<u>Recall</u>: L language such that

$\mathcal{E}_L \quad \left(= \{ L_w : w \in \Sigma^* \} \right)$

is finite.

Defined

$D := \langle \mathcal{E}_L, \Sigma, \delta, q_0, F \rangle$

$q_0 = L_\varepsilon = L$

$F := \{ L_w : \varepsilon \in L_w \}$

$\delta(L_w, a) = L_{wa}$

Show that $L = L(D)$

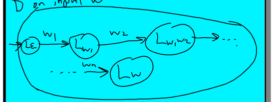
($\therefore$ L is regular).

Let $w \in \Sigma^*$ be any string,

$w = w_1 w_2 \cdots w_n$

(each $w_i \in \Sigma$)

D on input w



D accepts $w \iff$

$L_w \in F \iff$

$\varepsilon \in L_w \iff$

$w\varepsilon \in L \iff$

$w \in L$.

$\therefore$ D recognizes L. //

Pumping Lemma example

$L := \{ w \in \{0,1\}^* : \text{all 0's in } w\text{'s first half} \}$

$= \{ 0^n x : x \in \{0,1\}^* \text{ and } |x| \le n \}$

<u>Prop</u>: L is not pumpable.

<u>Proof</u>: (following the template)

Given $p > 0$,

let $s := 0^p 1^p$.

also possible: $0^p 1 0^{p-1}$

Given x, y, z such that

$s = xyz$, $|xy| \le p$, $|y| > 0$

($y = 0^k$ for some $k > 0$).

Let $i := 0$.

Then $xy^0z = xz = 0^{p-k} 1^p \notin L$ //

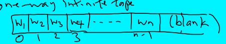
<u>Computability & reducibility</u>

<u>Turing machines</u> (TMs)

Picture of a TM computation

Input $w = w_1 \cdots w_n$ ($w_i \in \Sigma$)

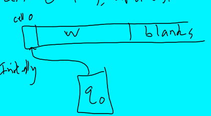
one-way infinite tape



discrete cells, labeled $0, 1, 2, \ldots$

Each cell contains a single symbol

Initially, $w_1 \cdots w_n$ are in cells 0–$(n-1)$, respectively.



<u>Def</u>: A Turing machine is a tuple

$\langle Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej} \rangle$

where

Q is a finite set (<u>state set</u>)

Σ is an alphabet (the <u>input alphabet</u>)

Γ is an alphabet (the <u>tape alphabet</u>)

$q_0 \in Q$ (the <u>start state</u>)

$q_{acc} \in Q$ (the <u>accept state</u>)

$q_{rej} \in Q$ (the <u>reject</u> state)

We fix a symbol $\sqcup$
(or just)
as the <u>blank symbol</u>,

Require: $\sqcup \in \Gamma \sim \Sigma$

$$\Sigma \subseteq \Gamma$$

$$q_{acc}, q_{rej} \in Q$$

$q_{acc} \neq q_{rej}$ and

$$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$$

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