



UNIVERSITY OF  
**SOUTH CAROLINA**

# CSCE 574 ROBOTICS

## Locomotion

# Vehicle Locomotion

---

- Objective: convert desire to move  $A \rightarrow B$  into an actual motion:
  - How to arrange actuators (mechanical design)
  - actuator output  $\leftarrow \rightarrow$  Incremental motion: *Forward kinematics* and *inverse kinematics*



# Vehicle Locomotion

---

- *Forward Kinematics:*
  - (actuators actions)  $\rightarrow$  pose
- *Inverse Kinematics (inverse-K):*
  - pose  $\rightarrow$  (actuators actions)

$$\text{pose} = \{x, y, \theta\}$$



# Design Tradeoffs with Mobility Configurations

---

1. Maneuverability
2. Controllability
3. Traction
4. Climbing ability
5. Stability
6. Efficiency
7. Maintenance
8. Navigational considerations



# Navigational considerations

---

- Some mechanisms are more accurate and reliable.
- Some are mathematically more easily predicted and controlled.



# Wheeled Vehicles



# Differential Drive

- 2 wheels
- 2 points of contact
- 2 degrees of freedom



- Translation and rotation are *coupled*
  - “You can't have one without the other”.
  - F. Sinatra
  - Control is a "little bit" complicated.

# Differential drive

---

## Basic design:

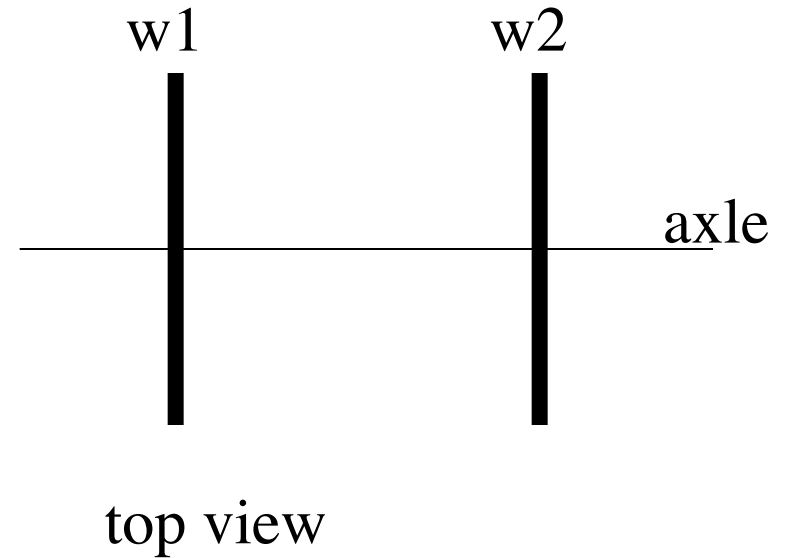
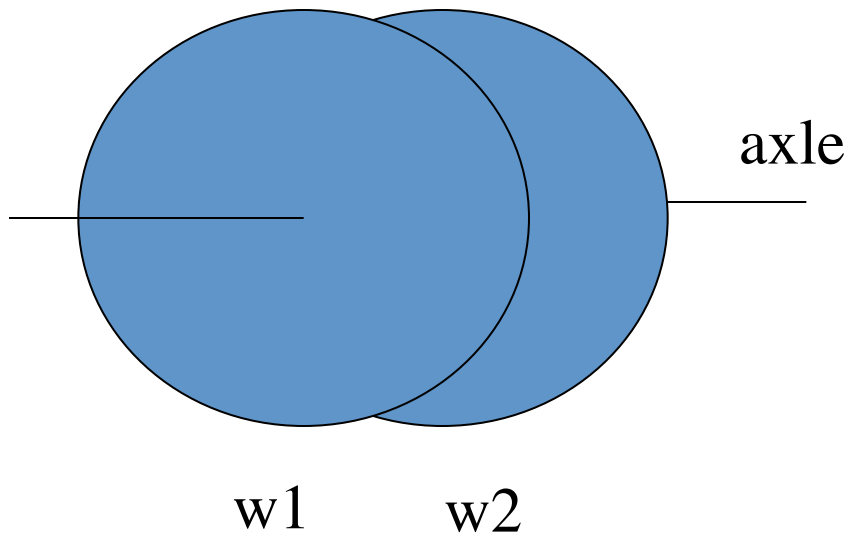
- 2 circular wheels
- infinitely thin
- same diameter
- mounted along a common axis
- vehicle body is irrelevant (in theory).





# Idealized differential drive

side view



# Differential Drive Intuition

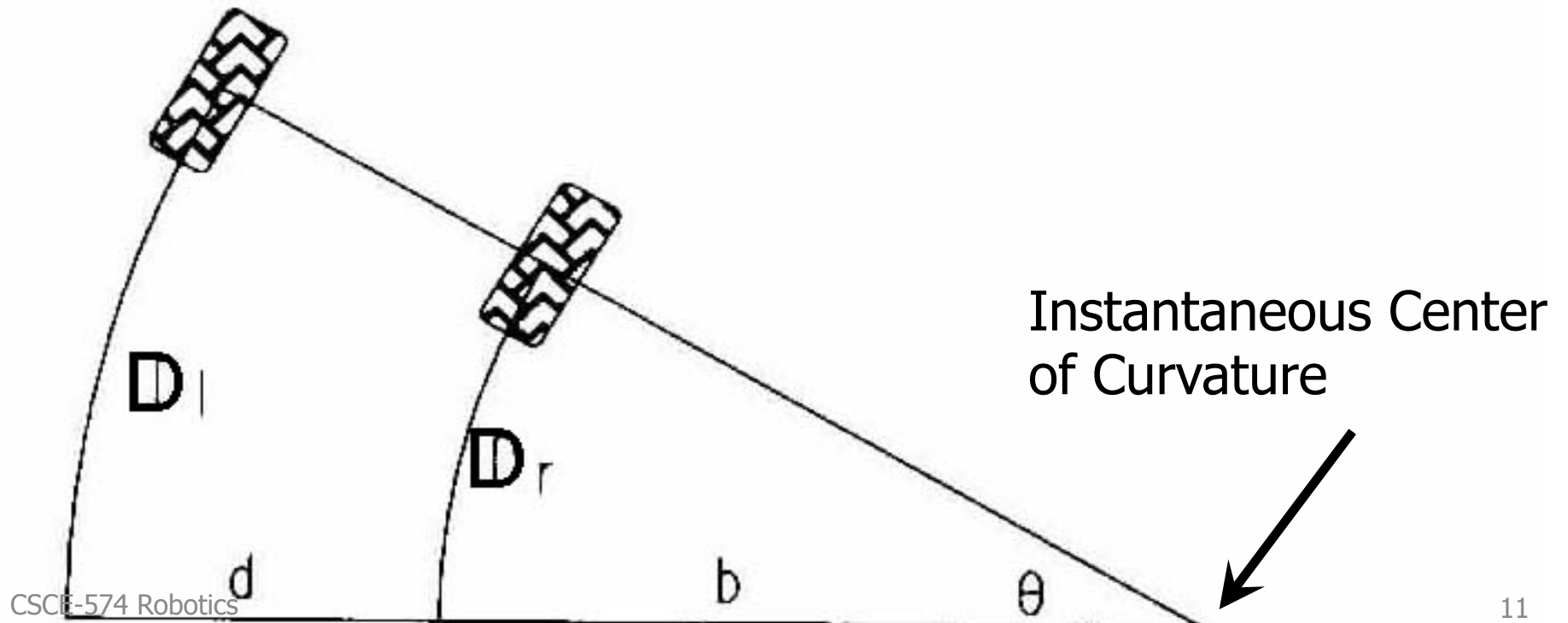
---

- Drive straight ahead?
- Turn in place?
- (these are questions of *kinematics*)



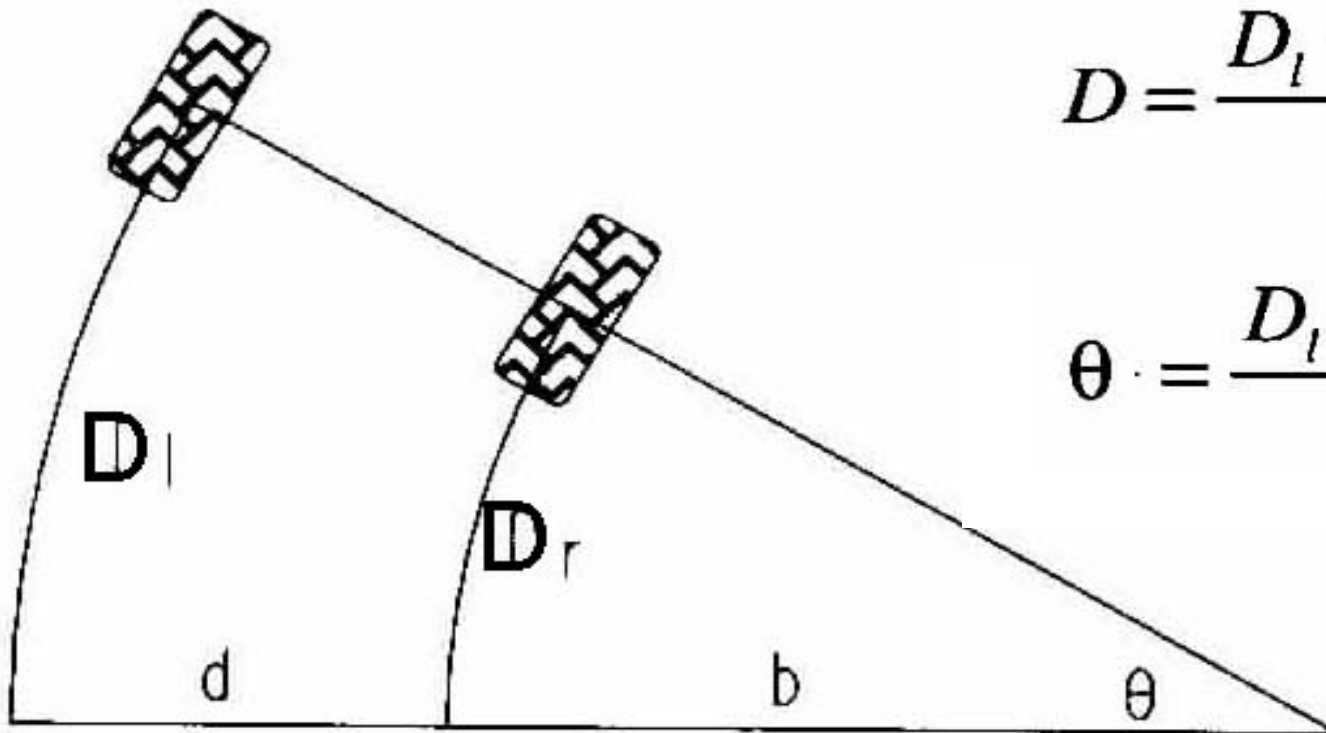
# Differential Drive Observation

- Vehicle rotation can be described relative to an axis running through the two wheels.



# Forward Kinematics of Differential Drive

- Wheel rotation by angle  $\phi_1, \phi_2$
- Distance of wheel motion  $D_i = \phi_i r$



$$D = \frac{D_l + D_r}{2}$$

$$\theta = \frac{D_l - D_r}{d}$$

# Forward Kinematics: Path Integration

- $D$ ,  $\theta$  determine *differential* motion:
  - the tangent and velocity of the vehicle motion.
- To get the path followed, you have to integrate over *time*.

$$x(t) = \frac{1}{2} \int_0^t [v_r(t) + v_l(t)] \cos[\theta(t)] dt$$

$$y(t) = \frac{1}{2} \int_0^t [v_r(t) + v_l(t)] \sin[\theta(t)] dt$$

$$\theta(t) = \frac{1}{d} \int_0^t [v_r(t) - v_l(t)] dt$$



# Non-Holonomic Constraints

---

- Cannot change robot pose arbitrarily
- In D.D: Robot cannot move sideways
- Complicates planning:
  - Parallel parking...



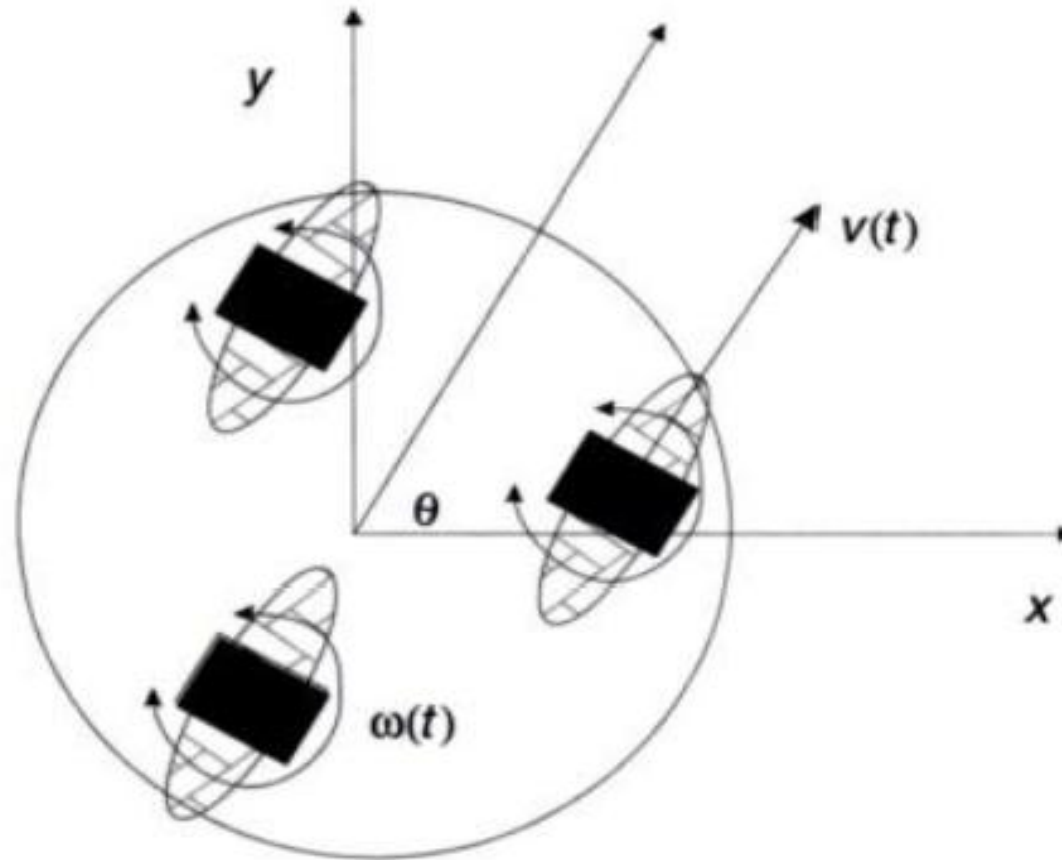
# Differential Drive Issues

---

- Matching of drive mechanisms
  - Tire wear ( $r$  is wrong)
  - Motors ( $\phi$  is wrong)
  - Ground traction (rotation  $\phi r$  is not motion of  $\phi r$ )
  - Net result: motion  $\phi r$  is actually wrong
- Balance
  - Castor (caster) wheel



# Synchronous Drive





# Forward Kinematic - Synchronous Drive

---

- Simpler:

$$x(t) = \frac{1}{2} \int_0^t v(t) \cos[\theta(t)] dt$$

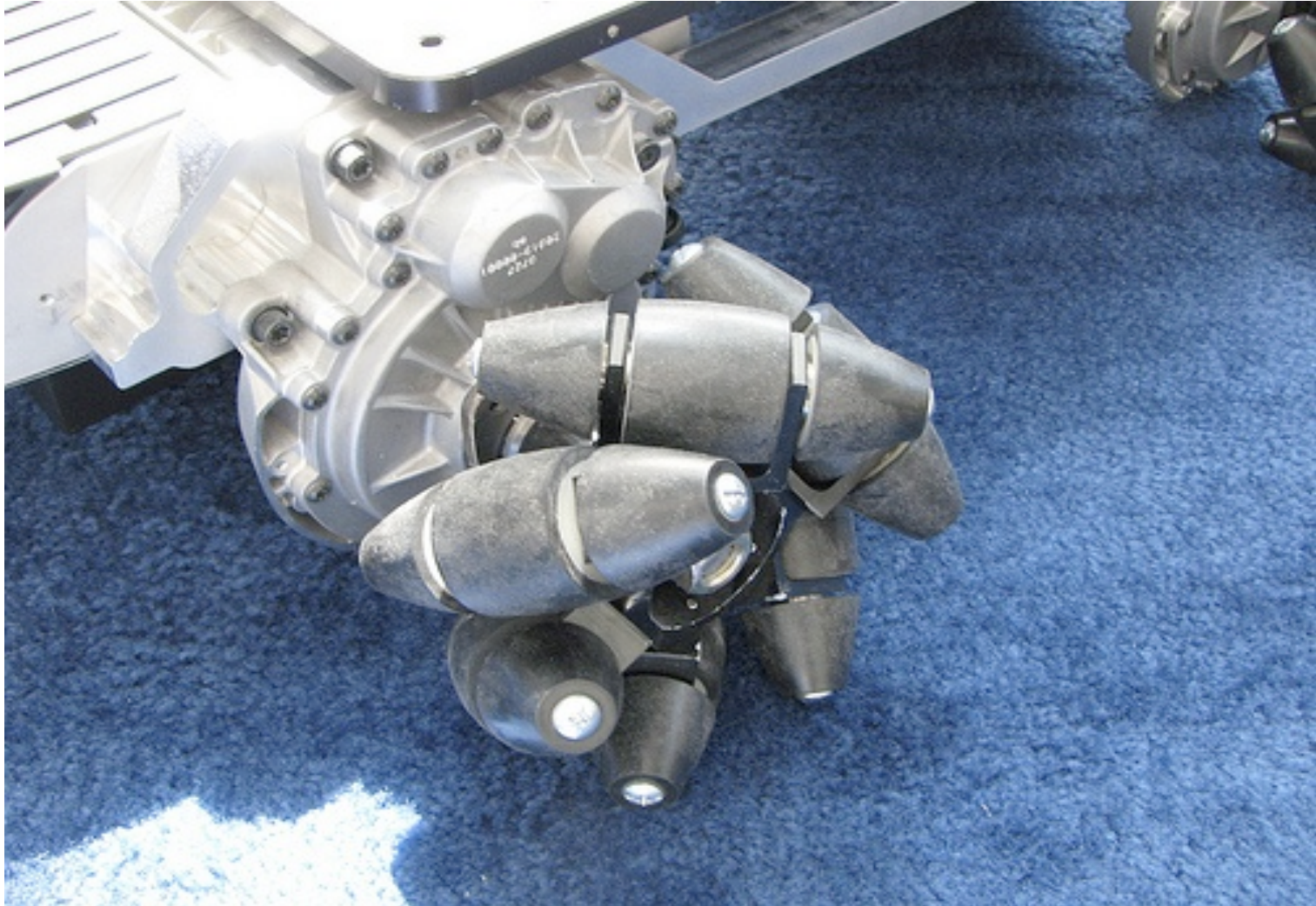
$$y(t) = \frac{1}{2} \int_0^t v(t) \sin[\theta(t)] dt$$

$$\theta(t) = \int_0^t \omega(t) dt$$

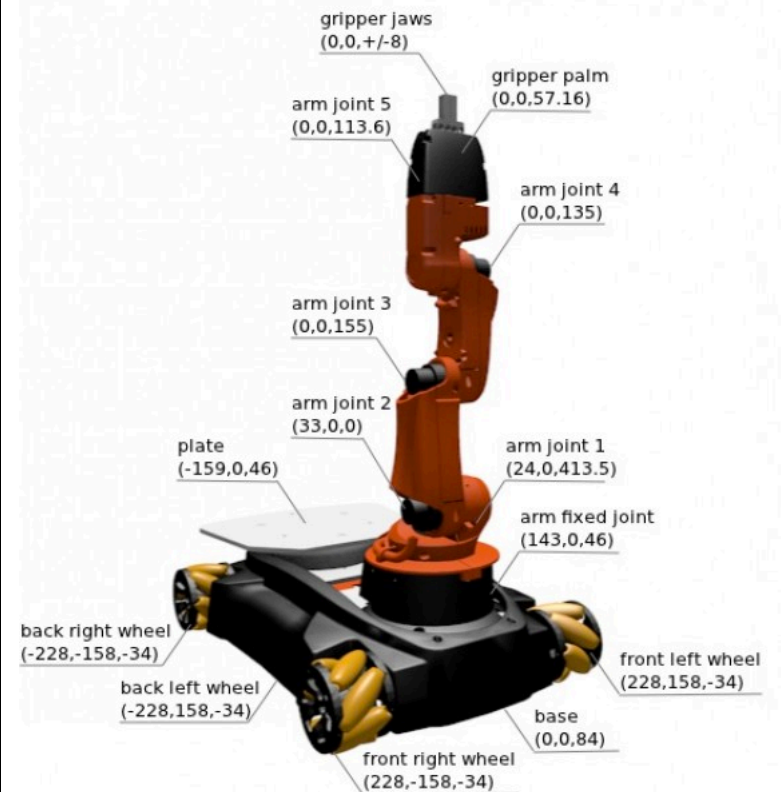
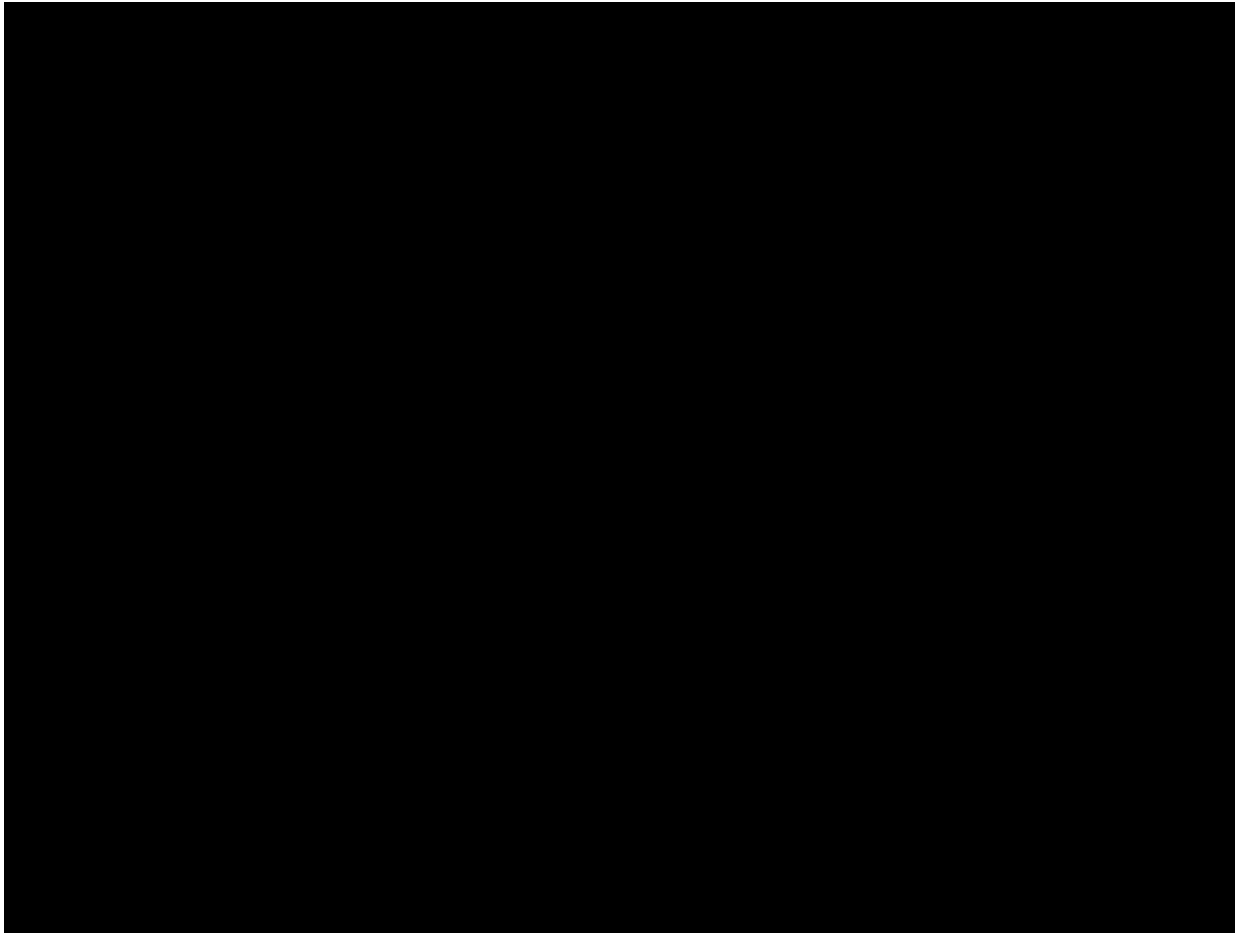
- Will not suffer from mechanical mismatch compared to Diff. Drive



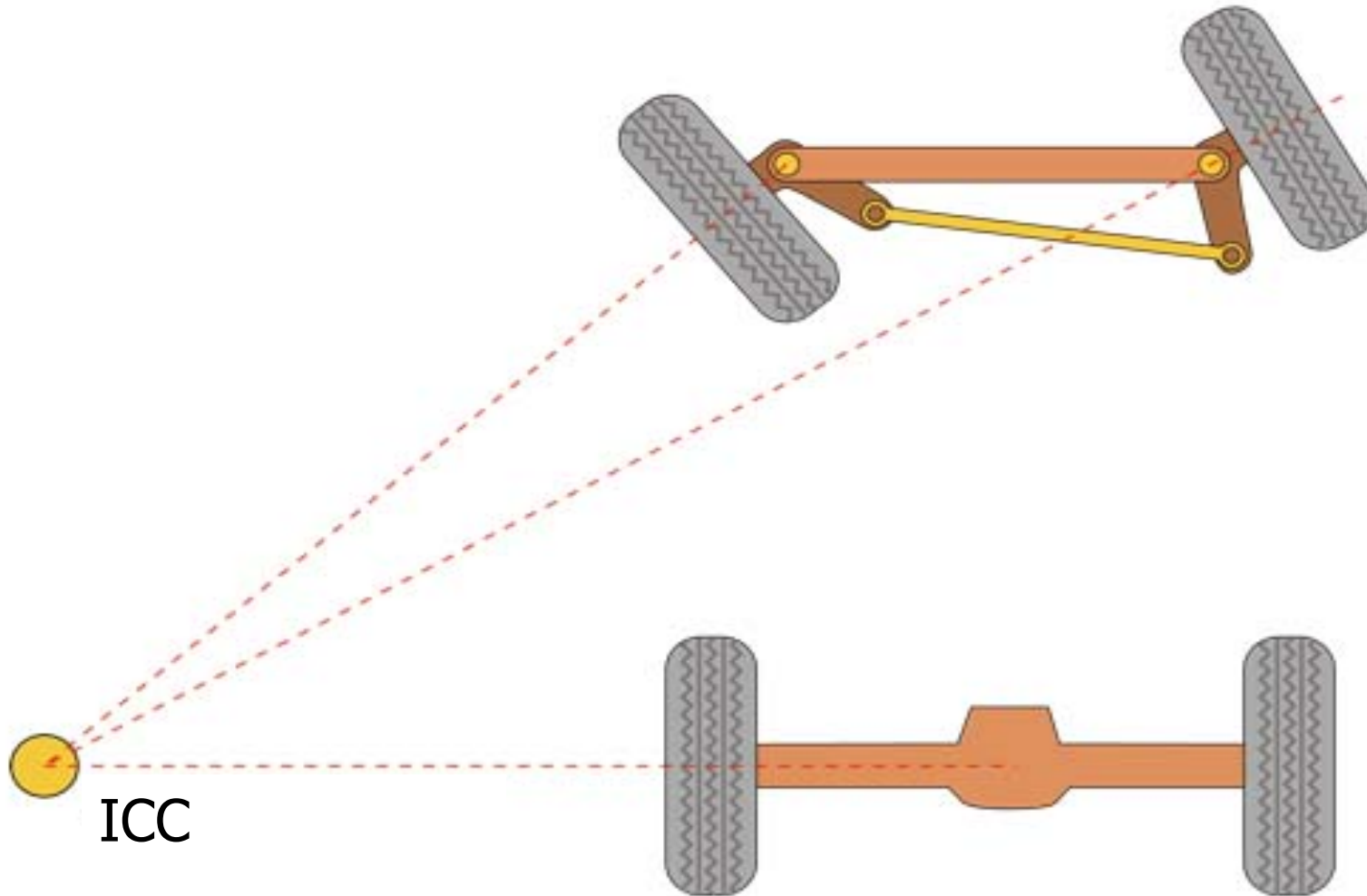
# Mecanum Wheels



# Mecanum Wheels



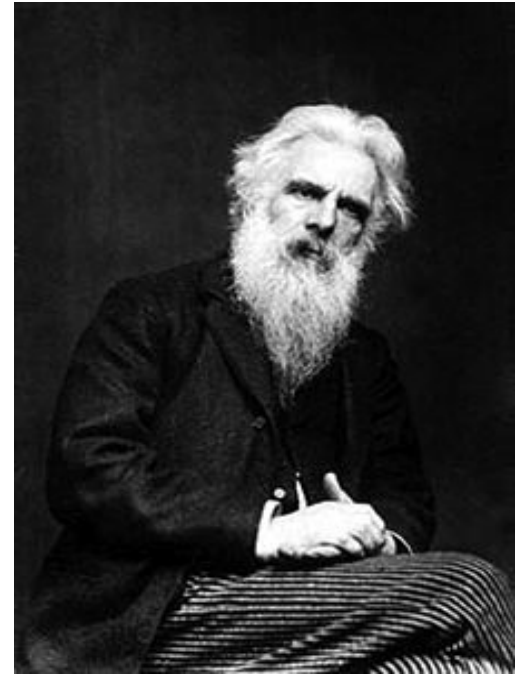
# Ackerman (Used in Cars)



# Legged Locomotion

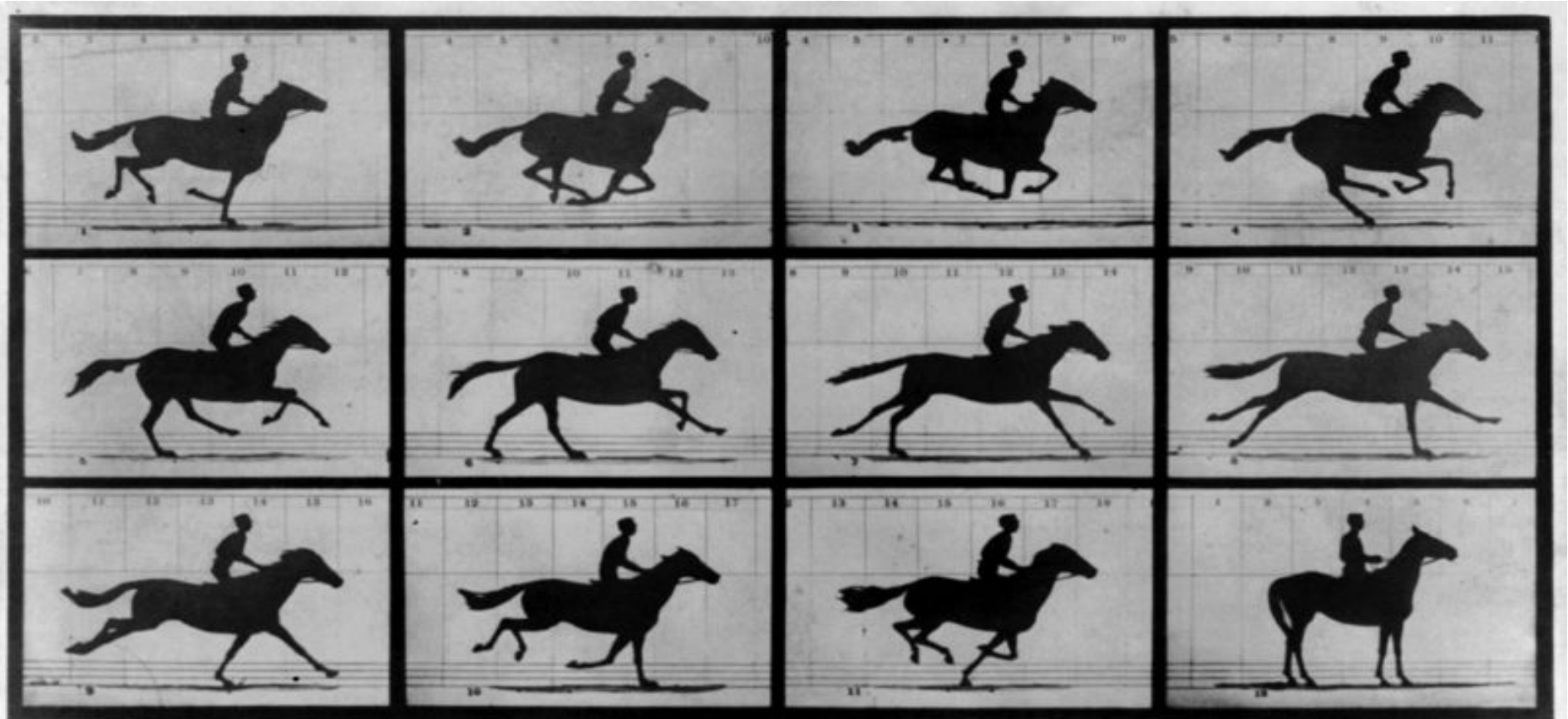
---

- Started to resolve a bet between Governor of California *Leland Stanford* and a friend, in 1872.
- Muybridge took the challenge



Eadweard Muybridge  
(*April 9, 1830 – May 8, 1904*)

# Legged Locomotion



Copyright, 1878, by MUYBRIDGE.

MORSE'S Gallery, 417 Montgomery St., San Francisco.

## THE HORSE IN MOTION.

Illustrated by  
MUYBRIDGE.

AUTOMATIC ELECTRO-PHOTOGRAPH.

"SALLIE GARDNER," owned by LELAND STANFORD; running at a 1.40 gait over the Palo Alto track, 19th June, 1878.

The negatives of these photographs were made at intervals of twenty-seven inches of distance, and about the twenty-fifth part of a second of time; they illustrate consecutive positions assumed in each twenty-seven inches of progress during a single stride of the mare. The vertical lines were twenty-seven inches apart; the horizontal lines represent elevations of four inches each. The exposure of each negative was less than the two-thousandth part of a second.



# Hildebrand Gait Diagrams

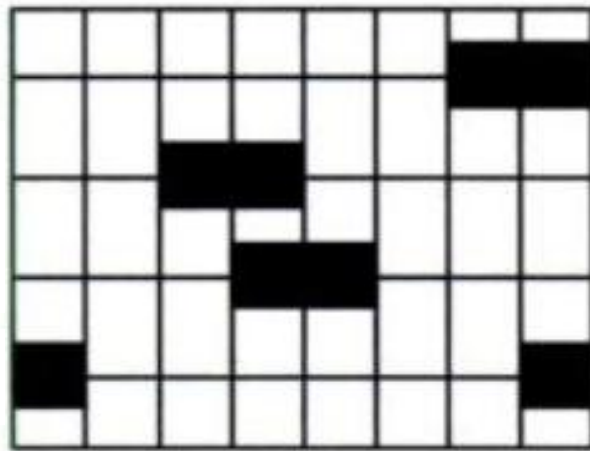
## Trot

Front Left

Front Right

Back Left

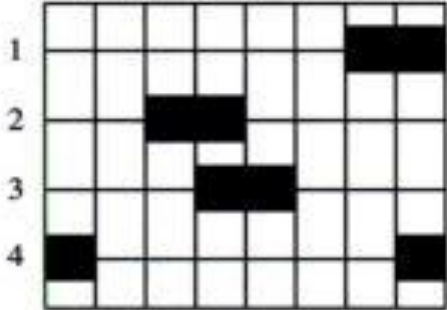
Back Right



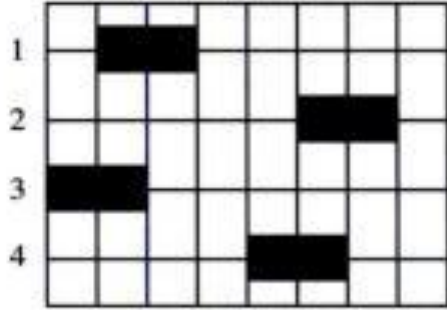
↑ Trot ↑  
Ballistic Phase



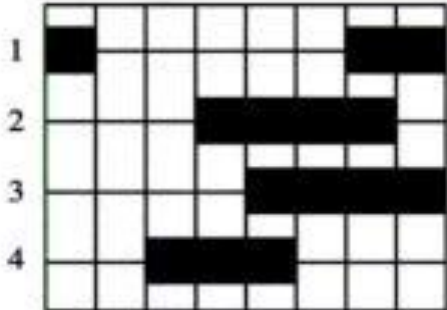
# Hildebrand Gait Diagrams



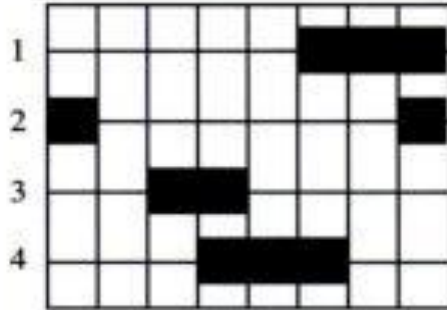
Trot



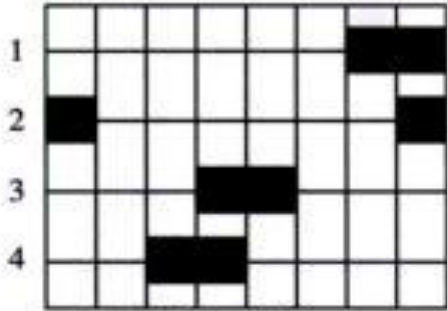
Rack



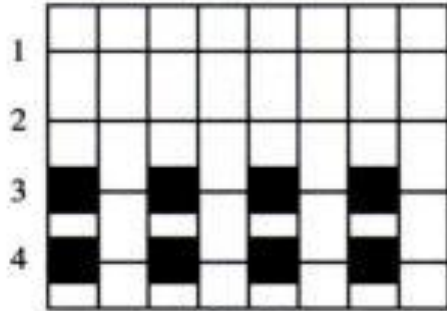
Canter



Transverse Gallop



Rotary Gallop



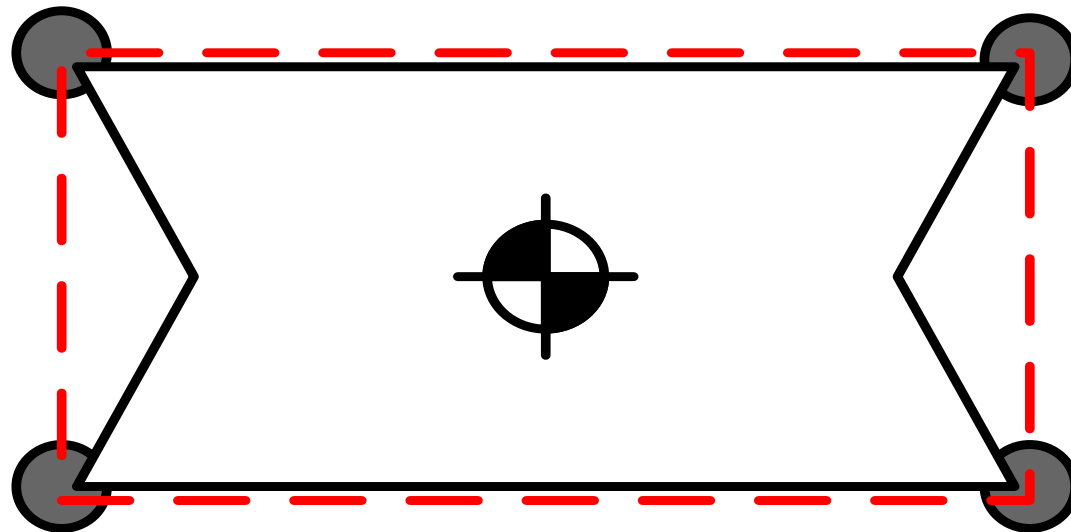
Ricochet





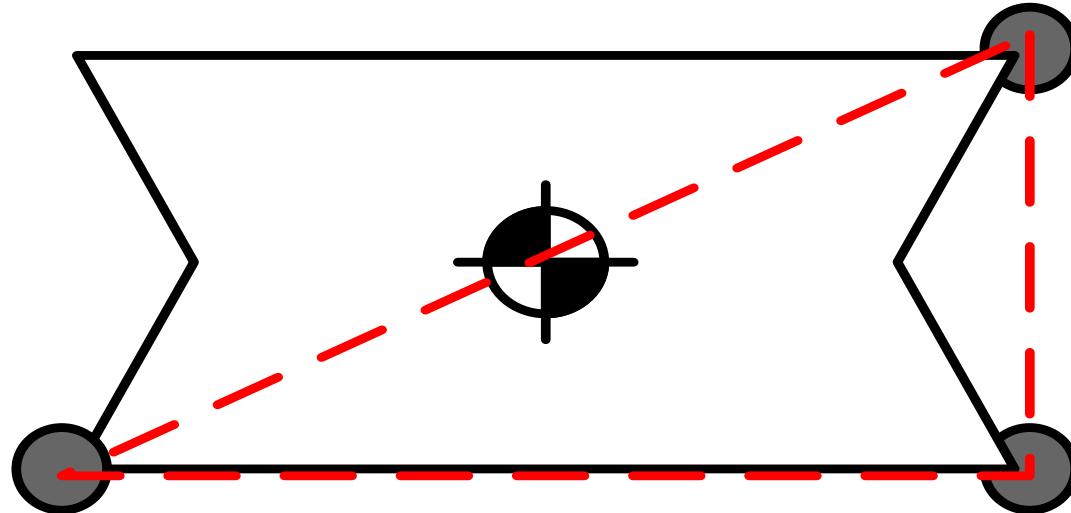
# Support Polygon

---

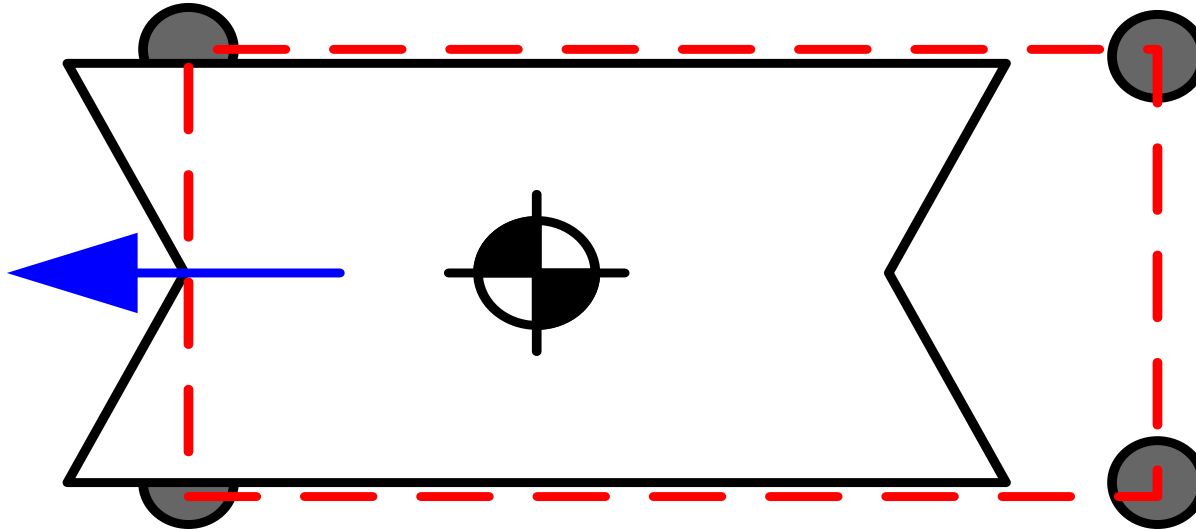


# Support Polygon

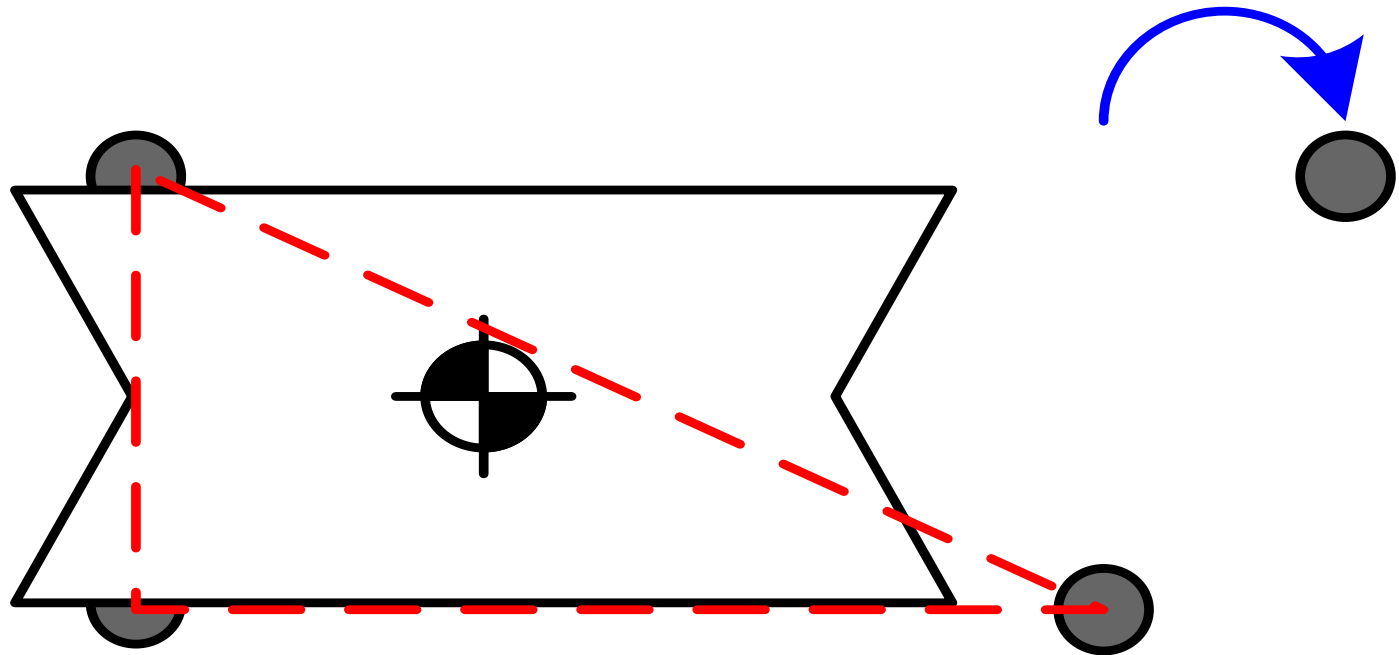
---



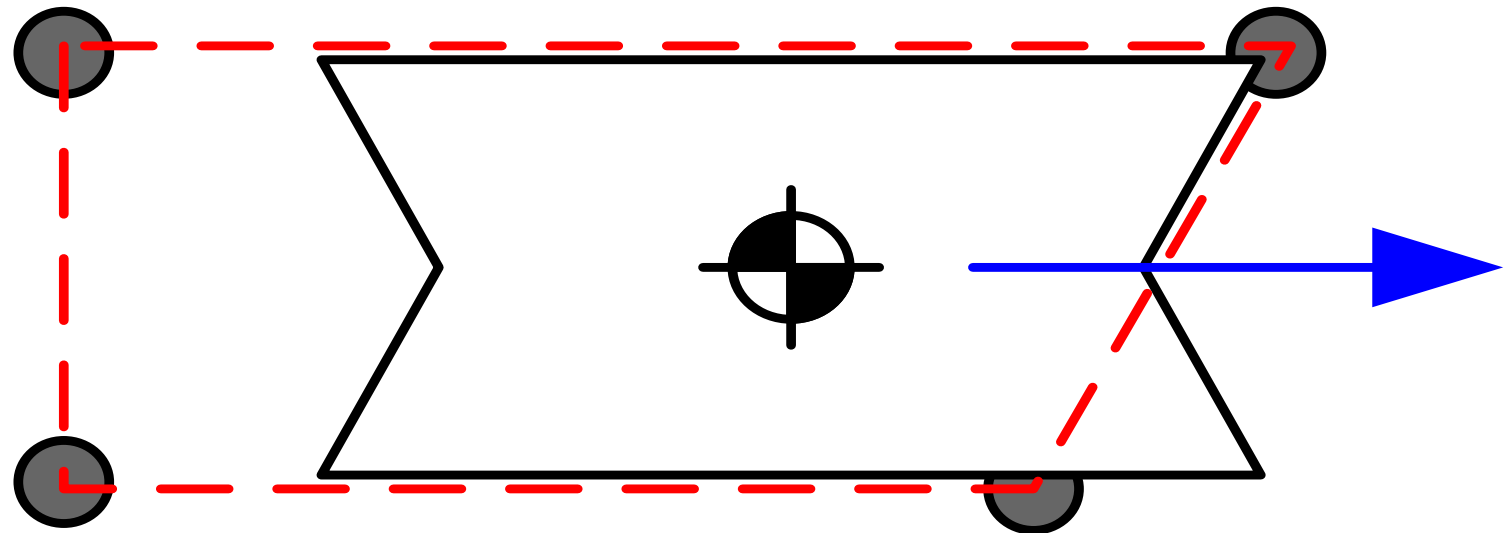
# Support Polygon



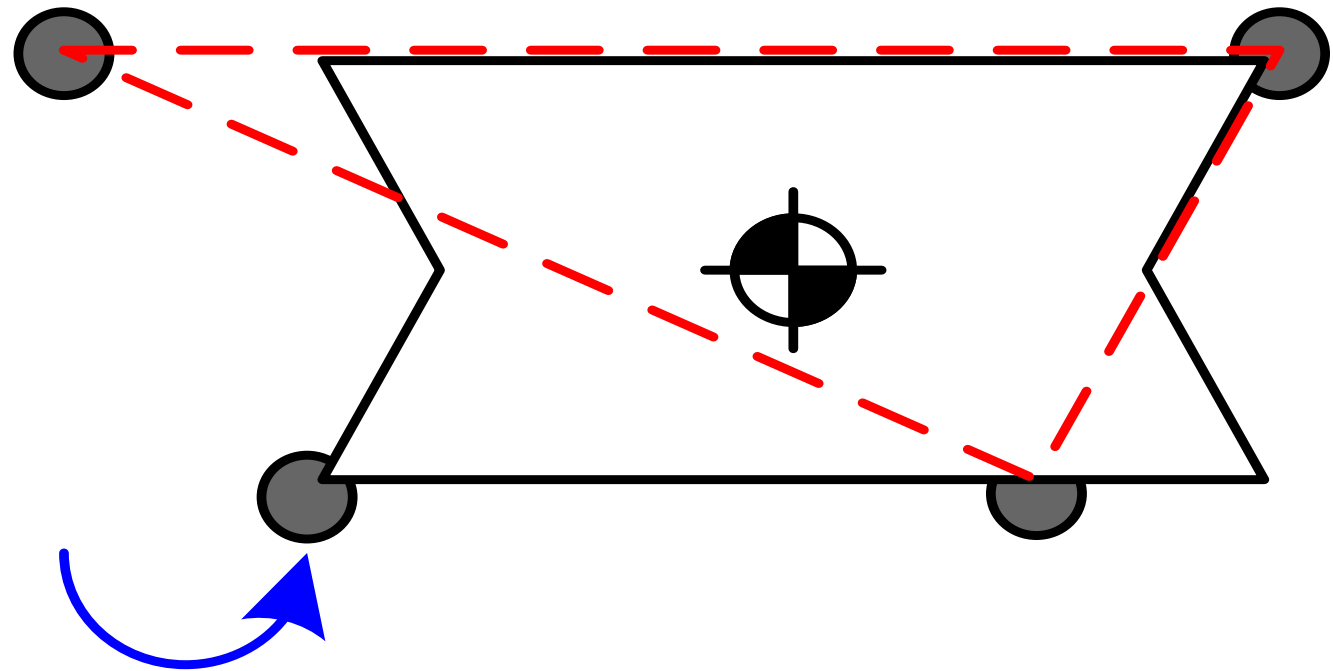
# Support Polygon



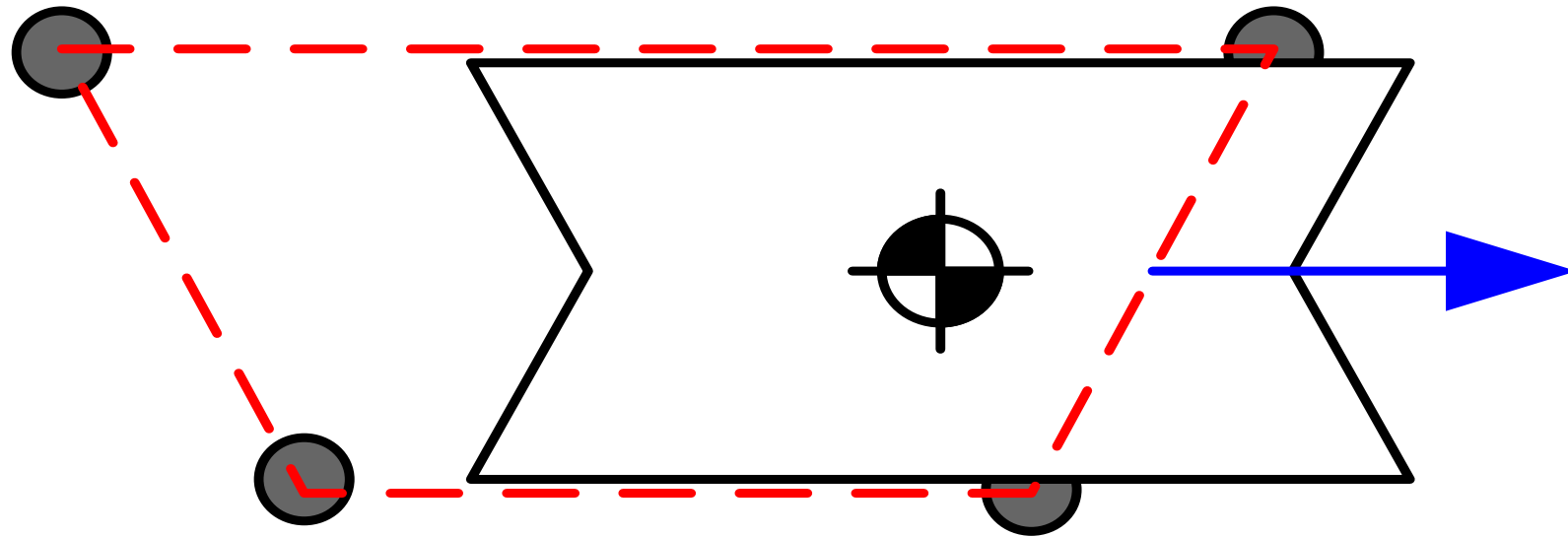
# Support Polygon



# Support Polygon

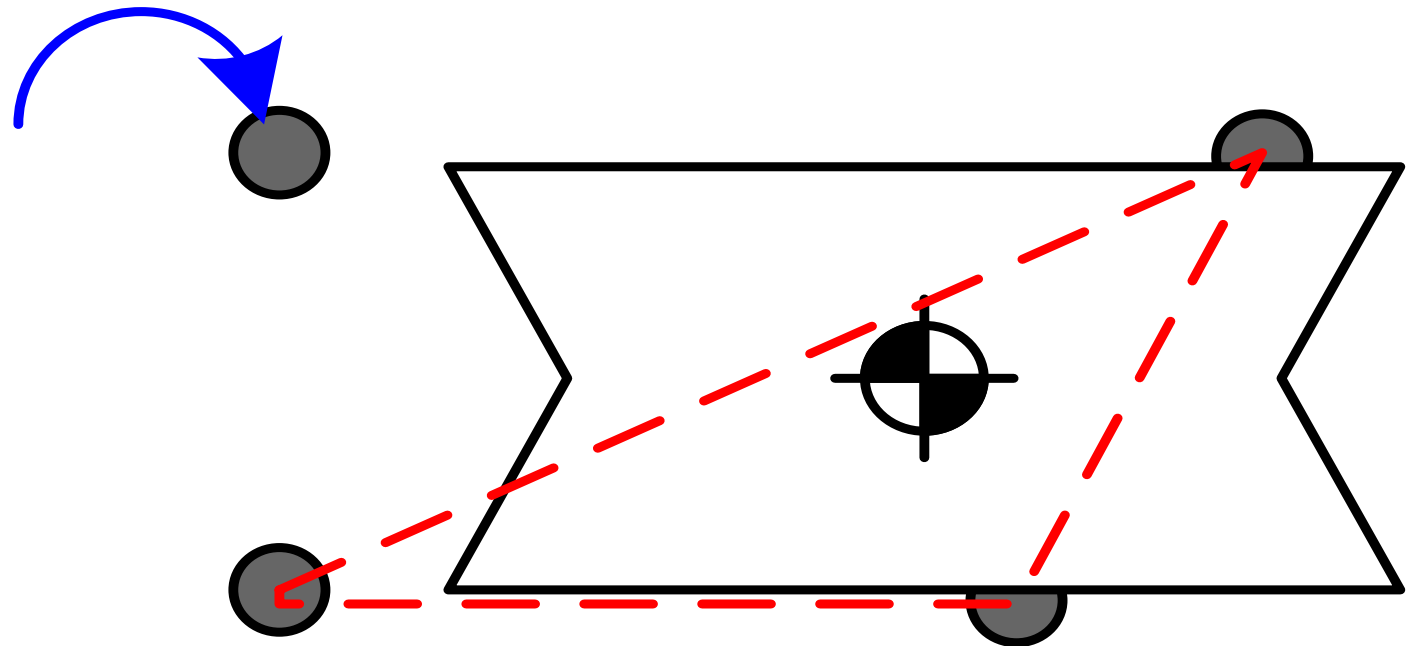


# Support Polygon



# Support Polygon

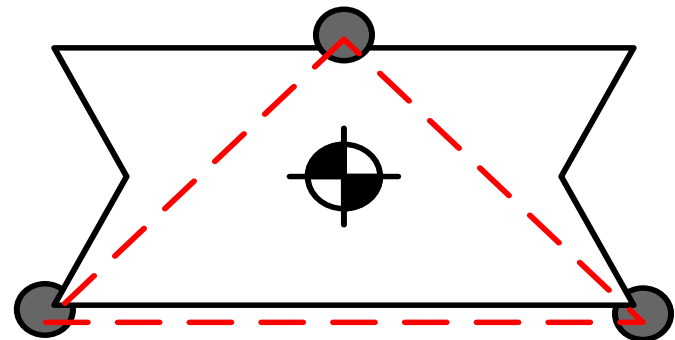
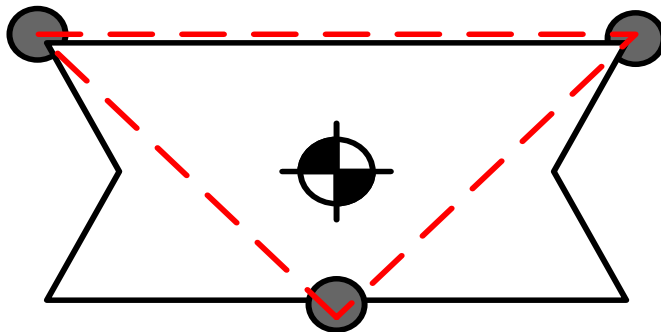
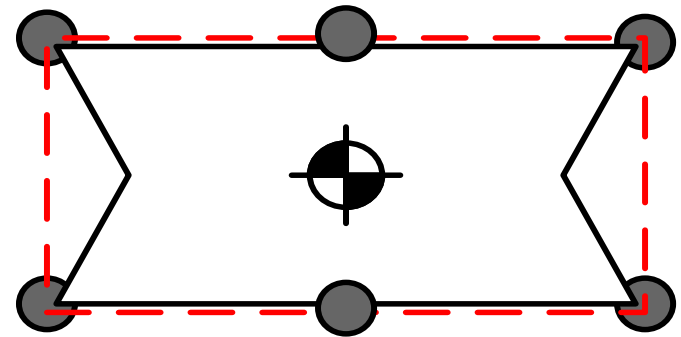
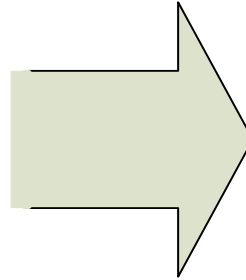
---



And so on...



# Hexapod RHex



# RHex: Tripod Gait



# Bi-Pedal: Zero Moment Point

---



# Dynamically Stable Gaits

---

- Robot is not always statically stable
- Must consider energy in limbs and body
- Much more complex to analyze
- E.G. Running:
  - Energy exchange:
    - Potential (ballistic)
    - Mechanical (compliance of springs/muscle)
    - Kinetic (impact)

