $A$   $486 \times 486$  binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes  $11 \times 11$ ,  $15 \times 15$ , and  $45 \times 45$ , respectively. The elements of the SEs were all 1s.

$$A \ominus B = \{z \mid (B)_z \subseteq A\} \quad \text{or} \quad A \ominus B = \{z \mid (B)_z \cap A^c = \emptyset\}$$

Shrink or thin objects and remove the details smaller than the SE



# Dilation



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



0	1	0
1	1	1
0	1	0

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$$

Grows or thickens objects and remove the gaps smaller than the SE



# Properties of Erosion and Dilation

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- Dilation is commutative

$$A \oplus B = B \oplus A$$

- Dilation is associative

$$A \oplus B \oplus C = A \oplus (B \oplus C)$$

- Dilation is distributive over the union operation

$$A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$$



# Properties of Erosion and Dilation

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- Erosion
- Erosion and dilation are duals of each other

$$A \oplus B = (A \uparrow c \ominus B) \uparrow c$$

- $A \subseteq (C \ominus B)$  if and only if  $(A \oplus B) \subseteq C$
- If  $A \subseteq C$ ,  $A \oplus B \subseteq C \oplus B$  and  $A \ominus B \subseteq C \ominus B$





# Opening

- Smooth the contour of an object
- Break narrow bridges
- Eliminate thin protrusions

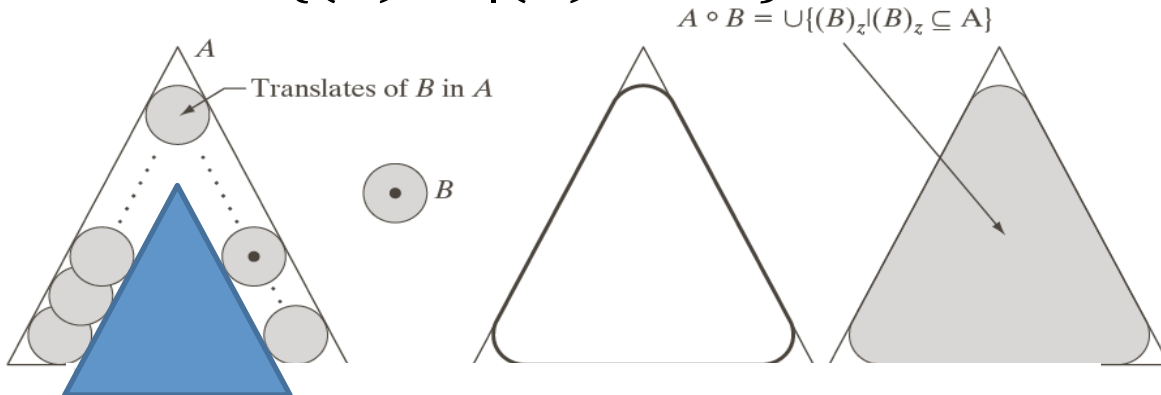
$$A \circ B = (A \ominus B) \oplus B$$

$$(A \circ B) \circ B = A \circ B$$

$$(A \circ B) \subseteq A$$

$$\exists \bar{C} \subseteq C \quad \forall \circ B \bar{C} \subseteq C \circ B$$

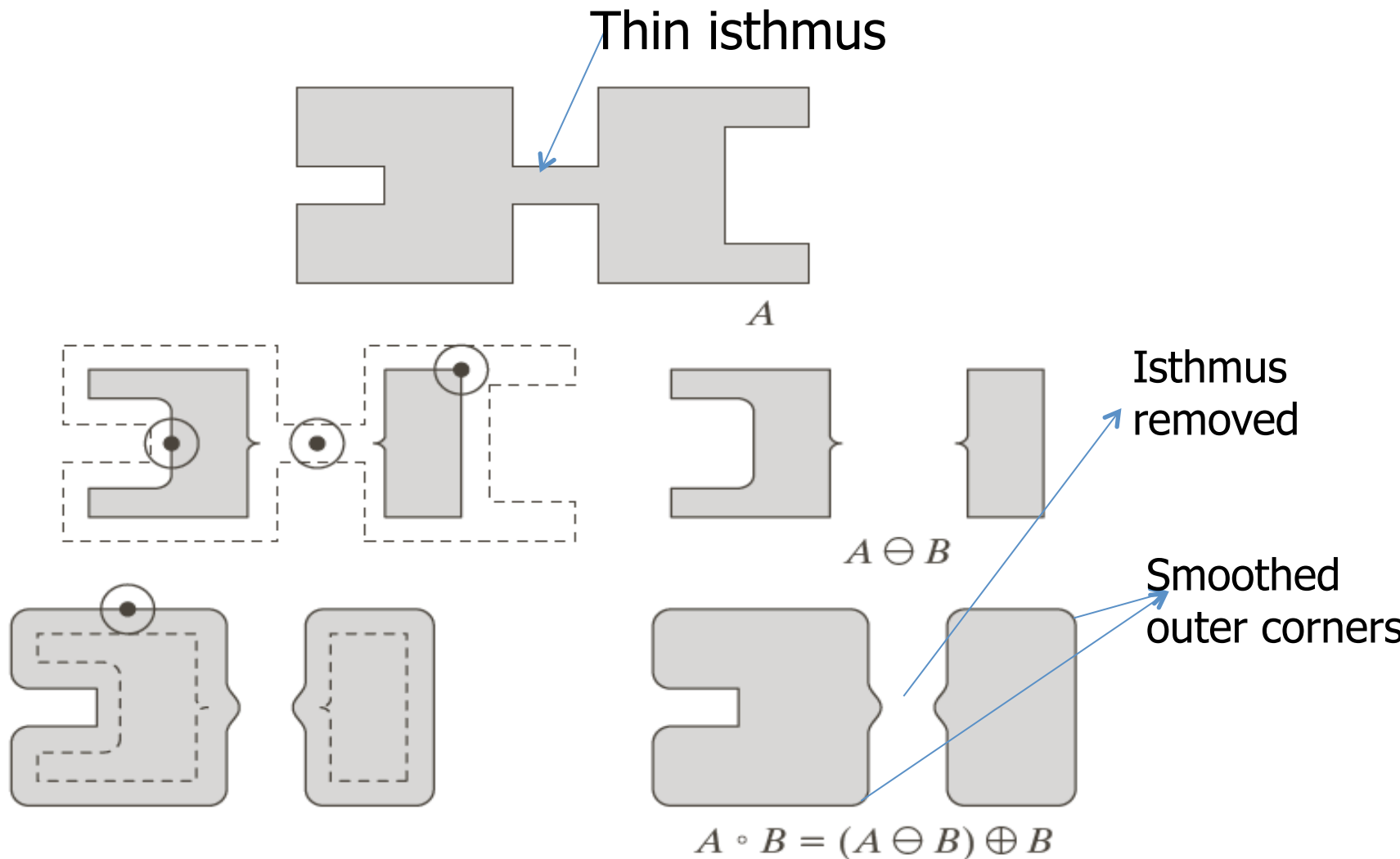
$$A \circ B = \cup \{ (B) \downarrow z \mid (B) \downarrow z \subseteq A \}$$



The SE rolls within the boundary of A.

**FIGURE 9.8** (a) Structuring element  $B$  “rolling” along the inner boundary of  $A$  (the dot indicates the origin of  $B$ ). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade  $A$  in (a) for clarity.

# Opening (Cont'd)



# Closing

- Smooth the contour of an object
- Fill narrow breaks and gaps
- Eliminate long and thin gulfs
- Eliminate small holes

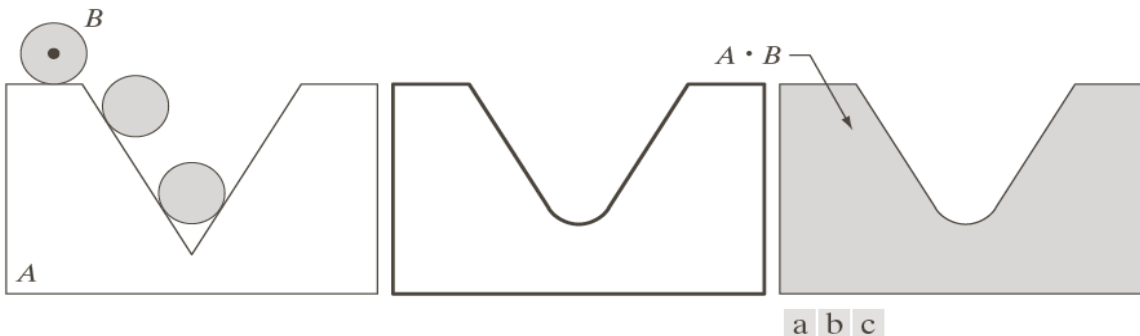
$$A \bullet B = (A \oplus B) \ominus B$$

$$(A \bullet B) \bullet B = A \bullet B$$

$$A \subseteq (A \bullet B)$$

$$\forall C \subseteq C \bullet B \quad \forall C \bullet B \subseteq C$$

Opening and closing are duals of each other  $A \bullet B = (A^c \circ \hat{B})^c$

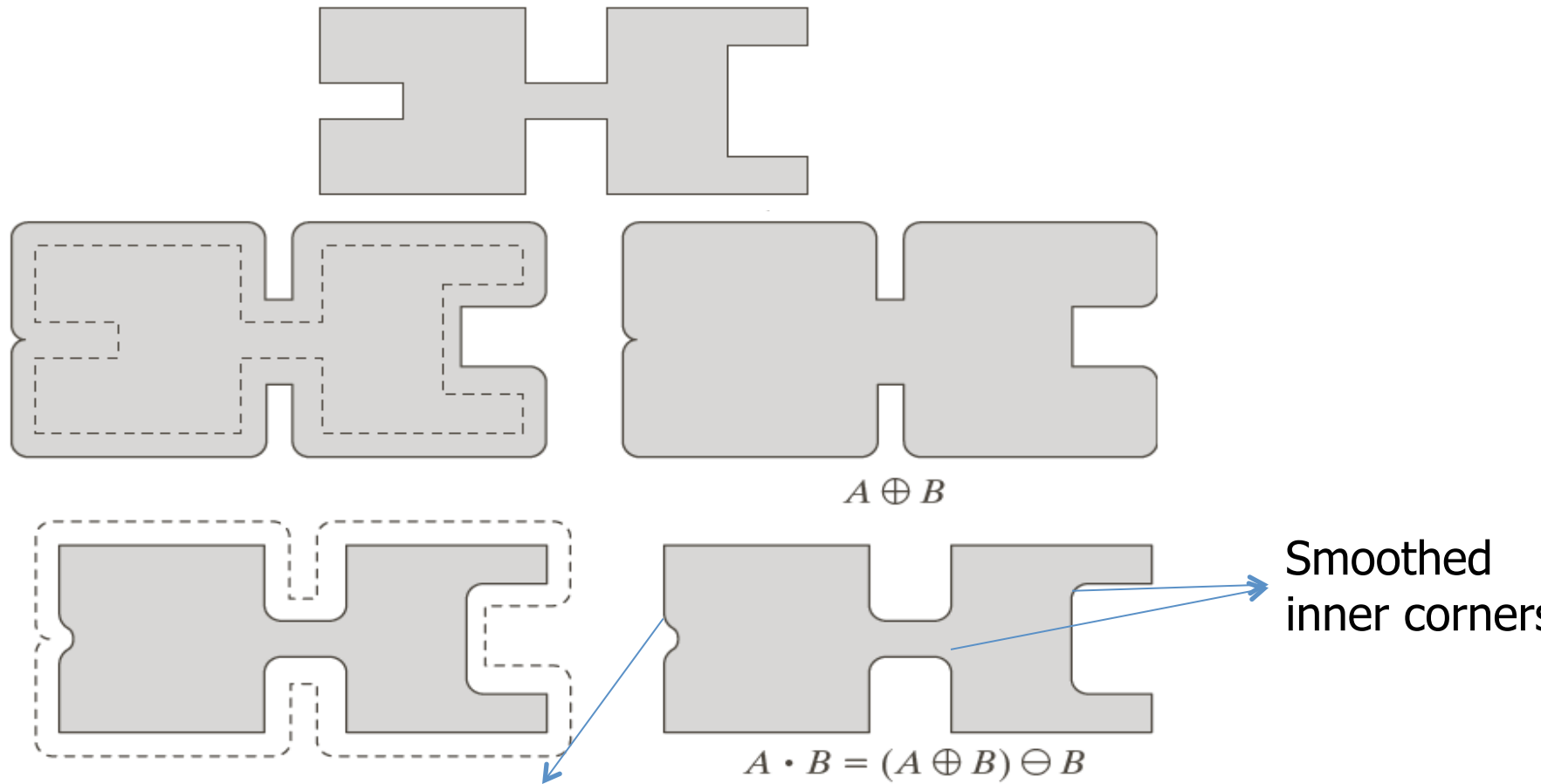


The SE rolls outside the boundary of  $A$ .

**FIGURE 9.9** (a) Structuring element  $B$  “rolling” on the outer boundary of set  $A$ . (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade  $A$  in (a) for clarity.



# Closing (Cont'd)

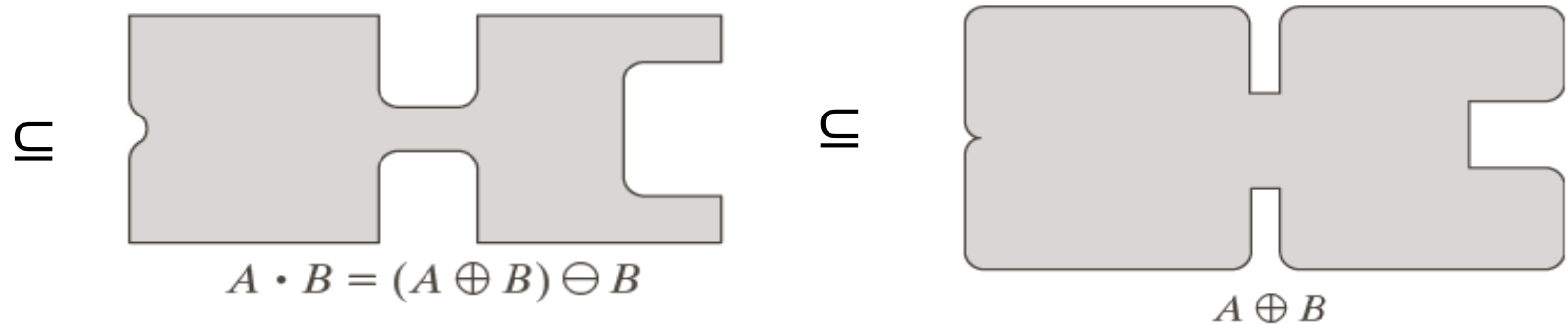
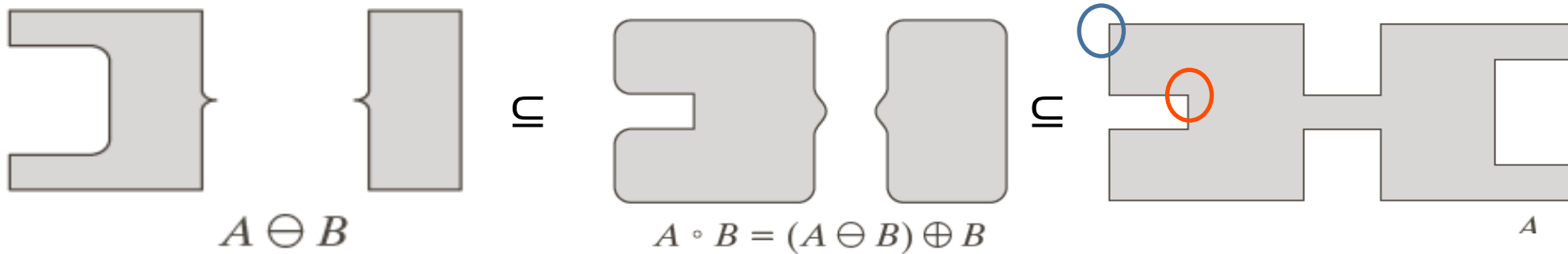


Eliminated thin gulf

Smoothed inner corners



# Opening & Closing



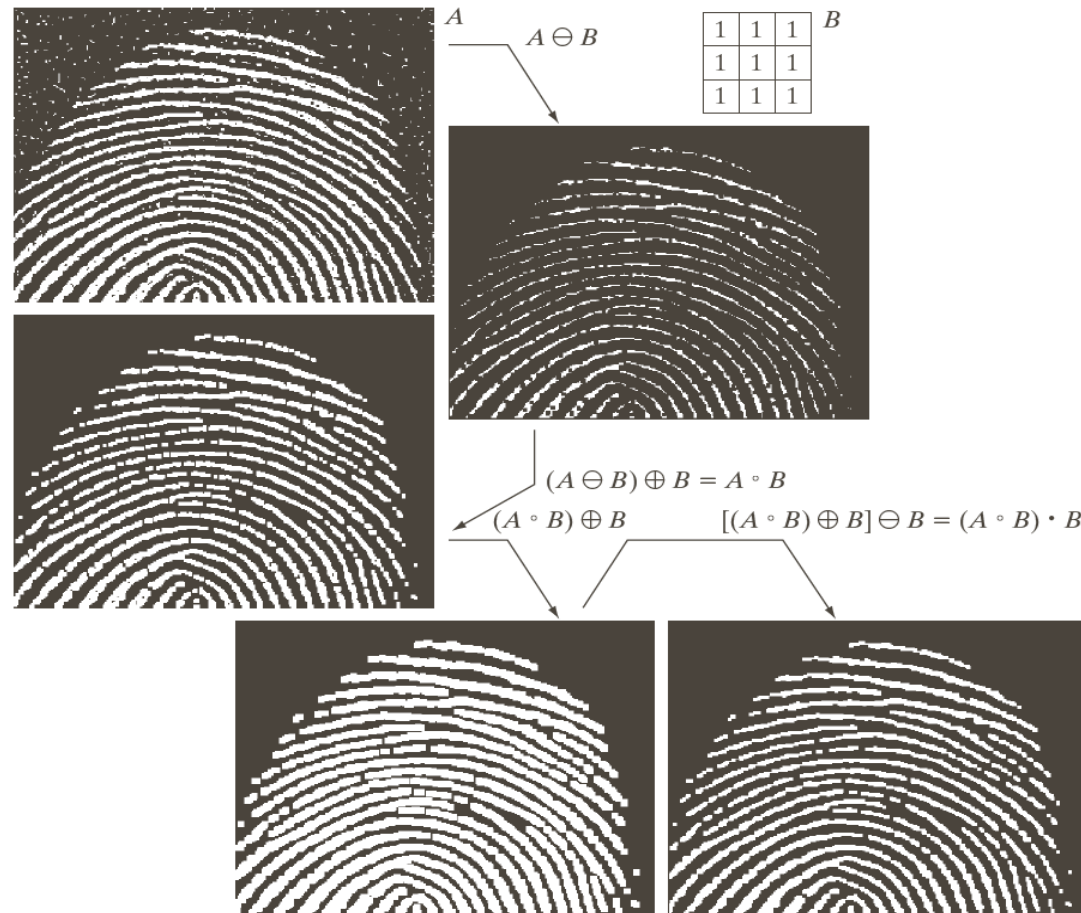
$$A \ominus B \subseteq A \circ B \subseteq A \subseteq A \oplus B$$

$$A \bullet B \subseteq A \oplus B$$



# An Example of Opening & Closing

- An opening removes all noise
  - removing the white noise by erosion
  - removing the black noise by dilation
- An additional closing fills the gaps



**FIGURE 9.11**  
 (a) Noisy image.  
 (b) Structuring element.  
 (c) Eroded image.  
 (d) Opening of  $A$ .  
 (e) Dilation of the opening.  
 (f) Closing of the opening.  
 (Original image courtesy of the National Institute of Standards and Technology.)



# Questions?

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