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3.2 Restoration for Binary Symmetric Channel Errors

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A previous report [1] has presented and analyzed a technique for restoring the output of a quantizer so that the result more accurately matches the quantizer's input with respect to a mean-square error criterion. The restoration is obtained by the use of

$$E\{x \mid x \in R\} = \frac{\int_R \underline{x}\, p(\underline{x}) d\underline{x}}{\int_R p(\underline{x}) d\underline{x}} \qquad (1)$$

where R is a region in N-space to which an N x 1 vector x is assigned during quantization, and $p(\underline{x})$ is the multidimensional probability density function of $\underline{x}$. The restoration is based essentially upon exact knowledge of the quantizer output. A similar, but more difficult problem results when the quantizer output is not known exactly. This could occur, for example, when the quantizer output is transmitted over a noisy channel. The first section in this report explores the effect of channel errors on the restorations obtained using eq.(1). The next section examines a technique that statistically compensates for the effect of channel errors.

Effects of Channel Errors on Quantized Signals: In this analysis, channel errors are assumed to be modelled by a binary symmetric channel (BSC) [2]. The characteristics of this type of channel are shown in figure 1. The channel is discrete and memoryless and can be specified by a transition probability assignment $P(j|k)$, for $j,k=0,1$, as

$$P = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix} \qquad (2)$$

Since the channel is memoryless, the probability of an output sequence $\underline{z}=(z_1,z_2,\ldots,z_N)$, given an input sequence $\underline{x}=(x_1,x_2,\ldots,x_N)$, is given by

$$p(z|x) = \prod_{i=1}^{N} p(z_i|x_i) \qquad (3)$$

Based on this definition, a BSC was computer simulated with the channel error probability, p, chosen to be 0.01. The simulated channel was then applied to transform coded images. Three images were zonal transform coded in 16 x 16 blocks and their quantized transform domain components were encoded by assigning each a binary code word. The resulting sequence of binary digits was operated on by the

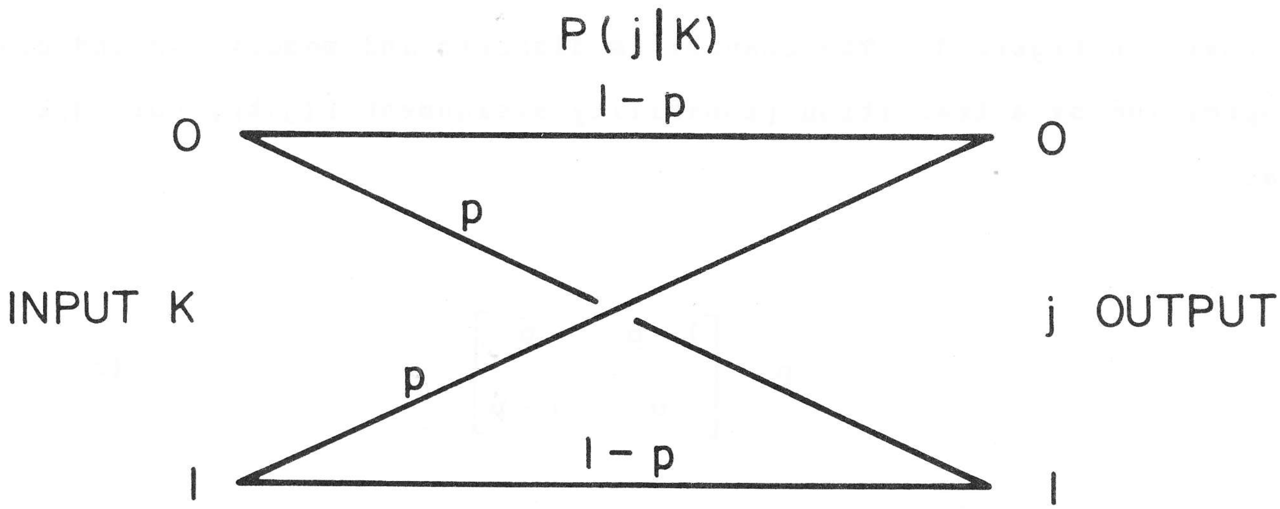


Figure 3. 2-1. Transition probabilities for a binary symmetric channel.

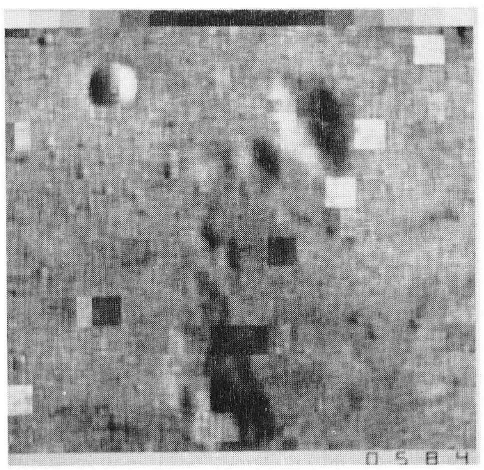
simulated channel. The error-corrupted bit stream was then either decoded directly, as shown in figures 2a, 2c, and 2e, or restored by the use of eq.(1) to reduce the effects of the quantization process. Figure 3 contains a schematic of this procedure. The decoded images with the quantization effects reduced are shown in figures 2b, 2d, and 2f.

Bit errors in transform coding that arise due to a binary symmetric channel are seen to result in an emphasis of the block structure and a subjective error that extends over the entire block. This latter effect occurs because inverse transforming a block containing an error distributes this error over all the resultant image domain components. The reconstruction technique implied by eq. (1) is thus insensitive to channel errors. Since it provides visual and mean-square error improvements in noise-free cases, it can be utilized equally well in noisy environments.

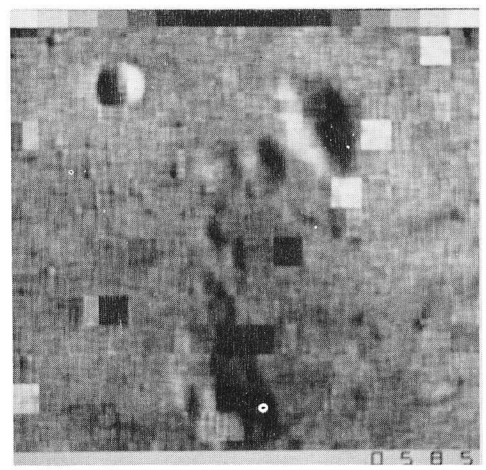
<u>Reconstruction of Quantized and Transmitted Signals:</u> The previous section demonstrated that channel errors do not adversely affect the performance of the restoration technique derived previously. However, this technique does nothing to ameliorate the effects of the channel errors. This is because the fundamental restoration formula presented in eq.(1) was derived without any consideration of channel structure. By including the channel structure in the derivation, the resultant restoration technique can simultaneously reduce the effects of the quantization process and mitigate the effects of channel errors.

(a) Quantized 0.5 bit/pixel
$P_e = 0.01$

(b) Restored 0.5 bit/pixel
$P_e = 0.01$



(c) Quantized 0.5 bit/pixel
$P_e = 0.01$



(d) Restored 0.5 bit/pixel
$P_e = 0.01$

(e) Quantized 0.5 bit/pixel
$P_e = 0.01$

(f) Restored 0.5 bit/pixel
$P_e = 0.01$

Figure 3.2-2. Minimum mean square error restoration of Hadamard transformed zonal quantized images.

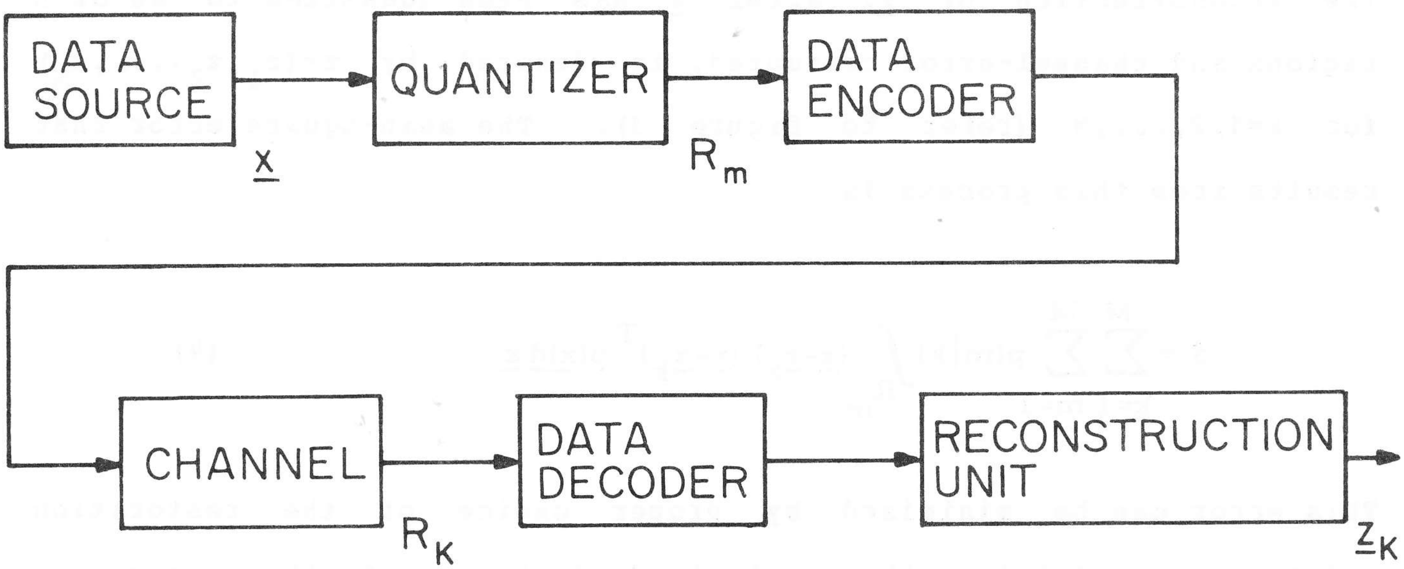


Figure 3.2-3. Data system used to model the effects of channel errors on the quantization restoration process.

Let the output of a data source (this output could consist of DPCM samples, PCM samples, or transform domain samples) be denoted by $\underline{x}=(x_1,x_2,\ldots,x_N)$ and described by a probability density function $p(\underline{x})$. The reconstruction of $\underline{x}$, after $\underline{x}$ has been quantized to one of M regions and channel-error corrupted, is denoted by $\underline{z}=(z_1,z_2,\ldots,z_N)$ for $k=1,2,\ldots,M$ (refer to figure 3). The mean-square error that results from this process is

$$\mathcal{E} = \sum_{k=1}^{M} \sum_{m=1}^{M} p(m|k) \int_{R_m} (\underline{x}-\underline{z}_k)\,(\underline{x}-\underline{z}_k)^T\, p(\underline{x})d\,\underline{x} \tag{4}$$

This error can be minimized by proper choice of the restoration points, z_k. Setting the partial derivatives of this error with respect to z_k equal to zero yields

$$z_k = \frac{\sum_{m=1}^{M} p(m|k) \int_{R_m} \underline{x}p(\underline{x})d\underline{x}}{\sum_{m=1}^{M} p(m|k) \int_{R_m} p(\underline{x})d\underline{x}} \tag{5}$$

for $k=1,2,\ldots,M$. This expression is the noisy channel version of eq.(1) and provides a minimum mean-square error estimate of the input to a quantizer based on the output of a noisy channel, the characteristics of the quantizer, and the a priori statistics of the input. This equation is also a multidimensional version of a result first derived in [3]. For a noiseless channel, the channel matrix P becomes the identity matrix and eq. (5) reduces to eq.(1). When the probability volume integrals in the denominator of eq.(5) are all equal, which is approximately true for Max quantization, the restoration equation simplifies to

$$z_k = \sum_{m=1}^{M} p(m|k) \frac{\int_{R_m} \underline{x}p(\underline{x})d\underline{x}}{\int_{R_m} p(\underline{x})d\underline{x}} \tag{6}$$

or

$$z_k = \sum_{m=1}^{M} p(m|k)y_m \tag{7}$$

where y_m is given by eq.(1). This result holds for maximum output entropy quantizers and two-level symmetrical quantizers, and is approximately correct for many other types.

A signal that has been quantized and then transmitted over a noisy channel can thus be optimally restored by utilizing eq.(5). The restoration solutions found earlier for Gaussian and Laplacian probability density functions (see [4] and [5], respectively) can be substituted directly into eq.(5) once the transition matrix for the channel has been determined. The resultant estimator can then be used to restore the outputs of transform and DPCM coders that have been degraded by channel errors.

References

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