UNIVERSITY OF SOUTH CAROLINA

CSCE 574 ROBOTICS

Coordinate Frames

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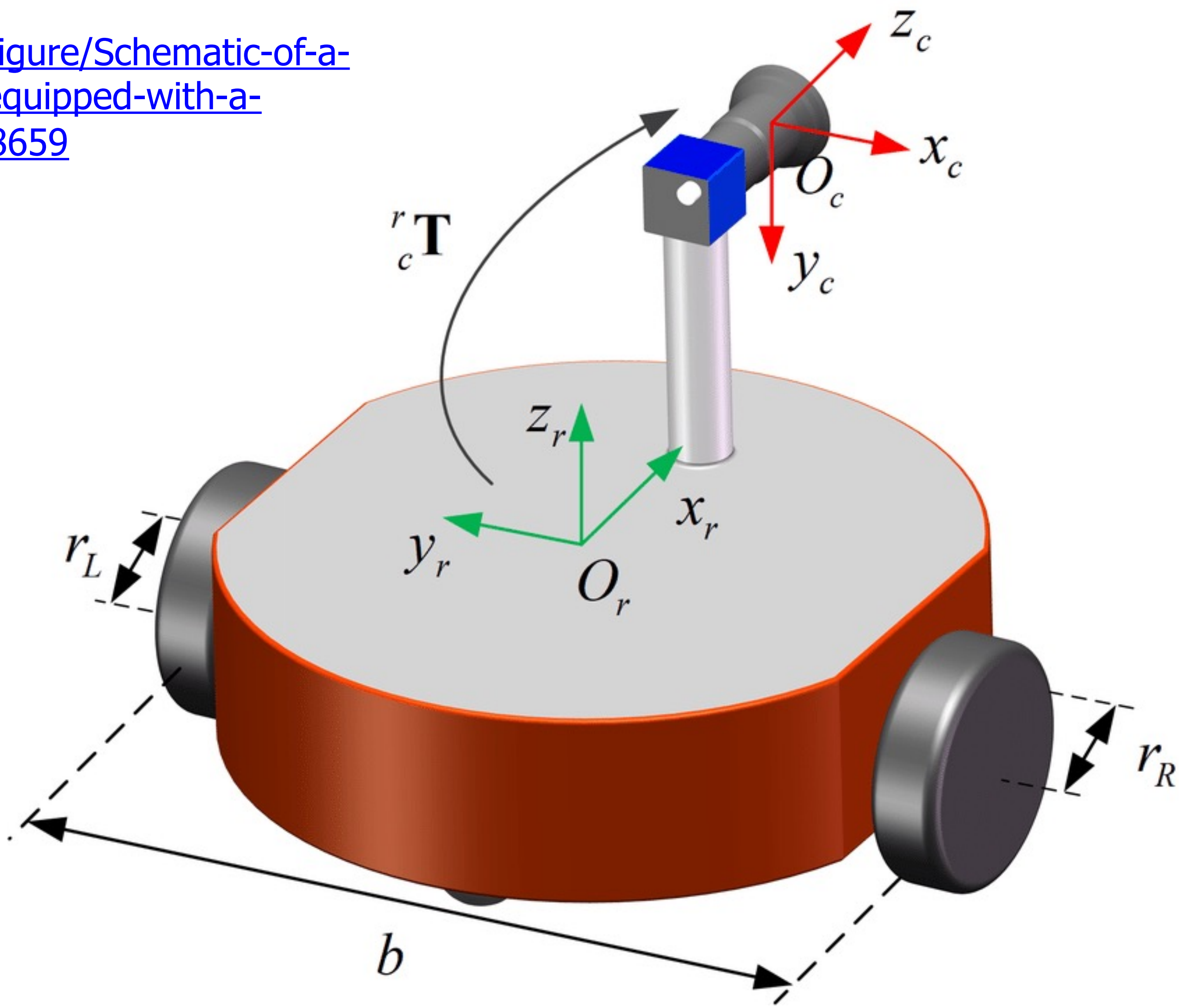
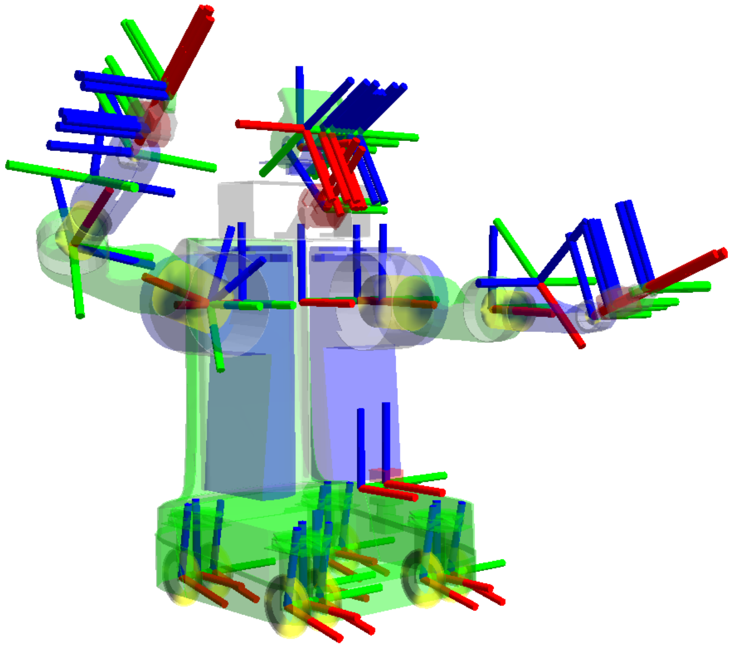
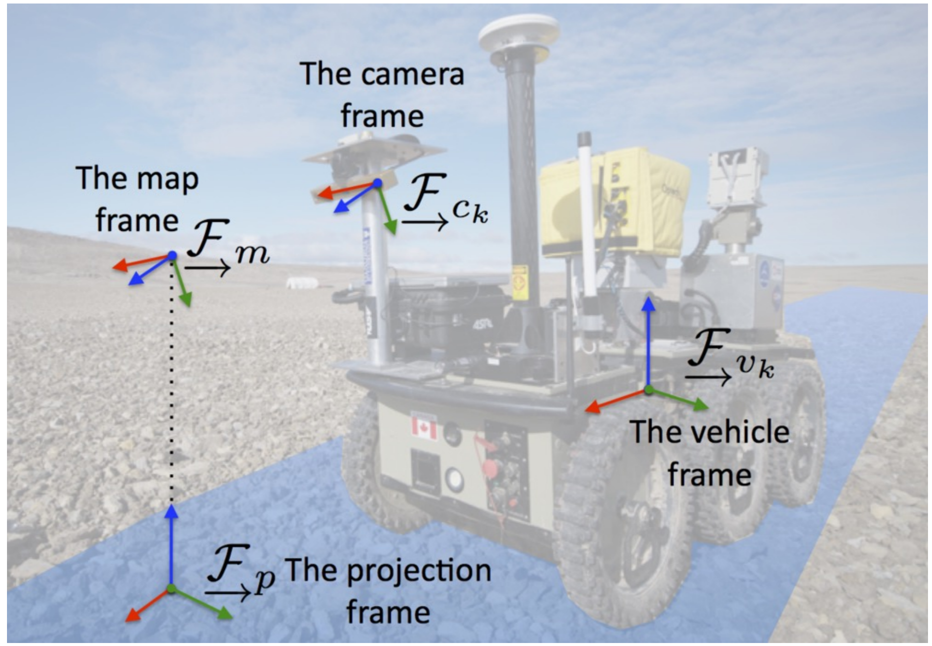
Coordinate Frames

From:
https://www.researchgate.net/figure/Schematic-of-a-differential-drive-mobile-robot-equipped-with-a-monocular-camera_fig1_327658659



3D reference frames

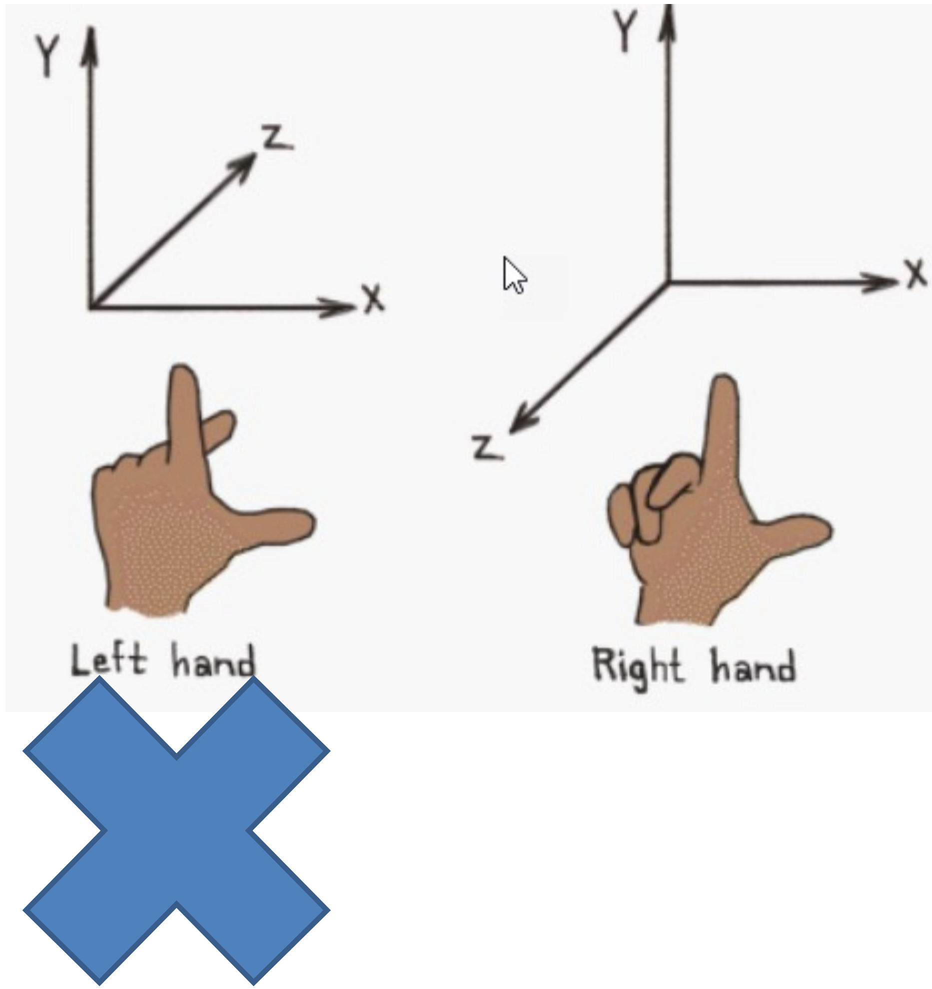




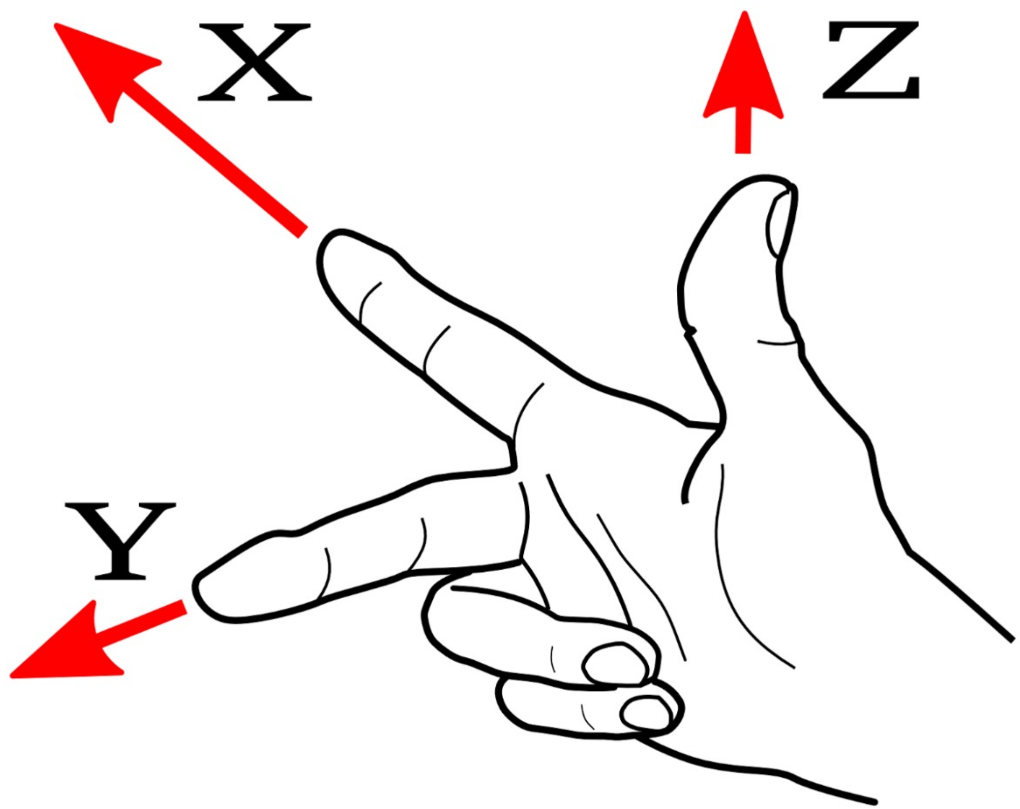
Why do we need many frames?

- Defining and representing frames of reference and reasoning about how to express quantities in one frame to quantities in the other.

Left vs right-handed

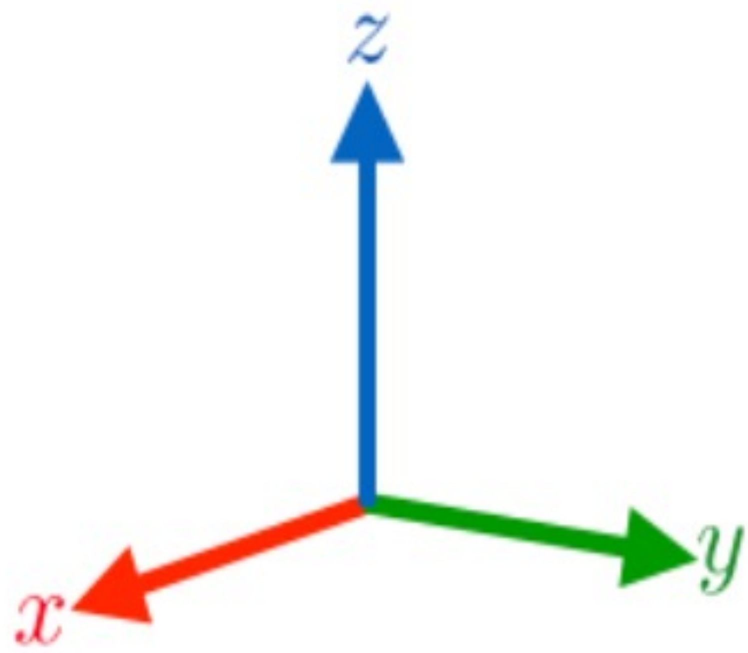


A convention for right-handed

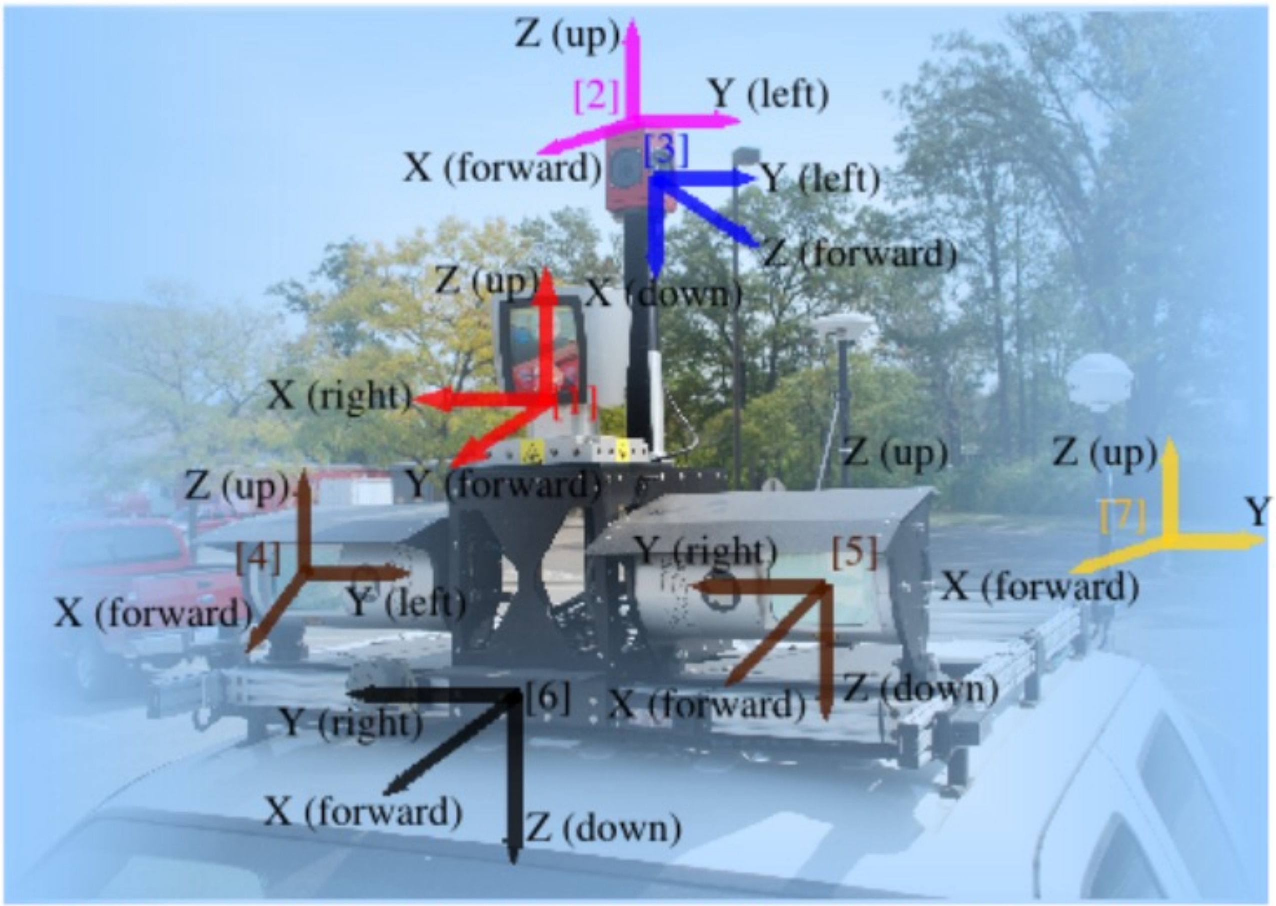


Colored convention

Color convention
for frames

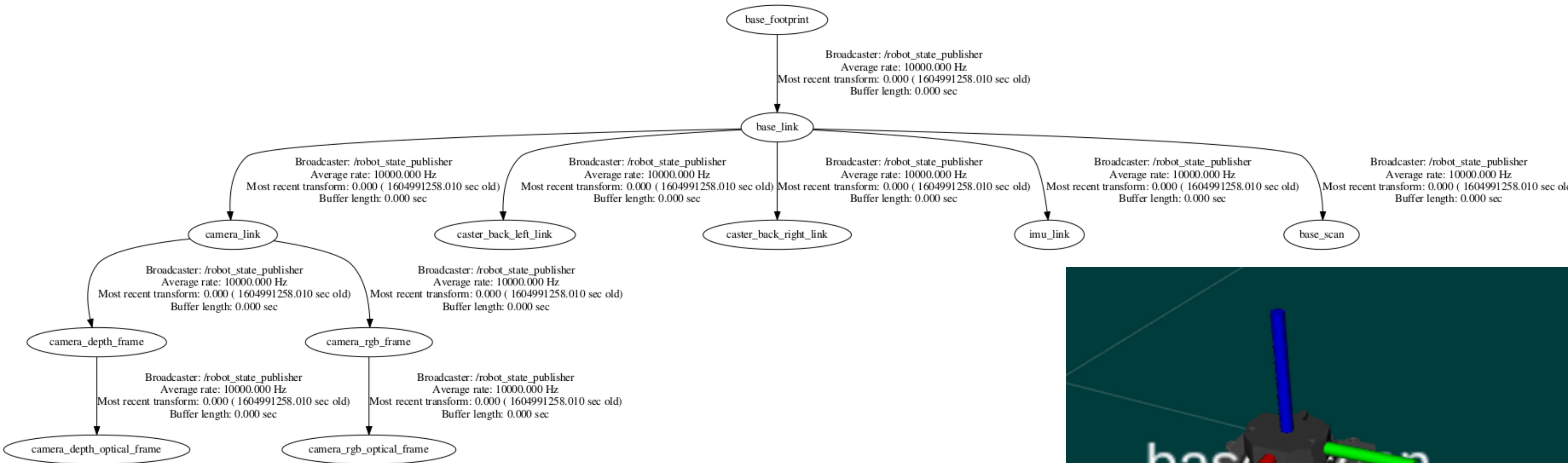


Always provide a frame diagram



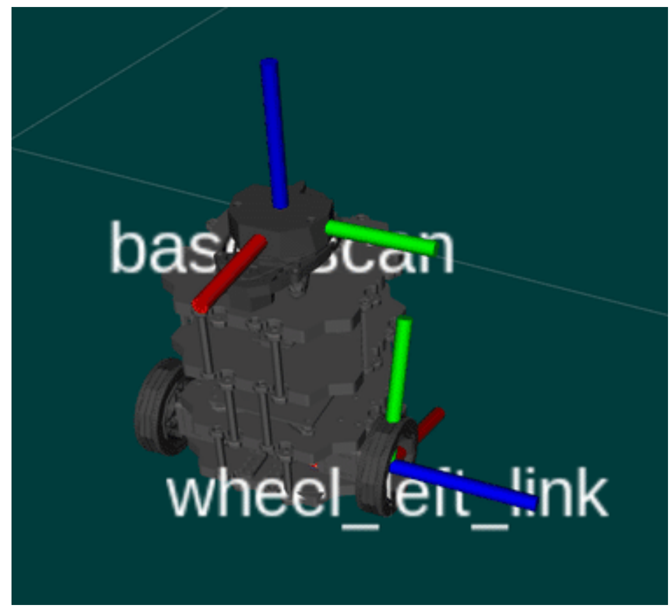
[1] Velodyne, [2] Ladybug3 (actual location: center of camera system),
[3] Ladybug3 Camera 5, [4] Right Riegl, [5] Left Riegl,
[6] Body Frame (actual location: center of rear axle)
[7] Local Frame (Angle between the X-axis and East is known)

Convention for mobile robot

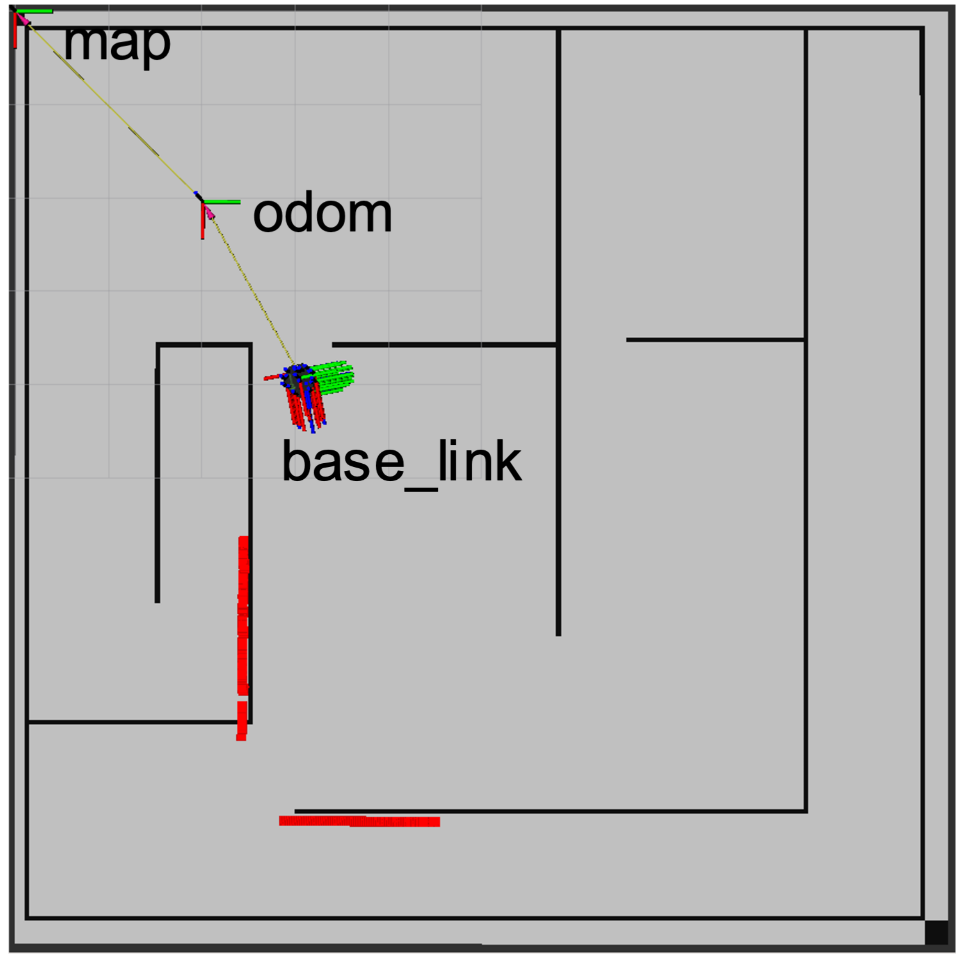


https://www.ros.org/reps/rep-0105.html

https://www.ros.org/reps/rep-0103.html



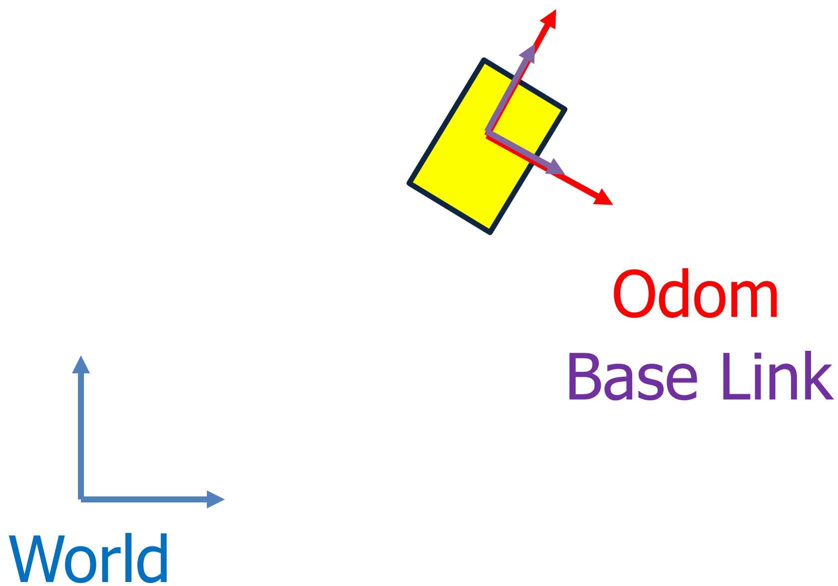
Convention for mobile robot



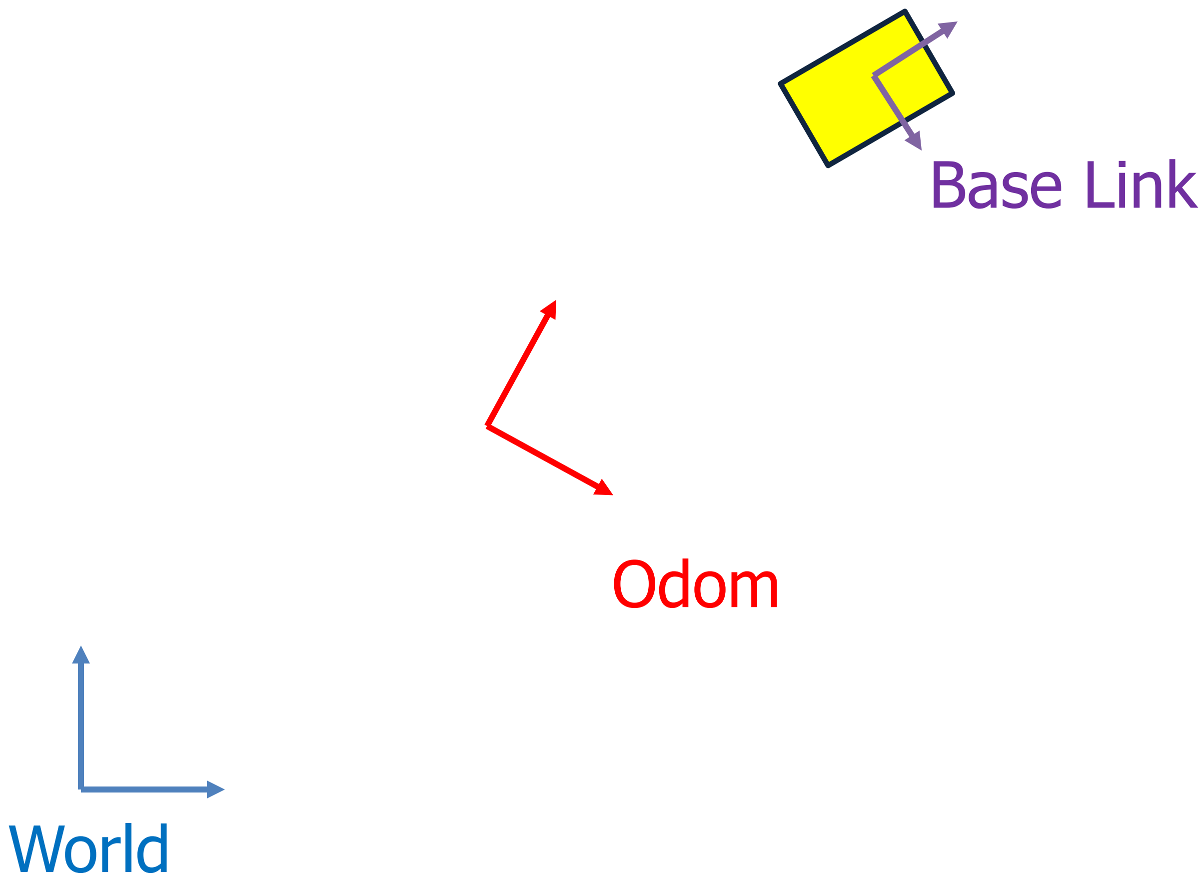
https://www.ros.org/reps/rep-0105.html

https://www.ros.org/reps/rep-0103.html

Example



Example



Rigid body motion

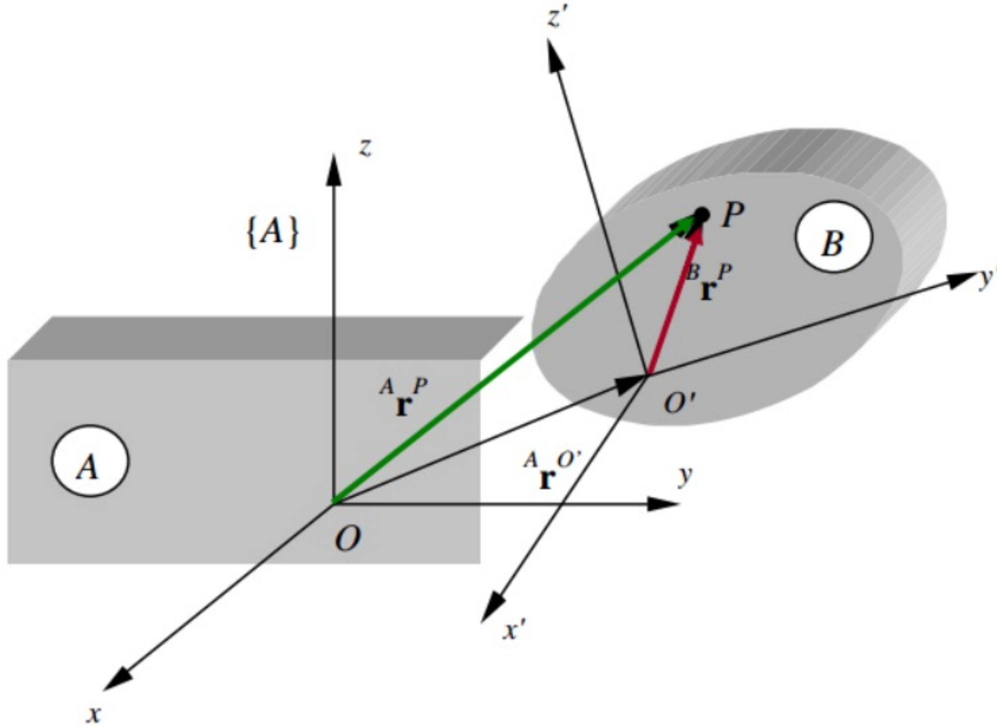
- Motion that can be described by a rotation and translation

- All the parts making up the body move in unison, and there are no deformations

- Representing rotations, translations, and vectors in a given frame of reference is often a source of frustration and bugs in robot software because there are so many options

Rigid body transformations in $\mathbb{R}^3$

- The most general coordinate transformation from B to A has the following form



$$ {}^{A}\mathbf{r}^{P} = {}^{A}\mathbf{R}_{B}\, {}^{B}\mathbf{r}^{P} + {}^{A}\mathbf{r}^{O'} $$

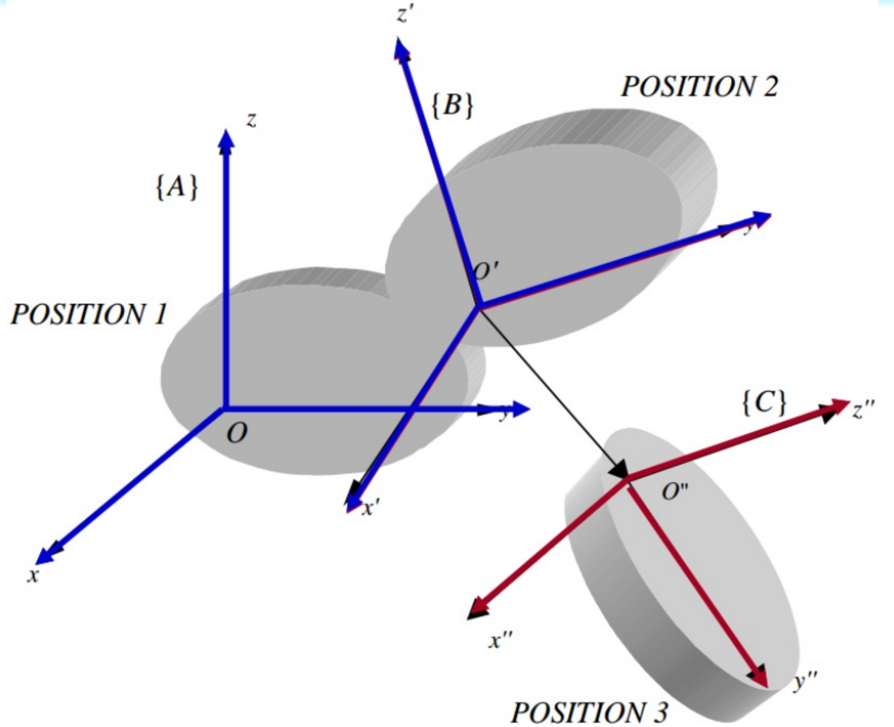
Rotation of B with respect to A

Translation of the origin of B wrt origin of A

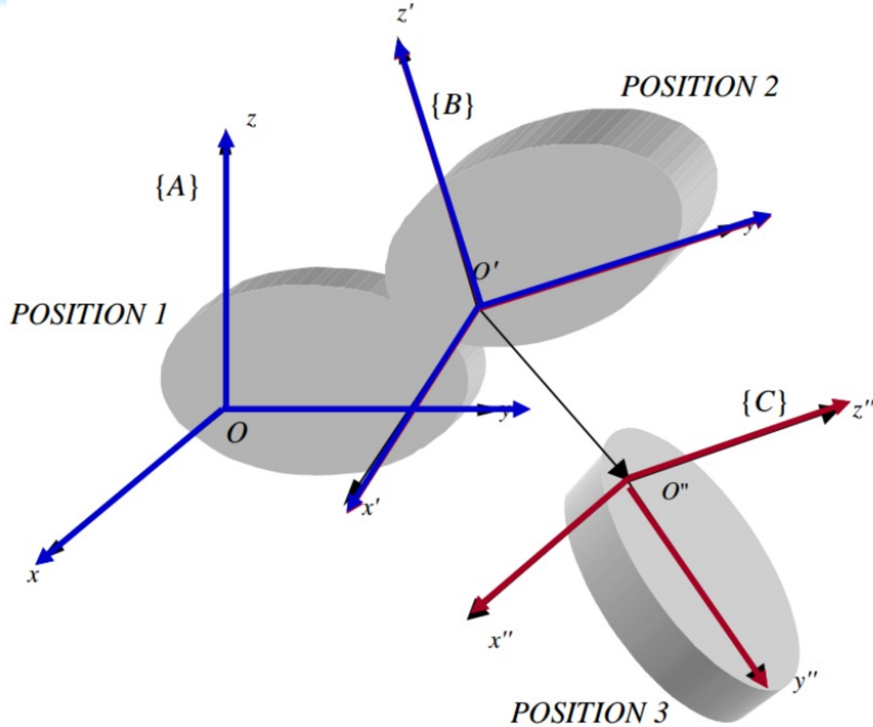
Homogeneous transformations

$$^{A}\mathbf{A}_{B} = \left[\begin{array}{c|c} ^{A}\mathbf{R}_{B} & ^{A}\mathbf{r}^{O'} \\ \hline \mathbf{0}_{1\times 3} & 1 \end{array}\right]$$

$$^{B}\mathbf{A}_{C} = \left[\begin{array}{c|c} ^{B}\mathbf{R}_{C} & ^{B}\mathbf{r}^{O''} \\ \hline \mathbf{0}_{1\times 3} & 1 \end{array}\right]$$



Composition of transformations



$$^{A}\mathbf{A}_{C} = {}^{A}\mathbf{A}_{B}\ {}^{B}\mathbf{A}_{C}$$

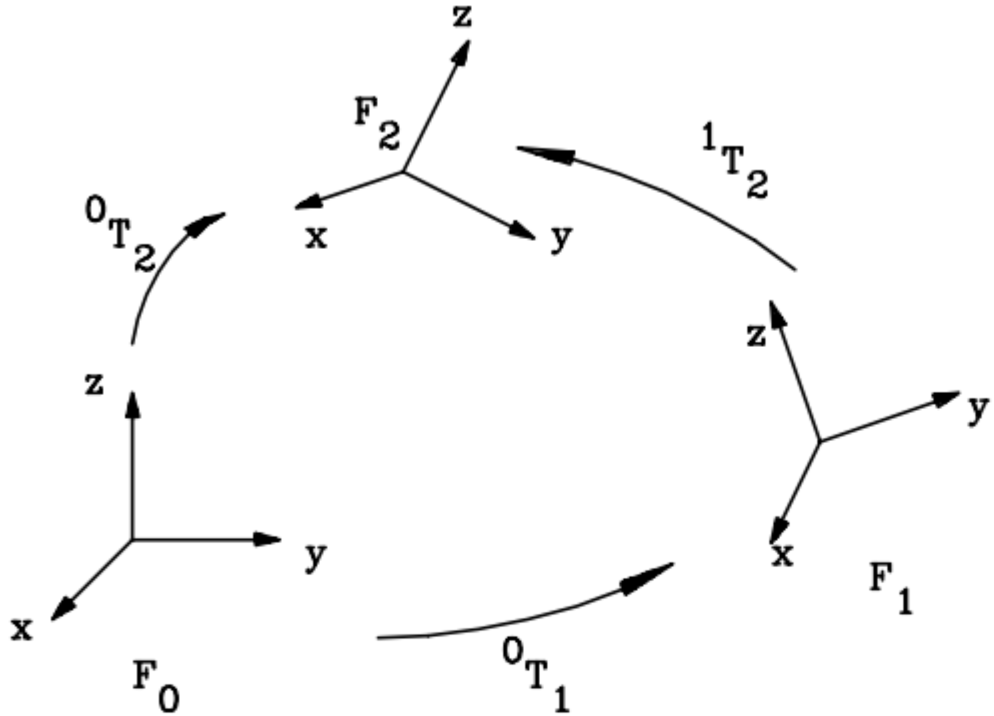
$$^{A}\mathbf{A}_{C} = \left[\begin{array}{c|c} {}^{A}\mathbf{R}_{C} & {}^{A}\mathbf{r}^{O''} \\ \hline \mathbf{0}_{1\times 3} & 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} {}^{A}\mathbf{R}_{B} & {}^{A}\mathbf{r}^{O'} \\ \hline \mathbf{0}_{1\times 3} & 1 \end{array}\right] \times \left[\begin{array}{c|c} {}^{B}\mathbf{R}_{C} & {}^{B}\mathbf{r}^{O''} \\ \hline \mathbf{0}_{1\times 3} & 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} {}^{A}\mathbf{R}_{B} \times {}^{B}\mathbf{R}_{C} & {}^{A}\mathbf{R}_{B} \times {}^{B}\mathbf{r}^{O''} + {}^{A}\mathbf{r}^{O'} \\ \hline \mathbf{0}_{1\times 3} & 1 \end{array}\right]$$

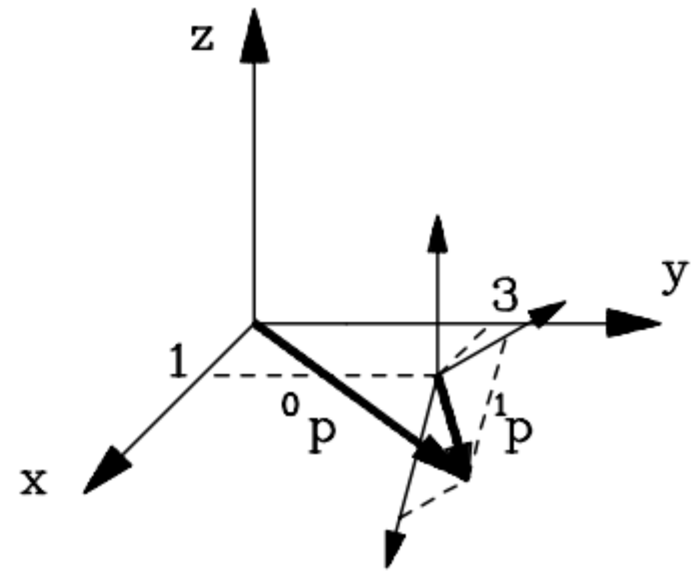
More generally

$$^0\mathbf{T}_n = {}^0\mathbf{T}_1\ {}^1\mathbf{T}_2 \ldots\ {}^{n-1}\mathbf{T}_n$$



Exercise: Example of transformations

- The frame F_1 with respect to F_0 is translated of 1 along x_0 and of 3 along y_0, moreover it is rotated by 30 degrees about z_0

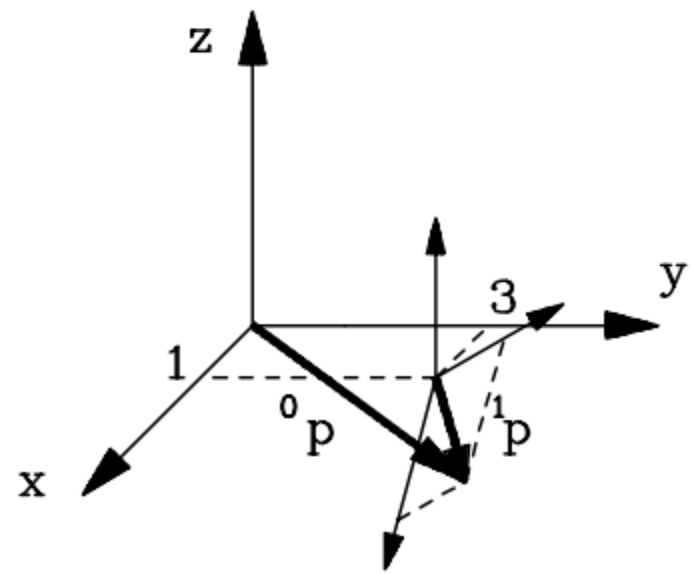


What is the transformation matrix from F_1 to F_0?

How the transformation matrix looks like?

Exercise: Example of transformations

- The frame F_1 with respect to F_0 is translated of 1 along x_0 and of 3 along y_0, moreover it is rotated by 30 degrees about z_0



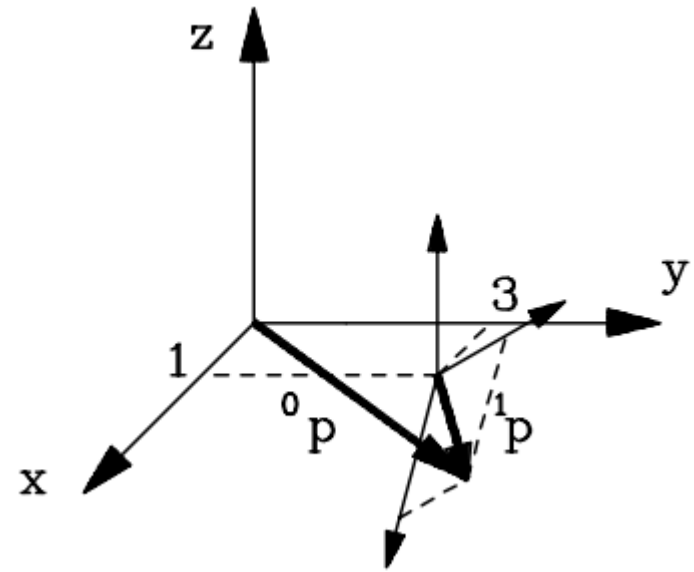
What is the transformation matrix from F_1 to F_0?

$$
{}^{0}\mathbf{T}_1 = \begin{bmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} & 0 & 1 \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Exercise: Example of transformations

- Consider now a point defined in F_1 by

$$^{1}\mathbf{p} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$



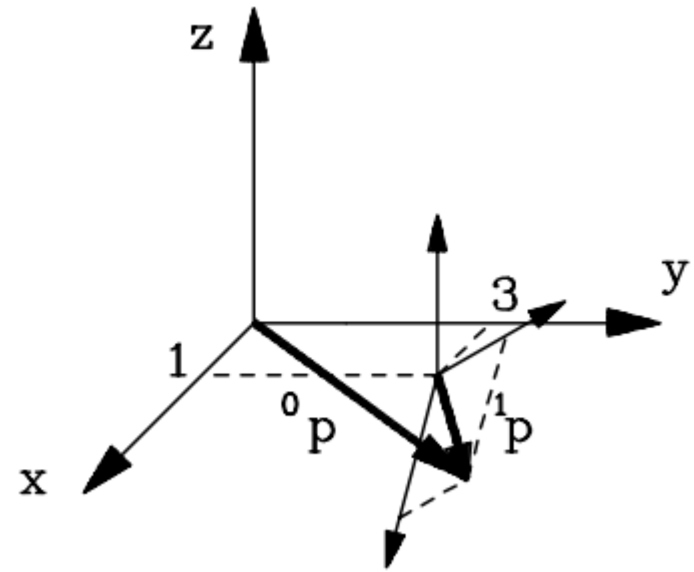
What are its coordinates in F_0?

What is the point's coordinate?

Exercise: Example of transformations

- Consider now a point define
  in F_1 by

$$^1\mathbf{p} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$



What are its coordinates in F_0?

$$^0\mathbf{p} = {}^0\mathbf{T}_1\ {}^1\mathbf{p} = \begin{bmatrix} 2.232 \\ 4.866 \\ 0 \\ 1 \end{bmatrix}$$

Inverse transformation

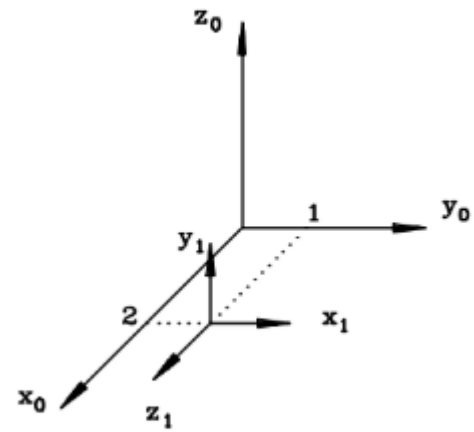
- Once the position/orientation of F_1 with respect to F_0 are known, defined by the homogeneous transformation matrix ${}^0\boldsymbol{T}_1$, it is simple to compute the inverse transformation ${}^1\boldsymbol{T}_0$, defining the position/orientation of F_0 with respect to F_1

$$
{}^1\mathbf{T}_0 = \begin{bmatrix} {}^0\mathbf{R}_1^T & -\,{}^0\mathbf{R}_1^T\,{}^0\mathbf{r}^{O'} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix}
$$

Exercise: inverse transformation

- Given

$$\mathbf{T} = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



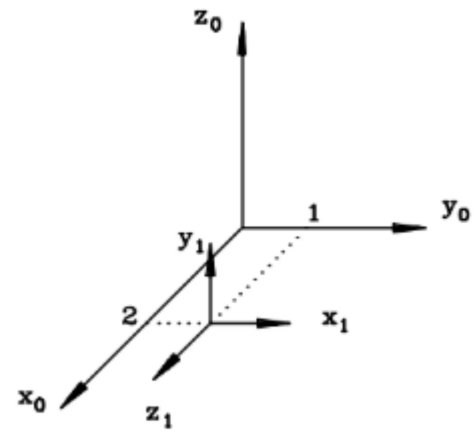
Compute its inverse transformation

What is the inverse transformation?

Exercise: inverse transformation

- Given

$$\mathbf{T} = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

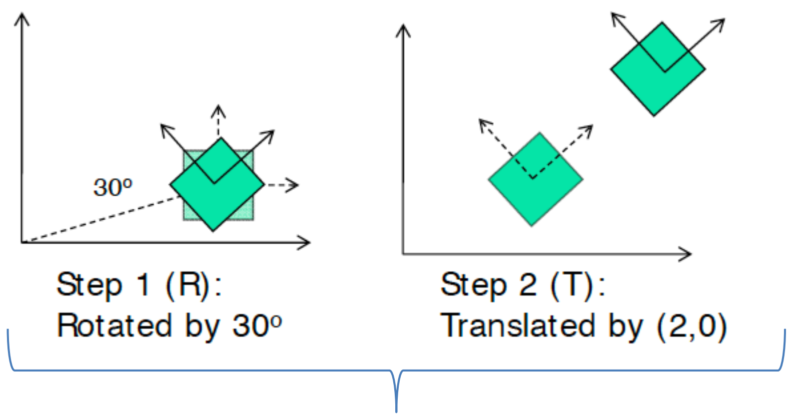


Compute its inverse transformation

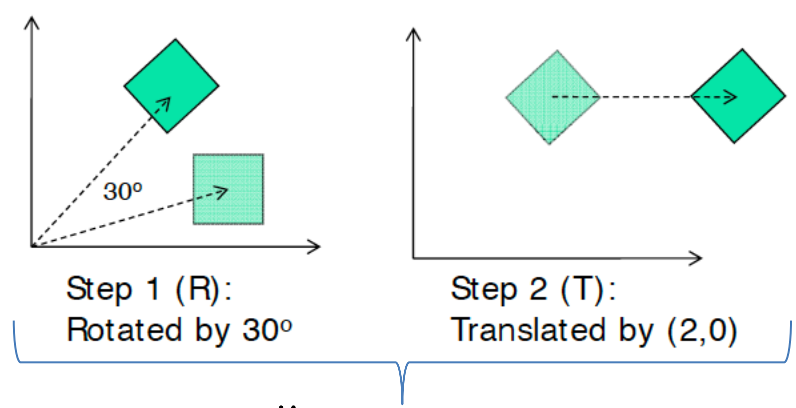
$$\mathbf{T}^{-1} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Post-multiplication vs. Pre-multiplication

- Transform with respect to local origin and basis (post multiplication)
- Transform with respect to global origin and basis (pre multiplication)



$$T' = R \times T$$



$$T'' = T \times R$$

Similar in concept as Euler mobile vs fixed angle

We use the post multiplication