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Note Title 2016-01-21

As observed on p. 32 [J07], the d-separation algorithm based on the definition can be very inefficient, because it requires checking every chain (path) between two variables.

There is an efficient algorithm based on the following theorem due to Steffen Lauritzen.

Thm. Let A, B, S be mutually exclusive sets of variables (nodes) in a causal network (DAG). Then, A and B are d-separated by S whenever A and B are separated (in the usual undirected graph sense) by S in the graph $\left(\mathcal{G}_{An(A \cup B \cup S)}\right)^m$, the moral graph of the smallest ancestral set containing $A \cup B \cup S$.

$\square$

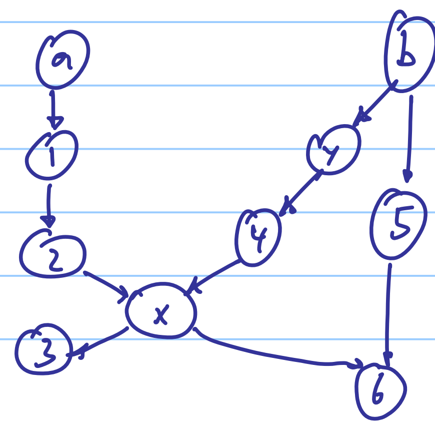
$An(U)$ is the smallest ancestral set of U, i.e., the smallest set containing U and its ancestors.

$\mathcal{G}_W$ is the induced subgraph of $\mathcal{G}$ that includes the vertices of W.

$\mathcal{G}^m$, where $\mathcal{G}$ is a directed graph, is the undirected graph with the same nodes as $\mathcal{G}$, (undirected) edges for each (directed) edge of $\mathcal{G}$ and additional edges between nodes that have a common child in $\mathcal{G}$. (Such nodes are said to "marry" the parents of a common child; hence the name moral graph)

Example

$\mathcal{G}$



$A = \{a\}$

$B = \{b\}$

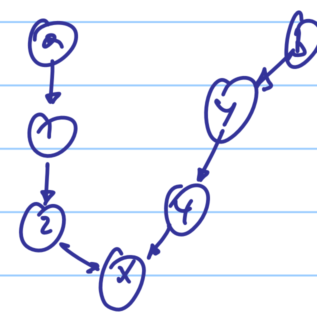
$S = \{x, y\}$

Are A and B d-seperated by S?

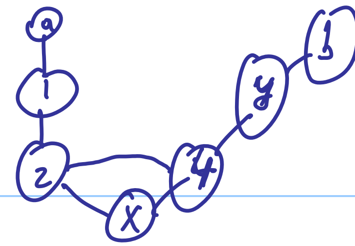
$$A \cup B \cup S = \{a\} \cup \{b\} \cup \{x, y\} = \{a, b, x, y\}$$

$$A_n(A \cup B \cup S) = A_n(\{a, b, x, y\}) = \{a, b, x, y, 1, 2, 4\}$$

$$\mathcal{G}_{A_n(A \cup B \cup S)} = \mathcal{G}_{\{a, b, x, y, 1, 2, 4\}} =$$



$$\left(\mathcal{G}_{An(A \cup B \cup S)}\right)^m =$$



In this graph, $\{x, y\}$ separates $\{a\}$ from $\{b\}$, so the answer is yes.