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3.3 Quantization Error Reduction for Image Coding

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Quantization is the process of representing continuously varying quantities by discrete intervals. This process is nonlinear and some of the information about the original data is irretrievably lost. The usual restoration procedure is to choose the midpoints of each quantization interval as the estimated values of the original data. However if it is known that the original data are correlated and are non-uniformly distributed, then improved restorations are possible using this information. As shown in a previous report [1], minimum mean square error estimates of correlated data require the solution of the following equation

$$x = E\{ x | x \in D \} = \frac{\int_D \underline{x} p(\underline{x}) \, d\underline{x}}{\int_D p(\underline{x}) \, d\underline{x}} \tag{1}$$

where $\underline{x}$ is the n-dimensional variable to be quantized, D is the particular region of n-space into which $\underline{x}$ is quantized, and $p(\underline{x})$ is the probability density function of $\underline{x}$. A partial solution to this equation has been obtained for data which have a jointly gaussian probability distribution. This solution has now been applied to the restoration of quantized one-dimensional random signals and two-dimensional transform domain zonal quantized images. The results reveal a decrease in mean square error in all cases. However, in spite of the error reduction, some images exhibit a degradation in subjective quality after restoration. Hence a nonlinear error criterion based on the human visual system and derived by Mannos and Sakrison [2] has been used in place of the mean square error function. Under this criterion a subjective image improvement as well as a numerical error reduction are obtained.

To demonstrate the utility of this restoration procedure, a randomly generated gaussian Markov signal has been quantized and restored. The results are shown in figure 1. A two bit per sample Max quantization scheme is employed to obtain the quantized approximation to the original signal. Using this quantized signal and the statistical knowledge about the original signal as inputs to the nonlinear estimator, the restoration decreases the mean square error by 33%. The average improvement in mean square error as a function of quantizing bit assignment for different correlation coefficients is shown in figure 2. It can be seen from this graph that, as the amount of correlation in the Markov process approaches zero, then the restoration provides no error improvement. There is also

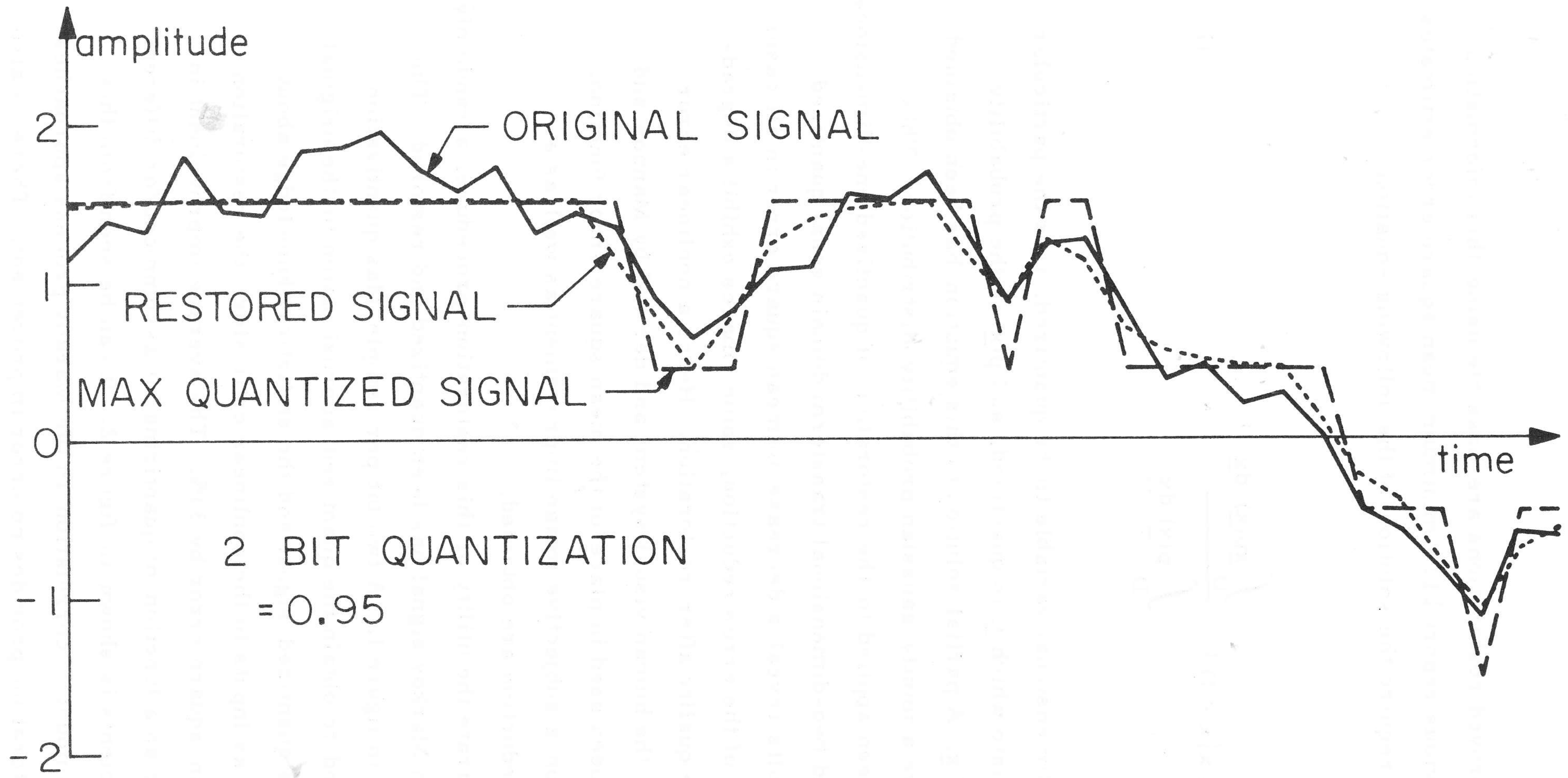


Figure 3.3-1. Restoration of a Quantized Random Markov Signal.

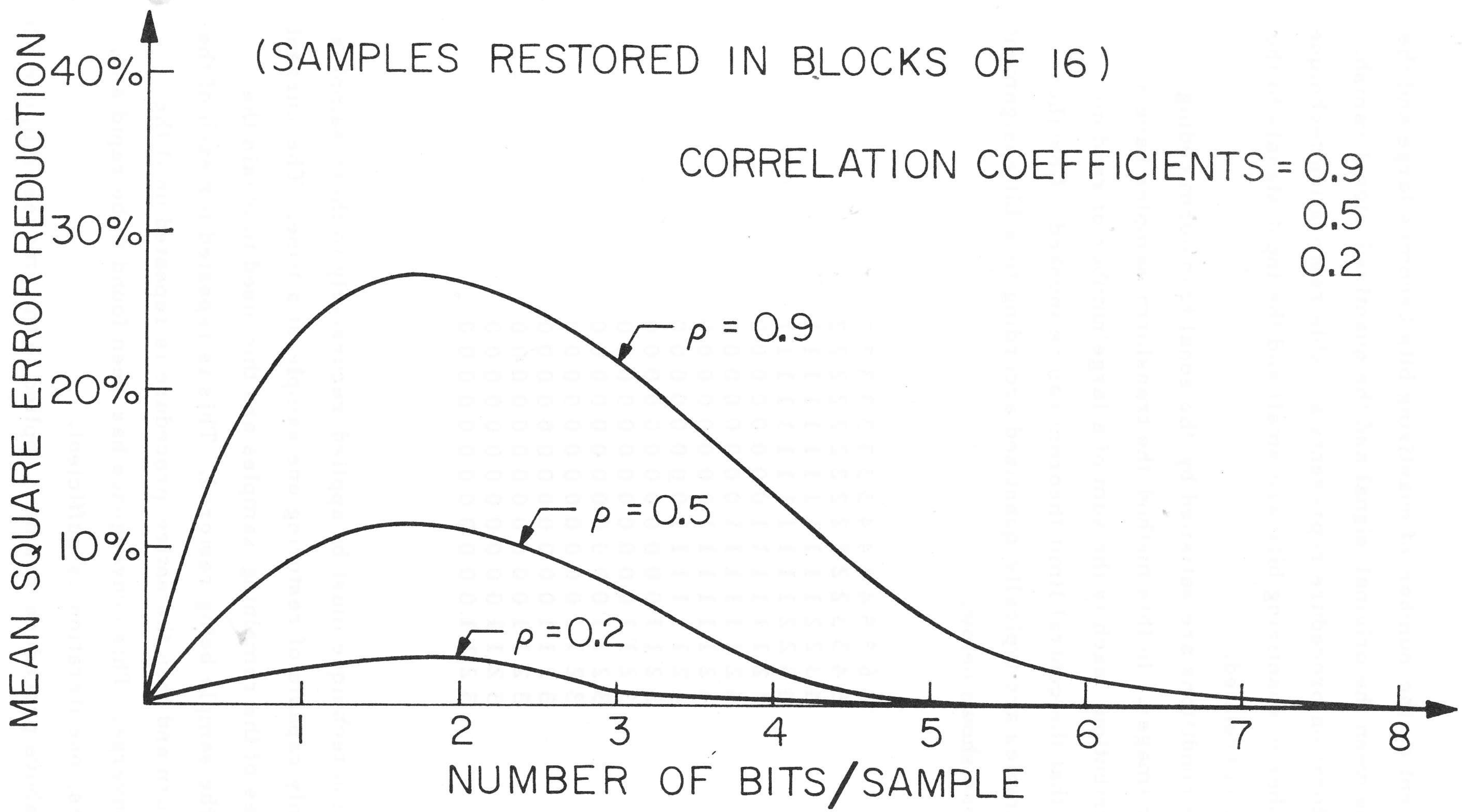


Figure 3.3-2. M.S.E. Improvement for Quantized Markov Signal.

no improvement as the number of quantizing bits becomes large and the differences between the original signal and the quantized signal vanish. Thus the restoration procedure represents a viable restoration technique when the number of quantizing bits are small and the input signals to the quantizer are correlated.

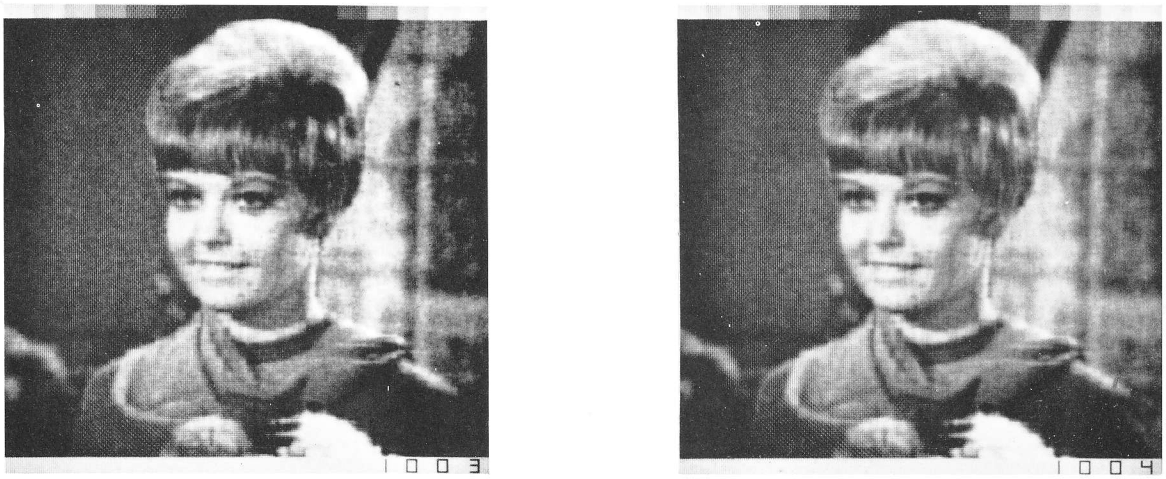
These conditions are satisfied by the zonal transform coding technique for images. In this method the transform samples have a gaussian distribution: each is the sum of a large number of random variables so that the central limit theorem can be invoked. Now these transform samples are typically quantized according to a bit assignment such as the one shown below.

8	6	5	5	4	4	4	4	3	3	3	3	3	3	3	3
5	4	3	3	2	2	2	2	2	2	2	2	2	2	2	2
4	3	2	2	1	1	1	1	1	1	1	1	1	1	1	1
4	3	2	2	1	1	1	1	1	1	1	1	1	1	1	1
4	2	1	1	1	1	1	1	0	0	0	0	0	0	0	0
4	2	1	1	1	1	1	1	0	0	0	0	0	0	0	0
4	2	1	1	1	1	1	1	0	0	0	0	0	0	0	0
4	2	1	1	1	1	1	1	0	0	0	0	0	0	0	0
3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0
3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0
3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0
3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0
3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0
3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0
3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0
3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0

The restoration technique must be applied recursively to these samples since it is only capable of restoring one sample at a time. The current best estimates of the remaining samples are then used to obtain the estimate of the sample being restored. This is repeated for each of the samples in turn and then the entire procedure is repeated until the estimates converge. This convergence has been found to be rapid and, in most cases, one iteration is sufficient.

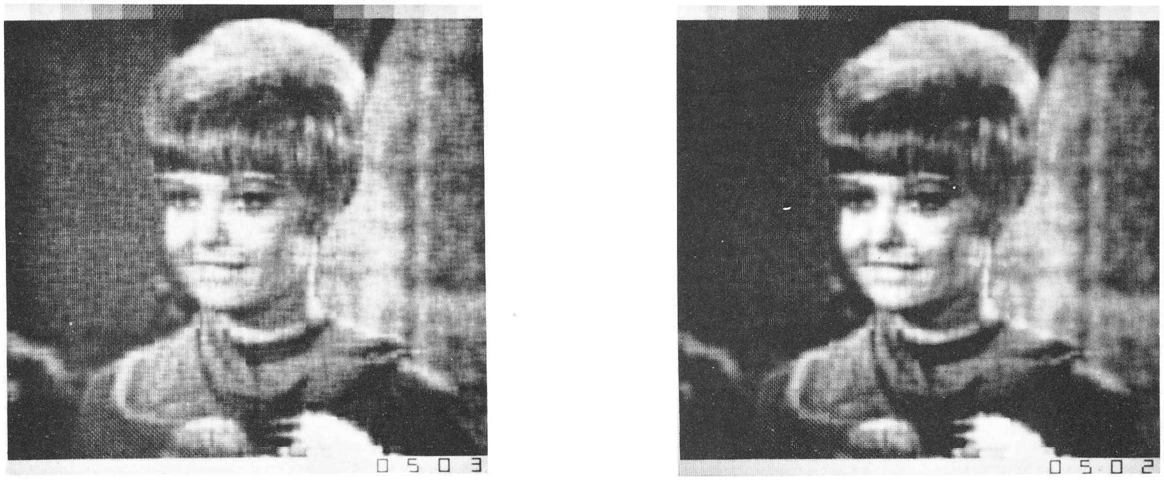
The above procedure has been applied to the images in figure 3 b and 3d. Figure 3b has been coded with an average of one bit per pixel

(a) Original Image 8 Bits/Pixel



(b) Quantized 1 Bit/ Pixel

(c) Restored 1 Bit/Pixel



(d) Quantized 0.5 Bit/Pixel

(e) Restored 0.5 Bit/Pixel

Figure 3.3-3. Restoration of Haar transform, zonal quantized images.

by using a Haar transform in 16 x 16 blocks, a zonal coding bit assignment and a Max quantizer. For the quantization and subsequent restoration, the original image samples are assumed to arise from a Markov source. Figure 3c is the restored version of this image, utilizing one iteration of the estimation procedure. The mean square error is reduced by 10% as a result of the restoration. Figures 3d and 3e, respectively, have been quantized to 0.5 bits and restored by means of the above technique. A reduction of 19% in mean square error is obtained in this case. Subjectively, the restored images appear much less noisy than the quantized images but more blurred, as is very evident in comparing figures 4b and 4c. Hence an error measure is required in which numerical results match subjective results.

This has been provided by modeling the error measure after the human visual system. It has been found that the human visual system is sensitive to approximately the cube root of incident light intensities. It is also most sensitive to middle spatial frequencies near eight cycles per degree. Hence to apply this error measure, an image is processed according to the block diagram in figure 5. The $(i, j)^{th}$ component of the filter function is chosen to be

$$T_{ij} = (.05 + .18525\, r) \exp\{-(.07125\, r)^{1.1}\} \tag{3}$$

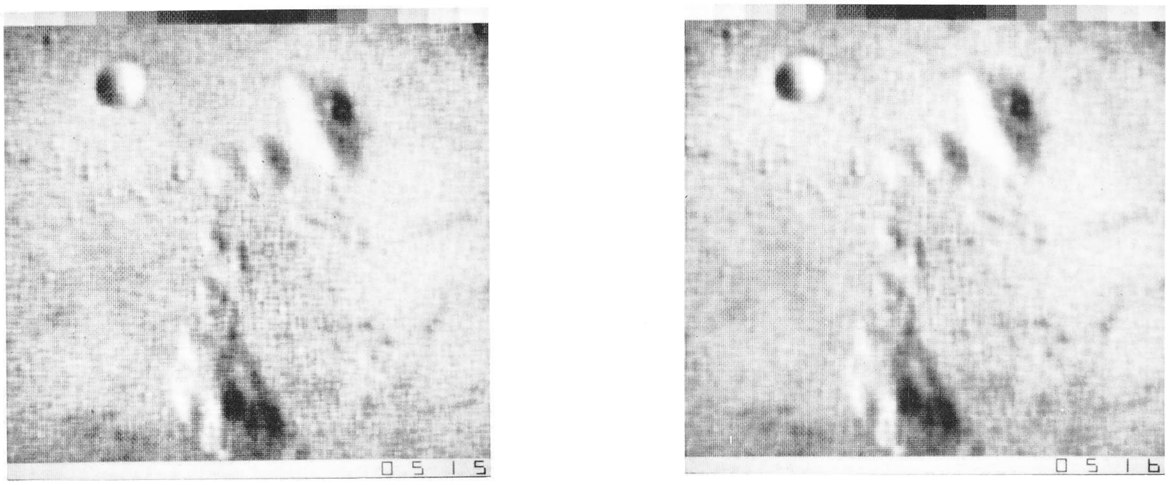
where

$$r = (i^2 + j^2)^{1/2} \tag{4}$$

Figures 4d and 4e show the results of this procedure for a Hadamard transform, with and without the restoration step, respectively. There are both subjective and numerical error improvements after restoration.

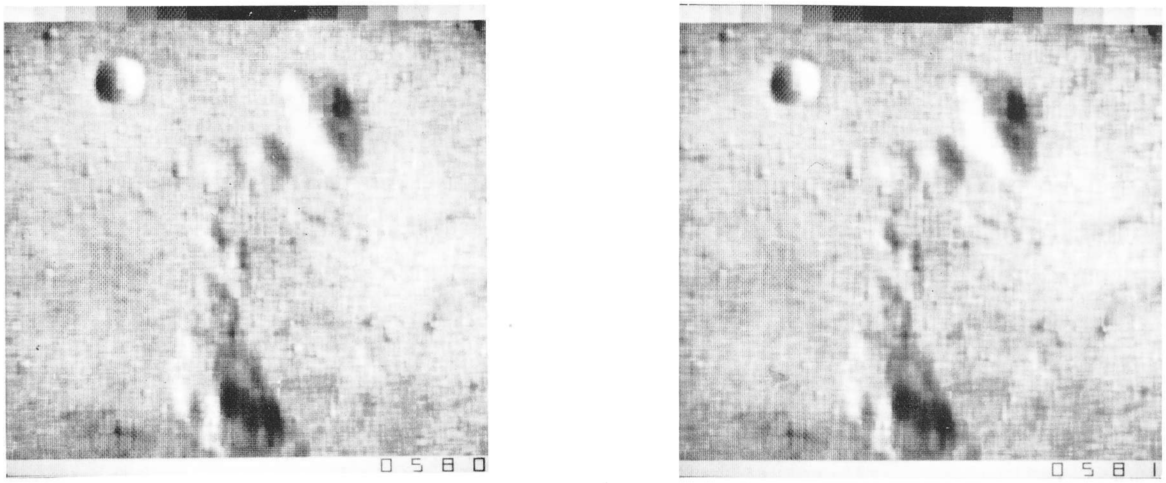
The restoration process has also been applied to a color coding experiment. In this experiment a color image is transformed to the YIQ coordinate system and then quantized according to the bit assignment indicated on the next page for a typical block of four pixels.

(a) Original Image 8 bits/pixel



(b) Zonal Haar Quantized

(c) Zonal Haar Restored



(d) Visual Hadamard Quantized

(e) Visual Hadamard Restored

Figure 3.3-4. Restoration of 0.5 bit/pixel transform coded images for two different error criteria.

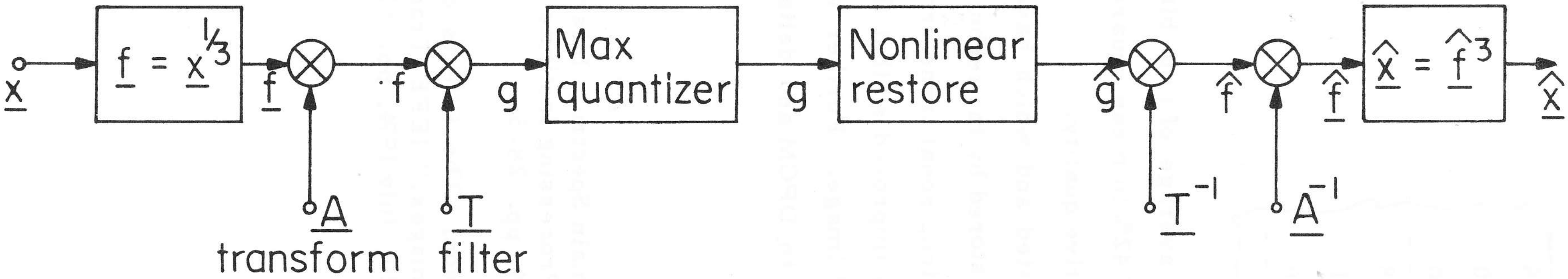


Figure 3.3-5. Coding and restoration technique for a nonlinear error criterion.

Y = 8 I = 1 Q = 0	Y = 8 I = 0 Q = 0
Y = 8 I = 0 Q = 1	Y = 8 I = 1 Q = 0

Each pixel is hence coded with an average of nine bits. Quantization restoration provides a decrease of 42% in mean square error in this case and an improvement in subjective quality.

Thus data which are correlated and which have been coarsely quantized are amenable to being restored by the techniques outlined above. By choosing a suitable error criterion, zonal transform coded images can be subjectively and analytically improved so that they more faithfully reproduce the details of an original image. Further work is expected to extend the restoration technique to DPCM and delta modulation coded images.

References

1. M. N. Huhns, "Transform Domain Spectrum Interpolation," University of Southern California Image Processing Institute Technical Report, USCIPI Report 530, March 1974, pp. 28-38.

2. J. L. Mannos and D. J. Sakrison, "The Effects of a Visual Fidelity Criterion on the Encoding of Images," IEEE Transactions on Information Theory, Vol. IT-20, July 1974, pp. 525-536.