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Thm (The chain rule for Bayesian networks)

Let $G = (V, E, P)$ be a BN. Then,

$$P(V) = \prod_{v \in V} P(v \mid c(v)).$$

[The joint prob. of all vars in a BN is the product of its CPTs.]

conditional probability table,

Let $|V| = n$. Reorder (if necessary) the variables in V, so that $\langle v_1, v_2, \ldots, v_{n-1}, v_n \rangle$ is a reverse topological order for the DAG (V, E).

$P(V) = P(v_1, v_2, \ldots v_{n-1}, v_n) =$ (by the fundamental rule) $= P(v_1 | v_2, \ldots v_{n-1}, v_n) P(v_2, \ldots v_{n-1}, v_n) =$

$=$ (by the definition of Bayesian network) $=$

$= P(v_1 | c(v_1)) \; P(v_2, \ldots v_{n-1}, v_n) =$ (by the f. rule) $=$

$= P(v_1 | c(v_1)) \; P(v_2 | v_3, \ldots, v_n) P(v_3, \ldots, v_n) =$ (by the defn. of BN) $=$

$= P(v_1 | c(v_1)) \; P(v_2 | c(v_2)) \; P(v_3, \ldots, v_n) =$

$$\ldots = P(v_1 | c(v_1)) \, P(v_2 | c(v_2)) \ldots P(v_{n-1} | c(v_{n-1})) \, P(v_n)$$