UNIVERSITY OF SOUTH CAROLINA

CSCE 574 ROBOTICS

Mapping

Ioannis Rekleitis

Introduction to Mapping

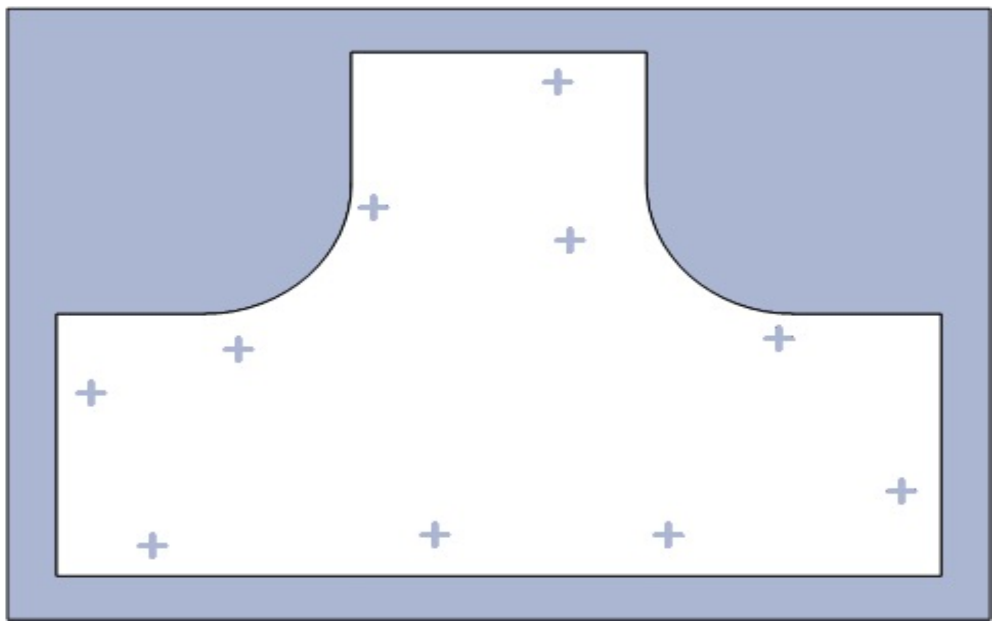
- What the world looks like?
- Knowledge representation
  - Robotics, AI, Vision
- Who is the end-user?
  - Human or Machine
- Ease of Path Planning
- **Uncertainty!**

Simultaneous Localization And Mapping

SLAM is the process of building a map of an environment while, at the same time, using that map to maintain the location of the robot.

- Problems for SLAM in large scale environments:
  - Controlling growth of uncertainty and complexity
  - Achieving autonomous exploration

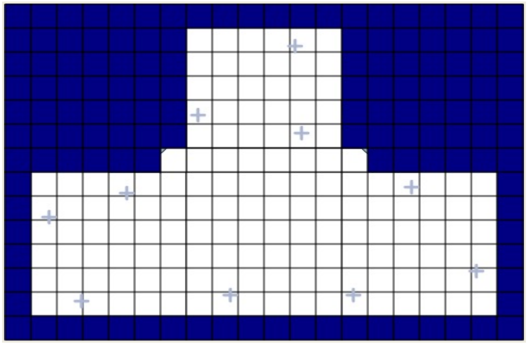
Consider this Environment:



Three Basic Map Types

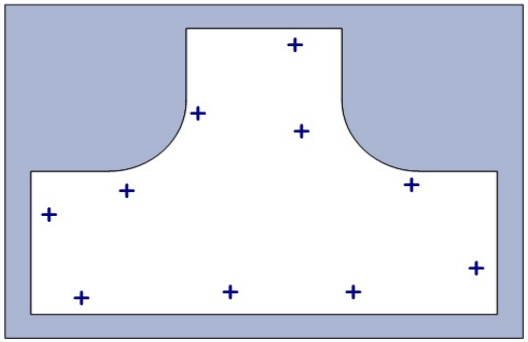
Grid-Based:

Collection of discretized obstacle/free-space pixels



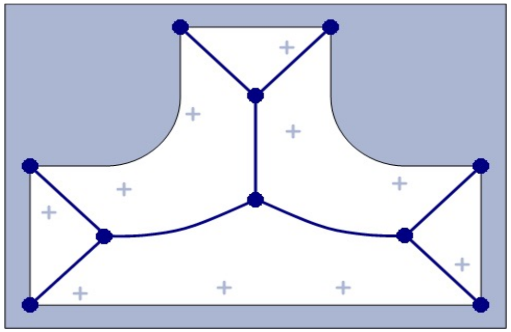
Feature-Based:

Collection of landmark locations and correlated uncertainty



Topological:

Collection of nodes and their interconnections



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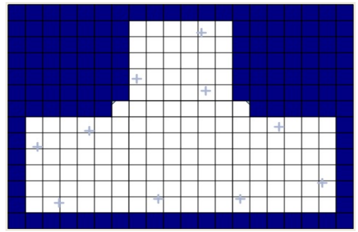

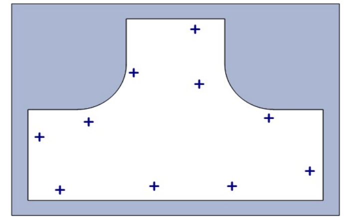

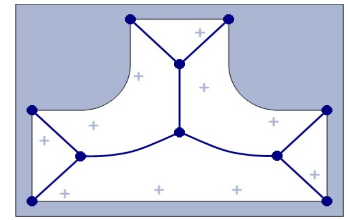

	Grid-Based	Feature-Based	Topological
Construction	Occupancy grids	Kalman Filter	Navigation control laws
Complexity	Grid size *and* resolution	Landmark covariance (N^3)	Minimal complexity
Obstacles	Discretized obstacles	Only structured obstacles	GVG defined by the safest path
Localization	Discrete localization	Arbitrary localization	Localize to nodes
Exploration	Frontier-based exploration	No inherent exploration	Graph exploration

Other Maps

	Appearance Based	Geometry Based	Mesh Based
Construction	Images	Lines, planes, etc	Mesh
Path Planning	N/A	Geometry based	Graph based
Localization	Arbitrary localization	Arbitrary localization	Arbitrary localization

Mapping

World

Robot

Map

Mapping

World

- Indoor/Outdoor
- 2D/2.5D/3D
- Static/Dynamic
- Known/Unknown
- Abstract (web)

Robot

Map

Mapping

World

Robot

Map

- Mobile
  - Indoor/Outdoor
  - Walking/Flying/Swimming
- Manipulator
- Humanoid
- Abstract

Mapping

World

Robot

Map

- Topological
- Metric
- Feature Based
- 1D,2D,2.5D,3D

Mapping

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Sonar sensing

"The sponge"

sonar timeline

0

a "chirp" is emitted into the environment

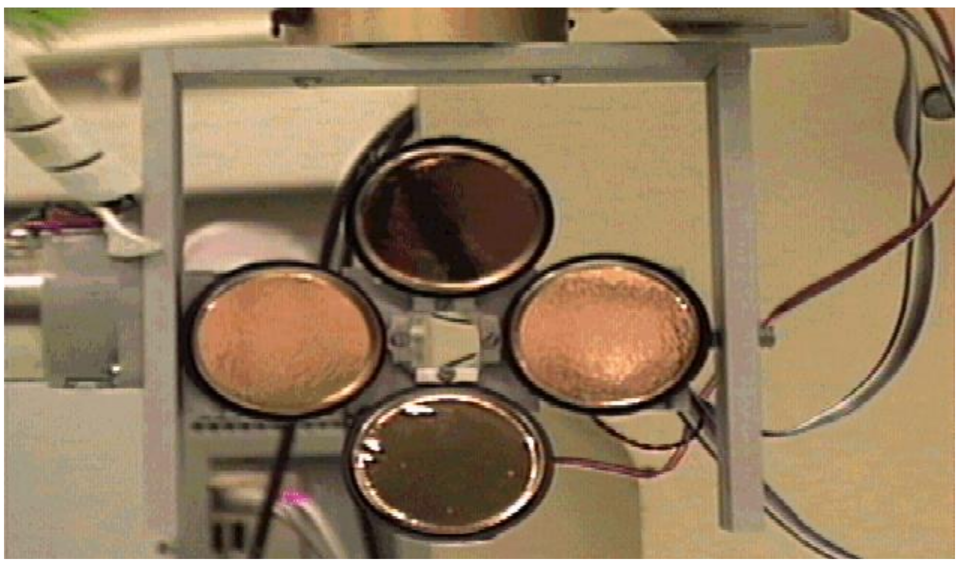
75μs

typically when reverberations from the initial chirp have stopped

the transducer goes into "receiving" mode and awaits a signal...

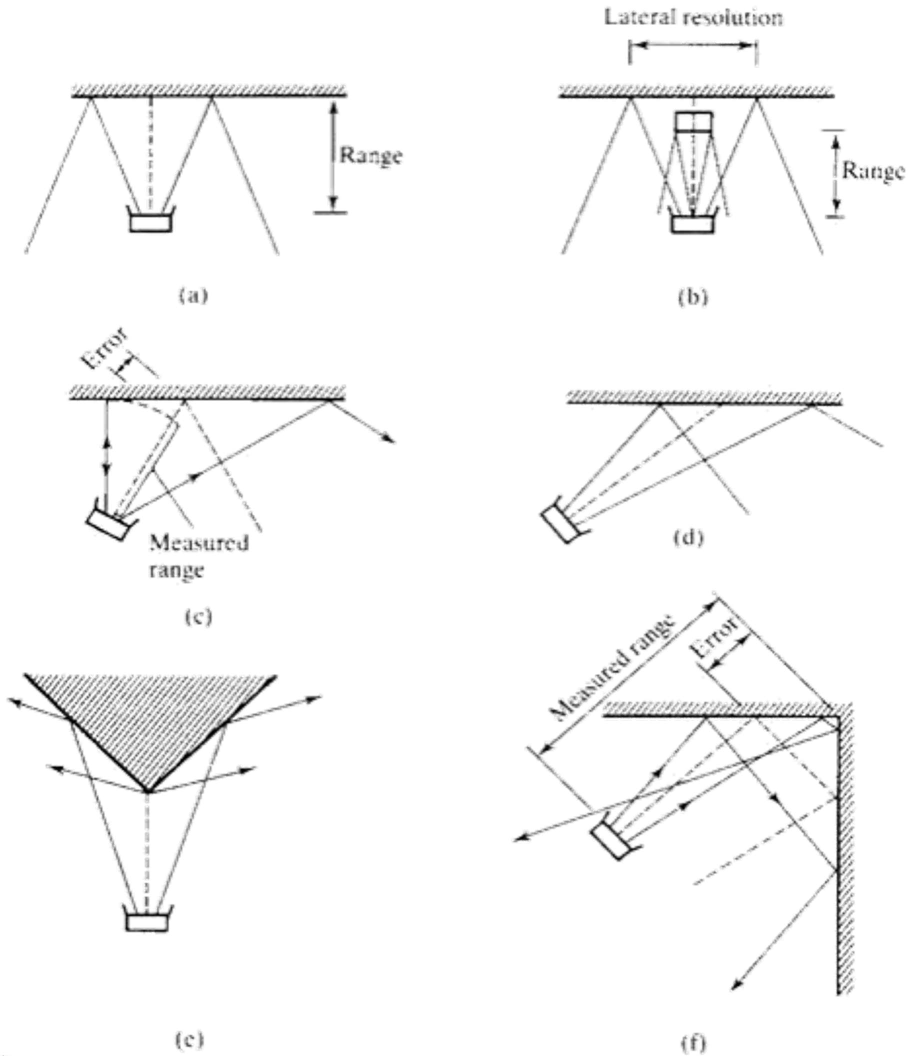
.5s

after a short time, the signal will be too weak to be detected



Polaroid sonar emitter/receivers

Sonar effects



(a) Sonar providing an accurate range measurement

(b-c) Lateral resolution is not very precise; the closest object in the beam's cone provides the response

(d) Specular reflections cause walls to disappear

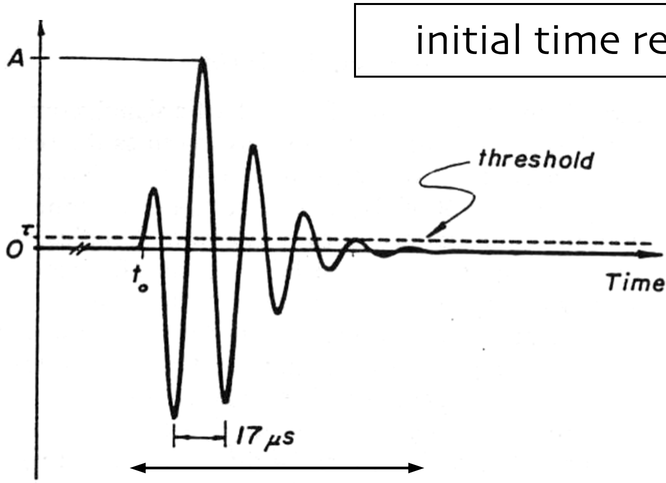
(e) Open corners produce a weak spherical wavefront

(f) Closed corners measure to the corner itself because of multiple reflections --> sonar ray tracing

resolution: time / space

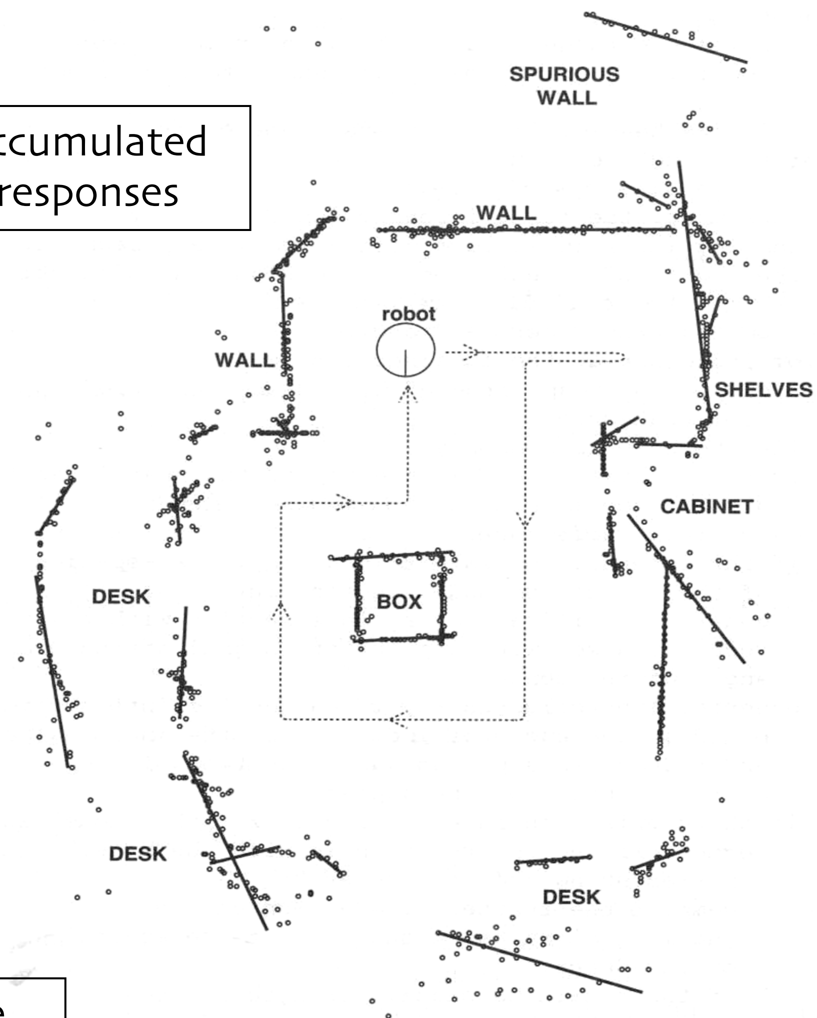
Sonar modeling

initial time response

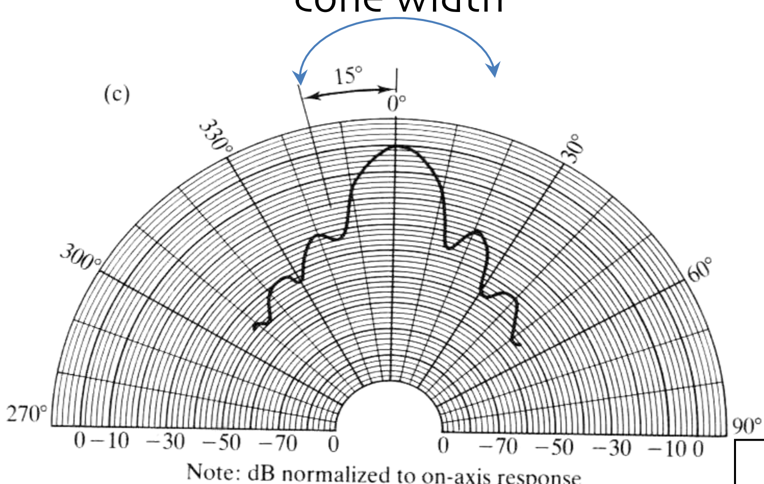


blanking time

accumulated responses



cone width

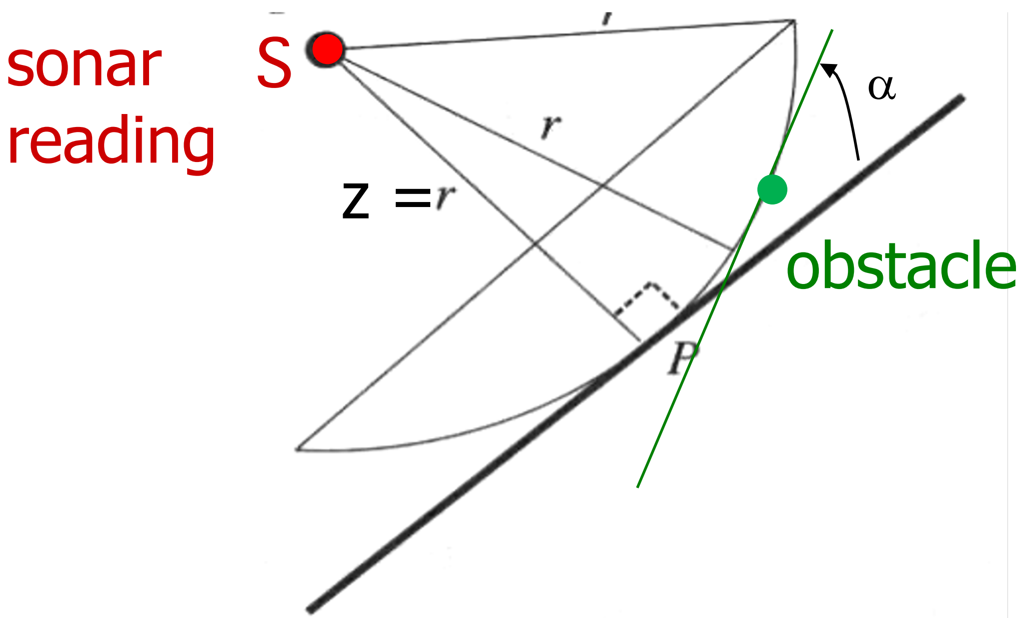


spatial response

Sonar Modeling

response model (Kuc)

$$h_R(t, z, a, \alpha) = \frac{2c\cos\alpha}{\pi a \sin\alpha}\sqrt{1 - \frac{c^2(t - 2z/c)^2}{a^2 \sin^2\alpha}}$$



- Models the response, h_R, with:

  c = speed of sound

  a = diameter of sonar element

  t = time

  z = orthogonal distance

  α = angle of environment surface

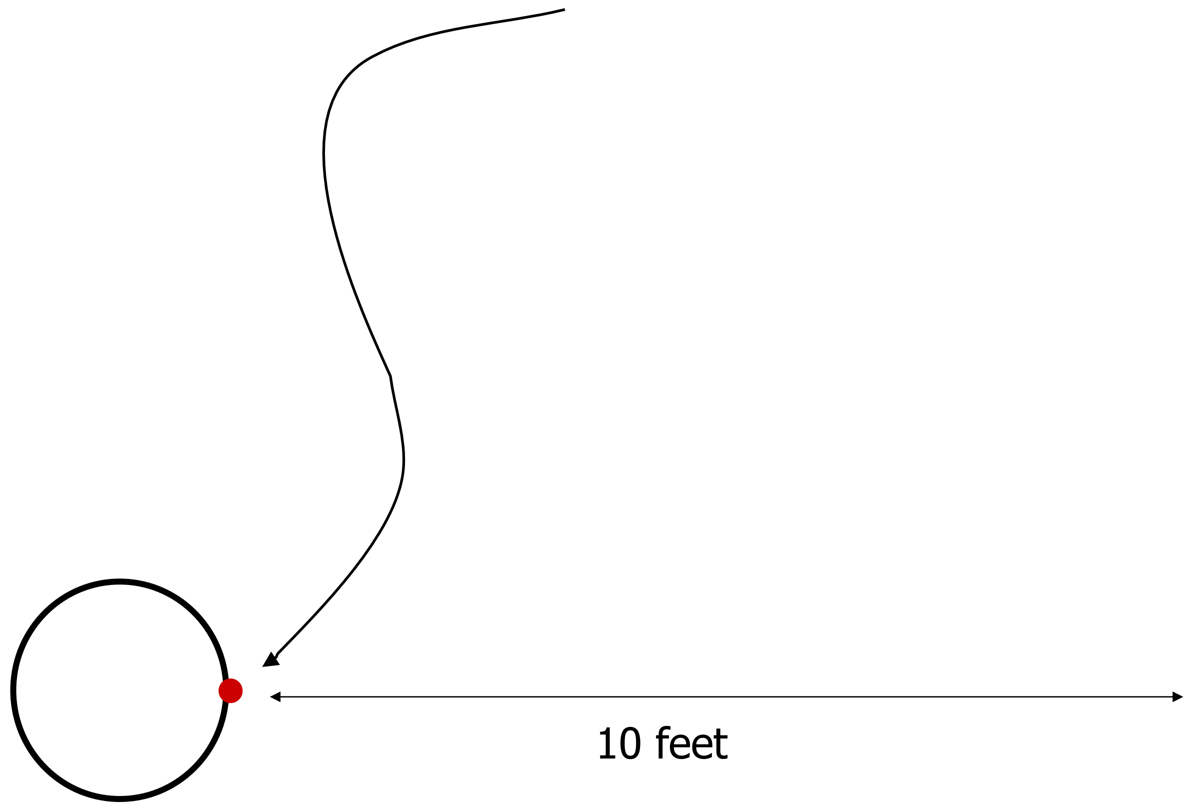
- Then, add noise to the model to obtain a probability: p(S | o)

chance that the sonar reading is S,

given an obstacle at location o

Using sonar to create maps

What should we conclude if this sonar reads 10 feet?



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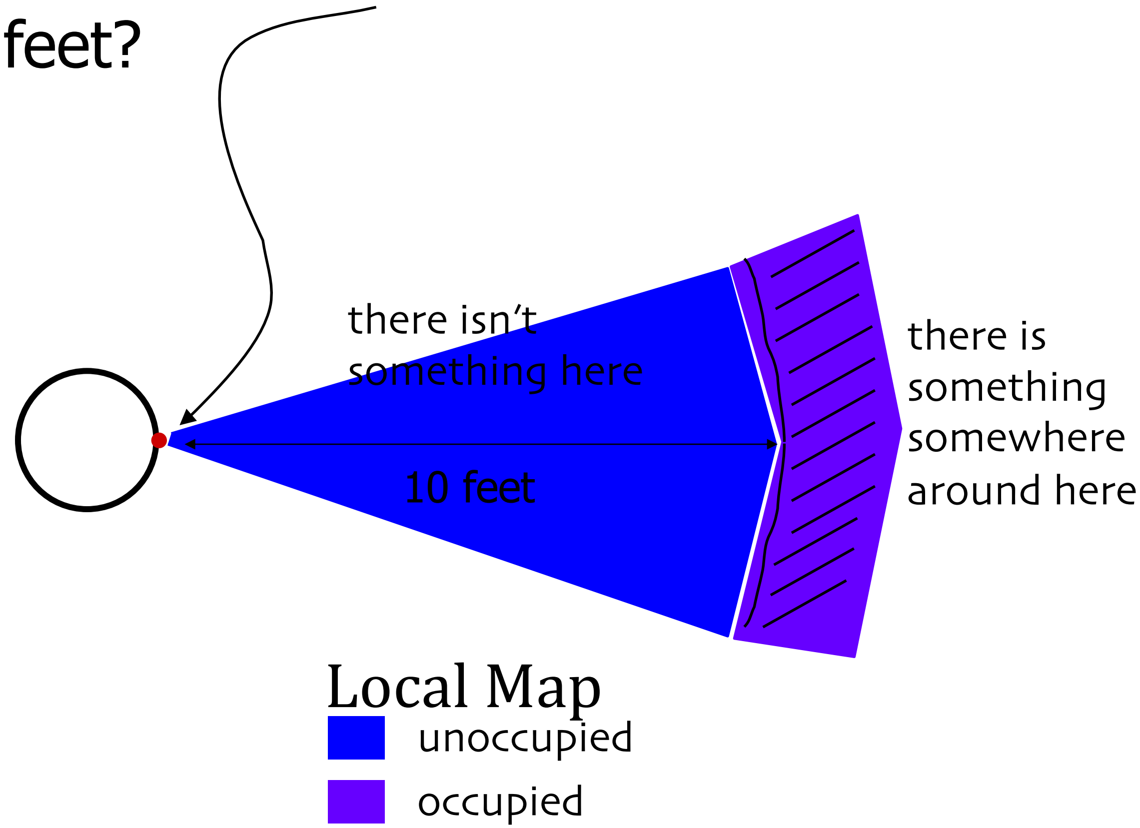

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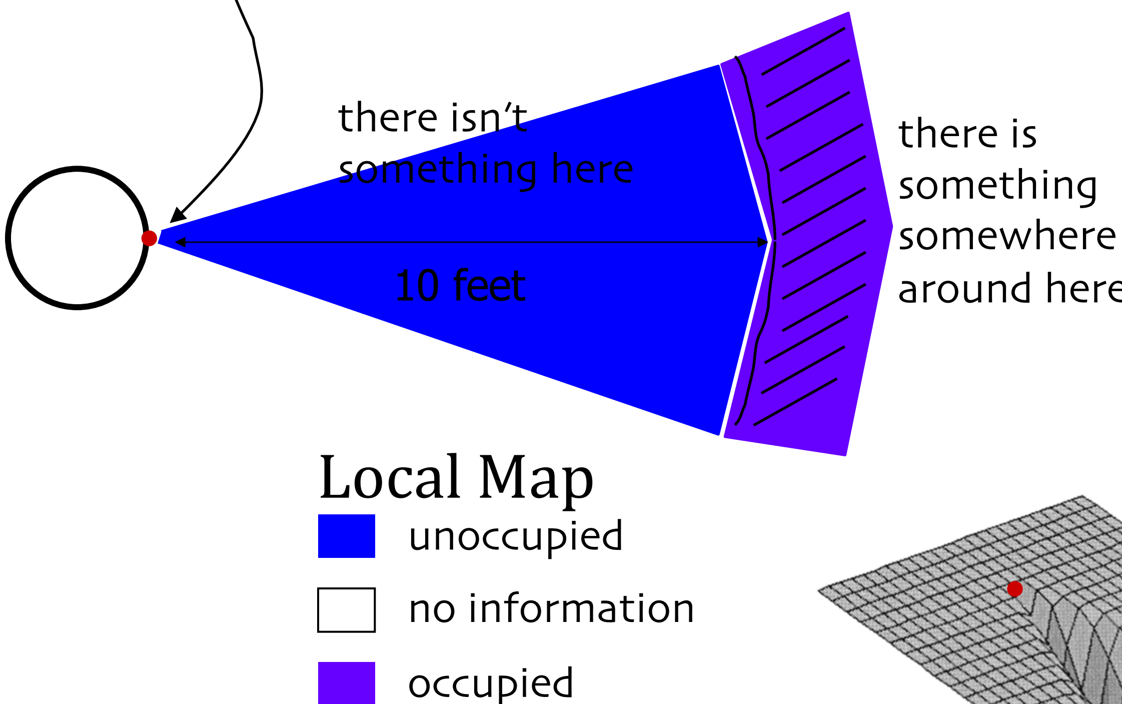
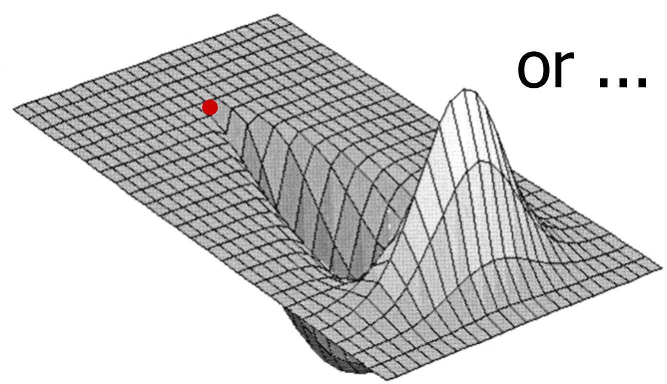


or ...



Using sonar to create maps

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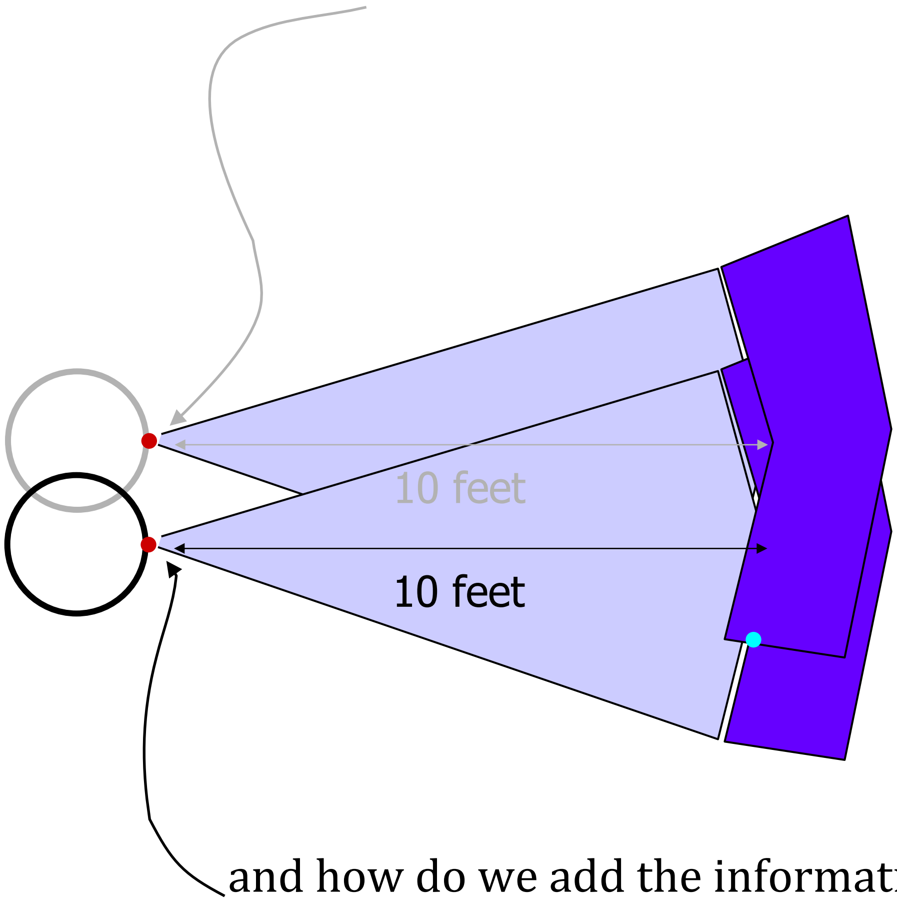
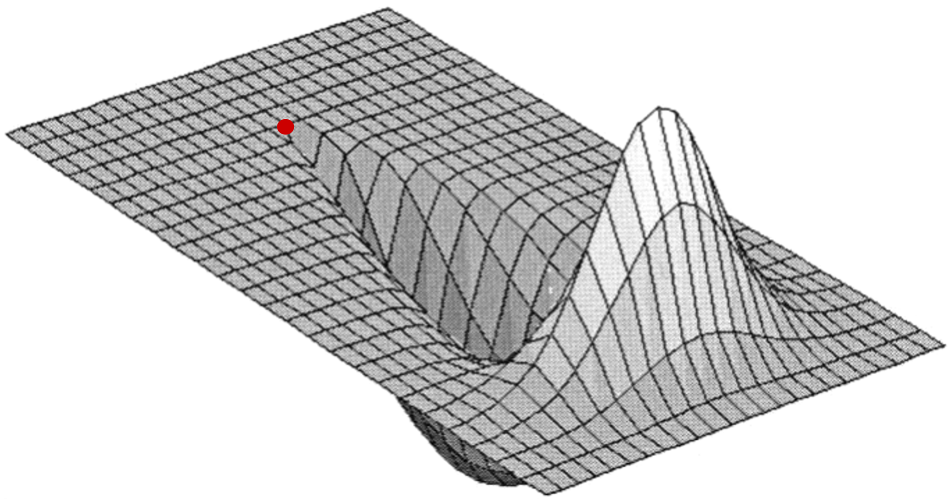


and how do we add the information that the next sonar reading (as the robot moves) reads 10 feet, too?

Combining sensor readings

- The key to making accurate maps is combining lots of data.
- But combining these numbers means we have to know what they are !



what is in each cell of this sonar model / map ?

What should our map contain ?

- small cells
- each represents a bit of the robot's environment
- larger values => obstacle
- smaller values => free

What is it a map of?

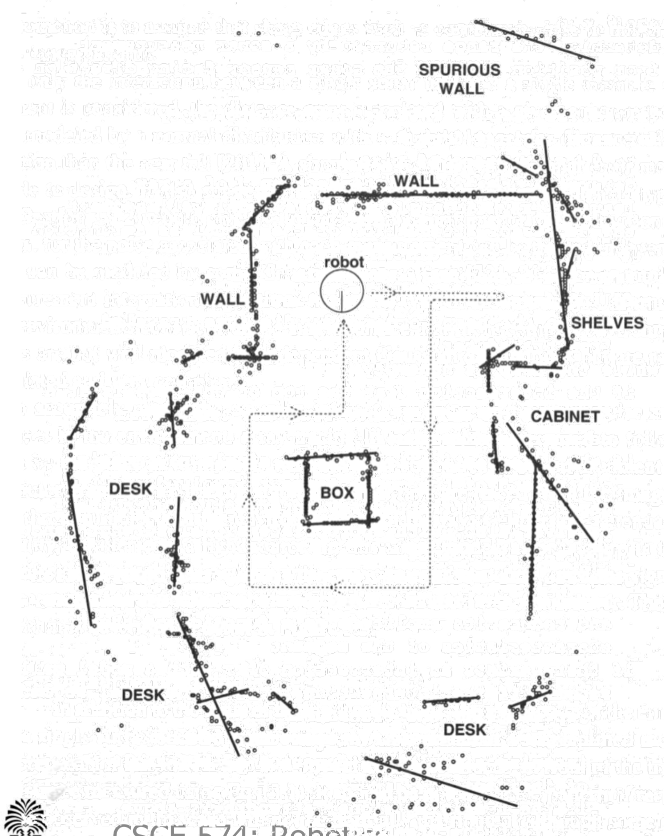
Several answers to this question have been tried:

It's a map of occupied cells.

pre `83

o_{xy} cell (x,y) is occupied

$\bar{o}_{xy}$ cell (x,y) is unoccupied



Each cell is either occupied or unoccupied -- this was the approach taken by the Stanford Cart.

What information **should** this map contain, given that it is created with sonar ?

What is it a map of ?

Several answers to this question have been tried:

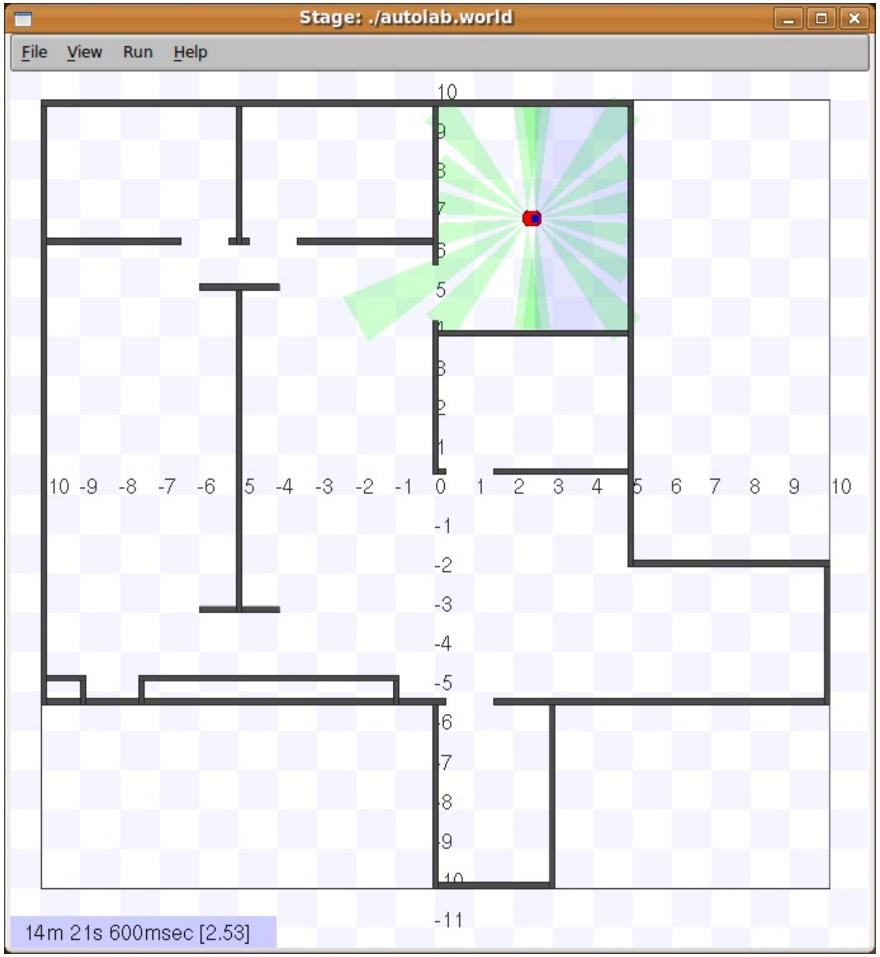
pre '83 It's a map of occupied cells. o_{xy} cell (x,y) is occupied $\overline{o}_{xy}$ cell (x,y) is unoccupied

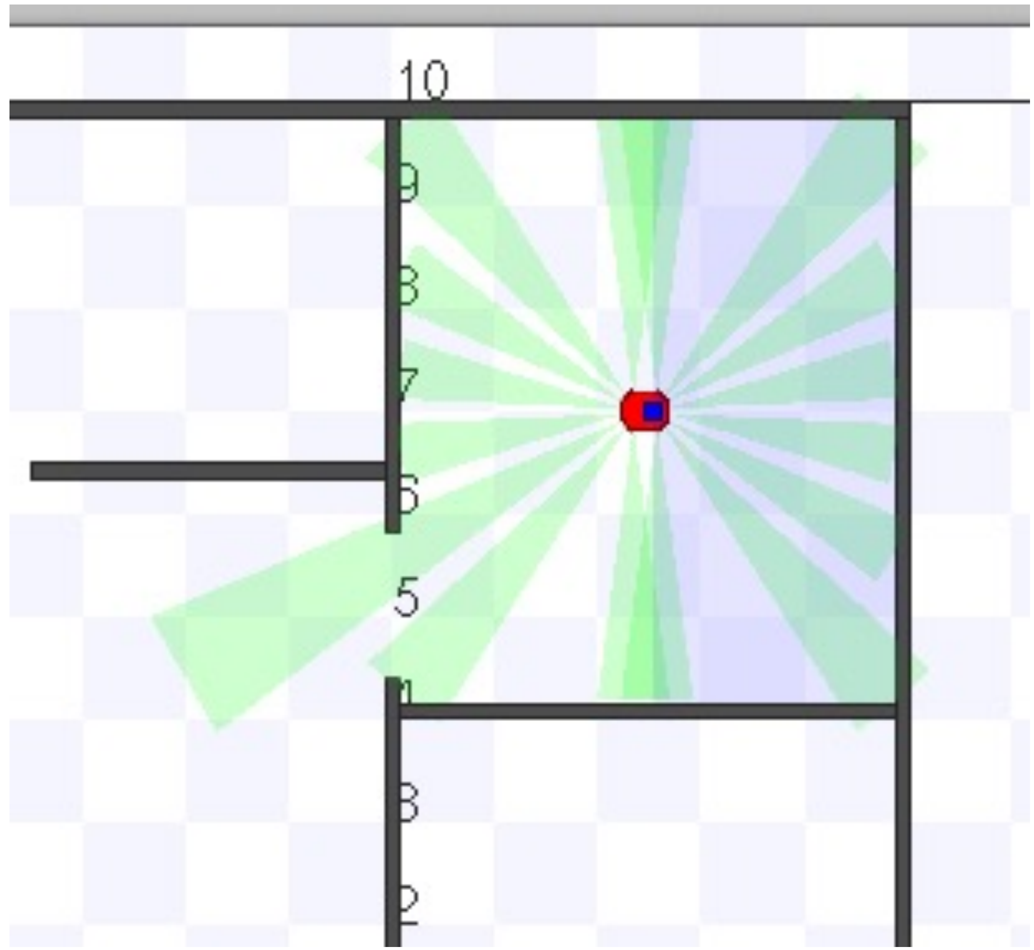
'83 - '88 It's a map of probabilities: $p(\, o \mid S_{1..i}\,)$ The certainty that a cell is **occupied**, given the sensor readings S_1, S_2, …, S_i

$p(\, \overline{o} \mid S_{1..i}\,)$ The certainty that a cell is **unoccupied**, given the sensor readings S_1, S_2, …, S_i

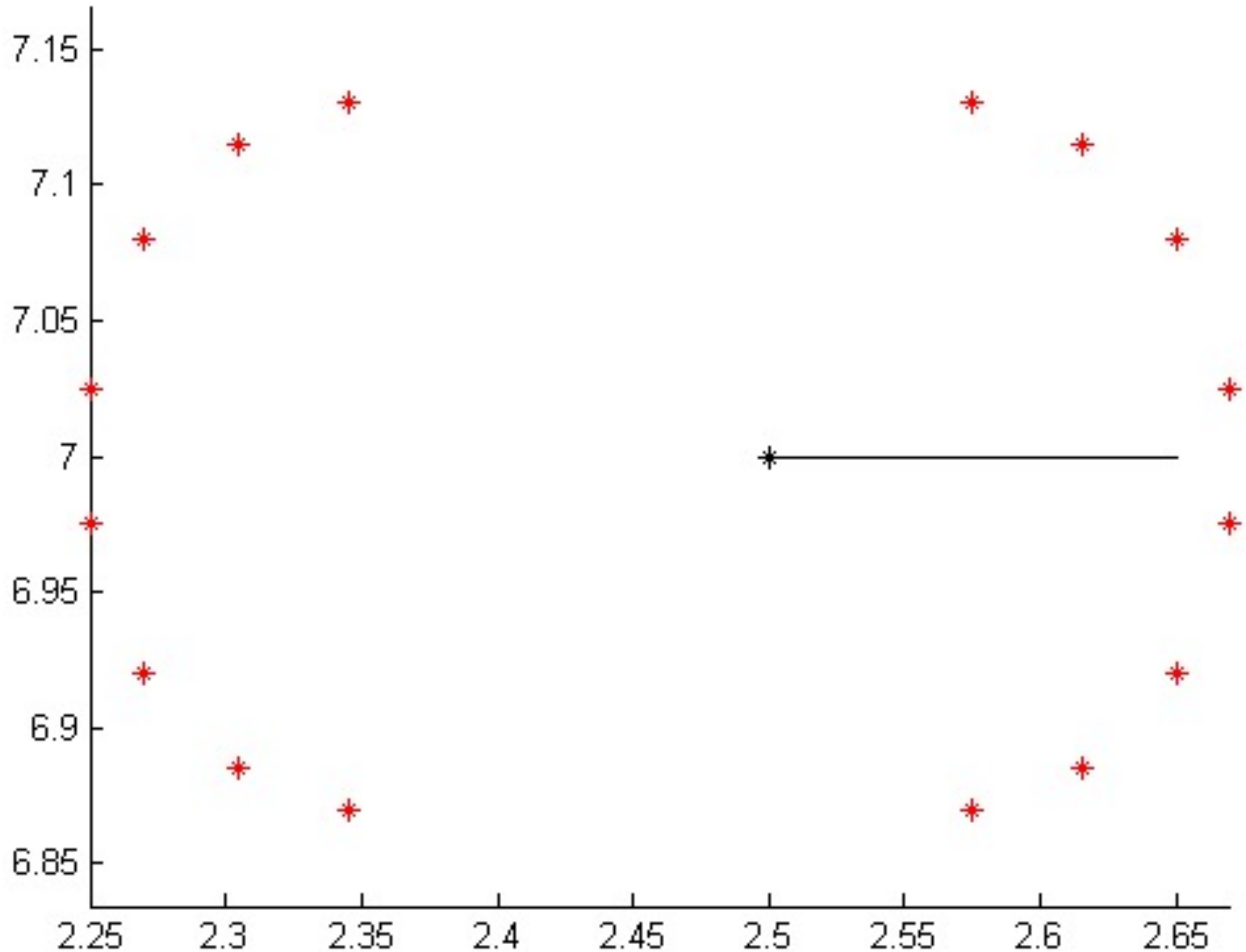
- maintaining related values separately?
- initialize all certainty values to zero
- contradictory information will lead to both values near 1
- combining them takes some work...

Sonars from P/S

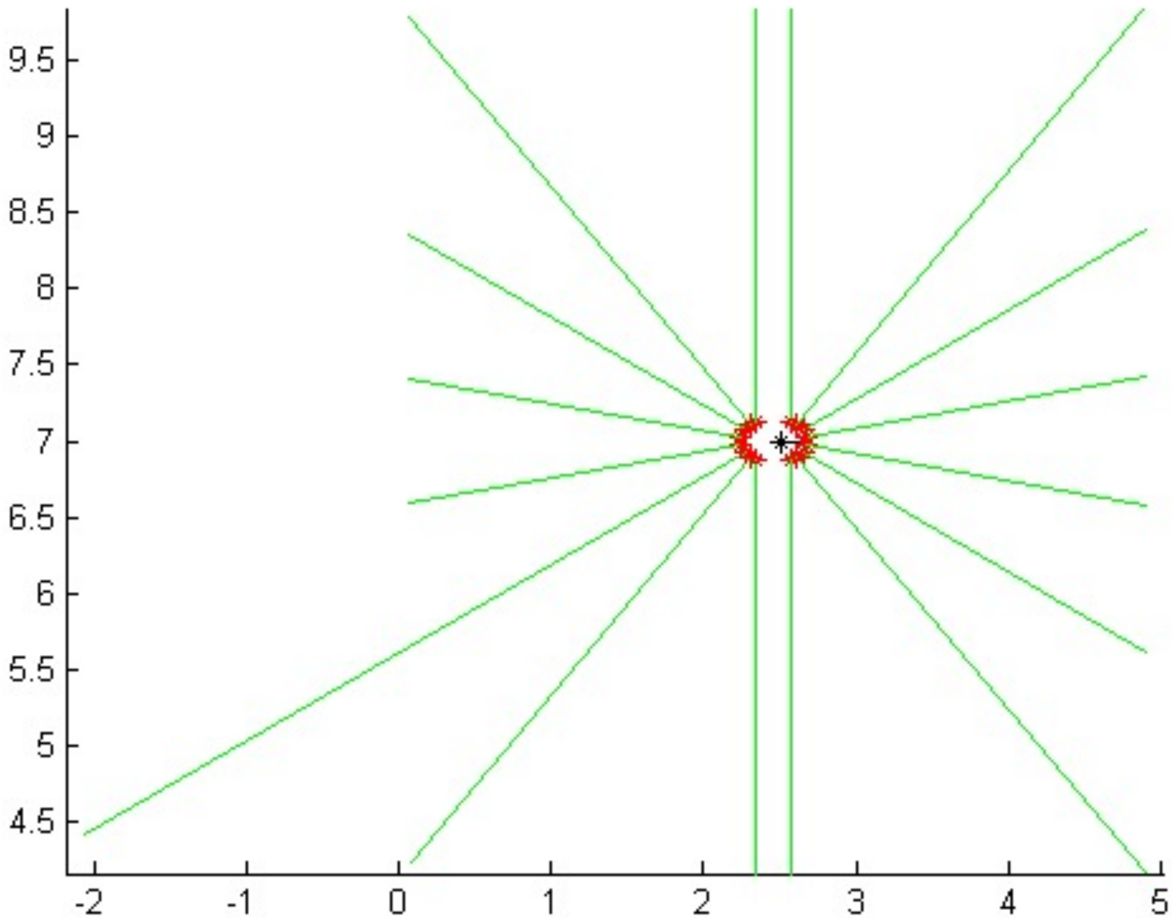


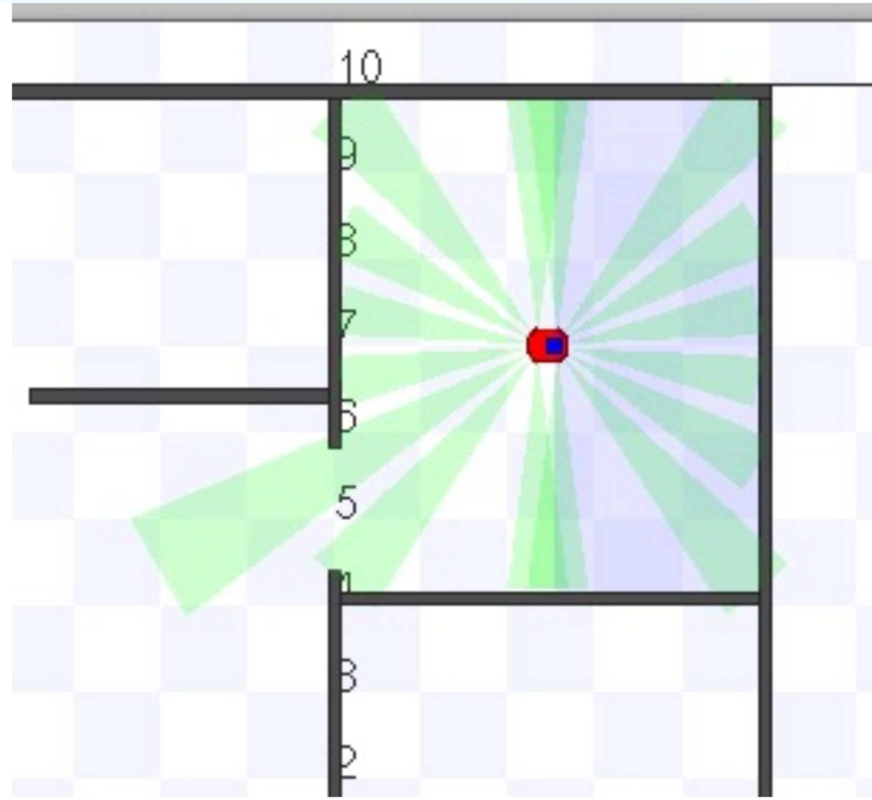


Sonar Locations Pioneer 3DX Robot

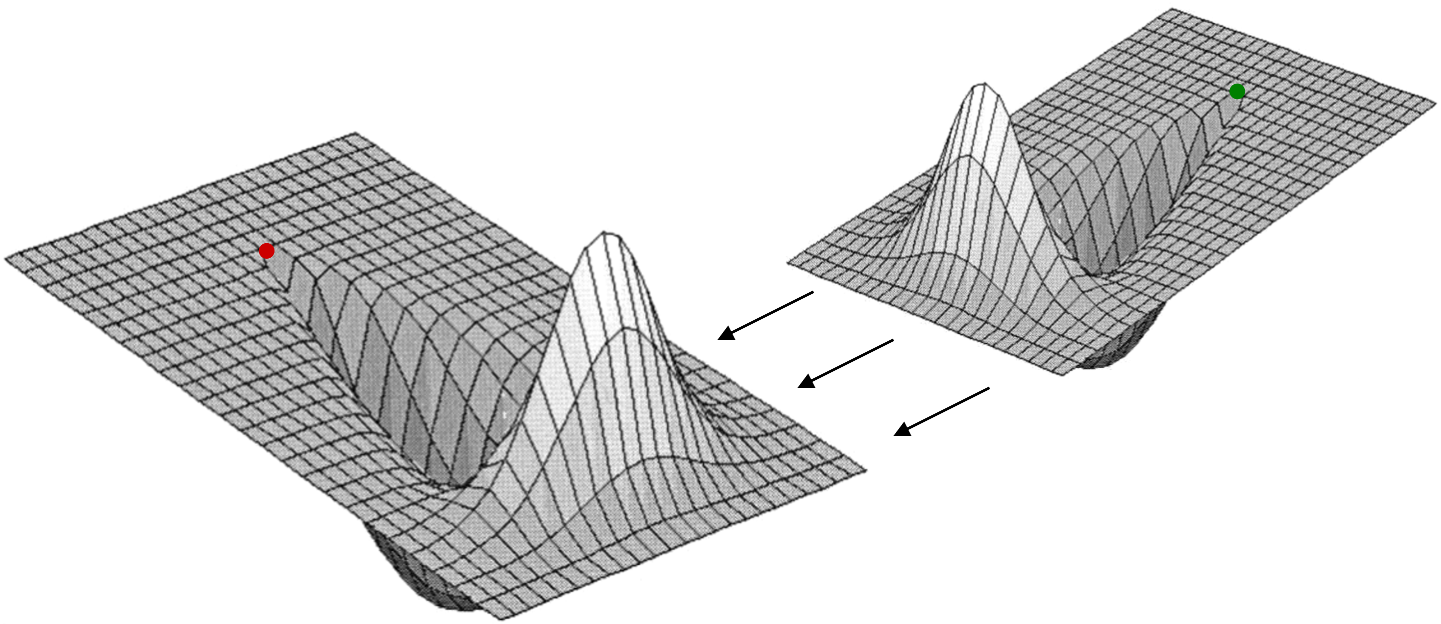


Sonar Data Calculation





Combining probabilities



How to combine two sets of probabilities into a single map ?

What is it a map of ?

Several answers to this question have been tried:

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$p(\overline{o} \mid S_{1\ldots i})$ The certainty that a cell is **unoccupied**, given the sensor readings $S_1, S_2, \ldots, S_i$

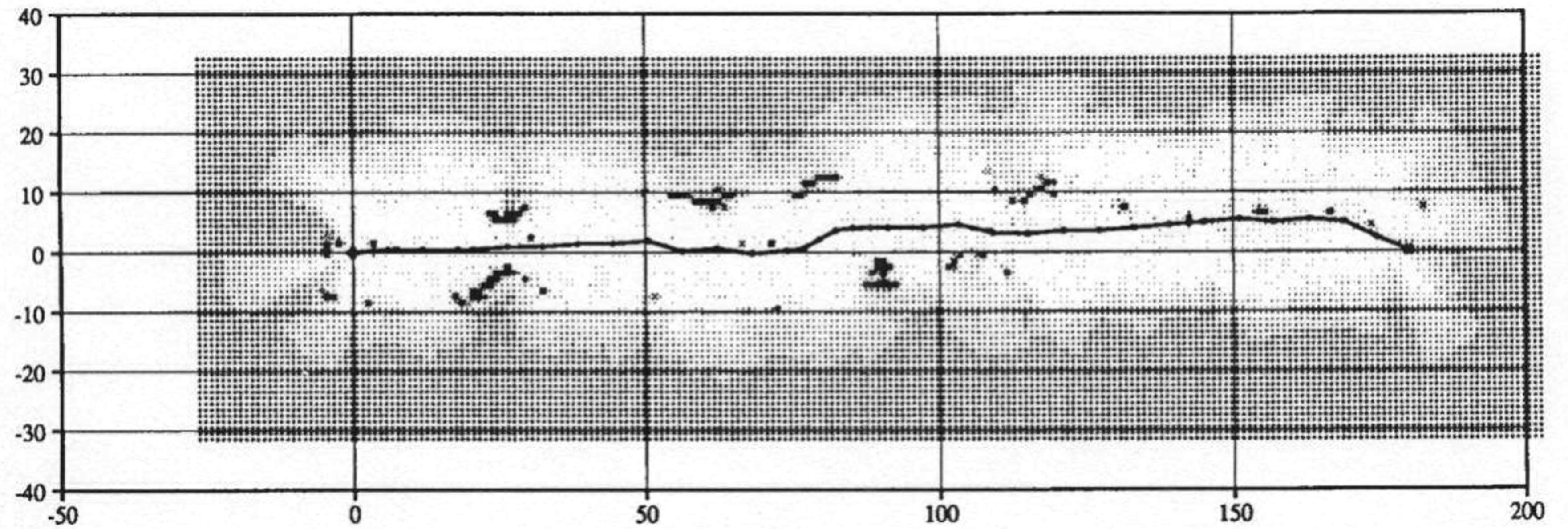
It's a map of *odds*. The odds of an event are expressed *relative to the complement* of that event.

probabilities

The odds that a cell is **occupied**, given the sensor readings $S_1, S_2, \ldots, S_i$

$$odds(o \mid S_{1\ldots i}) = \frac{p(o \mid S_{1\ldots i})}{p(\overline{o} \mid S_{1\ldots i})}$$

An example map



units: feet

Evidence grid of a tree-lined outdoor path

lighter areas: *lower* odds of obstacles being present

darker areas: *higher* odds of obstacles being present

how to combine them?

Conditional probability

Some intuition...

$$p(\ o\ |\ S\ ) \ = \ \frac{\text{The probability of event } \mathbf{o} \text{, given event } \mathbf{S}\text{ .}}{\text{The probability that a certain cell } \mathbf{o} \text{ is occupied, given that the robot sees the sensor reading } \mathbf{S}\text{ .}}$$

$$p(\ S\ |\ o\ ) \ = \ \frac{\text{The probability of event } \mathbf{S} \text{, given event } \mathbf{o}\text{ .}}{\text{The probability that the robot sees the sensor reading } \mathbf{S}\text{ , given that a certain cell } \mathbf{o} \text{ is occupied.}}$$

- What is really meant by conditional probability ?
- How are these two probabilities related?

Bayes Rule

- Conditional probabilities

$$p(o \wedge S) = p(o \mid S)\, p(S)$$

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- Conditional probabilities

$$p(o \wedge S) = p(o \mid S)\, p(S)$$

- Bayes rule relates conditional probabilities

$$\boxed{p(o \mid S) = \frac{p(S \mid o)\, p(o)}{p(S)}}$$

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Bayes rule

- So, what does this say about $\text{odds}(\ o \mid S_2 \wedge S_1\ )$?

Can we update easily ?

Combining evidence

So, how do we combine evidence to create a map?

What we want --

$$\text{odds}(\, o \mid S_2 \wedge S_1)$$

the new value of a cell in the map after the sonar reading S_2

What we know --

$$\text{odds}(\, o \mid S_1)$$

the old value of a cell in the map (before sonar reading S_2)

$$p(\, S_i \mid o\,) \;\;\&\;\; p(\, S_i \mid \overline{o}\,)$$

the probabilities that a certain obstacle causes the sonar reading S_i

Combining evidence

$$odds(o \mid S_2 \wedge S_1) = \frac{p(o \mid S_2 \wedge S_1)}{p(\overline{o} \mid S_2 \wedge S_1)}$$

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$$odds(o \mid S_2 \wedge S_1) = \frac{p(o \mid S_2 \wedge S_1)}{p(\bar{o} \mid S_2 \wedge S_1)}$$

definition of odds

$$= \frac{p(S_2 \wedge S_1 \mid o)\, p(o)}{p(S_2 \wedge S_1 \mid \bar{o})\, p(\bar{o})}$$

Combining evidence

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Bayes' rule (+)

$$= \frac{p(S_2 \mid o)\, p(S_1 \mid o)\, p(o)}{p(S_2 \mid \overline{o})\, p(S_1 \mid \overline{o})\, p(\overline{o})}$$

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conditional independence of S_1 and S_2

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conditional independence of S_1 and S_2

$$= \boxed{\frac{p(S_2 \mid o)}{p(S_2 \mid \bar{o})}} \boxed{\frac{p(o \mid S_1)}{p(\bar{o} \mid S_1)}}$$

Bayes' rule (+)

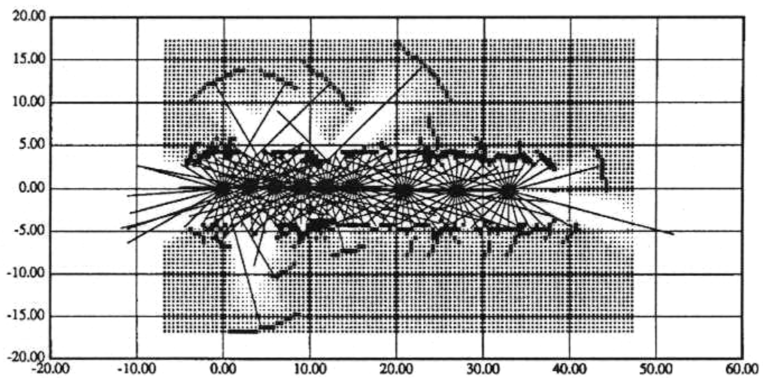
precomputed values

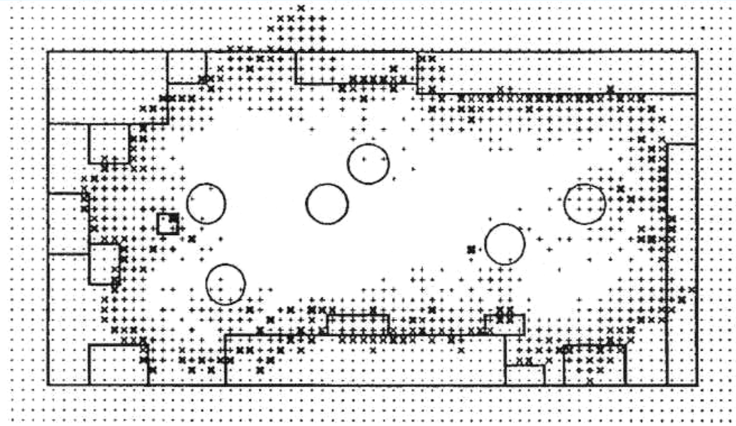
previous odds

the sensor model

Update step = multiplying the previous odds by a precomputed weight.

Evidence grids

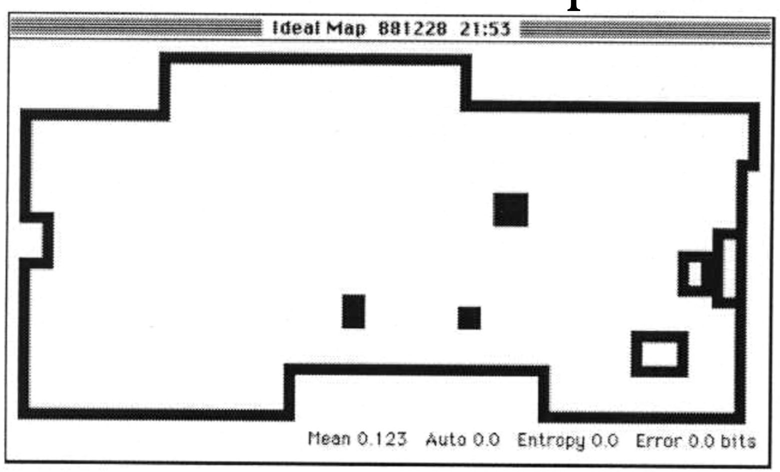


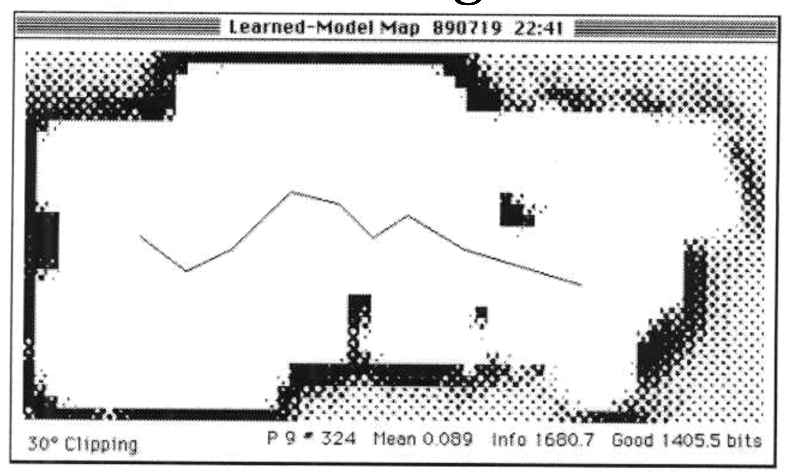


hallway with some open doors

lab space

known map and estimated evidence grid



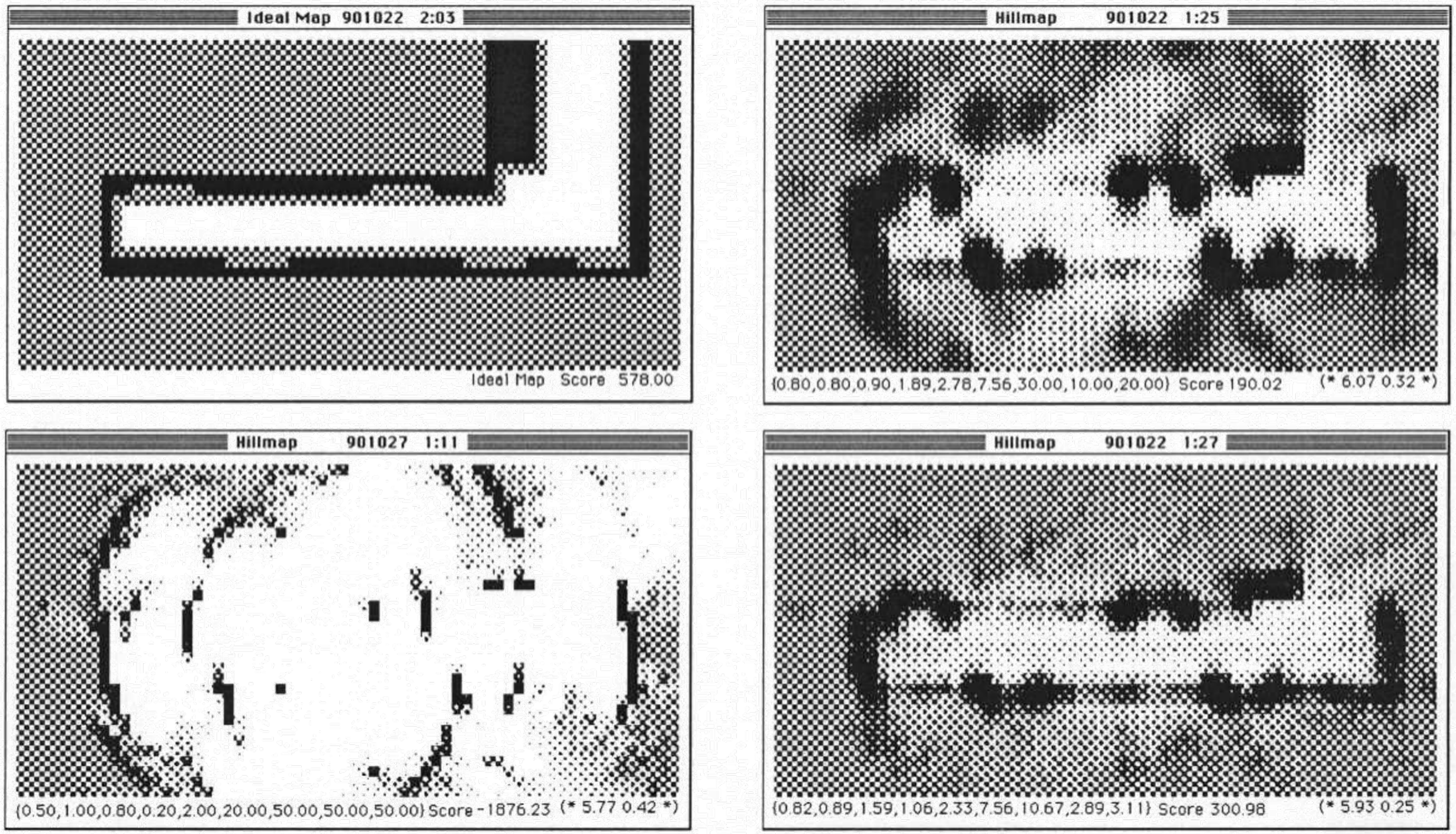


CMU -- Hans Moravec

Learning the Sensor Model

The sonar model depends dramatically on the environment -- we'd like to *learn* an appropriate sensor model

rather than hire Roman Kuc to develop another one...



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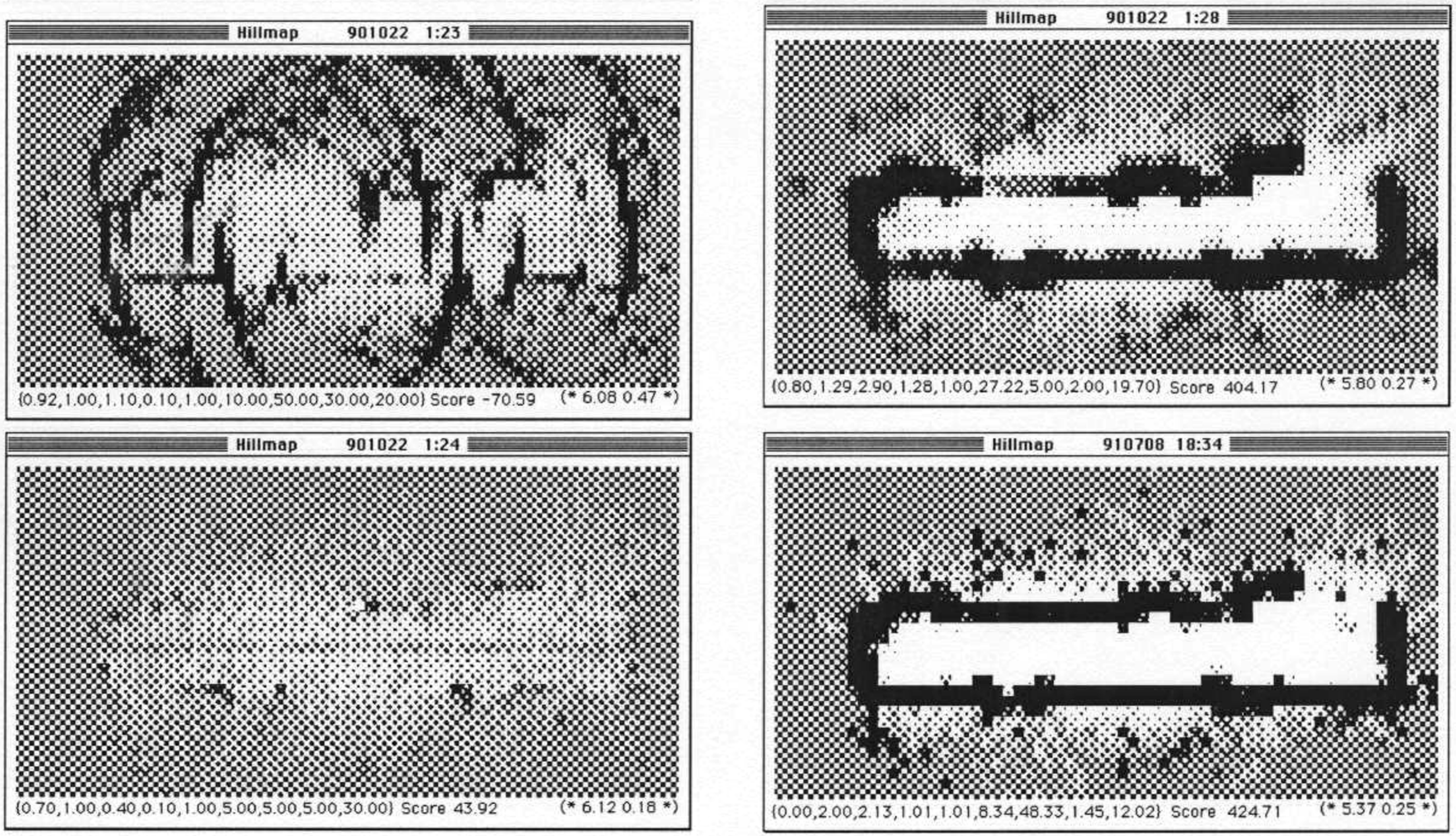
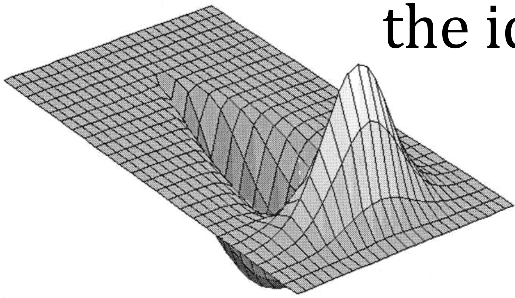
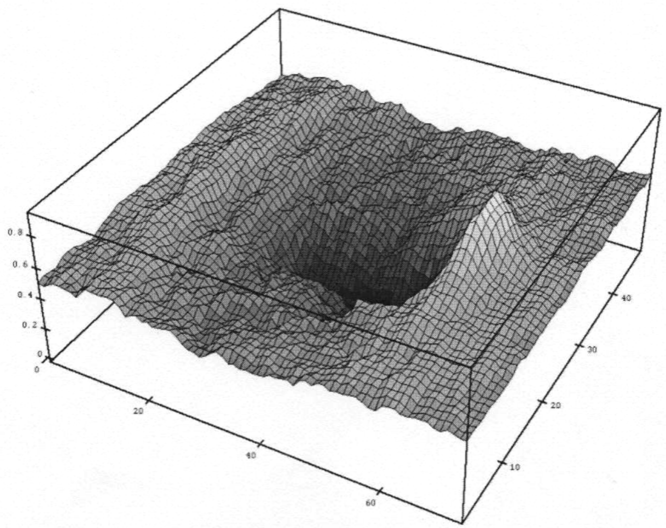


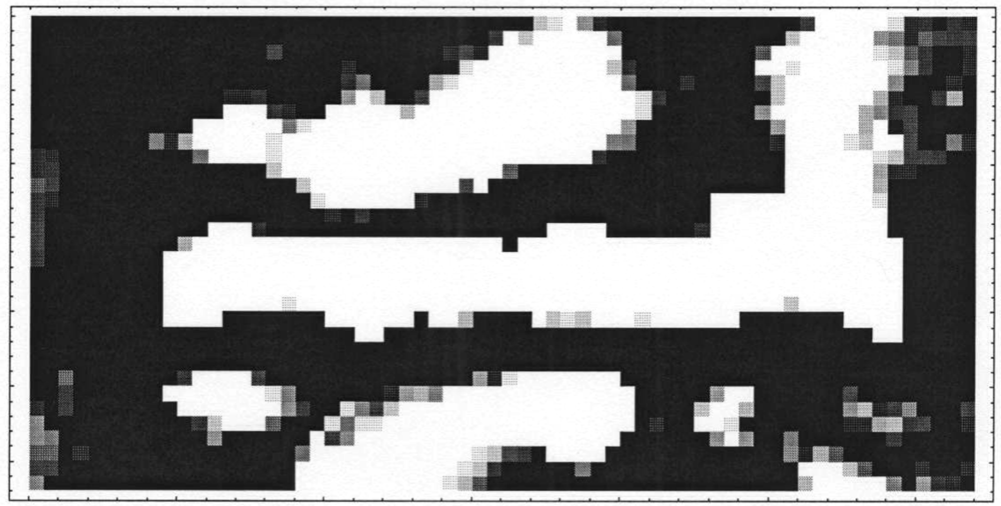
Learning the Sensor Model



the idealized model



part of the learned model



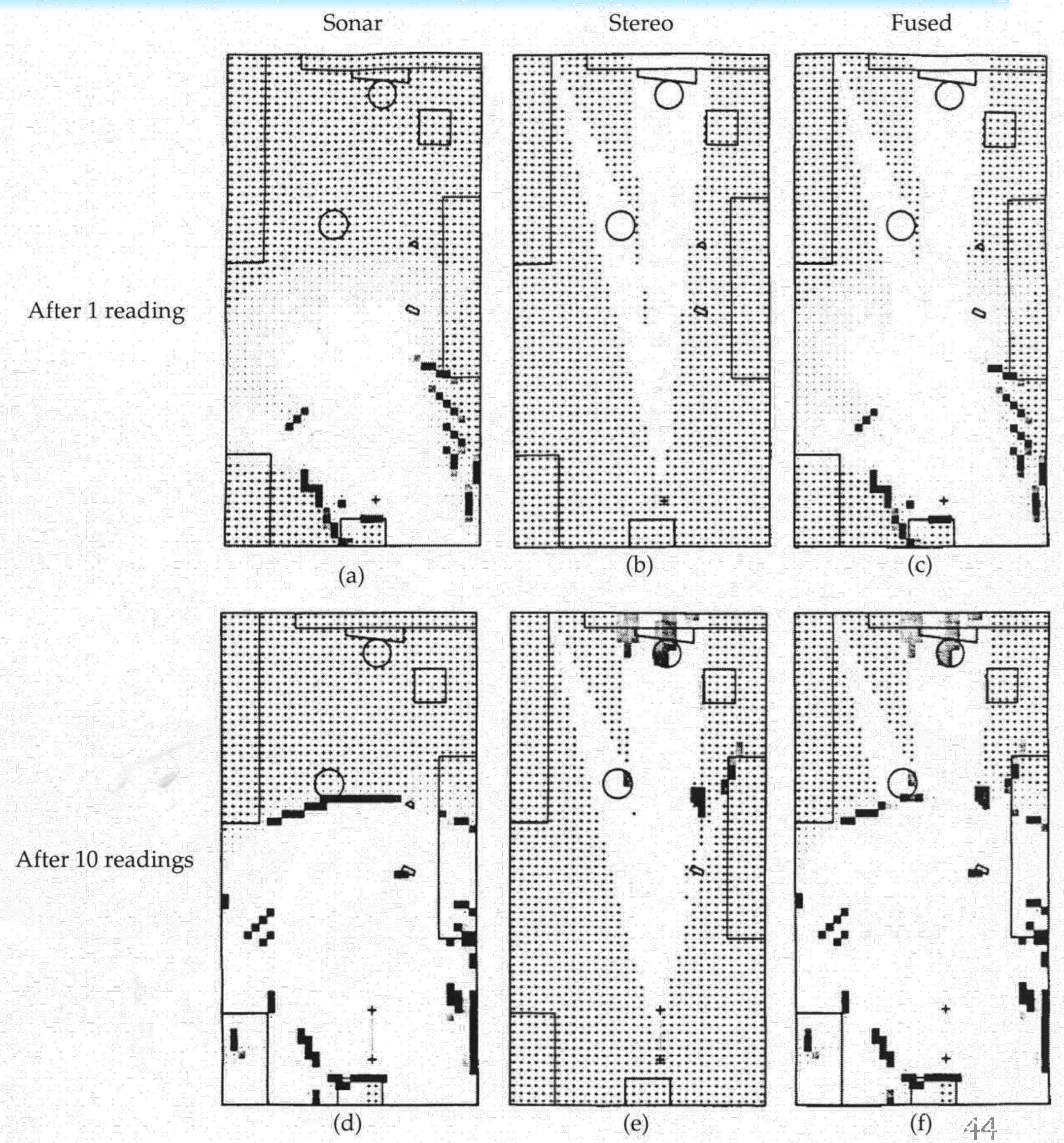
the mapping results of a model that had an even better match score (against the ideal map)

Sensor fusion

Incorporating data from other sensors -- e.g., IR rangefinders and stereo vision...

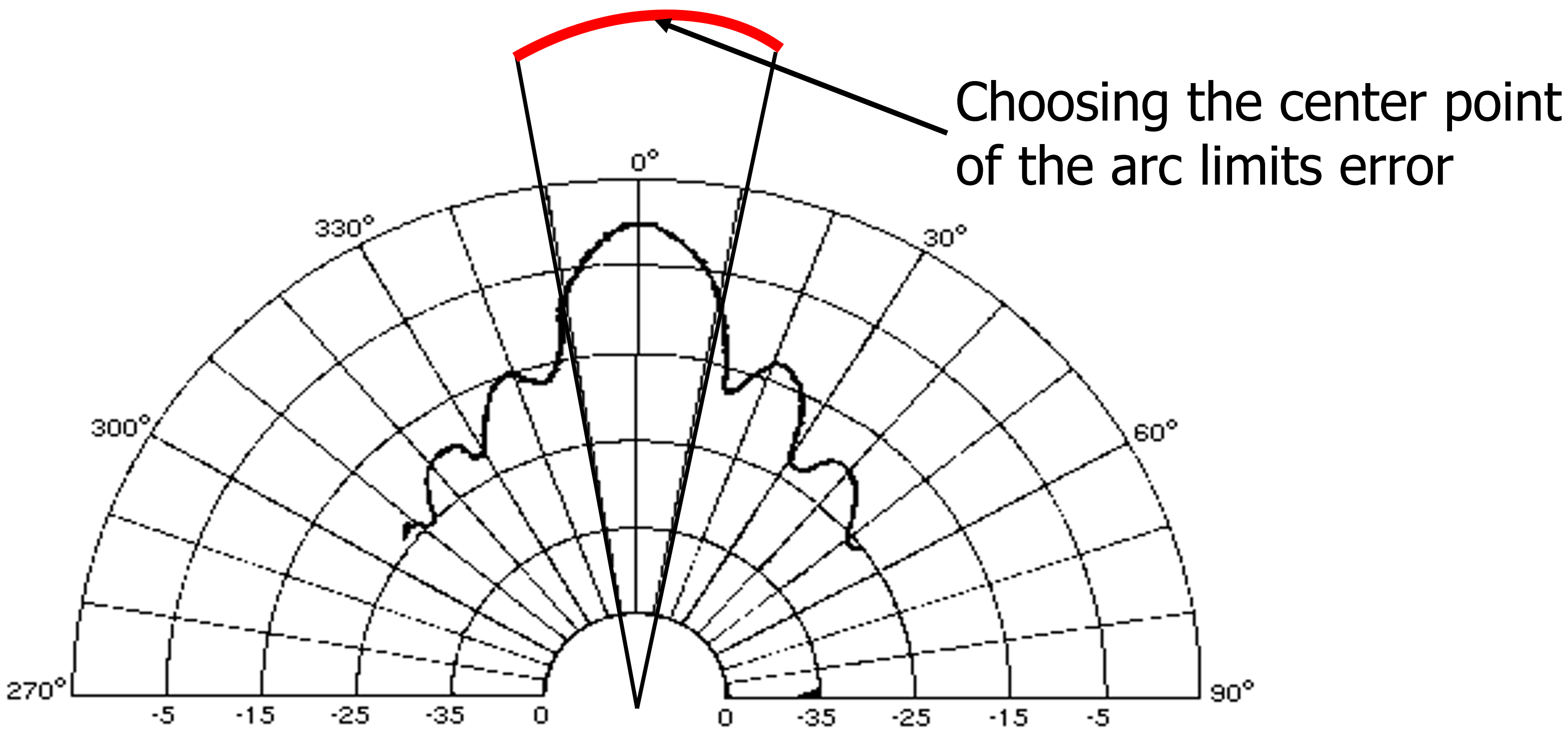
(1) create another sensor model

(2) update along with the sonar



Centerline

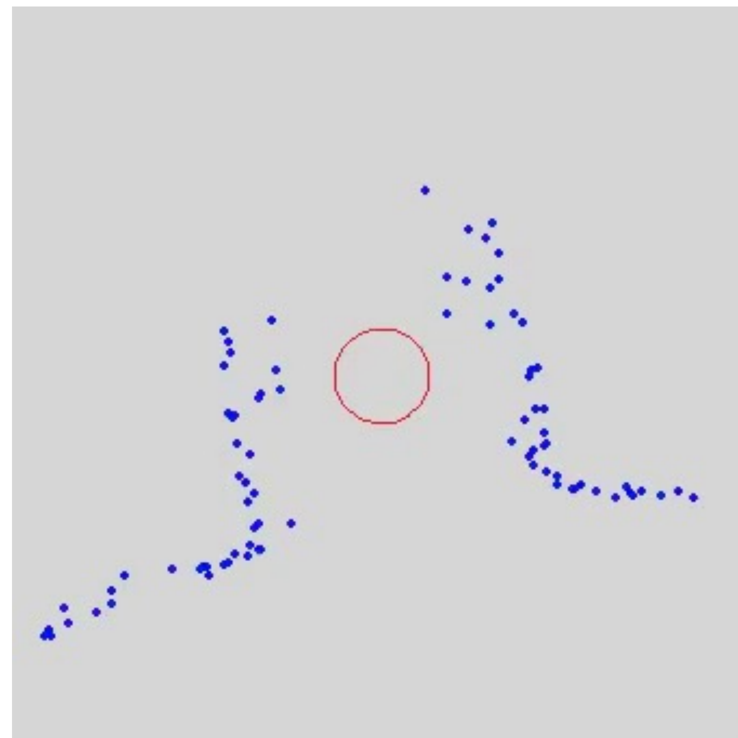
- Only consider region of significant response
- Approximate response with an arc of uniform probability



Centerline

- Advantages
  - Minimal computation required per sonar reading
  - Low latency
- Disadvantages
  - Inaccurate
  - Open areas may appear occluded

only centerline points displayed

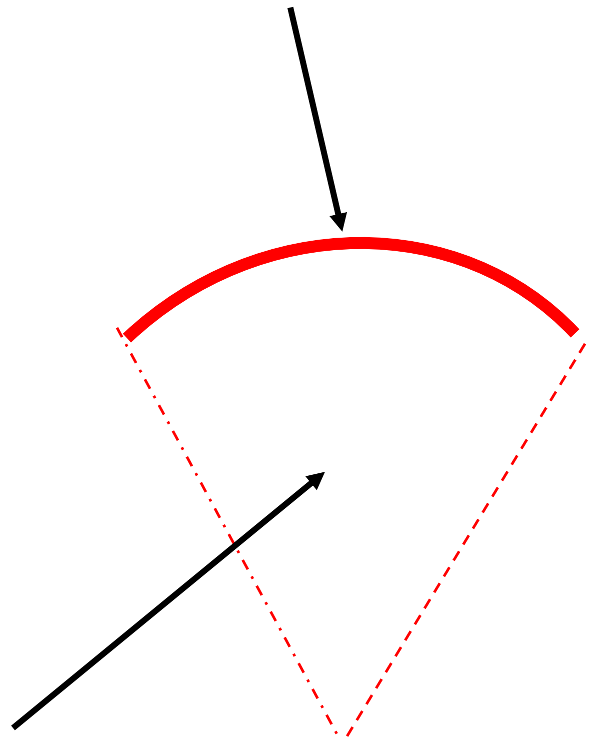


Fusing Multiple Readings

- Regions of Constant Depth (RCDs)
  - Leonard et al. 1995
- Arc Tangents
  - McKerrow 1993
- Arc Transversal Median (ATM)
  - Choset and Nagatani 1999
- Line Fitting
  - MacKenzie and Dudek 1994

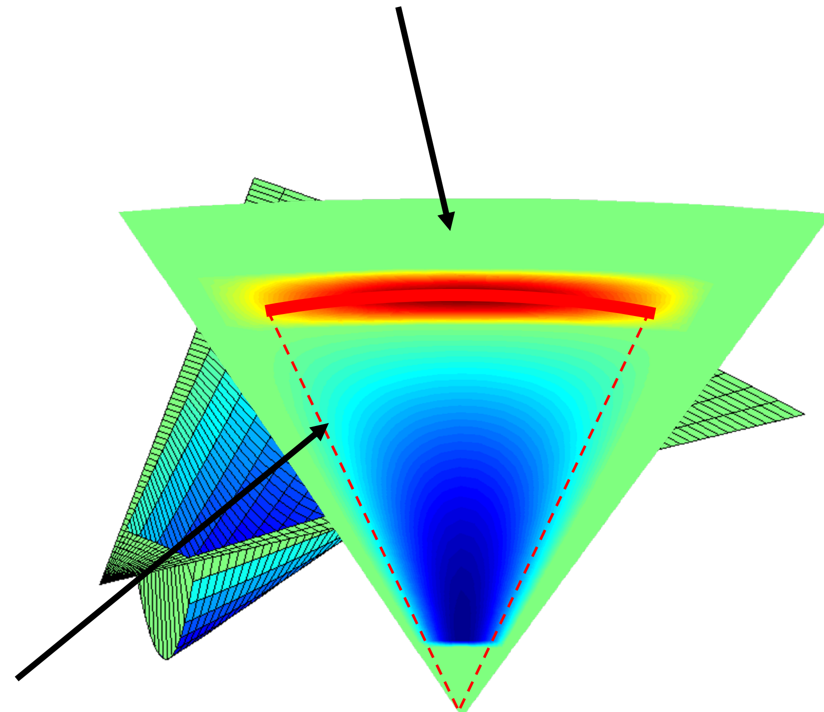
Arc Carving Sonar Model

- Represents a sonar return as a cone with an arc base
  - The arc approximates the sonar response
  - The interior of the cone represents a region of likely freespace



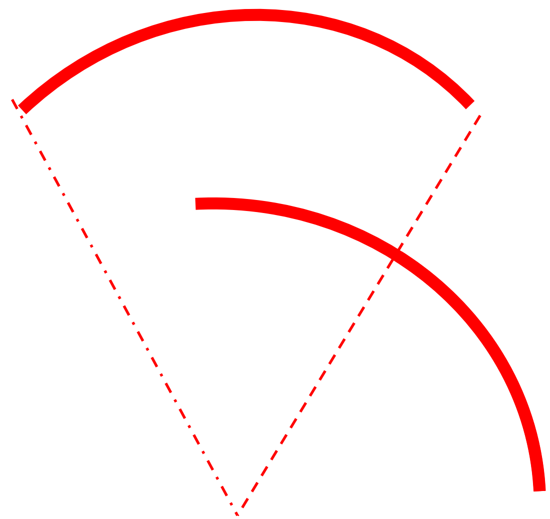
Occupancy Grid Sonar Model

- The arc carving model may be viewed as a binary approximation of the model used by Moravec and Elfes
  - An Arc with nonzero probability of occupancy
  - A cone with nonzero probability of freespace



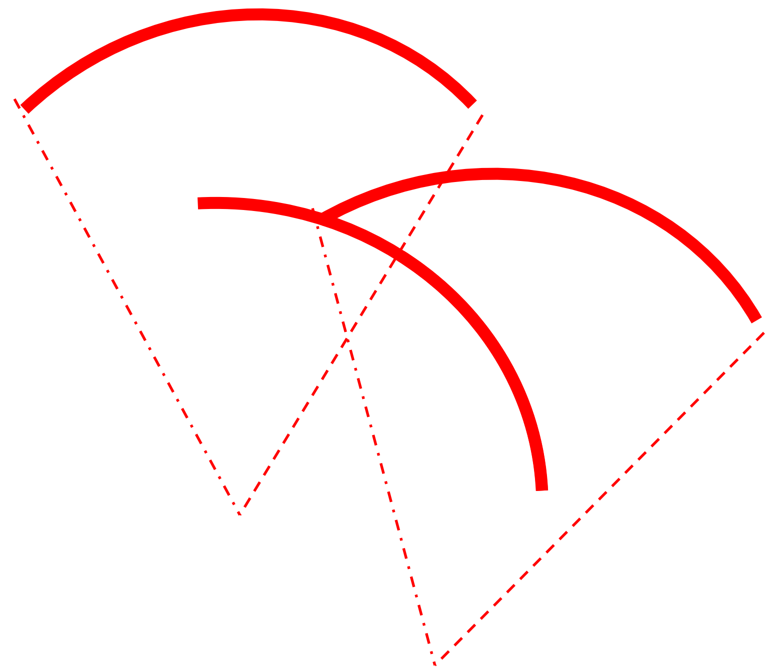
Arc Carving

- Each new sonar reading is checked against a history of previous readings
- If an arc is overlapped by the interior of a newer cone, the arc is "carved" to reflect this new information
- The updated arc is smaller, and therefore has a smaller bound on the error

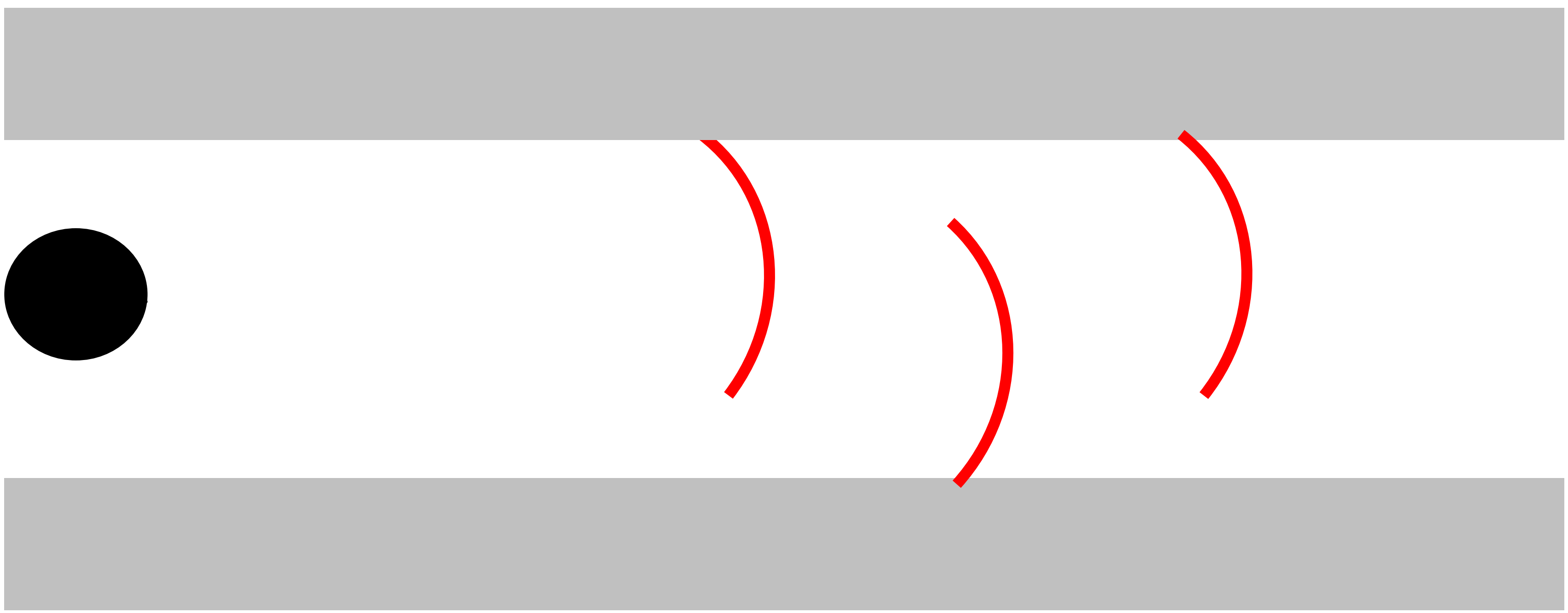


Arc Carving

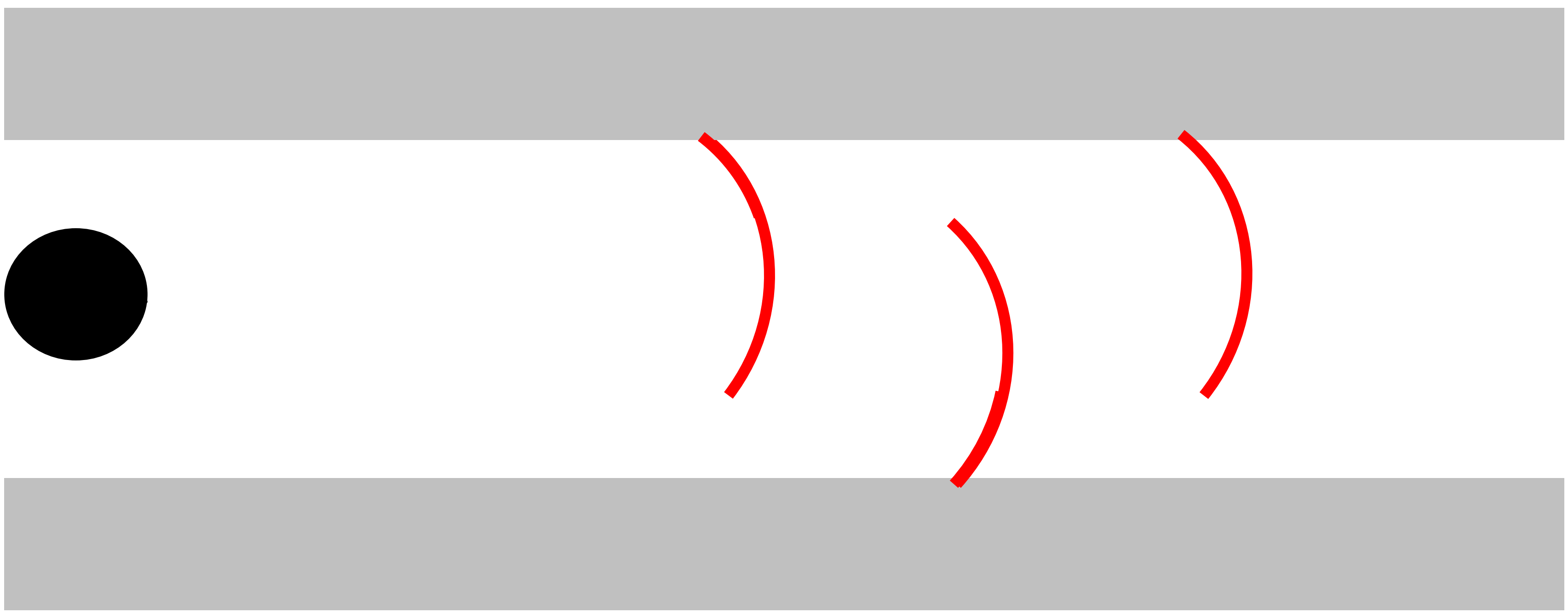
- Multiple passes of Arc Carving may completely remove an arc
  - Spurious sonar readings are removed
  - Response to dynamic environments is increased



Example – Ordinary Centerline



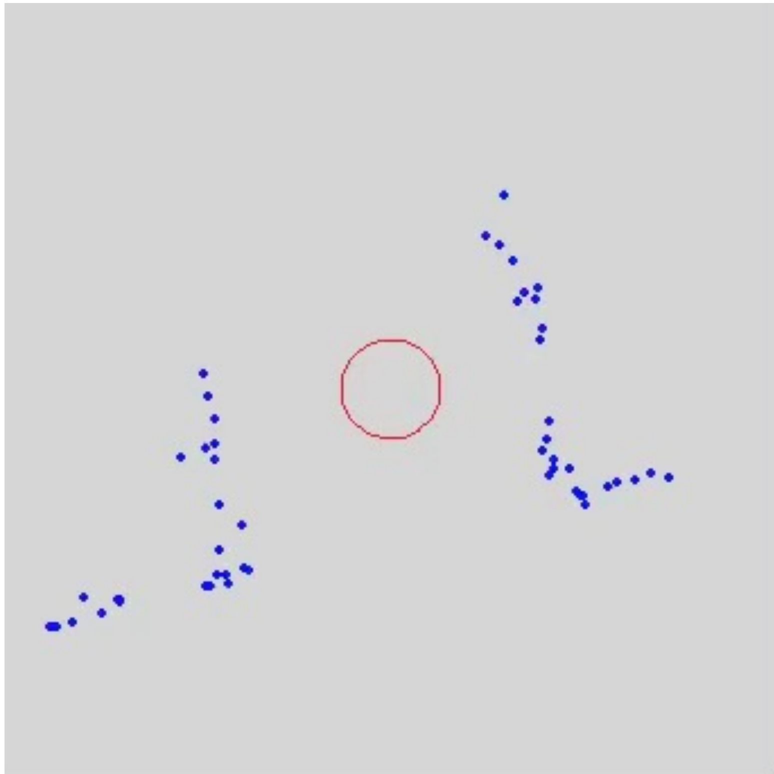
Example – Arc Carving



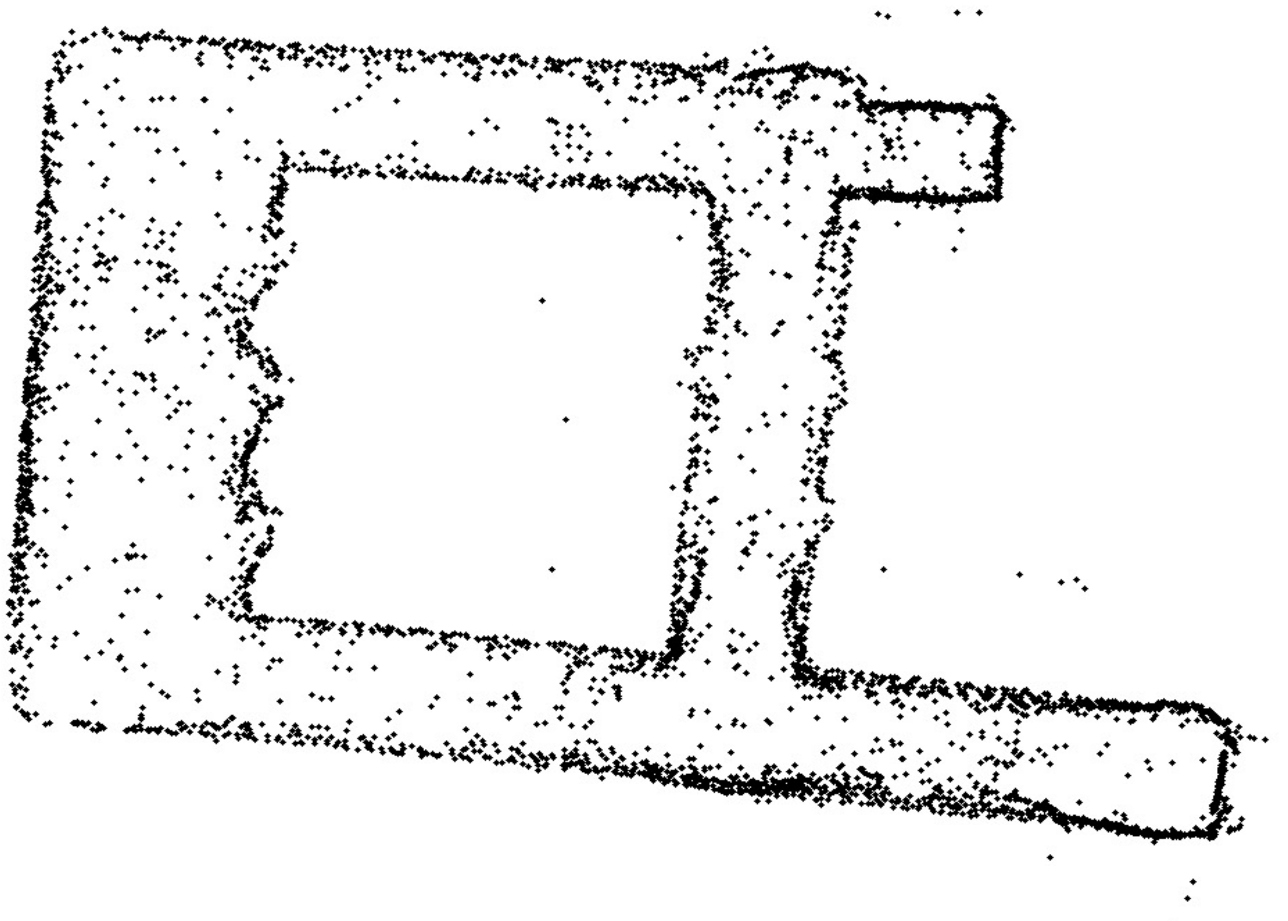
Arc Carving Video

- Latency issues are avoided
- The readings are more accurate than centerline
- Multiple reading approaches can be run off of the carved data

only carved points displayed



Experimental Results: Centerline Map



Experimental Results: Arc Carving Map