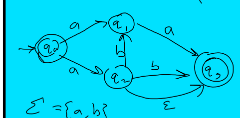
<u>Prop</u>: If L_1 & L_2 (langs over some Σ) are regular, then $L_1 \cap L_2$ is regular

<u>Proof</u>: If $L_1 = L(D_1)$
$L_2 = L(D_2)$
D_1 & D_2 are DFAs, Construct a product DFA
$\mathcal{D} = D_1 \wedge D_2$
such that $\underline{L_1 \cap L_2 = L(\mathcal{D})}$. //

<u>Cor</u>: For regular $L_1, L_2 \subseteq \Sigma^*$, any Boolean combination of L_1, L_2 is regular;

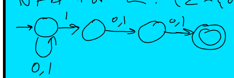
<u>Ex</u>:
$L_1 - L_2 = L_1 \cap \overline{L_2}$
$L_1 \cup L_2 = \overline{\overline{L_1} \cap \overline{L_2}}$
$L_1 \Delta L_2 = (L_1 - L_2) \cup (L_2 - L_1)$
$= (L_1 \cup L_2) - (L_1 \cap L_2)$
("symmetric difference")

Nondeterminism:
<u>non</u>deterministic finite automaton (<u>NFA</u>)



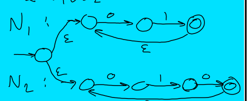
$\Sigma = \{a, b\}$
Input ab accepted:
$q_0 \xrightarrow{a} q_2 \xrightarrow{b} q_3$
aba acc: $q_0 \xrightarrow{a} q_2 \xrightarrow{b} q_1 \xrightarrow{a} q_3$
abb — rejected

NFA for L: ($\Sigma = \{0,1\}$



$L = \{ w \in \{0,1\}^* :$ 3rd last symbol of w is 1$\}$

$L_1 = \{(01)^n : n > 0\}$
$L_2 = \{(010)^n : n > 0\}$
NFAs for L_1 & for L_2 & $L_1 \cup L_2$

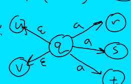


[$01010 \notin L_1 \cup L_2$]

<u>Notation</u>: Let S be any set. Write
$2^S := \{T : T \subseteq S\}$
the <u>powerset</u> of S
[$P(S)$ is also used for the powerset]

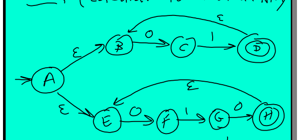
<u>Def</u>: An <u>NFA</u> is a 5-tuple $\langle Q, \Sigma, \delta, q_0, F \rangle$ where Q, Σ, q_0, F are same as with a DFA, and
$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \to 2^Q$

Idea:



$\delta(q, a) = \{r, s, t\}$
$\delta(q, \varepsilon) = \{u, v\}$

Ex: (tabular form of an NFA)



	0	1	ε
→ A	∅	∅	{B,E}
B	{C}	∅	∅
C	∅	D	∅
* D	∅	∅	B
E	F	∅	∅
F	∅	G	∅
G	H	∅	∅
* H	∅	∅	E

<u>NFA semantics</u>

Recall DFA semantics:

Given DFA $D = \langle Q, \Sigma, \delta, q_0, F\rangle$ and $w \in \Sigma^*$. D accepts w iff $\exists s_0, s_1, \ldots, s_n \in Q$ and $\exists w_1, w_2, \ldots, w_n \in \Sigma$ such that

- $w = w_1 w_2 \cdots w_n$
- $s_0 = q_0$
- $s_n \in F$
- $\forall i, 1 \le i \le n,\ s_i = \delta(s_{i-1}, w_i)$

<u>Def</u>: Let $N = \langle Q, \Sigma, \delta, q_0, F\rangle$ be an NFA and let $w \in \Sigma^*$ be a string.

We say that N accepts w <u>iff</u> $\exists k \ge 0$

$\exists s_0, \ldots, s_k \in Q$

$\exists w_1, \ldots, w_k \in \Sigma \cup \{\varepsilon\}$:

- $w = w_1 \cdots w_k$
- $s_0 = q_0$
- $s_k \in F$
- $\forall i, 1 \le i \le k,\ s_i \in \delta(s_{i-1}, w_i)$

Say that $s_0, \ldots, s_k$ is a <u>comp. path</u> of N.

$L(N) = \{ w \in \Sigma^* \mid N \text{ accepts } w \}$

"the lang <u>recognized</u> by N.

Is every reg. lang. (recog by a DFA) recog. by some NFA?

<u>Yes</u>:

Given DFA $D = \langle Q, \Sigma, \delta, q_0, F\rangle$ define NFA $N = \langle Q, \Sigma, \delta', q_0, F\rangle$ where $\forall q \in Q$ and $a \in \Sigma$,

$\delta'(q, a) = \{\delta(q, a)\}$

and

$\delta'(q, \varepsilon) = \emptyset$.

Then $\underbrace{L(N) = L(D)}$

say that N & D are <u>equivalent</u>

Given an NFA N is there an equivalent DFA D? Yes! (Next time)